

# A feedback real-time optimization strategy applied to an evaporator process

Dinesh Krishnamoorthy<sup>a</sup>, Esmaeil Jahanshahi<sup>b</sup>, and Sigurd Skogestad<sup>c,\*</sup>

Department of Chemical Engineering, Norwegian University of Science and Technology (NTNU), Trondheim, Norway

E-mail: <sup>a</sup>dinesh.krishnamoorthy@ntnu.no, <sup>b</sup>esmaeil.jahanshahi@hotmail.com, <sup>c</sup>skoge@ntnu.no (Corresponding author)

**Abstract.** This paper presents the application of a feedback based RTO using transient measurements for optimal operation of an evaporator process. The proposed method is based on estimating the steady-state gradient of the cost function by linearizing the nonlinear dynamic model around the current operating point. Any feedback control can then be used to control the estimated steady-state gradient to achieve optimal operation. Since the proposed method uses transient measurements, it avoids the steady-state wait time which is a huge limitation in the traditional steady-state RTO. Since the optimization is achieved via feedback, it does not require to solve numerical optimization problems. Hence it is computationally more efficient than dynamic RTO. Compared to self-optimizing control, the proposed method does not have any steady-state losses when operated far away from the nominal optimal point. Compared to model-free methods such as extremum seeking control, it is significantly faster and does not require external process excitation for steady-state gradient estimation. The performance of the proposed method for the evaporator process is compared with traditional static RTO, dynamic RTO, hybrid RTO, self-optimizing control and extremum seeking control.

**Keywords:** Real-time optimization, Optimal Control, Measurement Based Optimization

## 1. Introduction

Real-time optimization of chemical processes traditionally involves solving a steady-state optimization problem using rigorous steady-models of the process. Following any changes in the disturbance or the operating conditions, the optimization problem must be solved to re-compute the new optimal points. However, before the new optimal points can be recomputed, it is necessary to wait for the process to settle to a steady-state operating point. This steady-state wait time is a fundamental limitation of the traditional steady-state optimization of chemical processes [1]. Dynamic real-time optimization does not suffer from the steady-state wait time. However, the computation cost is prohibitively expensive in many applications even with today's computing power.

In order to address the steady-state wait time issue of the traditional static RTO and the computation cost of dynamic RTO, we recently proposed a hybrid RTO strategy, where the model adaptation is done using dynamic models and the numerical optimization is solved using the corresponding steady-state model. The hybrid RTO approach thus requires the maintenance of both the static and the dynamic models.

Recently, there has been an increasing interest in the so-called *direct input adaptation* methods, where optimal operation is achieved by means of feedback control. Self-optimizing control, extremum seeking control, NCO-tracking etc. are some well known approaches that belong to such a category. In self-optimizing control we control a combination of measurements to a constant setpoint such that the economic losses are minimized. However when the process is operated far from the nominal optimal region, this leads to large steady-state losses [2].

Model-free approaches such as extremum seeking control and NCO-tracking on the other hand are based on estimating the steady-state gradient directly from the measurements and controlling them to a constant

setpoint of zero. However, the main disadvantage of these methods is that it has a very slow convergence to the optimum. In addition, these methods also require additional perturbation for accurate gradient estimation [3]. Extremum seeking like approaches are also known to provide unwanted deviations in the presence of abrupt disturbances as motivated by [4].

In this paper, we apply a recently developed feedback-based RTO approach for the evaporator process, which is based on converting the hybrid RTO into a feedback control problem. The proposed method is based on estimating the steady-state gradient of the cost function by linearizing the nonlinear dynamic model around the current operating point. The estimated steady-state gradient can then be controlled to a constant setpoint of zero. Since the proposed method uses nonlinear dynamic models, it can use transient measurements and hence avoid the issue of steady-state wait time. It also does not require any additional dither to estimate the steady-state gradient.

## 2. Feedback RTO using steady-state gradient control

Consider a nonlinear dynamic system of the form,

$$\begin{aligned}\dot{x} &= f(x, u, d) \\ y &= g(x, u)\end{aligned}\tag{1}$$

where  $x \in \mathbb{R}^{n_x}$ ,  $u \in \mathbb{R}^{n_u}$ ,  $d \in \mathbb{R}^{n_d}$  and  $y \in \mathbb{R}^{n_y}$  are the states, inputs, disturbances and the measured outputs respectively. Note that the  $n_u$  control inputs considered here are the unconstrained degrees of freedom. In the proposed feedback RTO method, any state estimator such as an extended Kalman filter (EKF) [5] can be applied to estimate the states  $x$  of the system by using the measurements and the nonlinear dynamic model (1).

