A Dynamic Extremum Seeking Scheme Applied to Gas Lift Optimization ⋆

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Abstract: This paper presents the application of a data-driven optimization scheme using transient measurements to a gas-lift optimization problem. Optimal operation of a gas-lifted field involves controlling the marginal gas-oil ratio (mGOR), which is the steady-state gradient of the oil rate from the gas lift injection rate. In this paper we apply a dynamic extremum seeking scheme to estimate the marginal GOR online using transient measurements, which is based on identifying a local linear *dynamic* model around the current operating point instead of a local linear static model. By doing so, we can use the transient measurements and effectively remove the time-scale separation between the plant dynamics and the perturbation signal, that is typically required in the classical extremum seeking scheme. This results in significantly faster convergence to the optimum compared to classical extremum seeking scheme. The effectiveness of the proposed method is demonstrated using simulation results for a single gas lifted well, as well as a network of gas lifted wells.

Keywords: Production optimization, Measurement-based optimization, real-time optimization, extremum seeking control

1. INTRODUCTION

In oil production wells, when the reservoir pressure is sufficiently high, then the fluids from the reservoir flows naturally to the surface. Over time, the reservoir pressure drops and may no longer be sufficient to lift the fluids economically to the surface. In such cases, artificial lift methods are used to boost the production from the wells. One such commonly used artificial lift method is the gas-lift method, where compressed gases are injected into the well tubing via the well annulus. This reduces the fluid mixture density, hence reducing the hydrostatic pressure drop across the well tubing, leading to an increased oil production. However, injecting too much gas increases the frictional pressure drop, which has a detrimental effect on the oil production. The oil production rate starts to decrease if the effect of the frictional pressure drop becomes dominant over the effect of the hydrostatic pressure drop. Each gas-lifted well then has an optimal gas lift injection rate that maximizes the oil production. In addition, the amount of gas available for gas lift may be limited. The production optimization problem then deals with the problem of finding the optimal gas lift allocation for the gas lifted wells, in order to maximize the total oil production.

Daily production optimization is an important task for maximizing the daily operating revenue from a production network. Traditionally, production engineers use so-called *gas-lift performance curves* for daily production optimization, which maps the static relationship between the oil production and the gas lift injection rate for each well (Rashid, 2010). The gas lift performance curves are typically obtained using commercially available steady-state multiphase flow simulators. Steady-state nonlinear optimization tools may then be used to compute the optimal gas lift allocation among the different wells. Production engineers may also often use the gas-lift performance curves directly for production optimization by using a quantity known as *marginal gas-oil ratio* ($mGOR$). Marginal gas-oil ratio or simply marginal GOR, is a quantity that describes the increase in oil rate per unit change in the gas-lift injection rate. In other words, marginal GOR is given by the gradient of the gas-lift performance curves (Bieker et al., 2007).

The optimal allocation of the gas lift among the different wells is achieved when the marginal GOR is the same for all the wells (Urbanczyk et al., 1994). The principle of equal marginal cost has been proven to be the optimal solution for any parallel unit, e.g. by Downs and Skogestad (2011). Therefore, optimal operation of a gas lifted well network can be achieved by simply controlling the marginal GOR to be equal for all the wells. This is schematically represented for two wells in Fig.1.

The use of centralized dynamic optimization tools such as economic NMPC for production optimization has recently been gaining popularity. Codas et al. (2016) and Krishnamoorthy et al. (2016a) used economic MPC formulations to optimize production from a gas lifted well network. However, solving a numerical optimization prob-

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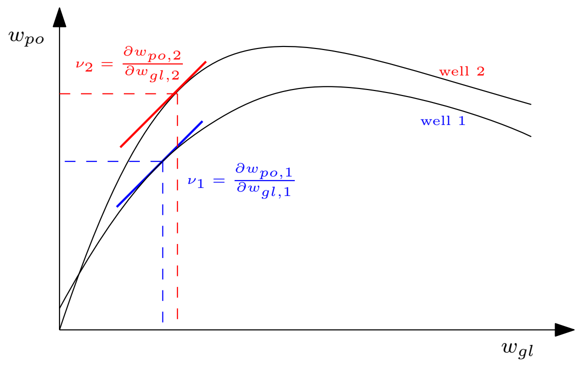

Fig. 1. Schematic representation of gas lift performance curves and the marginal GOR ν.

lem may be computationally intensive and can potentially lead to computational delays. Campos et al. (2009) point that many numerical issues need to be addressed before dynamic optimization can be used in practice for offshore oil and gas applications.

