

# Quantitative methods for Regulatory control layer selection

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## Abstract

In this paper, we apply the self-optimizing control ideas to find optimal controlled variables in the regulatory layer. The regulatory layer is designed to facilitate stable operation, to regulate and to keep the operation in the linear operating range. Its performance is here quantified using the state drift criterion and the method is evaluated on two distillation column case studies with one, two or more closed loops.

*Keywords:* Regulatory control layer selection, Self-optimizing control, State drift, Optimal controlled variables, Plantwide control

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## 1. Introduction

The plantwide control system for the overall plant is in most cases organized in a hierarchical structure (Figure 1), based on time scale separation between the layers. As shown in Figure 1, the control layer is usually divided in two parts; a supervisory (economic) layer and a faster regulatory (stabilization) layer. One may question the division of the control layer into two layers, but this paradigm is widely used and is the basis for this paper. The main justification is that the two tasks of regulation and economically optimal operation are fundamentally different. Of course, a single multivariable controller (e.g. using MPC with no lower-layer PID controllers) would be optimal from a theoretical point of view, but one would then need to include also regulatory objectives into the MPC design, so tuning would become

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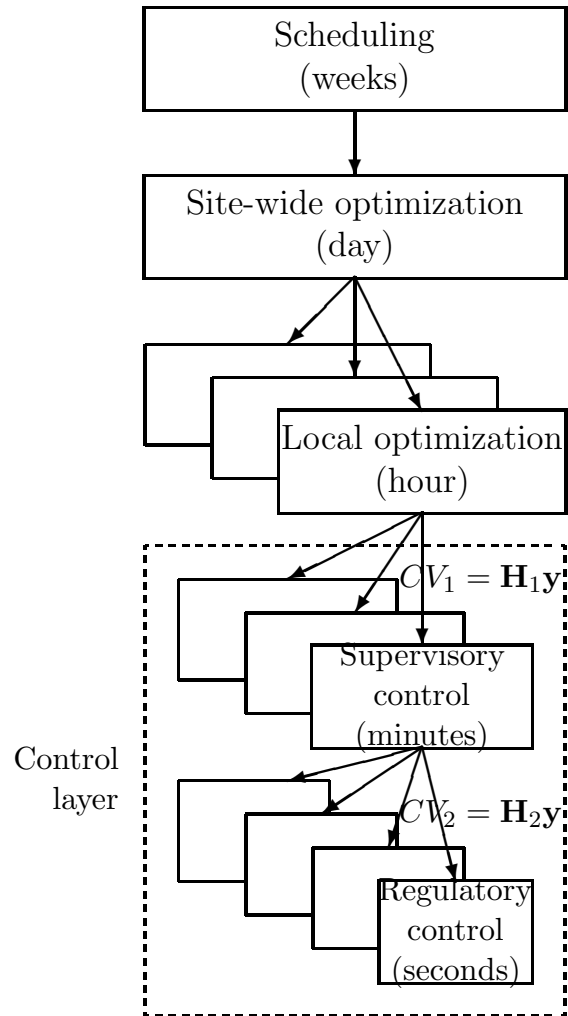


Figure 1: Control system hierarchy for plantwide control in chemical plants [1]

more difficult. The use of single multivariable controller would also require detailed models of all the dynamic interactions, whereas the PID controllers used in the regulatory layer may be designed based on much simpler models. In summary, trying to do both regulation and economic optimal operation at the same time is much more complex, both in terms of modeling and tuning, whereas it usually has limited performance benefit.

For both control layers, a key decision is the selection of the controlled variables,  $CV_1$  and  $CV_2$  in Figure 1. The controlled variables are usually a subset or combination of the measurements  $\mathbf{y}$ , and we can write  $CV_1 = \mathbf{H}_1\mathbf{y}$  and  $CV_2 = \mathbf{H}_2\mathbf{y}$ , where  $\mathbf{H}_1$  and  $\mathbf{H}_2$  are real-valued non-square matrices.

The supervisory or “advanced” control layer (often implemented using MPC) controls the primary (economic) variables  $CV_1$  at their set points on a long time scale. The variables  $CV_1$  should be selected such that when they are kept constant and there are disturbances, the economic cost  $J_1$  remains close to the truly optimal; this is the idea of self-optimizing control [2]. It is usually straightforward to define the economic cost, and typically we have

$$J_1 = \text{feed cost} + \text{utilities cost} - \text{products value} \text{ [}\$/\text{s]} \quad (1)$$

In process control, the economic cost  $J_1$  is often dictated mainly by the steady-state behavior, so a steady-state analysis may often be used when selecting the variables  $CV_1$ . Note that it is the *selection* of controlled variables (matrix  $\mathbf{H}_1$ ) for the supervisory layer which is based on the economic cost  $J_1$ ; the objective (cost) function for designing the supervisory controller (e.g. MPC) itself is different, typically it is to track the set points  $CV_{1s}$  while avoiding excessive input changes and satisfying constraints.

The regulatory or “basic” control layer (usually implemented using PID controllers) controls the secondary variables  $CV_2 = \mathbf{H}_2\mathbf{y}$ , which are kept constant on an intermediate time scale. The real-valued non-square matrix  $\mathbf{H}_2$  is often a selection matrix (consisting of 1’s and 0’s), but in this paper we consider the more general case where we also allow for  $\mathbf{H}_2$  to be a linear combination matrix of the measurements  $\mathbf{y}$ . However, to avoid undesirable combinations, we may impose limitations on the structure of  $\mathbf{H}_2$ .

Note that we do not “use up” any degrees of freedom in the regulatory layer because the set points  $CV_{2s}$  are left as manipulated variables (MVs) for the supervisory layer (see Figure 1). Furthermore, since the set points  $CV_{2s}$  are set by the supervisory layer in a cascade manner (Figure 1), the system approaches on a long time scale the same steady-state (as defined

by the choice of economic variables  $CV_1$ ) irrespective of the choice of  $CV_2$  in the regulatory layer. However, this does not mean that the choice of the variables  $CV_2$  is unimportant, because it determines the system’s initial response to disturbances (on a faster time scale). By allowing for cascade loops, the stabilization layer may in theory be designed independently of the supervisory (economic) control layer. However, when closing a stabilizing loop, we do “use up” some of the time window as given by the closed-loop response time (bandwidth) of the stabilizing loop. In addition, cascade loops add complexity. We should therefore try to simplify and reduce the use of cascade loops.

Ideally, we would like to have a tool that based on a process model, automatically selects the optimal structure of the regulatory control layer. This is a very difficult problem, both in terms of problem definition and solution. As a starting point, we first need to consider the objectives of the regulatory control layer. A list of nine objectives are given in Table 1 [1] and four additional objectives are:

- O10. The variables  $CV_2$  controlled in the regulatory layer, should be easily measurable and “robust” (e.g. composition measurements should typically be avoided).
- O11. The regulatory layer should be simple.
- O12. The regulatory layer should contribute to the overall operational objective as defined by the economic cost  $J_1$ , that is, it should contribute to good control of the economic variables  $CV_1$  on the fast time scale, whenever necessary.
- O13. The regulatory layer should preferably not be changed during operation.

This is only a partial list and one may easily add more objectives, like specifying robustness margins (gain margin, phase margin) to cope with variations and uncertainty.

To have a systematic approach to regulatory layer design, we would need to quantify these partially conflicting objectives in terms of a scalar cost function  $J_2$ . This is very difficult because the various objectives are difficult to quantify in terms of a single measure, like money. The intended contribution of this paper is therefore not find the “optimal” regulatory control layer, but

to develop a computationally tractable tool that can assist the designer, and to do this we need to define an appropriate cost function  $J_2$ .

One obvious choice, according to objective O12, is to select the variables  $CV_2$  so that we get good control of the economic variables  $CV_1$ , that is, select  $J_2 = \|CV_1\|$ , where the norm may measure the control error for  $CV_1$  in the time or frequency domain. This is the “indirect control” problem (e.g. [3]). However, we cannot usually simply select  $CV_2 = CV_1$ , because the variables  $CV_1$  may not be easily measurable (objective O10) and we want to avoid closing too many regulatory loops (O11).

In this paper, we consider a more general objectives, namely the weighted state drift away from the nominal operating point,

$$J_2 = \|\mathbf{W}\mathbf{x}\|_2^2 \quad (2)$$

Here,  $\mathbf{W}$  is a weighting matrix and  $\mathbf{x}$  are the states, which describe the system behavior. The indirect control can be included in (2) by selecting part of the weight matrix  $\mathbf{W}$  such that  $\mathbf{W}\mathbf{x} = CV_1$ , so the formulation in (2) is quite flexible. More precisely in (2), we should write  $\Delta\mathbf{x}$ , because  $\mathbf{x}$  denotes the deviation from the nominal value of the state, but we usually drop the  $\Delta$  to simplify notation.  $\mathbf{W}\mathbf{x}$  is a vector, which generally is a function of time or frequency. Many norms may be used, but we will consider the 2-norm where  $\mathbf{x}(j\omega)$  is evaluated at a selected frequency  $\omega$ . We will in the application consider steady-state,  $\omega = 0$ .

The problem studied in this paper is to find the controlled variables  $CV_2 = \mathbf{H}_2\mathbf{y}$  that minimize the cost  $J_2$  in (2) for the expected disturbances and measurement noise. To avoid the need to design the controller and select pairings, we assume that the variables  $CV_2$  are “perfectly” controlled. Since the regulatory control system is the fastest control layer, this assumption will hold well when viewed from the slower supervisory control later. A simplified steady-state version of this problem, without measurement noise, has been considered previously [1, 3]. The approach presented in this paper is much more practically useful, as it includes measurement noise and also allows us to control individual measurements in  $CV_2$  and not only combination. It also allows us to preselect variables in  $CV_2$  and to study the effect of the number of loops closed. The numerical problem to be solved is convex and thus numerically tractable and efficient.