Let the cost be modelled as,

$$J = h(x, u)\tag{2}$$

with  $h: \mathbb{R}^{n_x} \times \mathbb{R}^{n_u} \rightarrow \mathbb{R}$ . Note that the cost does not need to be measured in the proposed method. Using the updated states, the nonlinear model from the inputs to the cost can be linearized around the current operating point to get a local linear state-space model, given by (3) as described in [6].

$$\begin{aligned}\dot{x} &= Ax + Bu \\ y &= Cx + Du\end{aligned}\tag{3}$$

where the system matrices  $A, B, C$  and  $D$  are the Jacobians of the non-linear functions  $f$ , and  $h$  from (1) and (3), evaluated around the current operating point,

$$A = \left. \frac{\partial f}{\partial x} \right|_{x=\hat{x}} \quad B = \left. \frac{\partial f}{\partial u} \right|_{x=\hat{x}} \quad C = \left. \frac{\partial h}{\partial x} \right|_{x=\hat{x}} \quad D = \left. \frac{\partial h}{\partial u} \right|_{x=\hat{x}}\tag{4}$$

The steady state gradient can then be obtained by setting  $\dot{x} = 0$  to get,

$$\Delta J = (CA^{-1}B + D)\Delta u\tag{5}$$

The estimate of the steady-state gradient around the current operating point is then given by,

$$\hat{J}_u = CA^{-1}B + D\tag{6}$$

since  $\Delta J = J_u \Delta u$ .

To optimize the operation of the process, the estimated steady-state gradient is driven to a setpoint of  $\hat{J}_u = 0$  by using any feedback controller thus satisfying the necessary conditions of optimality [6].

It is important to note that by using a nonlinear state estimator and a dynamic model for estimating the steady-state gradient  $\hat{J}_u$ , we can use transient measurements, without the need to wait for steady-state, as in traditional RTO. The proposed method is schematically shown in Fig. 1.

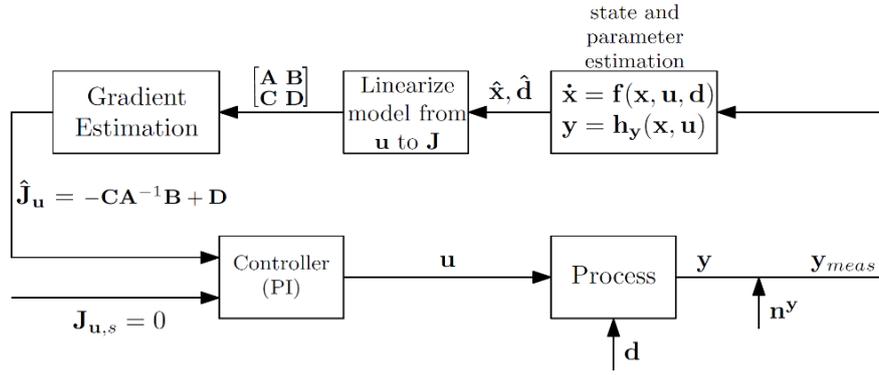


Fig. 1. Block diagram of the Feedback RTO approach

### 3. Optimal operation of an evaporator process

We now apply the proposed method on an evaporator process shown in Fig. 2 and compare it with the traditional static RTO (SRTO), dynamic RTO (DRTO) and the recently developed hybrid RTO (HRTO). In addition, the proposed method is also compared with two other direct-input adaptation methods, namely, self-optimizing control (SOC) and extremum seeing control (ESC). The purpose of the evaporator process is to increase the concentration of the dilute liquor by evaporating the feed solvent  $F_1$  while the liquor is circulated through the heat exchanger. The model is the same as the one used in [7] and for more detailed information on the model equations and the model parameters, the reader is referred to [7].

