Optimal PID-Control on First Order Plus Time Delay Systems

Verification of the SIMC rules

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Introduction

The Questions

- How much can we gain by using PID instead of PI control?
- Is there a simple PID tuning rule that gives close to optimal performance?

The system

We investigating the optimal tuning for the process

$$G(s) = \frac{k}{(\tau_1 s + 1)} e^{-\theta s}$$

Considering the cascade form PID controller

$$K(s) = K_C \left(\frac{\tau_I s + 1}{\tau_I s} \right) (\tau_D s + 1)$$

Quantifying the Optimal Controller

Trade-off between

- Output performance } High controller gain (Tight control)
- Robustness
- Input usage
- Noise sensitivity

(Robustness, Input usage, Noise sensitivity) } Low controller gain (Smooth control)

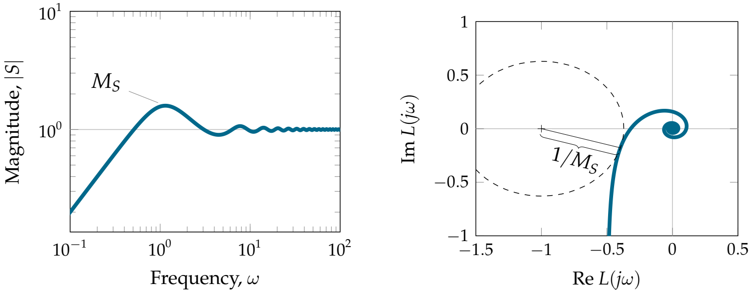
We focus on output performance and robustness.

- Output Performance: J – weighted average IAE
- Robustness: M_S – peak sensitivity

ROBUSTNESS – M_S

$$S = \frac{1}{1 + GK}$$

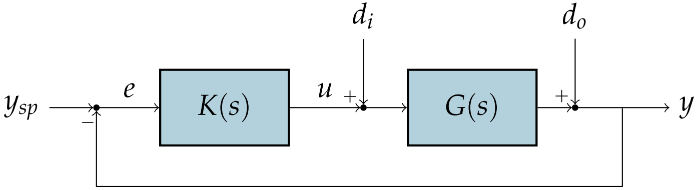
$$M_S = \max_{\omega} |S(j\omega)| = \|S\|_\infty$$



PERFORMANCE – J

We consider unit step disturbances at two different locations:

- at plant output
- at plant input



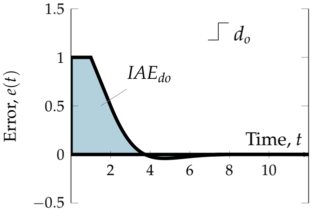
PERFORMANCE – J

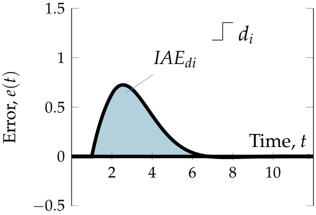
$$IAE = \int_0^\infty |e(t)|\,dt$$

$$J(K) = 0.5\left[\frac{\mathrm{IAE}_{do}(K)}{\mathrm{IAE}_{do}^\circ} + \frac{\mathrm{IAE}_{di}(K)}{\mathrm{IAE}_{di}^\circ}\right]$$

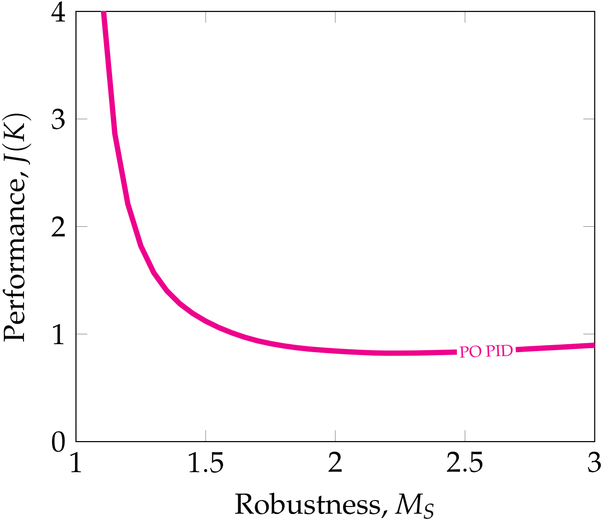
IAE_{do}° PID optimal controller for d_o (at $M_S = 1.59$)

