Dynamic compensation of static estimators from Loss method

Maryam Ghadrdan[*], **Ivar J. Halvorsen**[**], **Sigurd Skogestad**[*]

** Department of Chemical Engineering, Norwegian University of Science and Technology, N-7491 Trondheim, Norway, Emails: ghadrdan@nt.ntnu.no, skoge@nt.ntnu.no,*
*** SINTEF ICT, Applied Cybernetics, N-7465 Trondheim, Norway, Email: ivar.j.halvorsen@sintef.no*

Abstract: In this paper, we study different possibilities to overcome the inverse response problem which is caused by combining different measurements with fast and slow dynamics to form an estimator. Our goal is to obtain a response with "No inherent limitation".

Keywords: Combination of Measurements, Self-optimizing control, Inverse response

1. INTRODUCTION

Reliable and accurate measurement of product compositions is one of the important issues in distillation column control. On-line composition measurement devices are usually expensive and unreliable. In addition, there is usually a considerable time delay that may be a limitation for control performance. On the other hand, temperature measurements are fast, inexpensive and reliable and are used for distillation column control in industry instead of composition analyzers.

Commonly, simple linear relationships, $\hat{\mathrm{y}} = \mathbf{H}\mathrm{x}$, are used to estimate composition (y) based on temperature measurements (x).

Using a combination of measurements leads to a better steady-state estimate (compared to single measurements), but dynamically it may give rise to right-half plane zeros (inverse response behaviour) in the transfer function from input u to the estimate $\hat{\mathrm{y}}$, which limit the closed-loop performance for SISO systems. We have $\hat{\mathrm{y}} = \mathbf{H}\mathrm{x} = \mathbf{H}\mathbf{G}_x\mathrm{u}$ (see Figure 1). The appearance of a RHP zero in the square transfer function $\mathbf{G} = \mathbf{H}\mathbf{G}_x(s)$ from u to $\hat{\mathrm{y}}$ is common. The measurements have different dynamics ($\mathbf{G}_x(s)$) as they are located at different positions in the plant. This is noted by Alstad (2005) who considered a simple example with two measurements and one input.

$$\hat{\mathrm{y}}(s) = \mathbf{H}\mathbf{G}_x(s)\mathrm{u}(s) = h_1 g_1(s)\mathrm{u}(s) + h_2 g_2(s)\mathrm{u}(s) \qquad (1)$$

Here, $\mathbf{G}_x$ is modeled as a rational transfer function on the form $g_i(s) = \frac{n_{g_i}(s)}{d_{g_i}(s)}$, thus the resulting estimator is:

$$\hat{\mathrm{y}}(s) = (h_1 g_1(s) + h_2 g_2(s))\,\mathrm{u}(s) = \left(h_1 \frac{n_{g_1}(s)}{d_{g_1}(s)} + h_2 \frac{n_{g_2}(s)}{d_{g_2}(s)}\right)\mathrm{u}(s)$$
$$= \left(\frac{h_1 n_{g_1}(s) d_{g_2}(s) + h_2 n_{g_2}(s) d_{g_1}(s)}{d_{g_1}(s) d_{g_2}(s)}\right)\mathrm{u}(s) \qquad (2)$$

The poles of the resulting transfer function $\mathbf{G}$ are identical to the poles of the two subsystems, while the zeros are changed. For systems where $h_1 g_1$ and $h_2 g_2$ have opposing effects, this may lead to right-hand plane zeros.

We have studied three approaches to overcome this problem:

- Cascade Control:
  The idea is to close a fast inner loop based on a single measurement with no RHP-zero and adjust the setpoint on a time scale which is slower than the RHP-zero.
- Use of measurements from the same section of the process:
  If the dynamic behavior of the selected measurements are similar, then it is less likely to get RHP-zero. However, this gives a larger steady-state error.
- Filters:
  Low-pass filters will keep the system optimal at steady state. The idea is to filter the measurements before they are combined to give the estimate. The filtered measurements are $\hat{\mathrm{y}} = \mathbf{H}\mathbf{H}_F\mathrm{u}$

2. MOTIVATING EXAMPLE

We first illustrate the ideas with a simple example. Afterwards, we will give some guidelines for the case study of a distillation column.