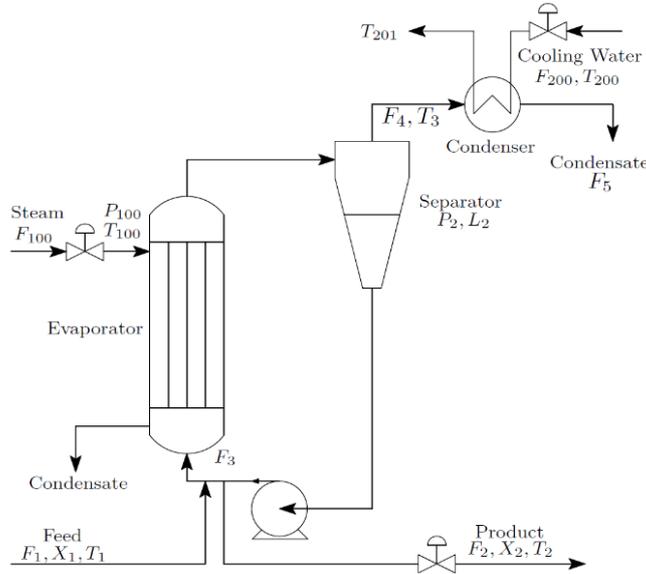


Fig. 2. Evaporator process

The objective is to minimize the operation cost, and the optimization problem is formulated as,

$$\begin{aligned}
 \min_u J &= 600F_{100} + 0.6F_{200} + 1.009(F_2 + F_3) \\
 \text{s.t. } \dot{x} &= f(x, u, d) \\
 X_2 &\geq 35.5\% \\
 P_{100} &\leq 400\text{kPa}
 \end{aligned} \tag{7}$$

There are four dynamic degrees of freedom, namely,  $F_2$ ,  $P_{200}$ ,  $F_3$  and  $F_{200}$ . At the optimal point, two of the inequality constraints are active, namely  $X_2 = 35.5\%$  and  $P_{100} = 400\text{kPa}$ . Therefore, these two active constraints are tightly regulated using PI controllers. The input  $F_2$  is used to control  $X_2$  at a constant

setpoint of  $X_2 = 35.5\%$ , and  $P_{100}$  is an input itself which is maintained at a constant of  $P_{100} = 400\text{kPa}$ . In addition, the separator level which does not have any steady-state effect is controlled by  $F_3$ . Therefore,  $F_{200}$  is the only remaining unconstrained degree of freedom that can be used for optimization. In other words, we only have one steady-state degree of freedom, i.e.  $u = F_{200}$ . By using  $F_{200}$  for control, we optimize  $P_2$ , thus optimizing the evaporator process. We assume there are four unmeasured disturbances  $d = [F_1 \quad X_1 \quad T_1 \quad T_{200}]$  and the available measurements are  $y = [F_2 \quad F_{100} \quad T_{200} \quad F_3]$ .

As mentioned earlier, the proposed method uses a state estimator. In this work, we use an extended Kalman filter (EKF) for combined state and disturbance estimation by augmenting the unmeasured disturbances with the states, as described in [5]. The steady state gradient of the cost is then estimated according to (6). In this work, we use a PI controller to control the estimated gradient to a constant setpoint of zero. The process is simulated for a total simulation time of 10h with variations in the unmeasured disturbances as shown in Fig.3. The measurements are assumed to be available with a sampling rate of 1s.

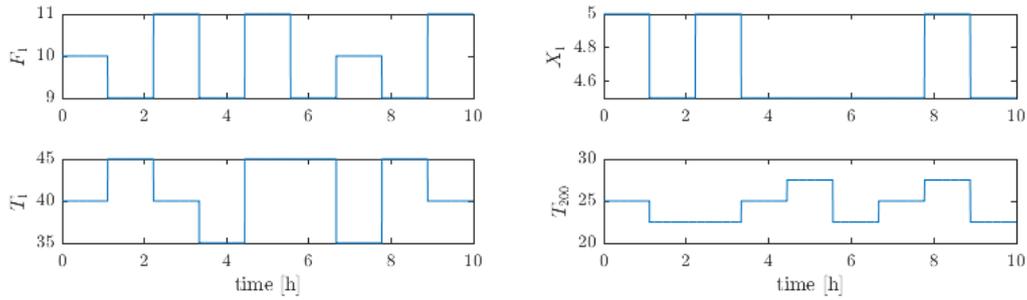


Fig. 3. Disturbance trajectories affecting the evaporator process.

### 3.1. Comparison with optimization based approaches

First, we compare the proposed method with the traditional steady state RTO (SRTO), where the disturbances are estimated using a static model of the system. The system is subject to disturbances as shown in Fig. 3. To use only the steady-state operating points, a steady-state detection (SSD) algorithm as described in [8] was used. The steady-state wait time due to the SSD is a fundamental limitation of the traditional SRTO and the plant is operated sub-optimally for significant periods before the model can be updated.

Then we apply the hybrid RTO (HRTO) approach and dynamic RTO (DRTO) to the evaporator process using the same extended Kalman filter as the one used in the proposed feedback RTO approach. As mentioned earlier, the hybrid RTO approach uses the dynamic model and the EKF to update the model using the transient measurements, but uses the corresponding updated static model to solve a static numerical optimization problem. The only difference between the hybrid RTO and the proposed feedback RTO is that the hybrid RTO solves a numerical optimization problem, whereas in feedback RTO, optimization is achieved via feedback control.

