Antonio Espuña, Moisès Graells and Luis Puigjaner (Editors), Proceedings of the 27th European Symposium on Computer Aided Process Engineering – ESCAPE 27
October 1st - 5th, 2017, Barcelona, Spain

Gas Lift Optimization under Uncertainty

Dinesh Krishnamoorthy[a], Bjarne Foss[b] and Sigurd Skogestad[a*]

[a]*Department of Chemical Engineering, NTNU, Trondheim, Norway*
[b]*Department of Engineering Cybernetics, NTNU, Trondheim, Norway*
skoge@ntnu.no

Abstract

In this paper, we consider the problem of production optimization under uncertainty applied to gas lifted well networks. Worst-case and scenario optimization methods are presented to explicitly handle the uncertainty. We also compare the performance and computation time of the presented methods with nominal and ideal cases using Monte Carlo simulations. We show that the scenario optimization method is able to reduce the conservativeness, however at the cost of computation time. We also show that the performance can be improved by parameter adaptation using an extended Kalman filter for combined state and parameter estimation.

Keywords: Real-time optimization (RTO), uncertainty, scenario tree, gas lift optimization, Extended Kalman filter

1. Introduction

In offshore oil and gas production processes, the problem of production optimization (equivalent to RTO) seeks to optimize some economic objective while satisfying process constraints. The models used in RTO are typically obtained either from fundamental knowledge of the system or from experimental data or a combination of both. However, lack of knowledge, model simplification and sparsity of experimental data (from well tests) makes the models used in production optimization to be inherently uncertain.

A significant amount of work on production optimization can be found in the literature. However, most of these considers a deterministic problem and the uncertainty is simply ignored. A few works that do consider the problem of optimization under uncertainty only considers static models, Hanssen and Foss (2015). In our recent paper, Krishnamoorthy et al. (2016), we considered the problem of gas lift optimization under uncertainty using dynamic models. We proposed the use of scenario-based optimization to provide a robust feasible solution whilst reducing the conservativeness from the worst-case optimization. The performance of scenario optimization was demonstrated using a simulation example, however, the computational times were not considered. In this paper, we give a more thorough comparison of nominal, worst-case and scenario optimization in terms of performance and computation time using Monte-Carlo simulations. Additionally, the plant model considered in our earlier work did not include pressure coupling in the well models and perfect state feedback of all the states was also assumed. This is rarely the case in the real applications. Therefore in this paper, we also extend the work by including pressure coupling in the well models and an Extended Kalman filter (EKF) for state feedback. In addition, we also show that the performance can be further improved by implementing an augmented Kalman filter for combined state and parameter estimation.

2. Problem statement

In this work, we consider a production network consisting of two gas lifted wells connected to a common riser manifold which produces to an offshore separator. The objective of the production optimization problem is then to find the optimal gas lift injection rate for each well such that the total oil production is maximized while satisfying the gas capacity constraints. Note that the common riser manifold introduces pressure coupling between the wells. This implies that any change made in one well also has an effect on the other well. The separator is assumed to be at a constant pressure. The dynamic model of a gas lifted well network was developed based on physical knowledge of the system. The dynamics are introduced in the form of mass holdups in each well and in the riser. The densities, pressures, flow rates in each well and riser are described by algebraic equations. Detailed information about the model can be found in Krishnamoorthy et al. (2016). The production network model can be expressed as a semi explicit index-1 DAE system of the form

$$\dot{\mathbf{x}} = f(\mathbf{x},\mathbf{z},\mathbf{u},\mathbf{p}) \tag{1}$$

$$g(\mathbf{x},\mathbf{z},\mathbf{u},\mathbf{p}) = 0 \qquad \mathbf{p} \in \mathscr{U} \tag{2}$$

where, $\mathbf{x}$ and $\mathbf{z}$ denote the differential and algebraic states respectively, $\mathbf{u}$ represents the control inputs, $\mathbf{p}$ represents the uncertain parameters belonging to a compact set $\mathscr{U}$. In this work, we assume that the gas-oil ratio (GOR) is the uncertain parameter and we assume that the expected value $\mathbb{E}(GOR)$ and the variance σ for each well is known a-priori, i.e. $\mathbf{p}_i \in \{\mathbb{E}_0(GOR_i) \pm \sigma_i\}\ \forall i \in \mathscr{N}$, where the subscript i denotes any individual well from a set of $\mathscr{N} = \{1,\cdots,n_w\}$ wells.