IAE_{di}° PID optimal controller for d_i (at $M_S = 1.59$)

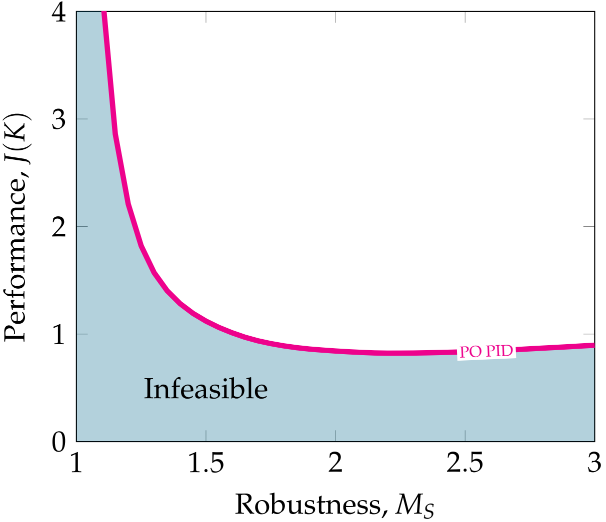




Pareto optimal plots – Optimal $J(K)$ vs. M_S



PARETO OPTIMAL PLOTS – OPTIMAL $J(K)$ VS. M_S



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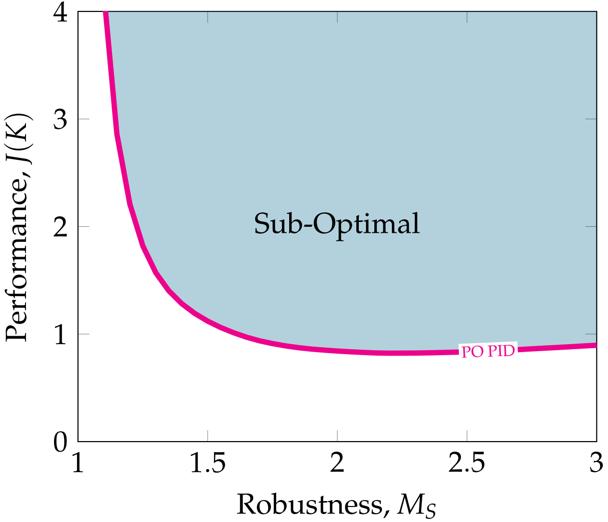

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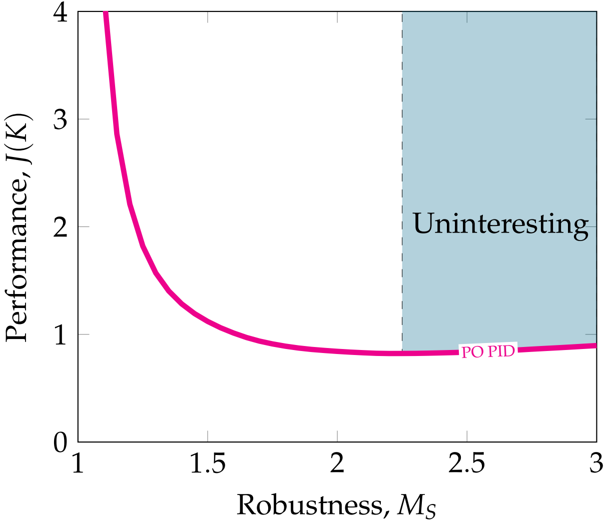