For the dynamic RTO, the updated dynamic model is directly used to solve a dynamic optimization problem with a prediction horizon of 30 min and a sampling time of 1s to compute the optimal input trajectory. As mentioned earlier, the main challenge with DRTO is the computation time.

The performance of the proposed feedback RTO (red solid lines) is compared with the traditional static RTO (black dash-dotted lines), dynamic RTO (green dashed lines) and hybrid RTO (blue dashed lines) in Fig.4. The cost function  $J$  is shown in Fig.4a along with the integrated loss in Fig.4b, which is given by the expression,

$$L_{int} = \int_0^t J_{opt,SS}(t) - J(t) dt \quad (8)$$

### 3.2. Comparison with self-optimizing control

For comparison with self-optimizing control, null-space method was used to compute the optimal selection matrix. The resulting self-optimizing variable  $c = 0.002F_2 - 0.0976F_{100} - 0.0081T_{201} + 0.0125F_3$  is controlled to a constant setpoint of  $c_s = -0.9951$ , as described in [7]. The simulations were performed with

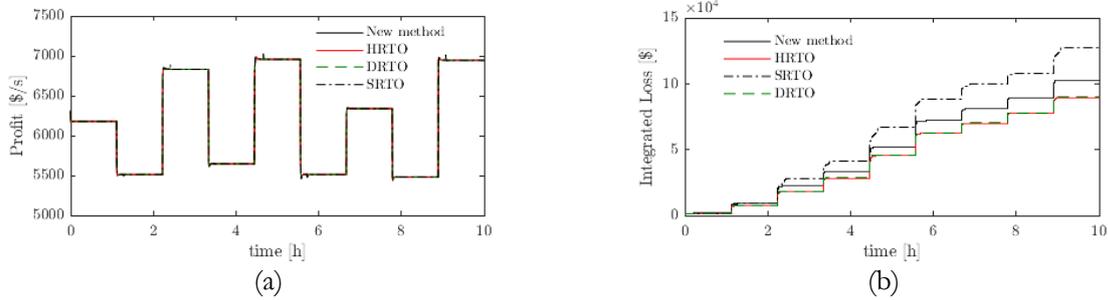


Fig. 4. Comparison of the proposed method with traditional static RTO, dynamic RTO and hybrid RTO (a)Profit (b) Integrated Loss given by (8)

the same disturbances as in the previous case. The performance of the proposed feedback RTO (red solid lines) is compared with self-optimizing control (solid blue lines) in Fig.5 where the cost function is shown in Fig.5a and the integrated loss is shown in Fig.5b

It can be clearly seen that when the disturbances move the operating point of the system away from the nominal optimal point, self-optimizing control leads to steady-state losses. This is not the case with the proposed feedback RTO. This is because, in the proposed method, the nonlinear model is linearized around the current operating point as opposed to linearizing around a nominal optimal point in self-optimizing control.

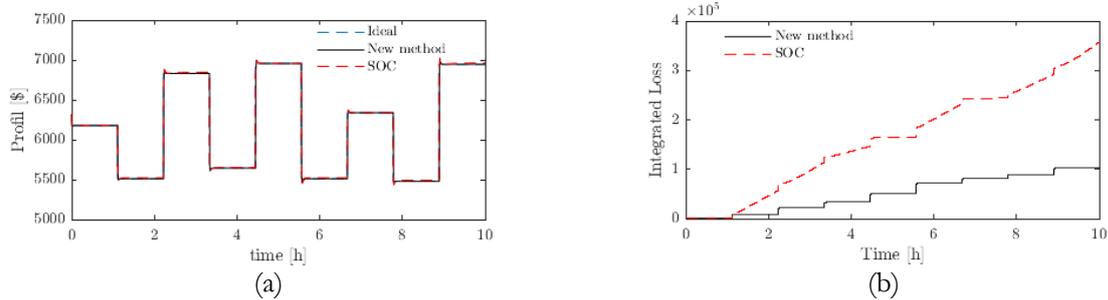


Fig. 5. Comparison of the proposed method with self-optimizing control (a) Profit (b) Integrated loss given by (8)

### 3.3. Comparison with extremum seeking control

In this subsection, we compare the performance of the proposed method with extremum seeking control, which is also based on estimating and controlling the steady-state gradients directly from measurements. In this section we use the least square based extremum seeking control as introduced in [9]. In this method, the steady-state gradient is estimated purely based on the measurements by constantly perturbing the system around the current operating point. The estimated steady state gradient is then driven to zero using integral action.