3. Daily Production Optimization

Before we can formulate the production optimization problem, the infinite dimensional problem is first discretized into a finite dimensional problem which is divided into N equally spaced sampling intervals. We use a third order direct collocation method for polynomial approximation of the system in Eq.(1) and the discretized system dynamics at any time instant k can be expressed as $\mathbf{F}(\tilde{\mathbf{x}}_k, \mathbf{x^0}_k, \tilde{\mathbf{z}}_k, \tilde{\mathbf{u}}_k, \mathbf{p}) = 0$, where $\tilde{\mathbf{x}}_k$ and $\tilde{\mathbf{z}}_k$ are the discretized differential and algebraic state vectors containing the three collocation points of the states at each sampling interval k. The vector of initial differential states at each interval given by $\mathbf{x^0}_k$, ensures state continuity by forcing the last collocation point of any given time interval $\mathbf{x}_{k,3}$ to be equal to the initial condition of the consecutive time interval $\mathbf{x}_{k+1,0}$. $\tilde{\mathbf{u}}_k$ denotes the vector of control inputs which is assumed to be piecewise constant in each interval. A detailed explanation on how the system is discretized into a nonlinear programming problem using direct collocation can be found in Krishnamoorthy et al. (2016).

The mathematical optimization problem can now be formulated as shown below,

$$\min_{\theta} J = -\sum_{k=1}^{N}\left(\alpha_o \sum_{i=1}^{n_w} w_{po_i} - \alpha_{gl}\sum_{i=1}^{n_w} w_{gl_i}\right) + \sum_{k=1}^{N}\|\Delta u\|_2 + \rho \sum_{k=1}^{N}\|s\|_2 \tag{3a}$$

$$\text{s.t.} \quad \mathbf{F}(\tilde{\mathbf{x}}_k, \mathbf{x^0}_k, \tilde{\mathbf{z}}_k, \tilde{\mathbf{u}}_k, \mathbf{p}) = 0 \tag{3b}$$

$$\sum_{i=1}^{n_w} w_{pg_i} \leq w_{g_{Max}} + s \qquad \forall k \in \{1,\cdots,N\}, \forall p \in \mathscr{U} \tag{3c}$$

where, $\theta_k = [\tilde{\mathbf{x}}_k, \tilde{\mathbf{z}}_k, \tilde{\mathbf{u}}_k]$ is the vector of decision variables, α_o is the price of oil, α_{gl} is the cost of compressing the gas for gas lift injection, w_{po} is the oil flow rate, w_{gl} is the gas lift rate w_{pg} is the produced gas rate and $w_{g_{max}}$ is the total gas capacity constraint as implemented in (3c). Soft constraints using slack variables and an exact penalty function is implemented for the total capacity constraint to ensure feasibility in the face of noise and estimation error. ρ is a tuning parameter that penalizes constraint violations. (3b) represents the discretized dynamic model equations. In addition, there are upper and lower bound constraints and rate of change constraints on the differential and algebraic states and decision variables, which are not shown due to page limitations.

The uncertain parameter $\mathbf{p} = GOR$ can take any value from a bounded uncertainty set, $\mathscr{U}$. When the uncertain parameters are assumed to be at their nominal values, the optimization problem (3) is solved with $GOR_i = \mathbb{E}_0(GOR_i)$. However, if the true realization of GOR is higher than the nominal value, then this leads to constraint violations when implemented as demonstrated in Krishnamoorthy et al. (2016). Robust optimization methods may be employed to ensure robust feasibility, where all the uncertain parameters are assumed to simultaneously take their worst-case realization. In this work, since the uncertainty is simple, the worst-case realization can be easily formulated a-priori without using the min-max formulation. The worst-case scenario occurs when the GOR of all the wells take their extreme value, $GOR_i = \mathbb{E}_0(GOR_i) + \sigma_i$. All the parameters taking its worst-case value will be an unlikely event. Therefore, the solution to the robust optimization problem will most likely be very conservative and suboptimal as shown in Krishnamoorthy et al. (2016). Robust optimization does not take into account that new information will be available in the future. To improve on this, closed-loop or feedback min-max MPC scheme was first introduced in Scokaert and Mayne (1998) and later extended in Lucia et al. (2013). The uncertainty is represented as a tree of discrete scenarios made up of M discrete models. The scenario tree represents how the uncertainty influences state propagation over time and the optimization problem is solved over the entire scenario tree. Typically, the M different models should be chosen to cover the entire uncertainty space. A common way is to select the a combination of values among the extreme and nominal values of all the uncertainties, (Martí et al. (2015)). For example, the vertices from the boundary and the nominal point as shown in Fig.2 can be chosen to cover the uncertainty space. In order to avoid an exponential growth of the optimization problem, the branching is stopped after a certain number of samples

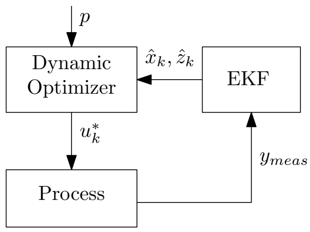

Figure 1: Block diagram of the implemented control structure including the EKF for state estimation.

Table 1: The GOR value used in the optimizer for nominal, worst-case and scenario based approach

	Nominal	worst-case	scenario-based				
GOR well 1	0.1	0.15	0.05	0.05	0.1	0.15	0.15
GOR well 2	0.12	0.13	0.11	0.13	0.12	0.11	0.13

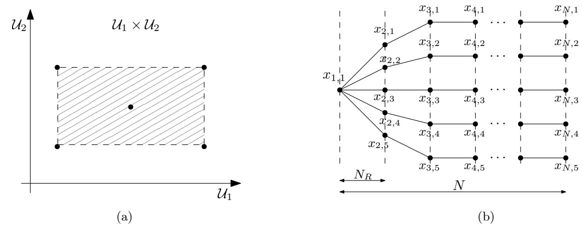

Figure 2: (a) Uncertainty subspace and the possible models for the scenario tree denoted by $\bullet$, (b) Scenario tree with $N_R = 1$ and $M = 5$ models $\Rightarrow S = 5^1 = 5$ scenarios.

$N_R \leq N$ (known as robust horizon). Each path from the root node to the leaf is called a scenario and the number of scenarios is then given by $S = M^{n_R}$. A typical scenario tree is shown in Fig.2. To reflect the fact that the optimization problem cannot anticipate the future, the decision variables that branch at the same node must be equal. This is known as non-anticipativity or causality constraints. For the scenario tree shown in Fig.2 with $n_R = 1$, the causality constraints are given by $\mathbf{u}_{2,1} = \mathbf{u}_{2,2} = \mathbf{u}_{2,3} = \mathbf{u}_{2,4} = \mathbf{u}_{2,5}$ at the first sample. This is collectively denoted by χ, see Lucia et al. (2013) and Krishnamoorthy et al. (2016). The scenario optimization can then be written as a sum of the optimization problem in Eq.(3) for each scenario subject to the constraints in Eq.(3) collectively denoted by $\mathscr{C}$ and the additional causality constraints.

$$\min_{\theta_j} \{\sum_{j=1}^{S} J_j \mid \sum_{j=1}^{S} \chi_j u_j = 0, \theta_j \in \mathscr{C}_j, \forall j \in \{1, \cdots, S\}\} \tag{4}$$

An Extended Kalman Filter (EKF) was implemented for state feedback. The annulus pressure, well head pressure, bottom hole pressure, wellhead choke flow rate and gas inflow rate are all assumed to be measured and used as measurements in the EKF. The original index-1 DAE system was converted into an explicit ODE system as shown in Eq.(5) by eliminating all the algebraic variables. The EKF was then implemented using Eq.(5) as shown in Simon (2006). An NMPC control structure was chosen together with an EKF for state estimation as shown in Fig.1.

$$\dot{\mathbf{x}} = f_{EKF}(\mathbf{x}, \mathbf{u}) \qquad \mathbf{y} = h_{EKF}(\mathbf{x}) \tag{5}$$

The model used in EKF uses the nominal GOR for the nominal optimization and worst-case GOR for the worst-case optimization. For the scenario optimization case, the model used in EKF uses nominal GOR values. The estimation error due to this assumption is handled using soft constraints as mentioned earlier.

4. Results and Discussion

The dynamic optimization problem was solved with a prediction horizon of $N = 15$ and a sampling time of $T_s = 5min$. In the case of scenario optimization, a robust horizon of $n_R = 1$ was chosen as shown in Fig.2. To compare the performance of the nominal, worst-case and scenario optimization methods, we now present a Monte Carlo simulation with 35 simulations. The GOR values used in the nominal, worst-case and scenario optimization are summarised in Table1.

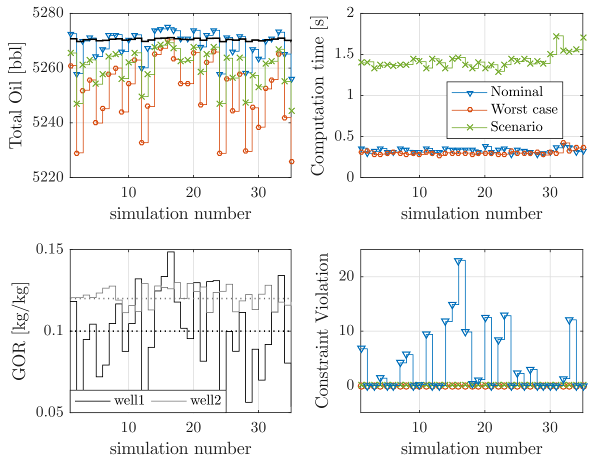

Figure 3: Monte Carlo Simulation results each with a simulation time of 75min

For each Monte Carlo simulation, the true GOR used in the simulator was randomly picked from the known uniform distribution $\{\mathbb{E}(GOR) + \sigma\}$ and is shown in Fig.3. Each Monte Carlo simulation was run for 75min (15 NMPC samples) and the total accumulated oil over 75min (integrated objective) is compared. As a benchmark, the results from the ideal case are also plotted, which corresponds to the case where we have perfect information about all the parameters. The nominal optimization seems to produce more oil than the ideal case in some simulations, however the total gas capacity constraints are violated in these simulations which demonstrates that nominal optimization can lead to infeasibility. The constraint violations for all the cases plotted in Fig.3 shows that the constraints are violated only in the nominal optimization case. However, it is important to note that this is not a general conclusion for scenario optimization. Scenario optimization can be constraint feasible if one of the scenarios corresponds to the worst-case scenario, which was the case in this simulation study (see Table.1). Fig.3 shows that the worst-case and scenario optimization were robust feasible at the cost of conservativeness. However, the scenario optimization was significantly less conservative than the worst-case optimization. On average, the scenario optimization produced 9.17 barrels of oil more than the worst-case optimization over the time period of 75min. The average computation time for nominal, worst-case and scenario optimization are also compared and we see that the improved performance of the scenario optimization comes at the cost of higher computation time. However, this can be improved by decomposing each scenario or a bundle of scenarios into smaller subproblem and solved using parallel computing as described in Martí et al. (2015). With the reported computation times and a sampling time of 5 min, real-time implementation may not be an issue for the application considered.

The worst-case and scenario optimization were shown to have a certain degree of conservativeness. This is due to the inclusion of the worst-case realization of all the parameters in the optimization problem. Using the model and the measurements the parameters could be estimated online instead of waiting for experimental data from well tests to update the models. The GOR can also be estimated using an augmented EKF to improve the performance by parameter adaptation. The estimated GOR is then used in the optimization model (Adaptive Optimization). Results from the scenario optimization are compared with the results from the optimization run with the estimated GOR. The system was simulated for 5h (60 samples) using a time varying GOR. Since we use the estimated GOR, we are closer to the true GOR and hence the computed optimum solution is close to the active constraint. Therefore the spare capacity that was not utilized by the scenario optimization is fully utilized by estimating the GOR. This is reflected in the produced oil rate which is higher than the scenario optimization. This is clearly seen in Fig.4. The results from nominal optimization with parameter estimation using EKF are shown in solid lines, whereas the scenario optimization are shown in dashed lines. However, note that the solution is not robust feasible when the GOR suddenly increases to its worst-case realization. The total gas capacity constraint is violated dynamically for a short period of time since this is a reactive strategy. The EKF updates the GOR based on the measurements and the constraints are satisfied after a few samples. In the case of hard constraints, a combination of robust and adaptive optimization method may be implemented which could be a future research direction.

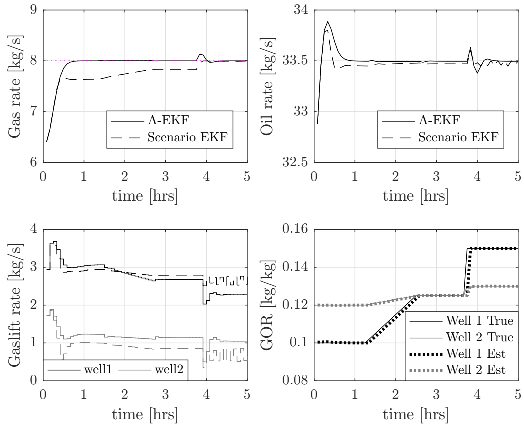

Figure 4: Simulation with Augmented EKF

Acknowledgement - The authors gratefully acknowledge the financial support from SUBPRO, which is financed by the Research Council of Norway, major industry partners and NTNU.

References

Hanssen, K. G., Foss, B., 2015. Production optimization under uncertainty-applied to petroleum production. IFAC-PapersOnLine 48 (8), 217–222.

Krishnamoorthy, D., Foss, B., Skogestad, S., 2016. Real-time optimization under uncertainty applied to a gas lifted well network. Processes 4 (52).

Lucia, S., Finkler, T., Engell, S., 2013. Multi-stage nonlinear model predictive control applied to a semi-batch polymerization reactor under uncertainty. Journal of Process Control 23 (9), 1306–1319.

Martí, R., Lucia, S., Sarabia, D., Paulen, R., Engell, S., de Prada, C., 2015. Improving scenario decomposition algorithms for robust nonlinear model predictive control. Computers & Chemical Engineering 79, 30–45.

Scokaert, P., Mayne, D., 1998. Min-max feedback model predictive control for constrained linear systems. IEEE Transactions on Automatic control 43 (8), 1136–1142.

Simon, D., 2006. Optimal state estimation: Kalman, H infinity, and nonlinear approaches. John Wiley & Sons.