Traditionally, the regulatory layer decisions are based on heuristic methods using process insight (e.g., [4] and references therein). Typical variables

Table 1: Objectives of regulatory control layer [1]

- O1. Provide sufficient quality of control to enable a trained operator to keep the plant running safely without use of the higher layers in the control system.
- O2. Allow for simple decentralized (local) controllers (in the regulatory layer) that can be tuned on-line.
- O3. Take care of “fast” control, such that acceptable control is achievable using “slow” control in the layer above.
- O4. Track the set points ( $CV_{2s}$ ) set by the higher layers in the control hierarchy.
- O5. Provide for local disturbance rejection.
- O6. Stabilize the plant (in the mathematical sense of shifting RHP-poles to the LHP).
- O7. Avoid “drift” so that the system stays within its “linear region” which allows the use of linear controllers.
- O8. Make it possible to use simple (at least in terms of dynamics) models in the higher layers.
- O9. Do not introduce unnecessary performance limitations for the supervisory control problem.

that are selected for control ( $CV_2$ ) are inventories such as liquid levels, and other “sensitive” variables such as selected accumulating components, pressures and certain temperatures. All of these variables are related to the “drift” in the process and thus may be captured with the state drift criterion in (2).

The rest of the paper is organized as follows: Section 2 defines the problem for optimal regulatory layer selection. Section 3 shows the application of self-optimizing control ideas to state drift and various cases to find optimal  $\mathbf{H}_2$  are described. Section 4 applies the method to a distillation column case study with 41 stages to find the regulatory layer with optimal  $CV_2$  as individual/combinations of measurements. The dynamic simulations in Section 5 are included to describe the ease of implementing the controlled variables in practice. Section 6 presents a Kaibel column case study with 71 stages. In Section 7, we discuss the alternative approach of finding a single controller and also provide a summary of our method where we assume two control layers. The conclusions are given in Section 8.

## 2. Minimization of state drift (Problem definition)

### 2.1. Regulatory layer

In general, the design of the regulatory layer involves the following decisions:

1. Selection of outputs  $\mathbf{y}_2$
2. Selection of inputs  $\mathbf{u}_2$  to control these outputs
3. Pairing of inputs and outputs (since decentralized control is normally used)
4. Controller design (normally PID controllers)

In this paper, we focus on the first two decisions, which can be combined into a single decision by defining

$$CV_2 = \begin{bmatrix} \mathbf{y}_2 \\ \mathbf{u}_1 \end{bmatrix} \quad (3)$$

where  $\mathbf{u}_1$  denotes the inputs that are *not* used by the regulatory control layer. Note that specifying  $\mathbf{u}_1$  will indirectly determine the inputs  $\mathbf{u}_2$  used

to control  $\mathbf{y}_2$ . This is because we here assume that the number of variables to be selected in  $CV_2$  is equal to number of physical (dynamic) degrees of freedom (denoted  $\mathbf{u}_0$ ), that is,  $\mathbf{u}_0 = \begin{bmatrix} \mathbf{u}_1 \\ \mathbf{u}_2 \end{bmatrix}$ , where  $\mathbf{u}_0$  is a given set.

The cost function  $J_2 = \|\mathbf{W}\mathbf{x}\|_2^2$  in (2) may include weights on the internal states  $\mathbf{x}$  of the system, for example, to limit the drift away from its steady state (objective O7). The “indirect control” problem (objective O12) can be included in (2) by selecting  $\mathbf{W}$  such that  $\mathbf{W}\mathbf{x} = CV_1$ . However, the economically optimal controlled variables ( $CV_1$ ) may change during operation (e.g., change in active constraint), whereas we want the regulatory layer to remain unchanged (objective O13). The cost function  $J_2$  in (2) is flexible in this respect, and it allows us to include more control variables than there are degrees of freedom, so that many variables are controlled acceptably. This is related to the “partial control” idea, where we control only a subset of the process variables [5, 6], but we nevertheless aim at achieving acceptable control also of other important variables.

An alternative and more direct way of including economics into the regulatory layer (objective O12), is to specify that some of the variables in  $CV_1$ , typically active constraints which may require tight control to reduce the “backoff”, should be included in the set  $CV_2$  in (3); this may be done in the present approach by preselecting parts of  $\mathbf{H}_2$ .

Another objective is that the regulatory layer should be “simple” (O9). One reason is that regulatory control system implies cascaded loops, which adds complexity. In this paper, we quantify this objective by the number of regulatory loops closed, that is, simplicity means that we want to maximize the number of unused inputs ( $\mathbf{u}_1$ ) when we select  $CV_2$  in (3).

## 2.2. Classification of variables

- $\mathbf{x}$ : States (usually deviation variables)
- $\mathbf{W}\mathbf{x}$ : Weighted states, which characterize the desired behavior of the regulatory control system, e.g. the drift of the system away from its steady state.
- $\mathbf{u}_0$ : Set of  $n_{u_0}$  physical degrees of freedom (inputs, manipulated variables (MVs)), which may or may not be used in the regulatory layer.

We write  $\mathbf{u}_0 = \begin{bmatrix} \mathbf{u}_1 \\ \mathbf{u}_2 \end{bmatrix}$ , where  $\mathbf{u}_1$  denotes the inputs not used in the



regulatory layer and  $\mathbf{u}_2$  the remaining inputs which are manipulated by the regulatory layer. For example, for a distillation columns with given feed and given pressure, the physical degrees of freedom are the two product flows ( $D, B$ ) plus the reflux ( $L$ ) and boilup ( $V$ ), that is  $\mathbf{u}_0 = \{L, V, D, B\}$ .

- $\mathbf{u}$ : Set of  $n_u$  independent variables (“inputs”) used when solving the regulatory layer problem.

Note 1: In our approach, it does not really matter what the variables  $\mathbf{u}$  are as long as they form an independent set, e.g. one may close loops and instead introduce the new set points as the variables  $\mathbf{u}$ . The reason is that we assume perfect control of the selected controlled variables and closing lower-level loops will not change the problem.

Note 2: One may select  $\mathbf{u} = \mathbf{u}_0$ , but this is not required. In particular, at steady state there may be degrees of freedom with no steady-state effect and these may be eliminated (i.e.,  $n_u < n_{u_0}$ ) to simplify the problem.

- $\mathbf{y}_m$ : Set of measurements in regulatory control layer (in addition to measured or known values of  $\mathbf{u}_0$ ).
- $\mathbf{y} = \begin{bmatrix} \mathbf{y}_m \\ \mathbf{u}_0 \end{bmatrix}$ : Combined set of measurements and physical inputs that we consider as candidates for including in  $CV_2 = \mathbf{c} = \mathbf{H}_2\mathbf{y}$ .
- $\mathbf{d}$ : Set of disturbances considered for the regulatory control layer problem.
- $\mathbf{c} = CV_2 = \mathbf{H}_2\mathbf{y}$ : Selected set of  $n_c = n_u$  independent controlled variables in the regulatory layer. The selection or combination matrix  $\mathbf{H}_2$  is here assumed to a constant real-valued matrix.

Note 3: Since  $n_c = n_u$  the specification of  $\mathbf{c}$  will uniquely determine  $\mathbf{u}$ .

Note 4: Since  $\mathbf{u}_0$  is included in the candidate set  $\mathbf{y}$ , controlling  $\mathbf{c} = \mathbf{H}_2\mathbf{y}$  also includes the possibility for open-loop and partially controlled systems. To show this more explicitly we may write  $\mathbf{c} = CV_2 = \begin{bmatrix} \mathbf{y}_2 \\ \mathbf{u}_1 \end{bmatrix}$  (see (3)), where  $\mathbf{y}_2$  denotes the variables that are actually controlled in the regulatory layer and  $\mathbf{u}_1$  denotes the “unused” inputs.

### 2.3. Assumptions

- A linear model is used to represent the nominal operating point. This model may at each frequency  $\omega$  be written

$$\mathbf{x} = \mathbf{G}^x(j\omega)\mathbf{u} + \mathbf{G}_d^x(j\omega)\mathbf{d} \quad (4a)$$

$$\mathbf{y} = \mathbf{G}^y(j\omega)\mathbf{u} + \mathbf{G}_d^y(j\omega)\mathbf{d} + \mathbf{n}^y \quad (4b)$$

where  $\mathbf{G}^x(j\omega), \mathbf{G}_d^x(j\omega)$  and  $\mathbf{G}^y(j\omega), \mathbf{G}_d^y(j\omega)$  are frequency-dependent gain matrices.

- To avoid the need to explicitly design the controller, we assume that the selected variables in  $\mathbf{c}$  are perfectly controlled at the frequency  $\omega$ , i.e.  $\mathbf{c}(j\omega) = 0$ . At steady state ( $\omega = 0$ ) this is not a limitation since perfect control can always be achieved by using integral action, provided the system is operable in the first place. At other frequencies, we may assume perfect control, but the feasibility of this, including closed-loop stability, then assumes that there are no controllability limitations at this frequency, e.g. caused by an effective time delay.
- Unstable and integrating modes should be stabilized using any stabilizing controller before performing the selection of  $CV_2$ . This may seem to be a severe restriction, but actually it does not affect our problem if the set points for the stabilizing controllers are introduced as degrees of freedom (in  $\mathbf{u}$ ). This is related to the perfect control assumption in Note 2.

### 2.4. Problem formulation

The objective is to find what to control in the stabilizing layer,

$$CV_2 = \mathbf{c} = \mathbf{H}_2\mathbf{y} \quad (5)$$

given that we want to minimize the weighted state drift  $J_2$  in (2) for the expected disturbances ( $\mathbf{d}$ ) and implementation error (measurement noise,  $\mathbf{n}^y$ ), and that we want to close  $k$  loops,  $\forall k = 1, 2, \dots, n_u$ . This is explored in more detail next.

In the frequency domain, the problem can be stated as follows (Figure 2): Assuming perfect control of the selected  $\mathbf{c}$  (5), i.e.  $\mathbf{c}(j\omega) = 0$ , we want to find the optimal  $\mathbf{H}_2$  that minimizes  $J_2(\mathbf{x}(j\omega))$  for a given frequency range  $\omega \in [\omega_{B_1}, \omega_{B_2}]$ , when there are disturbances. In the rest of this paper, we

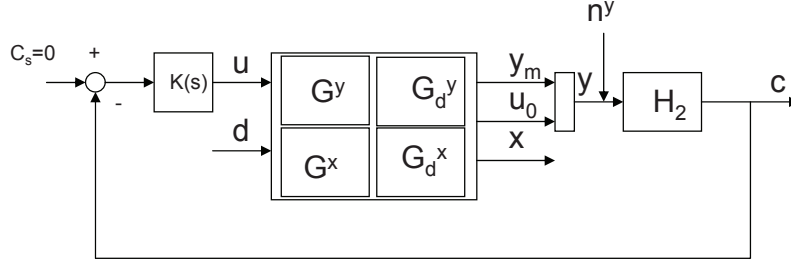


Figure 2: Regulatory control layer with control of variables  $\mathbf{c} = \mathbf{H}_2\mathbf{y}$

consider steady state only ( $\omega_{B_1} = \omega_{B_2} = 0$ ), but more generally it will be the frequency range over which we need regulatory control.

We use the self-optimizing control concepts [2, 7] and we consider minimization of the loss rather than the cost, because loss minimization can be formulated as a convex optimization problem in  $\mathbf{H}_2$  [8]. The loss is  $L = J_2 - J_{2,opt}(\mathbf{d})$ , where  $J_{2,opt}(\mathbf{d})$  is the minimum state drift achievable with the given degrees of freedom. In our case, this gives the same optimal  $\mathbf{H}_2$  as minimizing the cost  $J_2$ , because minimizing the state drift loss  $L$  on an average basis, e.g. using the Frobenius norm, is exactly the same as minimizing the cost  $J_2$ . In Figure 2,  $K(s)$  is the regulatory controller, but since we make the assumption of perfect control ( $\mathbf{c}(j\omega) = 0$ ), it does not actually matter what  $K(s)$  is. Our task is to select what to control,  $\mathbf{c} = \mathbf{H}_2\mathbf{y}$ , where  $\mathbf{H}_2$  is a constant real matrix.

We want to close as few loops as possible, that is we want to select in  $\mathbf{c} = \mathbf{H}_2\mathbf{y}$  as many variables as possible from the set  $\mathbf{u}_0$  of physical degrees of freedom (valves). Let

$$\begin{aligned} \mathbf{H}_2 &= [\mathbf{H}_y \quad \mathbf{H}_u] \\ \mathbf{c} = \mathbf{H}_2\mathbf{y} &= \mathbf{H}_y\mathbf{y}_m + \mathbf{H}_u\mathbf{u}_0 \end{aligned} \tag{6}$$

and we want to find the best controlled variables for various possibilities for closing loops

- Close 0 loops: In the set  $\mathbf{c}$ , select  $n_c$  variables from the set  $\mathbf{u}_0$  ( $\mathbf{H}_y = \mathbf{0}$ , where  $\mathbf{0}$  is a zero matrix,  $n_c$  columns in  $\mathbf{H}_u$  are nonzero)

- Close 1 loop : In the set  $\mathbf{c}$ , select  $n_c - 1$  variables from the set  $\mathbf{u}_0$  (one column in  $\mathbf{H}_y$  is nonzero, the rest are zero)
- Close 2 loops : In the set  $\mathbf{c}$ , select  $n_c - 2$  variables from the set  $\mathbf{u}_0$  (two columns in  $\mathbf{H}_y$  are nonzero, the rest are zero)
- Close  $k$  loops : In the set  $\mathbf{c}$ , select  $n_u - k$  variables from the set  $\mathbf{u}_0$
- Close all  $n_c$  loops: In the set  $\mathbf{c}$ , select 0 variables from the set  $\mathbf{u}_0$

In addition, we can have restrictions on the set  $\mathbf{c}$  such as selecting only single measurements (each column in  $\mathbf{H}_2$  containing one 1 and the rest 0's).

We can make use of mixed integer quadratic programming methods [9] or partial branch and bound methods [10] to find the optimal  $\mathbf{H}_2$  to arrive at optimal regulatory layer with 1, 2 or more closed loops.

### 2.5. Selection of the base variables $\mathbf{u}$

We mentioned in Note 1 that it does not really matter what the “base” variables  $\mathbf{u}$  are as long as they form an independent set. Mathematically, the requirement of an independent set is that  $\text{rank}(\mathbf{H}_2\mathbf{G}^y) = n_c$ , so that  $\mathbf{H}_2\mathbf{G}^y$  is invertible. It may seem surprising that it does not matter what the variables are, and this is because we consider the frequency domain and assume perfect control at a given frequency  $\omega$ ,  $\mathbf{c}(j\omega) = 0$ . With given  $\mathbf{c}(j\omega)$  and given  $\mathbf{d}$ , all other variables are then uniquely determined, including  $\mathbf{u}(j\omega)$ .

To show this, let the linear model for the effect of  $\mathbf{u}$  and  $\mathbf{d}$  on the selected states  $\mathbf{x}$  and  $\mathbf{y}$  be

$$\mathbf{y} = \mathbf{G}^y\mathbf{u} + \mathbf{G}_d^y\mathbf{d} \quad (7a)$$

$$\mathbf{x} = \mathbf{G}^x\mathbf{u} + \mathbf{G}_d^x\mathbf{d} \quad (7b)$$

$$\mathbf{c} = \mathbf{H}_2\mathbf{y} \quad (7c)$$

with the  $\mathbf{c} = 0$  we find  $\mathbf{u} = -(\mathbf{H}_2\mathbf{G}^y)^{-1}(\mathbf{H}_2\mathbf{G}_d^y)\mathbf{d}$  and with this input the states are

$$\mathbf{x} = \mathbf{P}_d\mathbf{d} \quad (7d)$$

where  $\mathbf{P}_d = (\mathbf{G}_d^x - \mathbf{G}^x(\mathbf{H}_2\mathbf{G}^y)^{-1}\mathbf{H}_2\mathbf{G}_d^y)$ . With  $\mathbf{d}$  given and  $\mathbf{c} = 0$ ,  $\mathbf{u}$  and  $\mathbf{x}$  are uniquely determined, so  $\mathbf{P}_d$  is independent of the choice for  $\mathbf{u}$ . However, we note that we must select  $\mathbf{u}$  so that  $\mathbf{H}_2\mathbf{G}^y$  is invertible.

### 2.6. Stabilization of integrators (levels)

As mentioned, any integrating modes, for example caused by liquid levels, need to be stabilized. For example, one may use  $u = k(y - y_s)$ , where  $y$  is integrating mode (e.g. liquid level) and  $k$  is proportional controller gain, and the set points  $y_s$  (e.g. level set points) are then chosen as the new independent variable in the set  $\mathbf{u}$ .

Importantly, for many liquid levels, the set point  $y_s$  may have no steady-state effect and we may use this to reduce the number of independent variables in  $\mathbf{u}$ . Formally, we may view this as including the liquid levels in the set  $CV_2$  of regulatory control variables, and we then have fewer degrees of freedom left in  $\mathbf{u}$ . For example, a distillation column with the LV-configuration has  $\mathbf{u}_0 = \{L, V, D, B\}$  and by controlling the two liquid levels, we have two remaining degrees of freedom which we may select as  $\mathbf{u} = \{L, V\}$ . One may use the steady-state model to obtain the linearized effect of  $\mathbf{u}$  and the  $\mathbf{d}$  on the original degrees of freedom in  $\mathbf{u}_0$ . This is explained in more detail in the distillation column case study.

## 3. Minimizing the state drift (optimal $\mathbf{H}_2$ )

Assume we want to find the matrix  $\mathbf{H}_2$  that minimizes at a given frequency the state drift,  $J_2 = \|\mathbf{W}\mathbf{x}\|_2^2$ , where  $\mathbf{x}$  is the deviation of states from the desired operating point and  $\mathbf{W}$  is a weighting matrix selected by the user.

### 3.1. Finding optimal $\mathbf{H}_2$ for case with no noise (Previous work)[1, 11]

The optimal choice for  $\mathbf{H}_2$  that counteracts the effect of disturbances on  $J_2$ , in the absence of measurement noise ( $\mathbf{n}^y$ ), when the number of measurements  $n_y \geq n_u + n_d$  is [1, 11]

$$\mathbf{H}_2 = (\mathbf{W}\mathbf{G}^x)^T [\mathbf{W}\mathbf{G}^x \quad \mathbf{W}\mathbf{G}_d^x] [\mathbf{G}^y \quad \mathbf{G}_d^y]^\dagger \quad (8)$$

where  $\mathbf{W}$  is state weighting matrix and  $\dagger$  represents the pseudo inverse of the matrix.

In the following, we use newer results from self-optimizing control [8, 9, 10, 12] to generalize (8). This generalization includes measurement noise and eliminates the requirement  $n_y \geq n_u + n_d$  and allows for  $CV_2$  as individual measurements and not only combinations.

### 3.2. Loss as a function of $\mathbf{d}$ , $\mathbf{n}^y$ and control policy $\mathbf{H}_2$

To include measurement noise, we need to quantify its expected magnitude. Let the linear model be

$$\mathbf{y} = \mathbf{G}^y \mathbf{u} + \mathbf{G}_d^y \mathbf{W}_d \mathbf{d}' + \mathbf{W}_n \mathbf{n}^{y'} \quad (9a)$$

$$\mathbf{x} = \mathbf{G}^x \mathbf{u} + \mathbf{G}_d^x \mathbf{W}_d \mathbf{d}' \quad (9b)$$

$$\mathbf{c} = \mathbf{H}_2 \mathbf{y} \quad (9c)$$

where the usually diagonal matrices  $\mathbf{W}_d$  and  $\mathbf{W}_n$  represent the magnitudes of disturbances and measurement noises, and  $\mathbf{d}'$ ,  $\mathbf{n}^{y'}$  denote normalized disturbances and noises.

The average or expected loss resulting from keeping (5) at a constant set point for a normal distributed set  $\begin{bmatrix} \mathbf{d}' \\ \mathbf{n}^{y'} \end{bmatrix} \in \mathcal{N}(0, 1)$  is given by [12, 8]

$$L_{avg} = \mathbb{E}(L) = \frac{1}{2} \|\mathbf{M}_2\|_F^2 \quad (10)$$

where

$$\mathbf{M}_2(\mathbf{H}_2) = \mathbf{J}_{2uu}^{1/2} (\mathbf{H}_2 \mathbf{G}^y)^{-1} \mathbf{H}_2 \mathbf{Y}_2 \quad (11)$$

$$\mathbf{Y}_2 = [\mathbf{F}_2 \mathbf{W}_d \quad \mathbf{W}_n]; \quad \mathbf{F}_2 = \frac{\partial \mathbf{y}_{opt}}{\partial \mathbf{d}} = \mathbf{G}^y \mathbf{J}_{2uu}^{-1} \mathbf{J}_{2ud} - \mathbf{G}_d^y \quad (12)$$

where  $\mathbf{J}_{2uu} \triangleq \frac{\partial^2 J_2}{\partial \mathbf{u}^2} = 2\mathbf{G}^{xT} \mathbf{W}^T \mathbf{W} \mathbf{G}^x$ ,  $\mathbf{J}_{2ud} \triangleq \frac{\partial^2 J_2}{\partial \mathbf{u} \partial \mathbf{d}} = 2\mathbf{G}^{xT} \mathbf{W}^T \mathbf{W} \mathbf{G}_d^x$ ,  $\|\mathbf{M}_2\|_F = \sqrt{\sum_{i,j} \mathbf{M}_{2ij}^2}$  denotes the frobenius norm of the matrix  $\mathbf{M}_2$ .

### 3.3. Optimal full $\mathbf{H}_2$

Finding the optimal  $\mathbf{H}_2$  in (10) is a convex optimization problem for the case where  $\mathbf{H}_2$  is a full matrix [8, 9]. For the case when  $\mathbf{Y}_2 \mathbf{Y}_2^T$  is a full rank matrix, an analytical solution for  $\mathbf{H}_2$  is [8]

$$\mathbf{H}_2^T = (\mathbf{Y}_2 \mathbf{Y}_2^T)^{-1} \mathbf{G}^y \left( \mathbf{G}^{yT} (\mathbf{Y}_2 \mathbf{Y}_2^T)^{-1} \mathbf{G}^y \right)^{-1} \mathbf{J}_{2uu}^{1/2} \quad (13)$$

However, when  $\mathbf{H}_2$  has a particular structure, the loss minimization (10) for  $\mathbf{H}_2$  is a non-convex optimization problem [13].

### 3.4. Optimal $\mathbf{H}_2$ with $CV_2$ as individual measurements

The optimal  $\mathbf{H}_2$  with  $CV_2$  as individual measurements, e.g.

$\mathbf{H}_2 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$ , gives an MIQP that require us to solve a convex QP at each node. This is because we may use a “trick” where  $\mathbf{H}_2$  is full in the selected measurements [9].

For a system where  $n_u < n_{u_0}$  (like the distillation example with  $n_u = n_c = 2$  and  $n_{u_0} = 4$ ), the regulatory layer with 0 loops closed is such a problem and with  $n_{u_0}$  inputs and  $n_u$  steady state degrees of freedom, we need to explore  $\binom{n_{u_0}}{n_c}$  possibilities. The control problem with 1 closed loop is to find (6) with one column in  $\mathbf{H}_y$  nonzero and  $n_c - 1$  columns in  $\mathbf{H}_u$  nonzero, and we need to explore  $\binom{n_{u_0}}{n_c-1} \binom{n_y}{1}$  possibilities. The control problem with 2 closed loops is to find (6) with two columns in  $\mathbf{H}_y$  nonzero and  $n_c - 2$  columns in  $\mathbf{H}_u$  nonzero, and we need to explore  $\binom{n_{u_0}}{n_c-2} \binom{n_y}{2}$  possibilities. The regulatory layer with  $k$  closed loops is to find (6) with  $k$  columns in  $\mathbf{H}_y$  nonzero and  $n_c - k$  columns in  $\mathbf{H}_u$  nonzero, and we need to explore  $\binom{n_{u_0}}{n_c-k} \binom{n_y}{k}$  possibilities. The regulatory layer with  $n_c$  closed loops is to find (6) with  $n_c$  columns in  $\mathbf{H}_y$  nonzero, and we need to explore  $\binom{n_y}{n_c}$  possibilities. The total possibilities are  $\binom{n_{u_0}}{n_c} + \binom{n_{u_0}}{n_c-1} \binom{n_y}{1} + \binom{n_{u_0}}{n_c-2} \binom{n_y}{2} + \dots + \binom{n_{u_0}}{n_c-k} \binom{n_y}{k} + \dots + \binom{n_y}{n_c}$ . For a case with  $n_{u_0} = 4$ ,  $n_u = n_c = 2$ , and  $n_y = 41$ , the total possibilities are 990.

In the regulatory layer with  $i$  loops closed require us solve (10) to find the best CV using mixed integer quadratic programming (MIQP) [9]. Hence, the regulatory layer with 0, 1, 2 and more closed loops can be obtained by solving  $(n_u + 1)$  mixed integer quadratic programming problems. For example, for a case with  $n_{u_0} = 4$ ,  $n_u = 2$ , and  $n_y = 45$ , the total MIQP problems that need to be solved are 3. Even though the number of MIQP problems need to be solved increase with  $n_u$ , the regulatory layer selection problem is tractable as it is an offline method. Later, depending on the allowable state drift threshold (typically ten times the  $J_{2,opt}$ ) set by the user, the minimum regulatory layer is obtained. Generally, in the regulatory layer,  $CV_2$  are individual measurements, if the state drift loss is very large then  $CV_2$  can be selected as measurement combination.

### 3.5. Optimal $\mathbf{H}_2$ for partial control with $CV_2$ as measurement combinations

We now consider the partial regulatory control problem where we allow for measurement combination for the controlled variables in  $CV_2$ . This can be viewed as solving (10) with a particular structure in  $\mathbf{H}_2$ , which is generally a non-convex problem. For example, a partially controlled system with 3

process measurements and 2 inputs, resulting in 5 candidate measurements in  $\mathbf{y}$ , is

$$\mathbf{H}_2 = \begin{bmatrix} h_{11} & h_{12} & h_{13} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \text{ or } \mathbf{H}_2 = \begin{bmatrix} h_{11} & h_{12} & h_{13} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \quad (14)$$

To solve (10) with this particular structure, we propose a two step approach which may not be optimal but which is convex. The first step is to partition the system inputs into two sets  $\mathbf{u}_1$  and  $\mathbf{u}_2$  ( $\mathbf{u}_0 = \{\mathbf{u}_1 \cup \mathbf{u}_2\}$ ) where we keep the inputs in the set  $\mathbf{u}_2 \in \mathbf{u}_0$  constant. The matrix for such a partial control system  $\mathbf{G}^{y,partial} \in \mathbb{R}^{n_y \times n_{u_1}}$  is obtained by picking the columns associated to input set  $\mathbf{u}_1$  and  $\mathbf{J}_{u_1,x}^{partial} \in \mathbb{R}^{n_{u_1} \times n_{u_1}}$ ,  $\mathbf{J}_{u_2,x}^{partial} \in \mathbb{R}^{n_{u_1} \times n_d}$  has elements associated to the inputs in the input set  $\mathbf{u}_1$ . The disturbance gain matrix  $\mathbf{G}_d^y \in \mathbb{R}^{n_y \times n_d}$ , disturbance magnitude matrix  $\mathbf{W}_d \in \mathbb{R}^{n_d \times n_d}$  and measurement noise magnitude matrix  $\mathbf{W}_n \in \mathbb{R}^{n_y \times n_y}$  will remain the same. The second step is to solve (10) with the matrices obtained in the first step as a convex optimization problem [9] to obtain  $\mathbf{H}_2^{partial}$  as a full matrix for the partially controlled system. For a case with  $n_u$  inputs, there are totally  $2^{n_u} - 2$  partially controlled systems.

As  $\mathbf{u}_2 \in \mathbf{u}_0$  varies in each partial controlled system, we cannot directly compare the losses obtained from different partial control systems. Hence, in order to compare the losses on an equivalent basis, the loss value is calculated for the full system with the optimal controlled variables  $CV_2^{partial}$  obtained for the partially controlled system and the constant inputs in  $\mathbf{u}_2$  as the other  $CV_2$ .

#### 4. Distillation column case study

The main purpose of this case study is to illustrate the proposed methods on a binary distillation column with 41 stages where we want to choose best temperature loop(s) to minimize state drift. The state drift  $J_2$  in the compositions on all 41 stages is

$$J_2 = \|\mathbf{W}\mathbf{x}\|_2^2 \quad (15)$$

where we select  $\mathbf{W} = \mathbb{I}^{41 \times 41}$  (identity matrix) to have equal weights on the mole fraction  $\mathbf{x}$ . The original problem has four physical inputs ( $\mathbf{u}_0$ ), but we need to control the two liquid levels ( $M_D, M_B$ ) which need to be included as regulatory controlled variables  $CV_2$ . The remaining problem has only



two degrees of freedom ( $\mathbf{u}$ ). The analysis is based on the LV-configuration [14, 15], where distillate flow (D) and bottoms flow (B) are used to control the integrating levels ( $M_D, M_B$ ) and reflux (L) and boilup (V) are the remaining steady-state degrees of freedom ( $\mathbf{u}$ ) (Figure 3). However, note that we would obtain identical results if we started with another configuration, e.g. the DV-configuration.

The considered disturbances are in feed flow rate ( $F$ ), feed composition ( $z_F$ ) and feed liquid fraction ( $q_F$ ), which can vary between  $1 \pm 0.2$ ,  $0.5 \pm 0.1$  and  $1 \pm 0.1$ , respectively. In summary, we have

$$\mathbf{u}_0 = \begin{pmatrix} L \\ V \\ D \\ B \end{pmatrix}, \quad \mathbf{u} = \begin{pmatrix} L \\ V \end{pmatrix}, \quad \mathbf{d} = \begin{pmatrix} F \\ z_F \\ q_F \end{pmatrix}, \quad \mathbf{y}_m = \begin{pmatrix} T_1 \\ T_2 \\ \vdots \\ T_{41} \end{pmatrix}$$

We consider a binary mixture with constant relative volatility  $\alpha$  between the components, constant pressure, negligible vapour hold up, equilibrium on each stage and constant molar flows. The column has 41 stages and the feed enters on stage 21. At the steady state operating point,  $L = 2.706 \text{ mol/min}$ ,  $V = 3.206 \text{ mol/min}$ ,  $F = 1 \text{ mol/min}$ ,  $z_F = 0.5$ ,  $q_F = 1$ ,  $\alpha = 1.5$ ,  $x_D = 0.99$ ,  $x_B = 0.01$ . The boiling points difference between the light key component (L) and heavy key component (H) is  $13.5 \text{ }^\circ\text{C}$ . For simplicity, the temperature  $T_i(^\circ\text{C})$  on each stage  $i$  is calculated as a linear function of the liquid composition  $x_i$  [14]

$$T_i = 0x_i + 13.5(1 - x_i) \quad (16)$$

The 41 stage temperatures ( $\mathbf{y}_m$ ) and the manipulated input flows  $\mathbf{u}_0 = \{L, V, D, B\}$  are taken as candidate measurements. The measurement error for temperatures is  $\pm 0.5^\circ\text{C}$  and it is  $\pm 10\%$  for the flows. The linearized relationship between the two base degrees of freedom  $\mathbf{u}$  and the four physical degrees of freedom  $\mathbf{u}_0$  can be obtained based on steady state mass balances (see Appendix A)

$$\mathbf{u}_0 = \mathbf{G}\mathbf{u} + \mathbf{G}^d\mathbf{d} \quad (17)$$

where

$$\mathbf{G} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 1 \\ 1 & -1 \end{bmatrix}, \quad \mathbf{G}^d = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 1 \end{bmatrix}$$

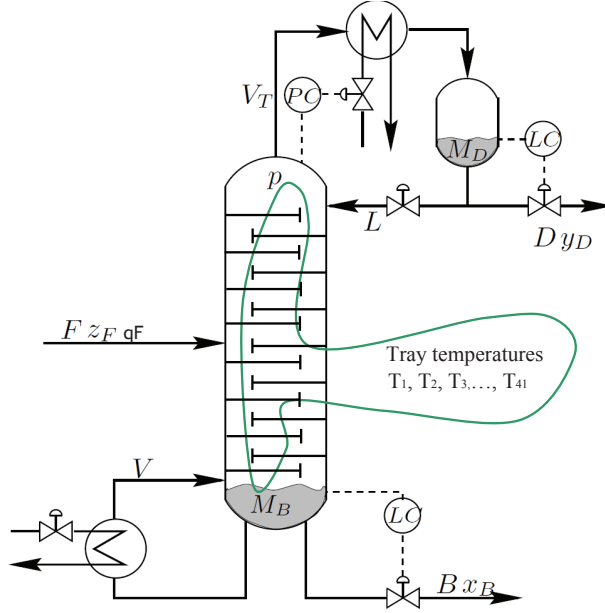


Figure 3: Distillation column using LV-configuration

#### 4.1. Justification for considering steady-state state drift

The frequency dependency of  $J_2(j\omega) = \|\mathbf{W}\mathbf{x}(j\omega)\|_2^2$  for the distillation column in LV-configuration (see Figure 3) is given in Figure 4. In Figure 4, the solid red curve ( $k = 0$ ) gives the expected state drift  $J_2$  as a function of frequency with no composition or temperature control ( $L$  and  $V$  constant) for combined disturbances in feed rate ( $F$ ), feed composition ( $z_F$ ) and feed liquid fraction ( $q_F$ ). The other curves show the effect when boil-up  $V$  is used for temperature control

$$V = V_0 + k(y - y_0) \quad (18)$$

for increasing value of the controller gain  $k$ .  $L$  remains constant. As  $k$  increases, we get tight control of  $y$ , temperature on stage 12, and the value of  $k = 10$  gives close to “perfect control”, where  $y(j\omega) = 0$ .

From Figure 4, it is clear that for a given controller gain  $k$ , the state drift is almost constant over the frequency band from 0.0001 to 0.02 rad/min. Also note that the state drift is reduced by a factor 27 (from 12 to 0.45) by closing the temperature loop. We conclude that for this example, a steady state analysis for state drift alone would be sufficient.

More generally, in process control  $J_2 = \|\mathbf{W}\mathbf{x}(j\omega)\|_2^2$  as a function of

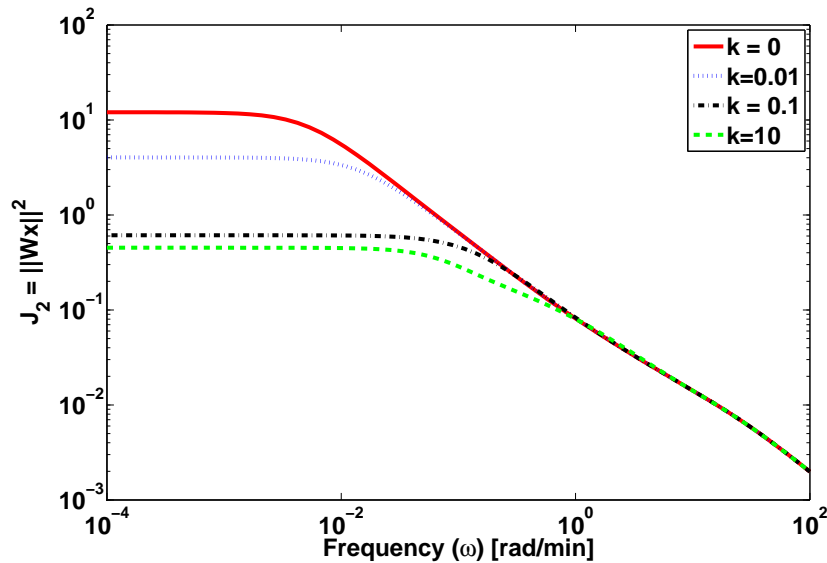


Figure 4: State drift  $J_2$  for distillation column as a function of temperature controller gain  $k$

frequency is often flat at lower frequencies and drops at higher frequencies similar to Figure 4, which means that we often get very good results by considering steady state ( $\omega = 0$ ). If  $J_2$  has a higher peak at a different frequency, then the methods of this paper can be used to find optimal  $CV_2$  to minimize the state drift by considering the frequency corresponding to the peak.

#### 4.2. Selection of controlled variables, $CV_2$

The distillation column case study has  $n_{u_0} = 4$  physical inputs, but since we have already decided to control  $M_D$  and  $M_B$  in the regulatory layer, we have  $n_u = 2$  steady state degrees of freedom and  $n_y = 45$  candidate measurements. For each  $i$  closed loops we need to solve an MIQP. With single measurements in  $CV_2$  with 0, 1 and 2 closed loops, there are  $n_u + 1 = 3$  MIQP problems. The MIQP problems are solved using IBM ILOG CPLEX solver in Matlab ®R2009a on a Windows XP SP2 notebook with Intel ®Core™ Duo Processor T7250 (2.00 GHz, 2M Cache, 800 MHz FSB). The QP that needs to be solved at each node in MIQP is convex and the initial conditions do not play any role.

The presence of integrating modes requires that we first close two loops for integrating levels ( $M_D, M_B$ ). Next, in addition to these, we want to close loops to minimize the state drift. In general, the loss decreases as we close more loops, but for simplicity we want to close as few loops as possible in the regulatory layer, that is we want to select in  $\mathbf{y}$  as many variables as possible from the set  $\mathbf{u}_0$ . This is a multi-objective problem involving a trade-off between the loss (magnitude of the state drift) and the number of loops closed. There is no simple mathematical solution to such problems, so the best is to provide the results and let the engineer make the decision. The loss with 0 loops closed (2 flows from  $\mathbf{u}_0$  are constant), 1 loop closed (one flow from  $\mathbf{u}_0$  is constant), 2 loops closed (no flows from  $\mathbf{u}_0$  are constant) are tabulated in Table 2 (upper part with 2 measurements used). From Table 2, the best system with zero temperature loops closed (that is with only liquid level loops closed) is to keep  $\{V, B\}$  constant, with a loss 109.669. However, this loss is not acceptable from operation point of view. We see from Table 2, closing a single temperature loop reduces the loss by almost a factor 1000. The best single temperature loop policy is to keep  $L$  constant and control tray temperature  $T_{18}$  with a lower loss of 0.188. The best policy with two temperature loops is to control tray temperatures  $T_{15}$  and  $T_{27}$  with a loss of 0.026. This should be compared with the minimal achievable state drift of 0.0204 obtained when allowing for measurement combinations. The loss reduction by closing one loop is very large (from 109.7 to 0.188), but the further reduction by closing two loops (from 0.188 to 0.026) may not be sufficient from a state drift  $J_2$  (regulatory) point of view. This is further illustrated by comparing the composition state drift profiles with an optimal, zero-loop, one-loop and two-loop policies are shown in Figure 5 (a), (b), (c) and (d) in the presence of disturbances  $F, z_F, q_F$ , respectively. Note that the comparison with zero loop closed is out of bound for the feed rate disturbance and also note that the contribution of one measurement noise with green + is included in Figure 5 (b),(c) and (d).

We next study the effect of using temperature measurement combinations. For the distillation case study, we have  $n_u = 2$  and the number of partial control systems are  $2^{n_u} - 2 = 2$  for each additional measurement and require us to solve  $2^{n_u} - 2 = 2$  more MIQP problems. The optimal  $CV_2$  for the partial control systems with  $CV_2$  as combination of 3, 4, 5 and 41 measurements while closing 1, 2 loops are also tabulated in Table 2. The reduction in loss with the number of measurements, when one loop, two loops are closed is shown as a bar chart in Figure 6. The reduction in loss when we use

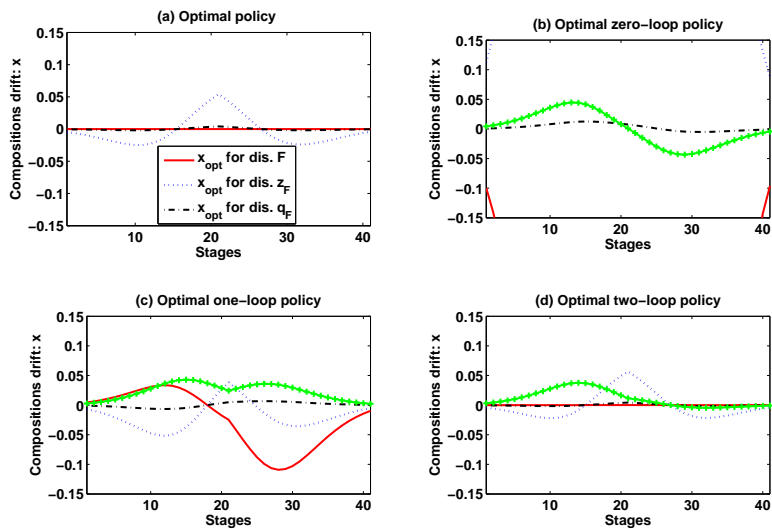


Figure 5: Distillation column state drift in the presence of disturbances  $F, z_F, q_F$ : (a) Optimal policy (minimum achievable state drift), (b) Optimal zero-loop policy, (c) Optimal one-loop policy, (d) Optimal two-loop policy. The effect of a measurement noise on state drift is shown with green + in subplots (b),(c) and (d)

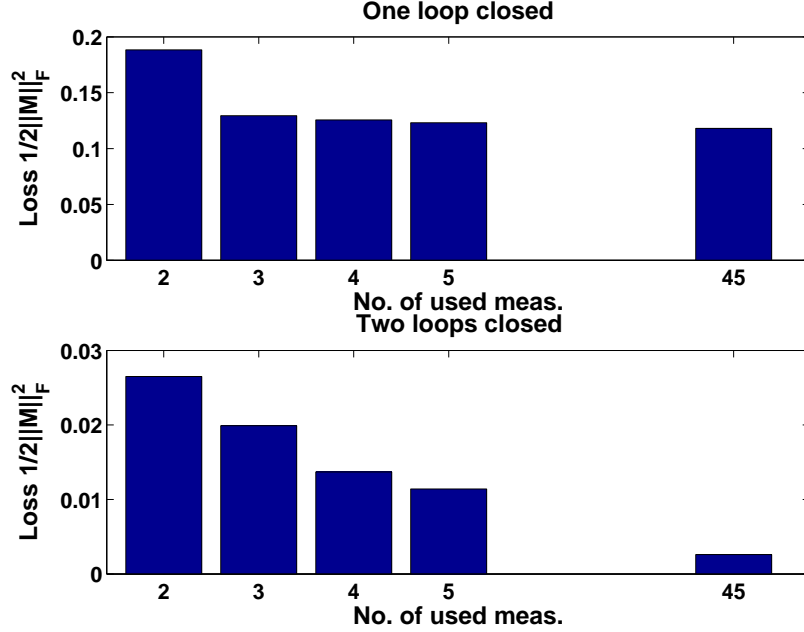


Figure 6: Distillation case study: The reduction in loss in state drift vs number of used measurements, Top: Loss with one loop closed, Bottom : Loss with two loops closed

more number of measurements for  $CV_2$  in two closed loops is higher than that with one closed loop. From Table 2, the best single loop control  $CV_2$  with 3 measurements is to control  $1.072T_{15} + T_{26}$  while keeping  $L$  constant. In conclusion, based on the acceptable steady state drift loss defined by the user, minimum regulatory layer can be obtained by finding  $CV_2$  as individual measurements or measurement combinations.

## 5. Dynamic simulations

The dynamic simulations for the distillation column study are included to show the ease of implementing the regulatory layer controlled variables obtained using the methods of this paper. The open loop gain and time constants with controlled variables  $c_1 = T_{27}, c_2 = T_{15}$  are obtained based on transient responses of +5% steps in  $L$  and  $V$ . A Proportional Integral (PI) controller between  $L$  and  $c_1$  with tuning parameters  $k_{c_1} = -0.5191, \tau_{I_1} = 8 \text{ min}$  and an another PI controller between  $V$  and  $c_2$  with tuning parameters

Table 2: Distillation Column case study: The self optimizing variables  $\mathbf{c}'$ 's as combinations of 2, 3, 4, 5, 41 measurements with their associated losses in state drift

No. of loops closed †	No. of meas.	Optimal CV	$Loss = \frac{1}{2}\ \mathbf{M}_2\ _F^2$ $J - J_{opt}(\mathbf{d})$	$J = \ \mathbf{W}\mathbf{x}\ _2^2$
0	2	$c_1 = V$ $c_2 = B$	109.669 <sup>††</sup>	109.690
1	2	$c_1 = L$ $c_2 = T_{18}$	0.188	0.209
2	2	$c_1 = T_{15}$ $c_2 = T_{27}$	0.026	0.047
1	3	$c_1 = L$ $c_2 = 1.072T_{15} + T_{26}$	0.129*	0.150
2	3	$c_1 = T_{15} - 0.1352T_{28}$ $c_2 = T_{26} + 1.0008T_{28}$	0.020	0.040
1	4	$c_1 = L$ $c_2 = 0.6441T_{15} + 0.6803T_{16} + T_{27}$	0.126*	0.146
2	4	$c_1 = T_{14} - 6.1395T_{26} - 6.3356T_{28}$ $c_2 = T_{16} + 6.2462T_{26} + 6.2744T_{28}$	0.014	0.034
1	5	$c_1 = L$ $c_2 = 1.1926T_{15} + 1.1522T_{16} + 0.9836T_{26} + T_{27}$	0.123*	0.144
2	5	$c_1 = T_{14} - 4.9975T_{26} - 5.0717T_{27} - 4.9813T_{28}$ $c_2 = T_{16} + 5.1013T_{26} + 5.0847T_{27} + 4.9166T_{28}$	0.011	0.032
1	41	$c_1 = L$ $c_2 = f_1(T_1, T_2, \dots, T_{41})$	0.118*	0.138
2	41	$c_1 = f_1(T_1, T_2, \dots, T_{41})$ $c_2 = f_2(T_1, T_2, \dots, T_{41})$	0.003	0.023

† In addition to two closed level loops

The loss is minimized to obtain  $\mathbf{H}_2$

The optimal state drift  $J_{opt}(\mathbf{d}) = 0.0204$

1 loop closed : 1  $\mathbf{c}$  from  $\mathbf{y}_m$ , 1  $\mathbf{c}$  from  $\mathbf{u}_0$

2 loops closed: 2  $\mathbf{c}$  from  $\mathbf{y}_m$

The loss is minimized to obtain  $\mathbf{H}_2$

†† Such a high value is not physical, but it follows because our linear analysis is not appropriate when we close 0 loops

\* used partial control idea to find optimal  $\mathbf{H}_2$  in two step approach

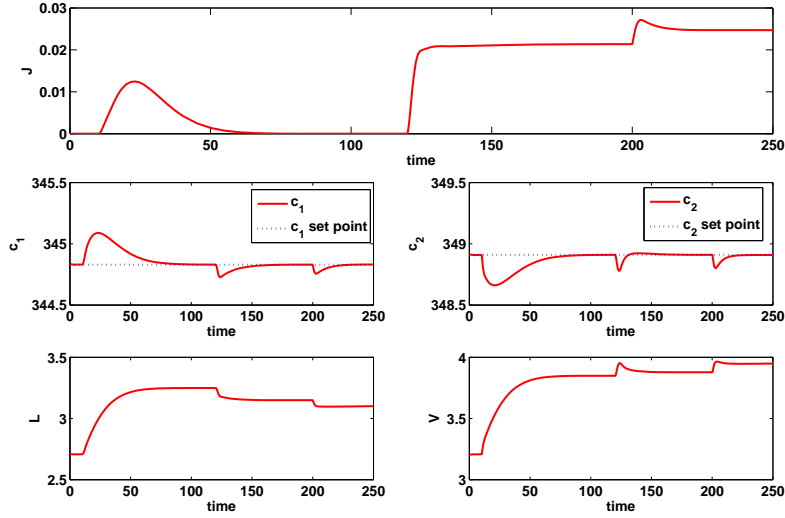


Figure 7: Distillation case study: Transient responses of state drift,  $J$  with two temperature loops closed ( $c_1 = T_{27}$ ,  $c_2 = T_{15}$ ) using inputs  $L$ ,  $V$  for +20% disturbance in  $F$  at time 10 min, +20% disturbance in  $z_F$  at time 120 min and +10% disturbance in  $q_F$  at time 200 min

$k_{c_2} = 0.6307$ ,  $\tau_{I_2} = 8 \text{ min}$  are obtained using SIMC tunings [16] with  $\tau_c = 2 \text{ min}$ .

Dynamic simulations are performed with these settings to evaluate the disturbance rejection performance with the controlled variables  $c_1 = T_{27}$  and  $c_2 = T_{15}$ . The disturbances are +20% disturbance in feed rate  $F$  at time 10 min, +20% disturbance in feed composition  $z_F$  at time 120 min, and +10% disturbance in feed liquid fraction  $q_F$  at time 200 min are shown in Figure 7. The transient responses of the state drift,  $J$ , selected controlled variables,  $c_1 = T_{27}$ ,  $c_2 = T_{15}$ , with their set points, manipulated variables,  $L$ ,  $V$ , are shown in each of the Figures 7 in the presence of disturbances **d**.



## 6. Kaibel column

The main purpose of the Kaibel column case study is to evaluate the proposed methods on a case with more inputs and states. The Kaibel column can separate four components into four products in a single column shell with a single reboiler [17]. The Kaibel column is an extension of the Petlyuk column [18]. The capital savings in the separation of four products with Kaibel column compared to conventional three columns in series makes it an attractive alternative [19, 20].

The given 4-product Kaibel column arrangement separates a mixture of methanol (A), ethanol (B), propanol (C), butanol (D) into almost pure components. The Kaibel column is modeled using a stage-by-stage model with the following simplifying assumptions: Constant pressure, equilibrium stages and constant molar flows. The vapor-liquid equilibrium is modeled using the Wilson equation. The Kaibel column is modeled with 7 sections and we indicate the temperature measurements of each section in Figure 8. Sections 1 and 2 make up the prefractionator, while the main column consists of sections 3 - 7. Each section has 10 stages and the reboiler is counted as an additional stage, which gives Kaibel column with 71 stages in total. Each stage has 3 compositions, 1 holdup and 1 temperature state resulting in a total of 355 states. The economic objective function  $J_1$  is to minimize the sum of impurities in the products.

$$J_1 = D(1 - x_{A,D}) + S_1(1 - x_{B,S_1}) + S_2(1 - x_{C,S_2}) + B(1 - x_{D,B}) \quad (19)$$

where  $D, S_1, S_2$  and  $B$  are the distillate, side product 1, side product 2 and bottom flow rates (mol/min) respectively.  $x_{i,j}$  is mole fraction of component  $i$  in product  $j$ .

The objective of the regulatory layer is to minimize the state drift in the 225 mole fractions of A,B and C components of the process streams (213 mole fractions for 71 trays plus 12 mole fraction states for streams L, D,  $S_1$  and  $S_2$ )

$$J_2 = \|\mathbf{W}\mathbf{x}\|_2^2 \quad (20)$$

where  $\mathbf{W} = \mathbb{I}^{225 \times 225}$  (identity matrix) to have equal weights on mole fractions of A,B and C components in process streams.

The considered Kaibel column then has 6 MVs,  $MV = \{L, S_1, S_2, R_L, D, B\}$  with 4 steady state degrees of freedom ( $\mathbf{u} = \{L, S_1, S_2, R_L\}$ ) and 71 temperature measurements (7 sections with each section having 10 tray temperatures

plus 1 temperature for reboiler). We included the 71 temperature measurements and the 6 inputs as candidate measurements ( $\mathbf{y}$ ) and  $n_y = 77$ . We assume that the temperatures are measured with an accuracy of  $\pm 1^\circ C$  and flows are measured with an accuracy of  $\pm 10\%$ . The considered disturbances are in vapor boil up (V), vapor split ( $R_V$ ), feed flow rate (F), mole fraction of A in feed stream ( $z_A$ ), mole fraction of B in feed stream ( $z_B$ ), mole fraction of C in feed stream ( $z_C$ ), liquid fraction in feed stream  $q_F$ , which vary between  $3 \pm 0.25, 0.4 \pm 0.1, 1 \pm 0.25, 0.25 \pm 0.05, 0.25 \pm 0.05, 0.25 \pm 0.05, 0.9 \pm 0.05$ , respectively. The reader is referred to [21] for further details. We optimize the system for the products impurity (19) and we operate the plant around that optimal operating point.

The Kaibel column has  $n_{u_0} = 6$  physical inputs,  $n_u = 4$  steady state degrees of freedom and  $n_y = 77$  candidate measurements. An MIQP needs to be solved for each  $i$  closed loops. To obtain  $CV_2$  as individual measurements with 0, 1, 2, 3 and 4 closed loops, we need to solve  $n_u + 1 = 5$  MIQP problems. These five MIQP problems are solved using IBM ILOG CPLEX solver in Matlab  $\text{\textcircled{R}}R2009a$  on a Windows XP SP2 notebook with Intel  $\text{\textcircled{R}}Core^{\text{TM}}$  Duo Processor T7250 (2.00 GHz, 2M Cache, 800 MHz FSB). The QP that needs to be solved at each node in MIQP is convex and initial conditions do not play any role.

The presence of integrating modes result in infinite state drift, so first we close 2 loops for integrating levels ( $M_D, M_B$ ). Next, we want to close additional loops to minimize the state drift. The loss with 0 loops closed (4 flows in  $\mathbf{u}_0$  constant), 1 loop closed (3 flows in  $\mathbf{u}_0$  constant), 2 loops closed (2 flows in  $\mathbf{u}_0$  constant), 3 loops closed (1 flow in  $\mathbf{u}_0$  constant), 4 loops closed (no flows in  $\mathbf{u}_0$  constant) are tabulated in Table 3. The best measurements for 0, 1, 2, 3 and 4 loops closed are shown in Table 3 (upper part with 4 measurements). From Table 3, the “best” system with zero loops closed is to keep  $\{S_1, R_L, D, B\}$  constant with a loss 8018.243. The best single-loop policy is to keep  $\{S_1, S_2, R_L\}$  constant and control  $T_{56}$  with a loss of 1628.773. The best two-loop policy is to keep  $\{S_1, S_2\}$  constant and control  $\{T_{13}, T_{42}\}$  with a loss of 469.037. The best three-loop policy is to keep  $S_1$  constant and control  $\{T_7, T_{39}, T_{51}\}$  with a loss of 33.150. Finally, the best four-loop policy is to control  $\{T_9, T_{31}, T_{51}, T_{66}\}$  with a loss of 0.089. These should be compared with the minimal achievable state drift of 0.347 obtained when allowing for measurement combinations. The loss reduces by every additional closed loop and the reduction ratio is very high when we close the final (4th) loop. This is further illustrated by the composition state drift profiles for the optimal,

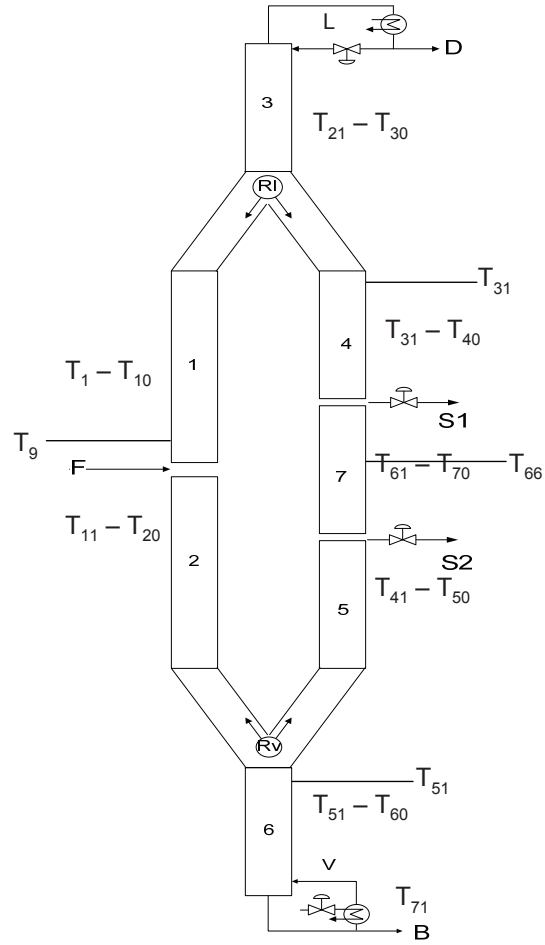


Figure 8: Kaibel distillation column with 7 sections

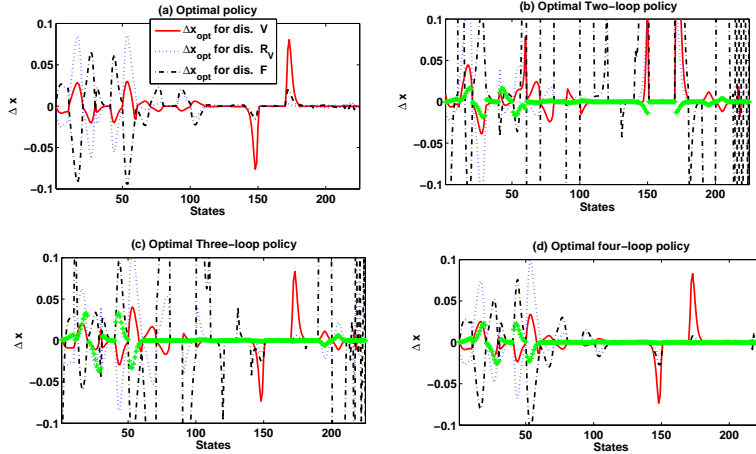


Figure 9: Kaibel column state drift in the presence of disturbances in  $V$ ,  $R_V$  and  $F$ , (a) Optimal policy (minimum achievable state drift), (b) Optimal two-loop policy, (c) Optimal three-loop policy, (d) Optimal four-loop policy. The effect of a measurement noise on state drift is shown with green + in subplots (b), (c) and (d)

two-loop, three-loop and four-loop policies are shown with in Figure 9 (a), (b), (c) and (d). Note that the contribution of one measurement noise with green + is also included in Figure 9 (b),(c) and (d).

We next study the effect of using temperature measurement combinations. For the Kaibel column case study, we have  $n_u = 4$  and to find  $CV_2$  as measurement combinations, the number of partial control systems (number of MIQP problems) increases by  $2^{n_u} - 2 = 14$  for every additional measurement included to obtain  $CV_2$  as a combination. The optimal measurements to find  $CV_2$  as combinations of 5, 6 and 71 measurements for 1, 2, 3 and 4 loops closed are also tabulated in Table 3. For the Kaibel column, we need to close four (temperature) loops as closing 1,2 or 3 loops give a state drift  $J_2$  that is 100 times greater than  $J_{2,opt} = 0.347$ . From a physical point of view this seems reasonable as we need to close one loop in the prefractionator ( $T_9$  in Table 3, and one loop for each of the three component splits A/B, B/C and C/D. In comparison, in the regular distillation column in Section 4, we only have one component split and it is sufficient to close only one loop. The reduction of loss with  $CV_2$  as combinations of 5,6 and 71 number of measurements, when one loop, two loops, three loops and four loops closed are also tabulated in Table 3.

Table 3: Kaibel column: The regulatory  $CV_2$  as combinations of 4, 5, 6 and 71 measurements with their associated losses

No. of loops closed †	No. of meas. used	Optimal CV	Loss $\frac{1}{2}\ \mathbf{M}_2\ _F^2$	Cost $J = \ \mathbf{W}\mathbf{x}\ _2^2$
0	4	$S_1 \ R_L \ D \ B$	8018.243 <sup>††</sup>	8018.590
1	4	$T_{56} \ S_1 \ S_2 \ R_L$	1628.773	1629.121
2	4	$T_{13} \ T_{42} \ S_1 \ S_2$	469.037	469.385
3	4	$T_7 \ T_{39} \ T_{51} \ S_1$	33.150	33.497
4	4	$T_9 \ T_{31} \ T_{51} \ T_{66}$	0.089	0.437
1	5	$f_1(T_{54}, T_{61}) \ S_1 \ S_2 \ R_L$	1605.107*	1605.455
2	5	$f_{1,2}(T_8, T_{39}, T_{51}) \ S_1 \ S_2$	454.122*	454.470
3	5	$f_{1,2,3}(T_9, T_{29}, T_{51}, T_{65}) \ S_1$	31.379*	31.727
4	5	$f_{1,2,3,4}(T_9, T_{29}, T_{31}, T_{51}, T_{66})$	0.075	0.422
1	6	$f_1(T_{15}, T_{39}, T_{51}) \ S_1 \ S_2 \ R_L$	1603.225*	1603.572
2	6	$f_{1,2}(T_9, T_{29}, T_{51}, T_{65}) \ S_1 \ S_2$	454.017*	454.364
3	6	$f_{1,2,3}(T_9, T_{28}, T_{31}, T_{51}, T_{66}) \ S_1$	31.368*	31.715
4	6	$f_{1,2,3,4}(T_9, T_{11}, T_{13}, T_{31}, T_{51}, T_{66})$	0.052	0.400
1	71	$f_1(T_1, T_2, \dots, T_{71}) \ S_1 \ S_2 \ R_L$	1603.161*	1603.508
2	71	$f_{1,2}(T_1, T_2, \dots, T_{71}) \ S_1 \ S_2$	453.975*	454.322
3	71	$f_{1,2,3}(T_1, T_2, \dots, T_{71}) \ S_1$	31.319*	31.666
4	71	$f_{1,2,3,4}(T_1, T_2, \dots, T_{71})$	0.013	0.360

† In addition to two closed level loops

The loss is minimized to obtain  $\mathbf{H}_2$

The optimal state drift  $J_{opt}(d) = 0.347$

1 loop closed : 1  $\mathbf{c}$  from  $\mathbf{y}_m$ , 3  $\mathbf{c}$  from  $\mathbf{u}_0$

2 loops closed: 2  $\mathbf{c}$  from  $\mathbf{y}_m$ , 2  $\mathbf{c}$  from  $\mathbf{u}_0$

3 loop closed : 3  $\mathbf{c}$  from  $\mathbf{y}_m$ , 1  $\mathbf{c}$  from  $\mathbf{u}_0$

4 loops closed: 4  $\mathbf{c}$  from  $\mathbf{y}_m$

The optimal state drift  $J_{opt}(d) = 0.347$

†† Such a high value is not physical, but it follows because our linear analysis is not appropriate when we close 0 loops

\* used partial control system idea to find optimal  $\mathbf{H}_2$  in to step approach

## 7. Discussion and summary

### 7.1. One control layer

The basis for this paper is that we have a two layer control system, where the upper “supervisory” control layer (e.g. using MPC) controls the economic variables ( $CV_1$ ) on a slow time scale and the lower control layer (e.g. using PID) controls the “drifting” variables ( $CV_2$ ) on a fast time scale. In this approach, the effect of economics on a fast time scale is largely ignored, or only included indirectly to the weight  $\mathbf{W}$  in the cost  $J_2$ , or by including active constraints as controlled variables in  $CV_2$ .

An alternative approach is to assume that there is only one control layer. One advantage is that one may then include economics more directly into the fast time scale, and obtain, for example, the required “back-off” for active hard constraints. The disadvantage is that this requires a detailed dynamic model and that the problem may be difficult to solve. Also, it does not yield a two-layer structure which is usually required. For the single control layer, one may either assume multivariable control [e.g. [22]] or decentralized control as studied by Perkins and co-workers [23, 24, 25, 26]. The multivariable control assumptions gives a simpler optimization problem, but may not give a controller suitable for practical implementation. Use of decentralized control [24] gives a more easily implementable control strategy, but the problem is very difficult to solve numerically. Furthermore, there may be an economic loss, also compared to the two-layer structure in Figure 1, which may be difficult to quantify.

### 7.2. Summary

The purpose of this section is to summarize the steps in the proposed quantitative method to arrive at regulatory layer with one, two, or more closed loops.

**Step 1** Define the objective function  $J_2 = \|\mathbf{W}\mathbf{x}\|_2^2$ .

**Step 2** Obtain the linear gain matrices from  $\mathbf{u}$  to  $\mathbf{y}$ ,  $\mathbf{G}^y$ , and  $\mathbf{d}$  to  $\mathbf{y}$ ,  $\mathbf{G}_d^y$  (7), and define the magnitudes for disturbances  $\mathbf{d}$  and implementation errors  $\mathbf{n}^y$  as  $\mathbf{W}_d$  and  $\mathbf{W}_n$ . The second derivatives of the weighted state drift with respect to  $\mathbf{u}$ , and  $\mathbf{u}$  and  $\mathbf{d}$  as  $\mathbf{J}_{2_{uu}}$  and  $\mathbf{J}_{2_{ud}}$ .

**Step 3** Use the new results of self-optimizing control [8, 9, 10, 12] to find an optimal  $\mathbf{H}_2$  that minimizes the loss (10) from the minimum cost.

This gives the optimal controlled variables as individual measurements in the partial regulatory control problem with one, two or more closed loops (Section 3.4).

**Step 4** If the loss is higher than what is acceptable, then find the regulatory layer  $CV_2$  as combination of measurements (Section 3.5).

## 8. Conclusions

The self-optimizing control concept is applied to select optimal controlled variables that minimize the weighted state drift in the presence of disturbances. In process control,  $\|\mathbf{W}\mathbf{x}(j\omega)\|_2^2$  is often flat at lower frequencies and drops at higher frequencies, so minimizing the state drift for steady-state ( $\omega = 0$ ) yields good results. The proposed method to find both optimal individual and combination of measurements as controlled variables was evaluated on a distillation column case study with 41 stages and a Kaibel column case study with 71 stages to arrive at optimal regulatory layer with 1, 2 or more closed loops. We included the dynamic simulations for the distillation column to show the ease of using these methods in practice.

## Appendix A.

Mass balances assuming for the condenser and reboiler at steady state yield [27]

$$\begin{aligned}\frac{dM_d}{dt} &= 0 = V_{top} - L - D \\ \frac{dM_b}{dt} &= 0 = L_{btm} - V - B\end{aligned}$$

Here, at steady state and assuming constant molar flows

$$\begin{aligned}V_{top} &= V + (1 - q_F)F \\ L_{btm} &= L + q_F F\end{aligned}$$

So we find at steady state

$$\begin{aligned}D &= V - L + (1 - q_F)F \\ B &= L + q_F F - V\end{aligned}$$

Note that there is no effect of  $z_F$  in this case. Linearizing gives  $\mathbf{G}$  and  $\mathbf{G}^d$  as given in (17).

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