Since the steady-state gradient are estimated directly from the measurements, it requires time scale separation between the system dynamics, perturbation and the convergence to the optimum. Therefore, the convergence of the extremum seeking control is prohibitively slow in many cases.

Due to the slow convergence, the process is simulated with a total simulation time of 100h (10 times longer than the previous simulation cases). In this simulation, an integral gain of  $K_{ESC} = 0.02$  was chosen. In this simulation, the disturbances vary over a period of 100h instead of 10h. The simulation results are shown in Fig.6. From the simulation results, it can be seen that the proposed method converges significantly faster than the extremum seeking control.

## 4. Conclusion

We have proposed a new method of utilizing transient measurements and a dynamic estimator to estimate the steady-state gradient and then using a simple PI controller for driving the process to its optimal operation. For an ammonia synthesis reactor with both disturbances and plant-model mismatch, the proposed method outperforms comparable control strategies. The industrial applicability is conceivable due to the usage of only seven measurements of the process besides the used dynamic model. An extended Kalman filter (EKF) allows the estimation of the steady-state gradients, even in case of plant-model mismatch by including unmeasured but modelled parameters in the estimator.

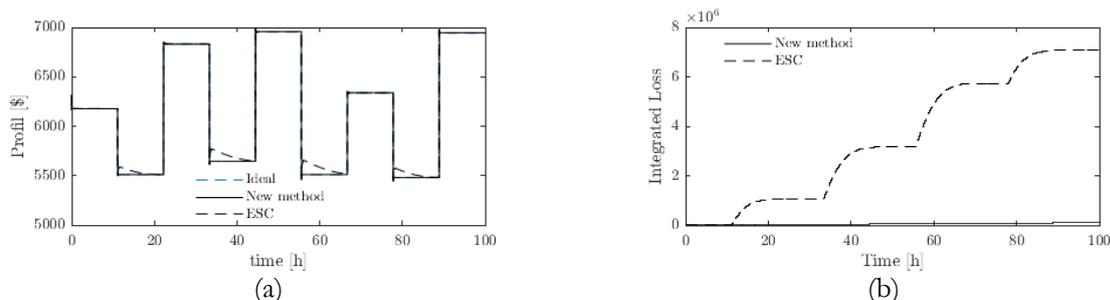


Fig. 6. Comparison of the proposed method with extremum seeking control (a) Profit (b) Integrated loss given by (8)

## 5. Acknowledgements

The authors acknowledge support from SFI SUBPRO which is financed by the research council of Norway, NTNU and major industry partners.

## References

- [1] M. L. Darby, M. Nikolaou, J. Jones and D. Nicholson, "RTO: An overview and assessment of current practice," *Journal of Process Control*, pp. 874-884, 2011.
- [2] S. Skogestad, "Self-optimizing control: The missing Link between steady-state optimization and control," *Computers and Chemical engineering*, pp. 569-575, 2000.
- [3] M. Krstic and H. Wang, "Stability of Extremum Seeking feedback for general nonlinear dynamic systems," *Automatica*, vol. 36, pp. 595-601, 2000.
- [4] D. Krishnamoorthy, A. Pavlov and Q. Li, "Robust extremum seeking control with application to gas lifted oil wells," *IFAC-papersOnline*, pp. 205-210, 2016.
- [5] D. Simon, *Optimal state estimation: Kalman, H infinity, and nonlinear approaches.*, John Wiley & Sons, 2006.
- [6] D. Krishnamoorthy, E. Jahanshashi and S. Skogestad, "Feedback Real time optimization approach using transient measurements," *Journal of process control*, in preparation.
- [7] L. Ye, Y. Cao, Y. Li and Z. Song, "Approximating Necessary Conditions of Optimality as Controlled Variables," *Industrial & Engineering Chemistry Research*, vol. 2, no. 52, pp. 798-808, 2012.
- [8] M. Câmara, A. Quelhas and J. Pinto, "Performance Evaluation of Real Industrial RTO Systems," *Processes*, vol. 4, no. 4, p. 44, 2016.
- [9] B. G. B. Hunnekens, M. A. M. Haring, N. van de Wouw and H. Nijmeijer, "A dither-free extremum-seeking control approach using 1st-order least-squares fits for gradient estimation," in *53rd IEEE Conference on Decision and Control (CDC)*, 2014.