On the other hand there have been developments in optimization approaches that do not require solving numerical optimization problems. Instead, optimal operation is achieved via feedback control. Self-optimizing control is one such method (Skogestad, 2000), where the objective is to find the right controlled variable, which when kept constant, leads to near optimal operation (i.e. minimum loss). The use of self-optimizing control using nullspace method for gas lift optimization was demonstrated by Alstad (2005). Since self-optimizing control is based on local linearization around a nominal optimal point, it may lead to steady-state losses if the disturbances moves the operation of the process far away from this nominal operating point. The ideal self-optimizing variable for the gas-lift problem would indeed be the marginal GOR, which is the slope of the gas-lift performance curve, see Fig.1. However, the major challenge is that the marginal GOR is not a readily available measurement for control.

The optimization approaches mentioned above rely on the use of complex physical models either online or offline. However, models are often uncertain due to lack of knowledge or simplification, which can affect the optimal operation point computed by these methods. To address the issues related to model uncertainty, purely data-driven optimization tools become an attractive alternative to optimize the process under plant-model mismatch. To this end, we will focus on data-driven optimization tools that do not require complex physical models in the reminder of the paper.

The use of data-driven methods such as extremum seeking control for oil and gas production optimization has recently been gaining steady interest. Peixoto et al. (2015) and Krishnamoorthy et al. (2016b) applied the classical extremum seeking scheme for gas lift optimization for a single gas lifted well. Extremum seeking control involves estimating the steady-state gradient (i.e. marginal GOR) directly using the gas lift rate and oil production rate measurements. The estimated marginal GOR is then controlled to a constant setpoint using simple integral action to drive the system to its optimum.

Since extremum seeking control involves estimating the steady-state gradient directly from the measurements, the use of transient measurements leads to erroneous gradient estimation. Therefore, such methods often require clear time scale separation between the plant dynamics and the perturbation and the convergence to the optimum, such that the plant can be approximated as a static map (Krstić and Wang, 2000). This results in very slow convergence to the optimum.

For processes such as gas lifted oil wells that have long settling times (typically in the range of minutes to hours), the convergence to the optimum can be prohibitively slow due to the time scale separation requirements. This impedes the direct applicability of the classical extremum seeking control scheme for oil and gas applications. Although Extremum seeking control was used for optimizing a gas lifted well by Peixoto et al. (2015), Krishnamoorthy et al. (2016b) and Pavlov et al. (2017) to name a few, the convergence time to the optimum was not the main focus of these works.

There have been several improvements in extremum seeking control to address the issue of slow convergence. For example, Hunnekens et al. (2014) proposed to use a least-square based method to improve the convergence. However, this method still assumes the plant as a static map, restricting the use of transient measurements. Trollberg and Jacobsen (2016) proposed the so-called greedy extremum seeking control to optimize during the transients for chemical and bio-processes with long settling times. However, the greedy extremum seeking control can be implemented only for a class of systems with a specific timescale structure. Peixoto et al. (2017), recently proposed a phase-lock-loop based extremum seeking control to account for the phase shift due to the plant dynamics and speed up the convergence to the optimum point.

In this paper, we investigate a different approach, where we directly use the transient measurements to identify a local linear *dynamic* model around the current operating point, instead of a local linear static model. For example, we can identify linear ARX models from the process measurements that are locally valid around the current operating point. The steady-state gradient can then be estimated from the identified local linear dynamic model.