Consider a system with two measurements x and one input u

$$\mathbf{G}_x = \begin{bmatrix} g_1 \\ g_2 \end{bmatrix} = \begin{bmatrix} \dfrac{1}{3s+1} \\ \dfrac{1}{s+1} \end{bmatrix}$$

Assume the estimator matrix $\mathbf{H}$ is

$$\mathbf{H} = [\,2 \;\; -1\,]$$

2.1 No dynamic compensation

The transfer function from u to $\hat{\mathrm{y}}$ is

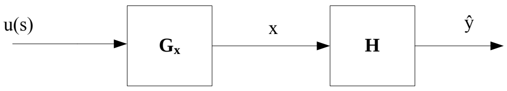

Fig. 1. Block diagram of the estimation

$$\mathbf{G} = \mathbf{H}\mathbf{G}_x = \frac{2}{3s+1} - \frac{1}{s+1} = \frac{1-s}{(3s+1)(s+1)}$$

The RHP zero in $\mathbf{G}(s)$ will limit the achievable closed-loop performance. Thus, we have introduced an unnecessary limitation on performance.

3. CASCADE CONTROL

Now we consider if we can avoid the effect of the RHP zero using cascade control. We assume that we control the faster measurement x_2 ($\frac{1}{s+1}$) in an inner loop. The desired closed-loop time constant is assumed to be $\tau_c = 0.1$ (time units).

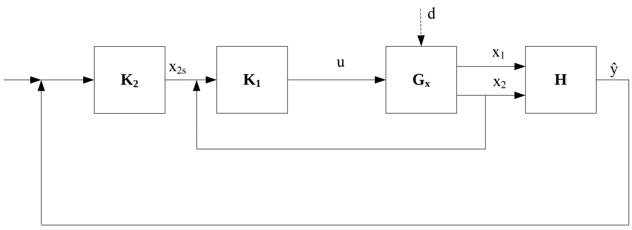

Fig. 2. Block diagram of the estimation with a cascade loop

Using the SIMC PI-tuning rules with $\theta = 0$ and $\tau_c = 0.1$, we have

$$K_c = \frac{1}{k}\frac{\tau_1}{\tau_c + \theta} = 10$$
$$\tau_I = \min(\tau_1, 4(\tau_c + \theta)) = 0.4$$

The resulting controller and loop transfer functions become

$$K_1(s) = \frac{K_c}{\tau_I s}(\tau_I s + 1) = \frac{10}{0.4s}(0.4s + 1)$$
$$L_1(s) = K_1(s).g_2(s) = \frac{25(0.4s+1)}{s(s+1)}$$

and

$$x_2 = \frac{L_1}{L_1 + 1} x_{2s} = \frac{0.4s + 1}{(0.1283s + 1)(0.3117s + 1)} \mathrm{x}_{2s}$$

and with

$$\mathrm{u} = g_2^{-1} \mathrm{x}_2$$

we can find x_1 and then

$$\hat{\mathrm{y}} = \mathbf{H}\mathrm{x} = \frac{(0.4s+1)(-s+1)}{(0.1283s+1)(0.3117s+1)(3s+1)} \mathrm{x}_{2s}$$

We see that the RHP zero still remains. This is explained from the following theorem.

Theorem 1. Cascade (inner-loop) control can not move the zero of $\mathbf{H}\mathbf{G}_x$

Proof. The expression for the estimated primary variable is

$$\hat{\mathrm{y}} = h_1 \mathrm{x}_1 + h_2 \mathrm{x}_2$$

where

$$\mathrm{x}_1 = g_1 \mathrm{u}$$
$$\mathrm{x}_2 = g_2 \mathrm{u}$$

Assume we control x_2 in an inner cascade loop.

$$\mathrm{u} = K(s)(\mathrm{x}_{1s} - \mathrm{x}_1)$$

So,

$$\mathrm{x}_2 = \frac{K(s) g_2}{1 + K(s) g_2} \mathrm{x}_{2s}$$
$$\mathrm{x}_1 = \frac{g_1}{g_2} \mathrm{x}_2$$

The transfer function from x_{2s} to $\hat{\mathrm{y}}$ is

$$\hat{\mathrm{y}} = (h_1 \frac{g_1}{g_2} + h_2) \frac{K g_2}{1 + K g_2} \mathrm{x}_{2s} \quad (3)$$

The term $(h_1 g_1 + h_2 g_2)$, which includes the RHP zero, is unchanged.