THE CASES

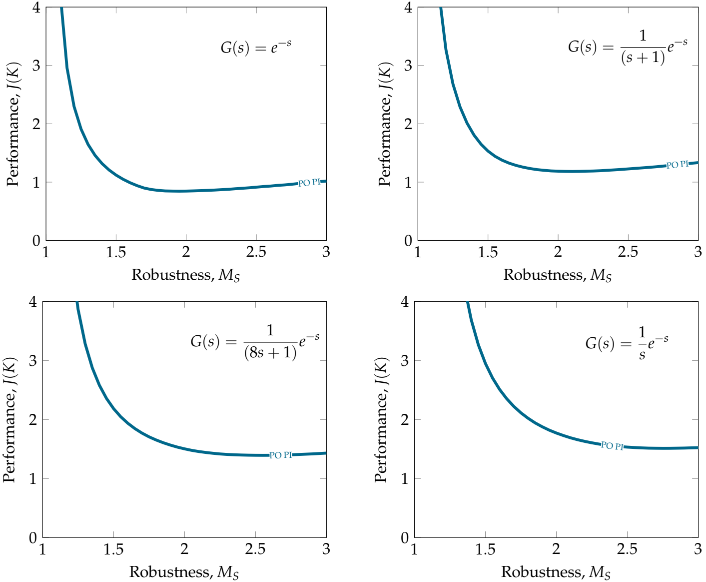
$$G_1(s) = e^{-s}$$

$$G_2(s) = \frac{1}{(s+1)}e^{-s}$$

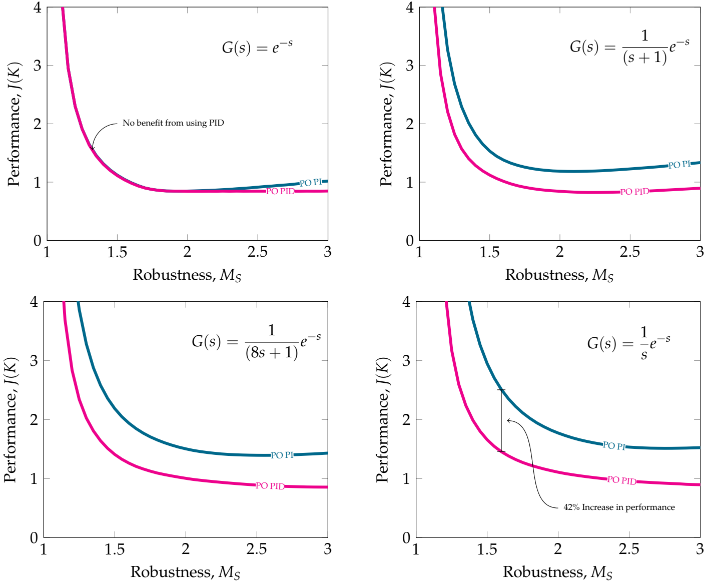
$$G_3(s) = \frac{1}{(8s+1)}e^{-s}$$

$$G_4(s) = \frac{1}{s}e^{-s}$$

Optimal PI control

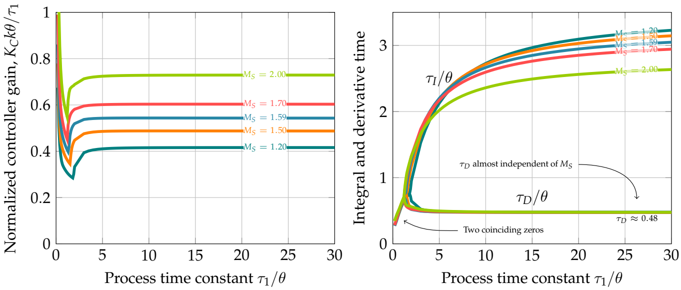


Optimal PID Compared with Optimal PI



Optimal PID tuning

- τ_D is independent of M_S and the time constant τ_1 for lag dominated processes ($\tau_1/\theta \gtrsim 2.5$).
- Optimal controller for delay dominated processes ($\tau_1/\theta \lesssim 2.5$) is on the boarder between cascade and ideal controller realization.

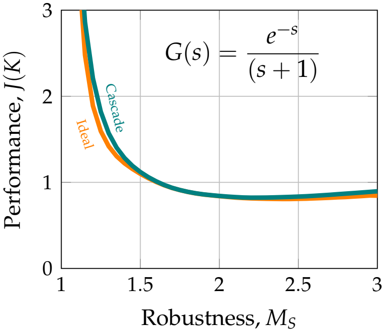


Optimal PID – Cascade vs. Ideal

Delay dominated processes $\tau_1/\theta < 3$:

Performance can be improved by using Ideal controller.

$$K_{\text{Ideal}} = K_C' \left(1 + \frac{1}{\tau_I' s} + \tau_D' s\right)$$



THE SIMC PI RULES FOR FOPTD

SIMC: Probably the best simple PID tuning in the world

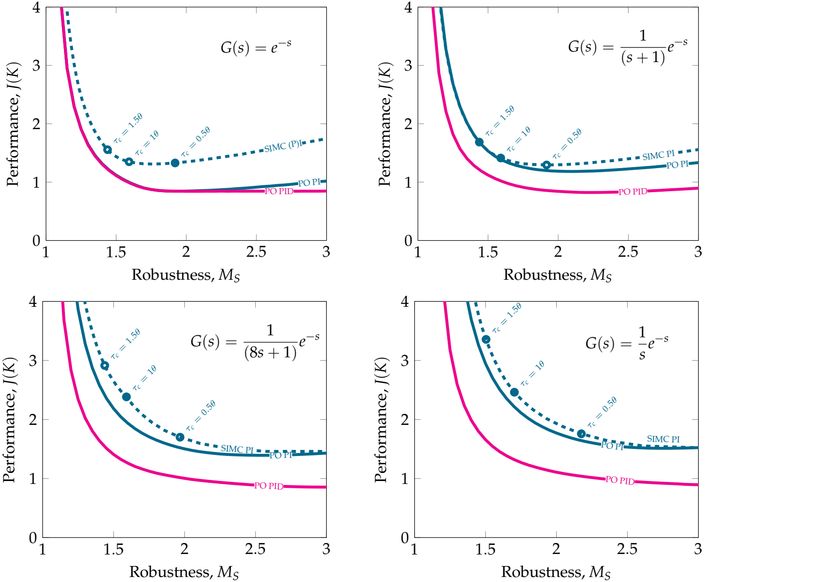
$$K_C = \frac{1}{k}\frac{\tau_1}{\tau_c + \theta} \quad (1)$$

$$\tau_I = \min\{\tau_1,\, 4(\tau_c + \theta)\} \quad (2)$$

where τ_c is the tuning constant.
$\tau_c = \theta$ is recommended for tight control.

However, only PI tuning for FOPTD

Optimal PID control Compared with SIMC



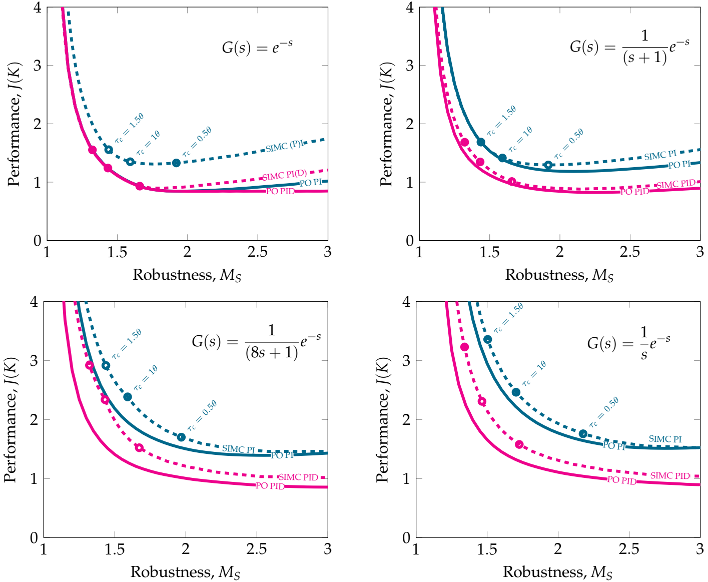
PROPOSED: SIMC PID RULES FOR FOPTD

$$K_C = \frac{1}{k}\frac{\tau_1}{\tau_c + \theta} \tag{3}$$

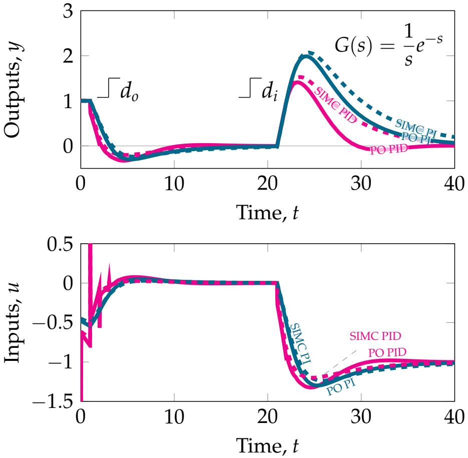
$$\tau_I = \min\{\tau_1, 4(\tau_c + \theta)\} \tag{4}$$

$$\tau_D = \theta/3 \tag{5}$$

Optimal PID control Compared with SIMC



Step response



Conclusions

- PID control can substantially increase performance for lag dominated processes.
- No Benefit of using PID control on pure time delay process
- SIMC PI with D: $\tau_D = \theta/3$ gives a good PID tuning.

Thank you ☺