In fact, the use of measurements to identify a dynamic model around the current operating point for optimization dates back to 1977 in the work by Bamberger and Isermann (1978). This was later extended by McFarlane and Bacon (1989), where the authors presented an empirical strategy for open-loop online optimization using ARX models. The main motivation for these works were indeed to optimize the steady-state behaviour of slow dynamic processes in a relatively short period of time. With the recent surge of interest in extremum seeking control for oil and gas applications, we reframe this old idea in the context of extremum seeking control in this paper and show that the proposed *dynamic* extremum seeking control addresses the convergence issues of classical extremum seeking control, especially for processes with long settling times. In addition, we also propose a simple control structure for distributed dynamic extremum seeking control scheme and apply to an oil and gas production network.

The reminder of the paper is organized as follows. Section 2 introduces the proposed dynamic extremum seeking scheme. Section 3 demonstrates the effectiveness of the proposed dynamic ESC compared to the classical ESC scheme for a single gas lifted well. A distributed dynamic ESC scheme is also proposed and applied in Section 3 to a production network with 6 gas lifted wells before concluding the paper in Section 4.

2. DYNAMIC EXTREMUM SEEKING SCHEME

Consider a nonlinear process, where the objective is to drive the cost J to its minimum by using the input u.

Assumption 1. The plant cost J can be measured.

Assumption 2. The plant cost J can be represented as Hammerstein model with a combination of a nonlinear time invariant mapping $f(\cdot) : \mathbb{R} \to \mathbb{R}$, with proper, stable, finite-dimensional, linear, time-invariant (FDLTI) dynamics $G(s)$, at its output, see Fig.2.

Assumption 3. $f(u)$ is sufficiently smooth and continuously differentiable such that

$$\frac{\partial f}{\partial u}(u^*) = 0 \tag{1}$$

$$\frac{\partial^2 f}{\partial u^2}(u^*) > 0 \tag{2}$$

Assumption 3 ensures that $f(u)$ has a unique minimizer at $u = u^*$ and the goal is to drive u to the neighborhood of u^*.

In this section, we propose a dynamic extremum seeking scheme which is based on identifying a local linear *dynamic* model around the current operating point using transient measurements. The cost and input measurements from a fixed moving window containing the last N data samples are used to continuously fit an ARX model of the form,

$$\begin{aligned} J(t) = &- a_1 J(t-1) - \cdots - a_{n_a} J(t-n_a) \\ &+ bu(t-1) + \cdots + b_{n_b} u(t-n_b) + e(t) \end{aligned} \tag{3}$$

Remark 1. The input and cost measurements in (3) are pre-processed such that they are mean-centered (Ljung, 1999).

We estimate the ARX polynomials,

$$\theta = [a_1 \ \cdots \ a_{n_a} \ b_1 \ \cdots \ b_{n_b}] \tag{4}$$

using linear least squares estimation

$$\hat{\theta} = arg \ \min_{\theta} \|J - \Phi^{\mathsf{T}}\theta\|_2^2 \tag{5}$$

where Φ is given by the expression

$$\Phi = [-J(t-1) \ \ldots \ -J(t-n_a) \ u(t-1) \ \ldots \ u(t-n_b)]^{\mathsf{T}} \tag{6}$$

Introducing the notation

$$A_{poly}(q) = 1 + a_1 q^{-1} + \cdots + a_{n_a} q^{-n_a}$$

and

$$B_{poly}(q) = b_1 q^{-1} + \cdots + b_{n_b} q^{-n_b}$$

yields a local linear dynamic system of the form,

$$J(t) = \frac{B_{poly}(q)}{A_{poly}(q)} u(t) + \frac{1}{A_{poly}(q)} e(t) \tag{7}$$

The steady-state gradient around the current operating point can then be estimated by,

$$\hat{\mathbf{J}}_{\mathbf{u}} = A_{poly}(q)^{-1} B_{poly}(q) \tag{8}$$

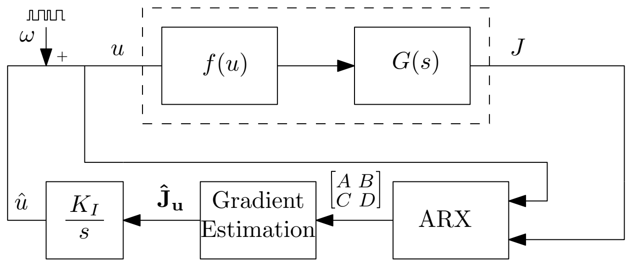

Fig. 2. A schematic representation of the proposed dynamic extremum seeking control scheme for a class of systems that can be modelled as Hammerstein models.