4. FILTERING

Here we use individual compensators (or filters) on the measurements as illustrated by the block $\mathbf{H}_F$ in Figure 3 The diagonal matrix $\mathbf{H}_F$ is applied on the measurements to improve the dynamic behavior. It is required that $\mathbf{H}_F(0) = I$. This means that the steady-state gain should not change, because it is already optimal. Each of the filters are simple first-order low-pass

$$\mathbf{H}_F = \begin{bmatrix} \frac{1}{\tau_{F1}s + 1} & 0 \\ 0 & \frac{1}{\tau_{F1}s + 1} \end{bmatrix}$$

or lead-lag,

$$\mathbf{H}_F = \begin{bmatrix} \frac{\tau_{F1n}s + 1}{\tau_{F1d}s + 1} & 0 \\ 0 & \frac{\tau_{F2n}s + 1}{\tau_{F2d}s + 1} \end{bmatrix}$$

Four different filters are used for the case-study, see Table 1. Figure 4 shows the step responses with the four filters. Note that we can make the transfer function from x to $\hat{\mathrm{y}}$ as fast as we want. From this example, it is seen that

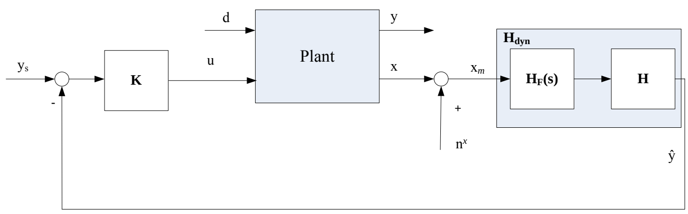

Fig. 3. Block diagram of the estimation system including filter ($\mathbf{H}$)

the lead-lag filters are performing better in making the response fast than the low-pass filters. One can optimize the filter parameters to get the best performance. In this particular example, it is not clear what the best performance means. The transfer function from x to $\hat{\mathrm{y}}$ is at most second order and there are no limitations on control performance.

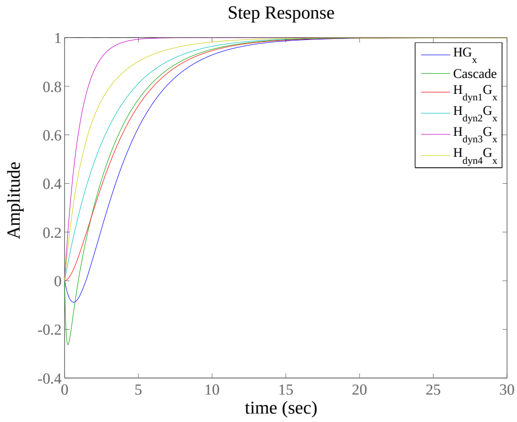

Fig. 4. Step response of all Gs

4.1 Distillation Case-study

We have designed a steady-state estimator **H** for a multi-component distillation column based on data from the simulator in UNISIM. The saturated liquid feed stream is an equi-molal mixture of methanol, ethanol, 1-propanol and 1-butanol. The column is separating the two light components from the two heavy components in a column with 36 stages. The concentration of 1-propanol in the distillate and the concentration of ethanol in the bottom product are 0.0124 and 0.0274 respectively. Figure 5 shows the value of **H** from each tray in the column for the two estimated values.

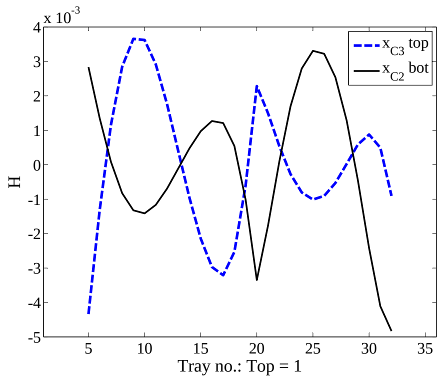

Fig. 5. Steady-state contribution of temperatures to the estimates (**H**), Dashed: top composition estimate, