$\mathcal{L}_1$ Adaptive Anti-Slug control

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***Abstract*—In many multiphase pipeline-riser systems, anti-slug control is necessary to ensure steady and optimal operation. In this paper, we propose using an $\mathcal{L}_1$ adaptive controller as an augmentation to standard PI control to stabilize the desired non-slugging flow regime. The $\mathcal{L}_1$ adaptive controller is designed based on a model identified from an experimental closed-loop step test. The proposed design involves fewer tuning parameters compared to other adaptive control methods and it does not need any observer. We have tested the controller by simulations in both MATLAB and OLGA, and by experiments in a small-scale laboratory. The results show that the proposed solution can stabilize the process outside the stability region of the traditional PI controller.**

I. Introduction

In a gas-liquid pipeline-riser system, slugging occurs if the liquid column blocks the gas flow. The gas will accumulate in the pipeline and the pressure will eventually exceed that of the liquid column in the riser, leading to a blow-out of the accumulated liquid into the topside separator.

Riser slugging causes several problems in pipeline-riser systems. The liquid level of the topside separator might exceed the maximum limit due to the large volume of the slug. There is a risk of shutting down production completely if the separator is unable to handle the slug. Another issue is the danger of equipment damage and stress caused by the high velocity of the slugs. [1].

Several methods exist to handle slugs, like installing passive slug-catchers or increasing the size of the topside separator. These are both expensive and space-demanding solutions and if slug-flow was not expected when building the production platform, it might even be impossible to install them. Another method to suppress the slugs is to reduce the opening of the topside choke valve. This ”open-loop” solution requires no control and is simple and inexpensive, but it will cause a higher backpressure through the production system and may therefore give large economic losses due to reduced production. [2].

A third solution is to use closed-loop feedback control based on measuring for example pressure and adjusting the valve position. The objective is to extend the region of the desired non-slug flow regime. The valve will move all the time, but its average value will be larger and thus give a lower pressure and higher production. Automatic control of the topside choke valve was investigated in [3] where they were able to suppress slugs by using automatic PI control.

Since then, several anti-slug control solutions have been developed. In 1996, Total presented an anti-slug control method that enables pipeline operations within the slugging region [4]. In 1999, in an independent study, ABB installed a similar anti-slug feedback solution on the Hod-Valhall platform operated by BP-Amoco [5].

In [2] and [6, Ch. 5], several PI control solutions and strategies are presented. These include both volumetric flow control and pressure control and the idea of using a cascaded control solution combining flow and pressure control. In [6, Ch. 6], model-based control solutions are presented and it is shown that a MISO H_∞ controller is not limited by RHP-zeros when both topside pressure and topside flow measurements are used. An LQG controller [6, Ch. 6] with an extended Kalman filter is designed, but this solution shows significantly worse performance than the H_∞ controller. Simple PI control is used in [7] where both the topside choke valve and a riser base valve are used to stabilize the riser base pressure.

In [8] an identified model of the unstable system is used to obtain an IMC (Internal Model Control) design. The IMC controller is used to obtain tuning parameters for PI and PID controllers at different operating points. Gain-scheduling can be implemented based on multiple IMC controllers for the different operating points. The resulting controllers show good robustness and are able to stabilize the system at different operating points.

Non-linear control solutions have also been used for anti-slug control. In [9], a non-linear state-feedback solution is proposed. The solution stabilizes the mass in the riser and it is able to control the process at operating points where the PI controller is unable to stabilize. It is, however, assumed that the full state is available for feedback, so a high gain observer is used. In [10], a feedback linearizing controller is proposed. This controller is able to stabilize the system at large valve openings without re-tuning and does not use an observer. It does, however, use both topside and riser-base pressure measurements.

More recently, autonomous approaches have been suggested to automatically change the riser base pressure setpoint. A lower setpoint implies a larger valve opening, lower back pressure, but stabilization becomes more difficult.

In [11], a slug detection calculation is performed and a supervisory system changes between two PID controllers.

One PID controller is designed to stabilize the slug flow and the other to slowly open the choke valve until slugging occurs.

In [12], the supervisory system detects oscillations and decreases the setpoint only if the system is considered stable. Otherwise the setpoint is increased to the last stable setpoint. An MRAC (Model Reference Adaptive Control) method that uses an observer-like reference model is applied. The solution uses an LQR controller as a baseline stabilizing controller and the adaptive controller is designed to assist the LQR controller if the performance degrades.

PI and PID controllers for anti-slug control seems to be the solution of choice in many real applications due to their simplicity and performance under nominal conditions, but in academia the focus is on higher order controllers, like non-linear and adaptive solutions. In this paper, we propose to use an $\mathcal{L}_1$ adaptive controller [13] with a PI controller as a baseline controller. The adaptation extends the stabilizable region enabling operation with larger valve openings.

We consider a system that has a PI controller installed and tuned for certain operating conditions, but the operating conditions change such that the PI controller fails. A change in operating conditions will occur in, e.g. a well that loses pressure over time. The proposed $\mathcal{L}_1$ adaptive controller depends only on feedback from the riser-base pressure and does not need an observer. This reduces the number of tuning parameters. The tuning of the $\mathcal{L}_1$ controller is based on a linear model of the dynamics from the valve opening to the bottom pressure. The model is identified from the response of the closed-loop system to an arbitrary small setpoint change. Once the system is rewritten in the appropriate form, the $\mathcal{L}_1$ adaptive controller method is straightforward to implement by following the guidelines of [13].

This paper is divided into the following sections: Section II describes the simulation model and experimental setup. The $\mathcal{L}_1$ adaptive control system is derived in Section III. Section IV contains the results from Matlab and OLGA simulations and from small-scale lab experiments. In Section V, the results are discussed and the concluding remarks are presented.

II. Slug model and system description

Several simplified models for pipeline-riser slugging exist [14]–[17]. In this paper we use the Jahanshahi model from [17]. The model contains four state variables as follows

- $m_{g,p}$: Mass of gas in pipeline
- $m_{l,p}$: Mass of liquid in pipeline
- $m_{g,r}$: Mass of the gas in riser
- $m_{l,r}$: Mass of the liquid in riser

The dynamic equations for the states are

$$\dot{m}_{g,p} = w_{g,in} - w_g \tag{1a}$$

$$\dot{m}_{l,p} = w_{l,in} - w_l \tag{1b}$$

$$\dot{m}_{g,r} = w_g - \alpha w_{out} \tag{1c}$$

$$\dot{m}_{l,r} = w_l - (1-\alpha) w_{out} \tag{1d}$$

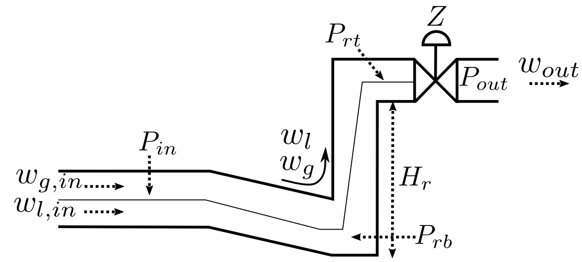

Fig. 1. Schematic of a pipeline-riser system.

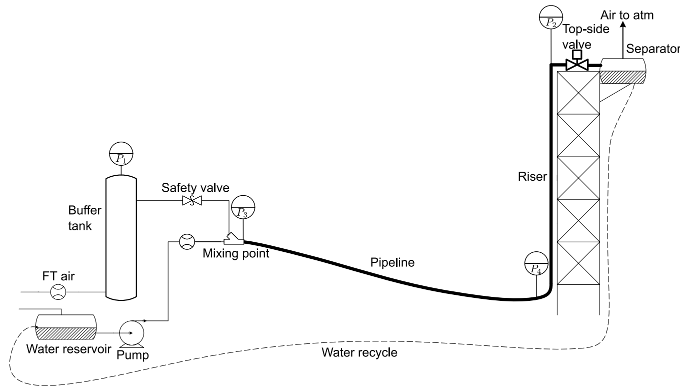

Fig. 2. The anti-slug experimental lab at NTNU. Figure adapted from [18].

where $w_{g,in}$ and $w_{l,in}$ are the gas and liquid mass inflow rates respectively. The inflows are considered to be constant, but could also be implemented as pressure-driven. The gas mass flow from the pipeline to the riser is denoted by w_g and w_l is the liquid mass flow from the pipeline to the riser. The outflow w_{out} is the total mass flow through the topside choke valve and α is the gas mass fraction at the top of the riser. The schematics of a pipeline-riser system is shown in Fig. 1. The mass outflow, w_{out}, is determined by the topside choke valve opening percentage, $Z \in [0, 100]$.

The model is tuned to fit the small scale anti-slug experimental lab at the Department of Chemical Engineering at the Norwegian University of Science and Technology (NTNU). This particular system enters unstable slugging flow at $Z \geq 15\%$ valve opening. A simulation model of the lab has also been implemented in the OLGA simulation software.

III. $\mathcal{L}_1$ Adaptive Anti-Slug control

PI and PID controllers have been showed to work well for anti-slug control. A well-known issue with linear controllers is that they might not work outside the region of operation for which they were designed.

In this paper we consider a situation where a PI controller is installed and tuned for a certain operating condition. We then decrease the setpoint for the riser-base pressure. This causes the valve opening to increase and the PI controller will eventually fail because of the change in operating point. By augmenting the PI controller with an $\mathcal{L}_1$ adaptive controller, we are able to stabilize the flow at setpoints the PI controller is unable to.

A. 1st order approximation of open-loop transfer function

Consider the transfer function from the valve opening to the riser bottom pressure for the four state model (1) linearized about an unstable operating point. In [8] this is found to be a fourth order transfer function on the form

$$G(s) = \frac{\theta_1(s+\theta_2)(s+\theta_3)}{(s^2-\theta_4 s+\theta_5)(s^2+\theta_6 s+\theta_7)} \tag{2}$$

It is shown, through a Hankel Singular Value decomposition, that the stable part of the system has little dynamic contribution. A reduced order model can be derived as

$$G_{ol}(s) = \frac{b_1 s + b_0}{s^2 - a_1 s + a_0} \tag{3}$$

with $a_0, a_1 > 0$. This model is further reduced to an unstable first order open-loop model

$$\bar{G}_{ol}(s) = \frac{b}{s-a} \tag{4}$$

where $a > 0$ and $b < 0$. It is shown in [8] that this model is incorrect, but we will show that the error in this simplification is handled by the $\mathcal{L}_1$ adaptive controller.

B. $\mathcal{L}_1$ adaptive controller

The overall system consists of the unstable process, a baseline PI controller and an adaptive controller that augments the baseline controller. The states of the system are

$$x_1 = \int_0^t y - r_p \, \mathrm{d}t \tag{5a}$$

$$x_2 = y - r_p \tag{5b}$$

where y is the measured output (i.e, the riser-bottom pressure) and r_p is the reference pressure which is assumed constant. All states are available through measurement or calculation. The dynamic equations describing this system in open-loop are

$$\dot{x}_1 = x_2 \tag{6a}$$

$$\dot{x}_2 = ay + b(\omega(t)u + \theta_0(t)y + \sigma_0(t)) \tag{6b}$$

where $a > 0$, b is unknown with known sign, $\omega(t)$ is a time-varying uncertainty in the input gain, $\theta_0(t)$ is a time-varying initial parameter uncertainty and $\sigma_0(t)$ models a time-varying initial input disturbance. The control input is

$$u = u_m + u_{ad} \tag{7}$$

where $u_m = k_p x_2 + k_i x_1$ with $k_p,\ k_i > 0$ is the baseline PI controller and u_{ad} is the adaptive control signal. Inserting the baseline controller and adding and subtracting $bk_i x_1$, $bk_p x_2$, ar_p and $\theta_0 r_p$ gives a system on the form

$$\dot{x}_1 = x_2 \tag{8a}$$

$$\begin{aligned}\dot{x}_2 = {} & a_m x_2 + bk_i x_1 + b(\omega(t)u_{ad} + (\omega-1)k_p x_2 \\ & + (\omega(t)-1)k_i x_1 + \frac{a}{b} r_p + \theta_0(t) x_2 + \sigma_0(t) + \theta_0 y)\end{aligned} \tag{8b}$$

where $a_m = a + bk_p$. The system is then cast into the form

$$\dot{x} = A_m x + B_m(\omega(t)u_{ad} + \theta(t)x_2 + \sigma(t)) \tag{9}$$

where

$$A_m = \begin{bmatrix} 0 & 1 \\ bk_i & a_m \end{bmatrix} \tag{10}$$

$$B_m = \begin{bmatrix} 0 \\ b \end{bmatrix} \tag{11}$$

$$\theta(t) = \theta_0 + (\omega(t)-1)k_p \tag{12}$$

$$\sigma(t) = \sigma_0 + (\omega(t)-1)k_i x_1 + \left(\frac{a}{b} + \theta_0\right) r_p. \tag{13}$$

A_m is Hurwitz because $a_m, bk_i < 0$.

Assumption 1: The unknown parameters and their derivatives are bounded

$$|\theta(t)| \le \Theta_0, \quad |\sigma(t)| \le \Delta_0, \ \forall t \ge 0$$

$$|\dot{\theta}(t)| \le d_\theta < \infty, \quad |\dot{\sigma}| \le d_\sigma < \infty, \ \forall t \ge 0$$

Assumption 2: Partial knowledge of the uncertain input gain

$$\omega \in \Omega_0 \triangleq \begin{bmatrix} \omega_{l0}, & \omega_{u0} \end{bmatrix}$$

where $0 < \omega_{l0} < \omega_{u0}$ are known upper and lower bounds on ω.

Consider the following predictor system

$$\dot{\hat{x}} = A_m \hat{x} + B_m(\hat{\omega}(t)u_{ad} + \hat{\theta}(t)\hat{x}_2 + \hat{\sigma}(t)) \,. \tag{14}$$

Looking at (9) and (11), it is clear that $B_m\omega$ is not affected by the PI controller gains k_p and k_i. The open-loop gain of the first order approximation in (4) is unknown, but we know that it is small and that the sign is negative.

The unknown parameters $\omega(t)$, $\theta(t)$ and $\sigma(t)$ are replaced by estimates $\hat{\omega}(t)$, $\hat{\theta}(t)$ and $\hat{\sigma}(t)$. The adaptation of the unknown parameters is governed by the projection-based adaptation laws

$$\dot{\hat{\theta}} = \gamma \mathrm{Proj}(\hat{\theta}(t), -\tilde{x}_2 pbx_2), \ \ \theta(0) = \theta_0, \tag{15a}$$

$$\dot{\hat{\omega}} = \gamma \mathrm{Proj}(\hat{\omega}(t), -\tilde{x}_2 pbu_{ad}), \ \ \omega(0) = \omega_0, \tag{15b}$$

$$\dot{\hat{\sigma}} = \gamma \mathrm{Proj}(\hat{\sigma}(t), -\tilde{x}_2 pb), \ \ \sigma(0) = \sigma_0 \tag{15c}$$

where $\tilde{x}_2 := \hat{x}_2 - x_2$, γ is the adaptation rate and p is the solution to the algebraic Lyapunov equation $a_m p + p a_m = -q$ for arbitrary $q > 0$. We have chosen $q = 1$ which gives $p = 5$ in all cases. The projection operator ensures that the unknown parameters are bounded.

The adaptive control signal is generated as

$$u_{ad}(s) = -kD(s)(\hat{\eta}(s) - k_g r_e(s)). \tag{16}$$

Since we are controlling the error dynamics, the reference r_e is zero and the term $k_g r_e(s)$ disappears. $\hat{\eta} \triangleq \hat{\omega}(t)u_{ad} + \hat{\theta}(t)x_2 + \hat{\sigma}(t)$, $k > 0$ is a feedback gain and $D(s)$ is chosen as $D(s) = \frac{1}{s}$. Stability proofs for the $\mathcal{L}_1$ adaptive controller can be found in [13].

TABLE I

Parameters used in MATLAB simulations.

Parameter	Value	Description
k_p	7.5	Proportional gain
τ_i	100	Integral time
k_i	k_p/τ_i	Integral gain
a_m	1/10	Prediction model natural frequency
b	-1/20	Prediction model static gain
γ	20	Adaptation gain
k	1	Feedback gain

IV. Results

The controller is implemented in both a MATLAB and OLGA simulation model and in the small-scale anti-slug laboratory test setup at NTNU. The lab operates with water and air as the liquid and gas phase respectively. The nominal inflow rate of water and air is approximately 4 l/min and 4.5 l/min respectively. The pressure in the topside separator is nominally constant at atmospheric pressure. This system is unstable at a valve opening of $Z \geq 15\%$ and is stabilized by a PI controller and with the $\mathcal{L}_1$ adaptive controller.

It is desirable to operate at lower pressure setpoints because this increases the production rate from the subsea oil wells. However, stabilization is more difficult at low pressure setpoints, which implies large valve openings, because the process gain decreases and approaches zero.

Disturbances such as changes in the inflow of gas and liquid are not considered as this would only shift the region of instability depending on the change. A higher gas flow or lower liquid flow implies more gas in the system and hence, a more stable system. The opposite is true for low gas flows and high liquid flows. The has been tested with the proposed solution in Matlab with positive results, but is not included in the paper due to space limitations.

A. Matlab simulations

The parameters used in the MATLAB simulations are listed in Table I. The prediction model time constant is chosen as $\tau = 10$ seconds giving a prediction model natural frequency $a_m = 1/10$. The prediction model gain is chosen as a relatively small, negative value, based on knowledge of the system.

Fig. 3 shows the result when only using the PI controller. The system is stable during the first setpoint change, but goes unstable when the setpoint is further reduced. It is clear that the PI controller cannot handle the setpoint change and would need re-tuning or augmentation by an adaptive controller to work. The $\mathcal{L}_1$ controller is activated and Fig. 4 shows the same setpoint changes as in Fig. 3. Now, the process is stabilized at the new setpoints.

The adapted parameters are shown in Fig. 5. Here we see that the estimated gain, $\hat{\omega}$, is reduced whenever the setpoint is changed. The value of $\hat{\omega}$ is larger or close to one, which indicates that our initial guess of b was too low. The unknown disturbance parameter $\hat{\sigma}$ is close to -5 after the setpoint changes. $\hat{\theta}$ is largely affected by measurement noise, causing it to act as an integrator.

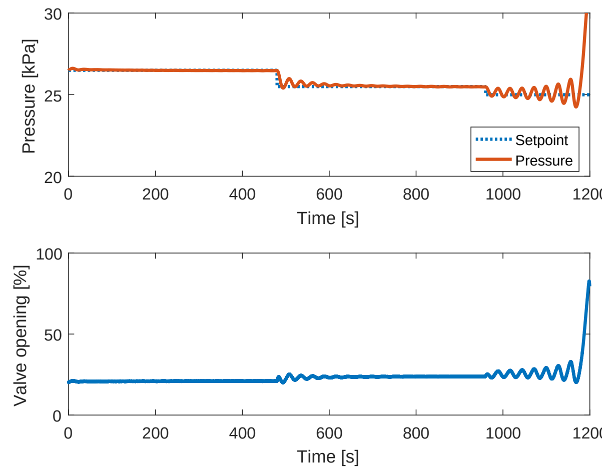

Fig. 3. Pressure at riser-bottom and valve opening when only using a PI controller. MATLAB simulation.

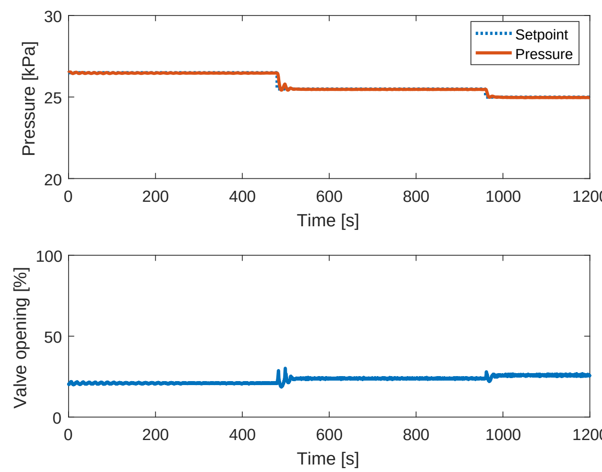

Fig. 4. Pressure at riser-bottom and valve opening when using a PI controller and a $\mathcal{L}_1$ adaptive controller together. MATLAB simulation.

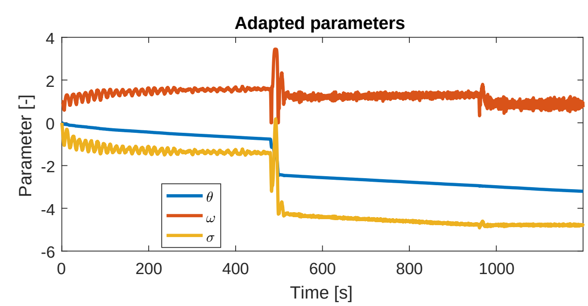

Fig. 5. The adapted parameters during the MATLAB simulation.

B. OLGA simulations

OLGA is an advanced dynamic multiphase flow simulator [19]. The parameters used in the OLGA simulations are listed in Table II. We have used the same values for the prediction model, but due to the slow sampling time, the adaptation gain, γ, and the feedback gain, k, had to be

TABLE II
PARAMETERS USED IN OLGA SIMULATIONS.

Parameter	Value	Description
k_p	7.5	Proportional gain
τ_i	100	Integral time
k_i	k_p/τ_i	Integral gain
a_m	1/10	Prediction model natural frequency
b	-1/20	Prediction model static gain
γ	1	Adaptation gain
k	0.5	Feedback gain

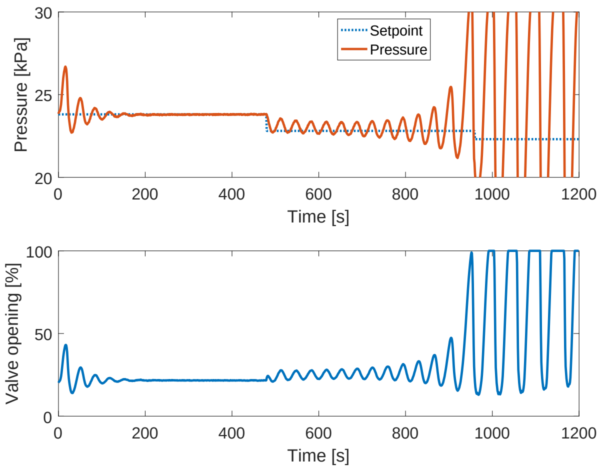

Fig. 6. Pressure at riser-bottom and valve opening when only using a PI controller. OLGA simulation.

reduced.

The same system is implemented in the oil and gas simulation software OLGA. The controllers are implemented in MATLAB, which is connected to OLGA via the OLGA OPC server. The OLGA OPC server only allows a sampling time of $T_s = 1$ second, hence the tuning of the $\mathcal{L}_1$ controller had to be changed slightly.

The results of only using the PI controller is shown in Fig. 6. It is clear that the PI controller is unable to stabilize the process when the setpoint moves too far away from the region of linearization. Fig. 7 shows the result of the same setpoint changes when the $\mathcal{L}_1$ adaptive controller is activated. The process is stabilized at lower setpoints. Looking at the adapted parameters in Fig. 8 it is clear that the gain is reduced when changing setpoints as $\hat{\omega}$ is reduced. $\hat{\sigma}$ reduces when the setpoint changes are initiated and $\hat{\theta}$ converges to a value close to -5.

C. Laboratory results

The parameters used in the lab experiments are listed in Table III. The controller was implemented in the small-scale anti-slug laboratory at NTNU, Fig. 2. Due to measurement noise, the controller needed some re-tuning. The lab is actually more stable than the model, and hence, the initial valve opening was moved to $Z = 30\%$.

As can be seen in Fig. 9, the PI controller is able to control the process during the first setpoint change, but the process goes unstable when the second setpoint change is introduced.

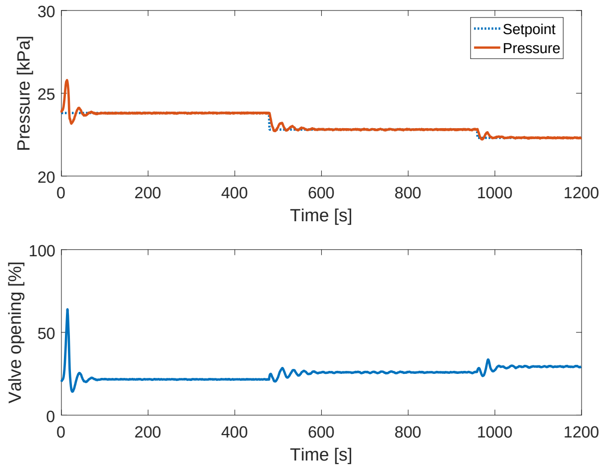

Fig. 7. Pressure at riser-bottom and valve opening when using a PI controller and a $\mathcal{L}_1$ adaptive controller together. OLGA simulation.

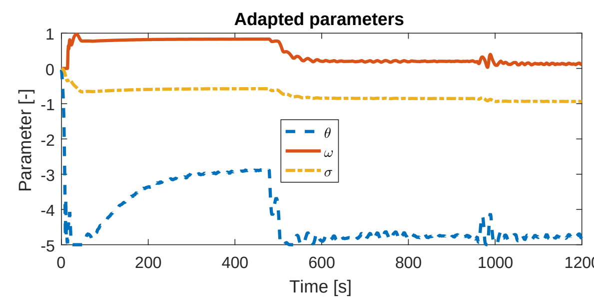

Fig. 8. The adapted parameters during the OLGA simulation.

TABLE III
PARAMETERS USED IN LAB EXPERIMENTS.

Parameter	Value	Description
k_p	7.5	Proportional gain
τ_i	100	Integral time
k_i	k_p/τ_i	Integral gain
a_m	1/10	Prediction model natural frequency
b	-1/20	Prediction model static gain
γ	10	Adaptation gain
k	1	Feedback gain

The $\mathcal{L}_1$ controller is able to stabilize the system at the second setpoint, as can be seen in Fig. 10. The adapted parameters, Fig. 11, are very noisy in this case. We believe this is caused by the measurement noise.

V. CONCLUDING REMARKS

The results show a clear improvement when using the $\mathcal{L}_1$ adaptive controller together with a PI controller. In a real setting, the tuning of the PI controller could of course be improved, but this can be time consuming and would require specially trained personnel. We believe that this retuning is unnecessary or at least less frequent when using the proposed adaptive method. Instead of replacing the whole control system with a purely adaptive solution, we augment the PI controller with the $\mathcal{L}_1$ adaptive controller because a PI controller with satisfactory performance at nominal conditions might be installed already. Our solution is designed

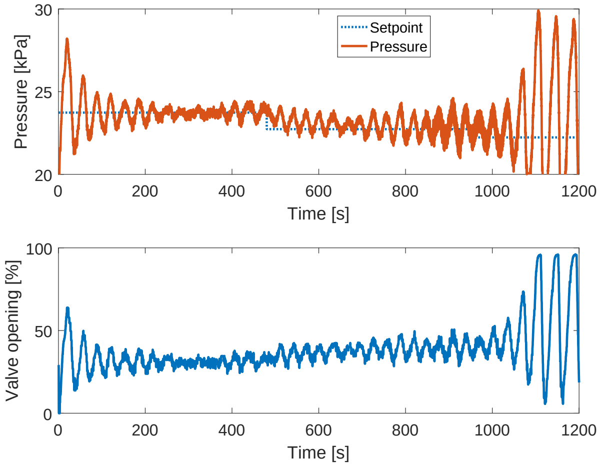

Fig. 9. Pressure at riser-bottom and valve opening when using a PI controller. Laboratory experiment.

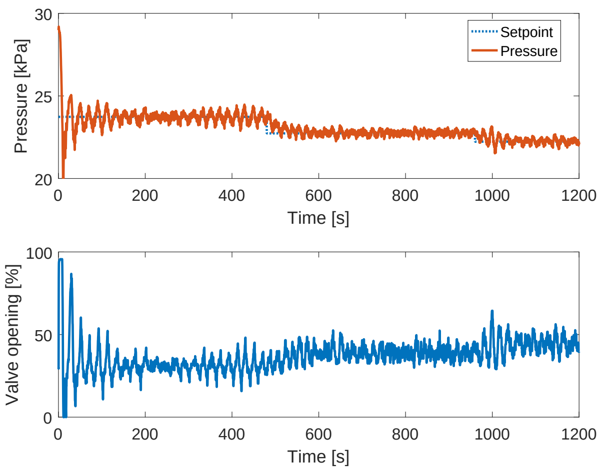

Fig. 10. Pressure at riser-bottom and valve opening when using a PI controller and a $\mathcal{L}_1$ adaptive controller together. Laboratory experiment.

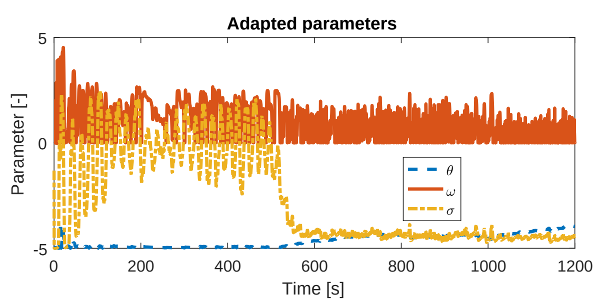

Fig. 11. The adapted parameters during the laboratory experiments.

to maintain this performance when the nominal conditions changes. The suggested $\mathcal{L}_1$ adaptive controller contains fewer tuning parameters compared to the proposed MRAC with observer-based reference model of [12].

We have made no qualitative comparison of our solution with other proposed anti-slug control solutions, but this could be looked into in the future.

The $\mathcal{L}_1$ adaptive controller can utilize fast adaptation by choosing large adaptation gain, γ. This will increase performance according to [13]. In our experiments, the adaptation gain had to be quite low $(1-20)$. This is thought to be because of measurement noise and a relatively slow sampling time of 0.1 second.

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