Alternatively, the identified ARX polynomials $A_{poly}(q)$ and $B_{poly}(q)$ can be converted to continuous time state-space system [1] as shown below,

$$\begin{aligned} \dot{x} &= Ax + Bu \\ J &= Cx + Du \end{aligned} \tag{9}$$

The steady state gain is then given by setting $\dot{x} = 0$ and eliminating the states x in (9),

$$J = \underbrace{\left(-CA^{-1}B + D\right)}_{\hat{\mathbf{J}}_{\mathbf{u}}} u \tag{10}$$

Once the steady-state gradient $\hat{\mathbf{J}}_{\mathbf{u}}$ is estimated, a simple integral action can be used to drive the system to its extremum. In discrete time, this can be expressed as,

$$\hat{u}(t+1) = \hat{u}(t) + \frac{K_I}{T_s} \hat{\mathbf{J}}_{\mathbf{u}} \tag{11}$$

where K_I is the integral gain and T_s is the sample time. Additional perturbation ω such as a pseudo random binary sequence (PRBS) signal is added to the input signal to provide sufficient excitation, $u(t+1) = \hat{u}(t+1) + \omega$. A schematic representation of the proposed dynamic extremum seeking scheme using ARX model identification is shown in Fig.2.

3. ILLUSTRATIVE EXAMPLE

3.1 Process description

We consider a production network with n_w gas lifted wells. The steady-state oil production rate for the i^{th} well $w_{po,i}$ is a function of the corresponding gas lift injection rate $u_i = w_{gl,i}$ and is given by the gas lift performance curve $w_{po,i} = f(w_{gl,i})$. Each gas lifted well is then modelled as a Hammerstein model with a proper stable first order linear dynamics $G_i(s)$

$$w_{po,i} = f(w_{gl,i}) G_i(s) w_{gl,i} \tag{12}$$

The use of such simplified Hammerstein models for gas-lifted well is justified in Peixoto et al. (2015) and Peixoto et al. (2017), where the authors show that the main features of the mechanistic model from Eikrem et al. (2006) can be sufficiently captured by using such a Hammerstein model. Plucenio et al. (2009) also use a Hammerstein model for gas lifted wells. Empirical models are also often used in practice (Hamedi et al., 2011).

[1] for example using `idss` and `d2c` command in `MATLAB`

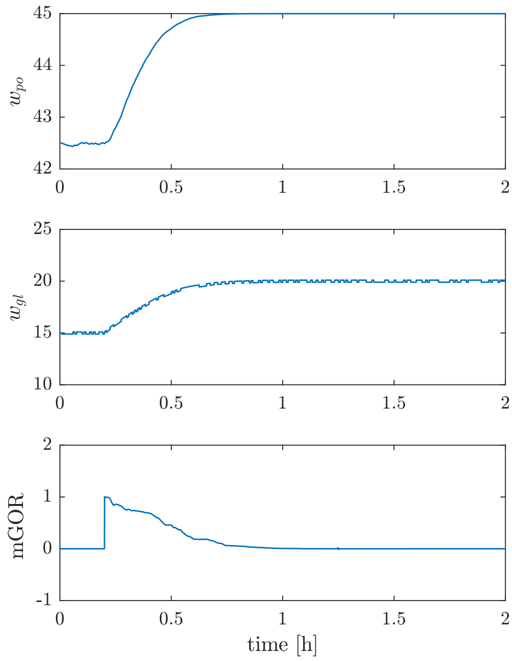

Fig. 3. Simulation results for the dynamic ESC for a single gas lifted well.

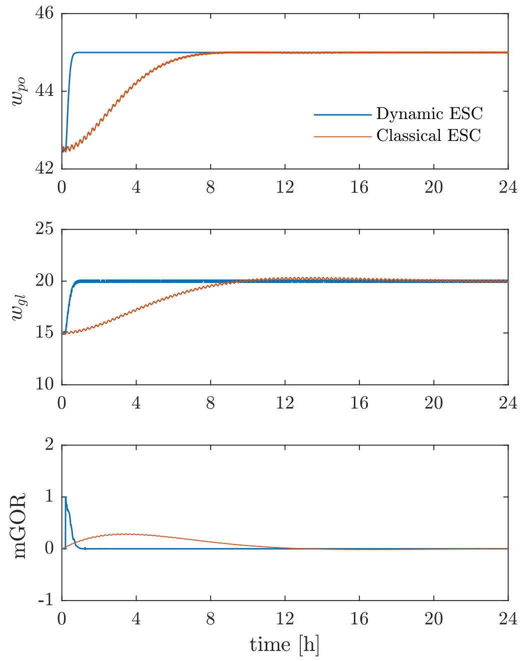

Fig. 4. Simulation results for the classical ESC for a single gas lifted well (red) compared to the dynamic ESC (blue).

We assume that the measurements for the oil production rate for each well $J_i = w_{po,i}$ is available. The objective is to maximize the total oil production rate w_{to} which is given by

$$\max J = w_{to} = \sum_{i=1}^{n_w} w_{po,i} \tag{13}$$

In many fields, the amount of gas available for gas lift injection is limited to w_{gl}^{max} and the total amount of the lift gas must be optimally allocated among the different wells. This is represented by the following constraint,

$$\sum_{i=1}^{n_w} w_{gl,i} \leq w_{gl}^{max} \tag{14}$$

Marginal gas oil ratio, which is defined as the change in the oil rate per unit change in the gas lift injection rate, is represented by the symbol ν

$$\text{mGOR}_i = \nu_i = \frac{\partial w_{po,i}}{\partial w_{gl,i}} \qquad \forall i \in \{1, \ldots, n_w\} \tag{15}$$

which is equivalent to the steady state gradient of (12) with respect to the gas lift injection rate.

3.2 Single gas lifted well

In the first simulation case, we demonstrate the effectiveness of the proposed dynamic extremum seeking scheme compared to the classical extremum seeking scheme using a single gas lifted well (i.e. $n_w = 1$), with special focus on the convergence time to the optimum. The gas lifted well model from Krishnamoorthy et al. (2016a) is captured by a Hammerstein system with the gas lift performance curve $w_{po} = -0.1(w_{gl} - 20)^2 + 45$ and first order linear dynamics

$$G(s) = \frac{1}{(1 + \tau s)} \tag{16}$$

where the time constant was set to $\tau = 174$s. For the dynamic extremum seeking scheme, we identify an ARX model with orders $n_a = 1$ and $n_b = 1$, using the past data points from a fixed moving window size of $N = 720$ samples. This implies that $A_{poly}(q) = 1 + a_1 q^{-1}$ and $B_{poly}(q) = b_1 q^{-1}$. Therefore two parameters, namely, a_1 and b_1 are fitted. The measurements are assumed to be available with a sample time of 1s. An additional PRBS perturbation with an amplitude of $0.1kg/s$ was added to the gas lift injection rate. An adaptation gain of $K_I = 0.005$ was used to drive the estimated steady state gradient (marginal GOR) to zero, since this is an unconstrained problem.

Fig.3 shows the simulation results for the proposed dynamic extremum seeking scheme for a single gas lifted well. Note that the extremum seeking controller was turned on after the first 720s (0.2h). It can be seen that the proposed scheme successfully drives the system to its optimal operating point within 1 hour.

We then compare the performance of the proposed method with the classical high-pass filter and low-pass filter based extremum seeking control (Krstić and Wang, 2000), where the system was perturbed with a sinusoidal signal with a time period of 800s and an amplitude of $0.1kg/s$. An adaptation gain of $K_I = 0.00075$ was used to drive the estimated steady state gradient to zero.

Fig.4 shows the simulation results of the classical extremum seeking scheme compared to the proposed dynamic extremum seeking scheme. It can be clearly seen that the classical extremum seeking scheme has a significantly slower convergence compared to the proposed dynamic extremum seeking scheme due to the static map

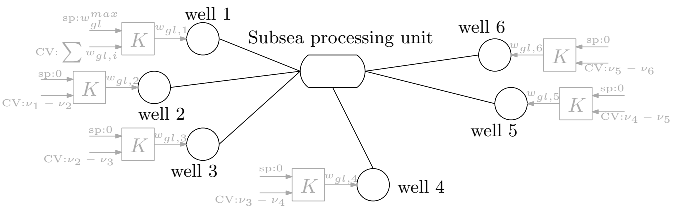

Fig. 5. Schematic representation of 6 gas lifted wells producing to a subsea processing unit. The proposed control structure is shown in grey blocks.

assumption. The classical ESC takes more than 15 hours to converge to the optimum, whereas the proposed dynamic ESC scheme converges only within 1 hour, making it more relevant for practical implementation. This example clearly demonstrates the effectiveness of the proposed dynamic extremum seeking scheme.

3.3 Gas lifted well network

In this simulation case, we now apply the proposed dynamic extremum seeking scheme to a production network consisting of $n_w = 6$ gas lifted wells producing to a common subsea processing unit. The total available gas for gas lift is limited to $w_{gl}^{max} = 56 kg/s$ during normal operation, which must be optimally allocated among the six wells. All the six wells are modelled as Hammerstein models with a polynomial function for the gas lift performance curve and linear first order dynamics with the time constants varying between 170 - 180s. See Ryu (2018) for detailed description of the models used for the six wells.

In the case of limited gas lift, the optimum operation happens when all the available gas is used for lifting (i.e. the constraint (14) is active at the optimum), which is typically the case in most gas lifted fields. According to good plantwide control practice (Skogestad and Postlethwaite, 2007), we then control the active constraint tightly using one of the wells. We use the remaining $(n_w - 1)$ unconstrained degrees of freedom to optimize the production from the well network. This is achieved by maintaining the marginal GOR for all the wells to be equal, according to the principle of equal slopes as described by Downs and Skogestad (2011).

We propose a simple decentralized control structure such that we have $(n_w - 1)$ feedback controllers to control the difference in the marginal GOR between two wells to a constant setpoint of zero and 1 feedback controller to control the active constraint tightly. In other words, the controlled variables for the $(n_w - 1)$ feedback controllers would be $(\nu_i - \nu_{i+1})$ for all $i \in \{1, \ldots, n_w - 1\}$ which is controlled to a constant setpoint of zero, thereby fulfilling the principle of equal slopes for optimal operation, and one feedback controller to control the total input usage $\sum_i^{n_w} w_{gl,i}$ to a constant setpoint of w_{gl}^{max}, as described by Krishnamoorthy et al. (2018). The marginal GOR ν_i for each well is estimated using the proposed dynamic extremum seeking scheme (10).

The simulation starts with normal operation with the total gas capacity constrained at $w_{gl}^{max} = 56 kg/s$. At time $t = 12$h, due to some unexpected topside disturbance, the processing capacity is reduced to $w_{gl}^{max} = 52 kg/s$. Fig.6 shows the simulations results for the 6 well case. The top subplot shows the oil production from the 6 wells and the second subplot shows the gas lift injection rates. From the principle of equal slopes, it is known that the optimal operation happens when then marginal GOR for all the wells are equal. It can be clearly seen that the marginal GOR for all the wells converge to a value of $1.5 kg/kg$ during normal operation and the marginal GOR of all the wells change to a value of $1.61 kg/kg$ when the processing capacity is reduced. Fig.7 shows the total oil production and the total gas lift injection rate.

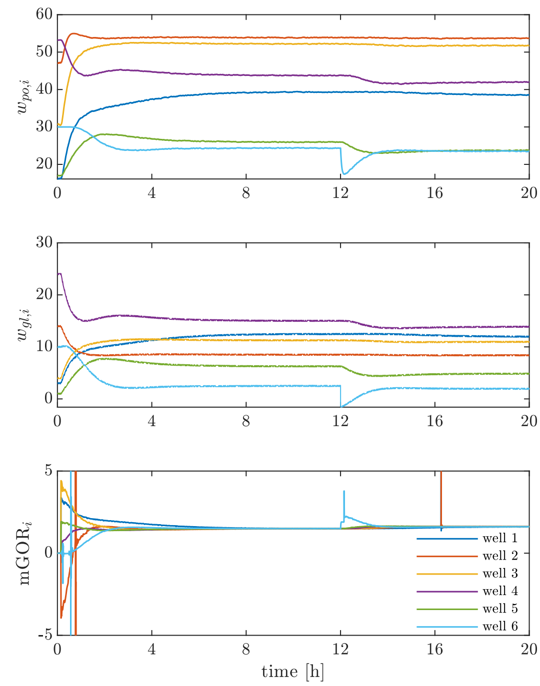

Fig. 6. Simulation results for the dynamic extremum seeking scheme applied to a network of 6 gas lifted wells.

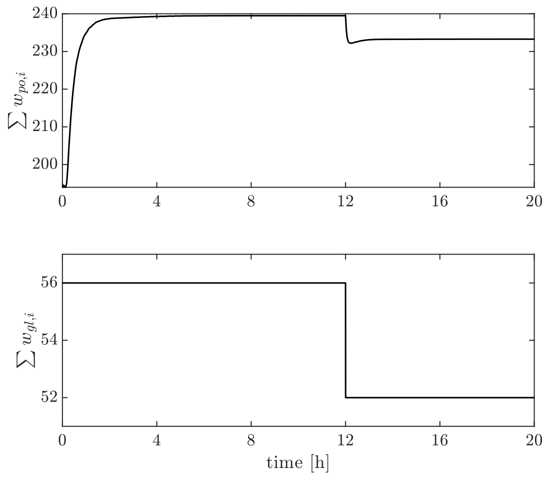

Fig. 7. Simulation results showing the total oil production rate from the well network (top subplot) and the total gas lift injection rate (bottom subplot).

Table 1 shows the oil production rate converged to steady-state using the proposed dynamic extremum seeking scheme compared to the true optimum which is computed by solving a nonlinear optimization problem (used as benchmark). This shows that the proposed scheme is able to drive the system to its true optimum.

Table 1. Oil production rates converged to the steady state using the proposed method compared to the true optimum .

w_{gl}^{max}	True optimum $56kg/s$	True optimum $52kg/s$	Converged solution $56kg/s$	Converged solution $52kg/s$
well 1	39.375	38.51	39.37	38.57
well 2	53.875	53.70	53.88	53.63
well 3	52.187	51.75	52.19	51.77
well 4	43.75	42.02	43.75	42.01
well 5	25.9375	23.78	25.93	23.78
well 6	24.375	23.51	24.4	23.49
Total	239.5	233.28	239.5	233.25

4. CONCLUSION

In this paper we proposed a dynamic extremum seeking scheme for a class of systems that can be modeled as Hammerstein models, which is based on identifying a local linear dynamic model around the current operating point. The steady-state gradient is estimated from the identified ARX model using (10). By using transient measurements for the gradient estimation, we have effectively eliminated the time scale separation required between the plant dynamics and the dither signal. This leads to a significantly faster convergence to the optimum, especially for systems with very long settling times, as demonstrated in Fig.3 and Fig.4. Additionally, for a network of gas lifted wells, we presented a simple decentralized framework for optimal allocation of lift gas in a network of gas lifted wells using the proposed dynamic extremum seeking scheme.

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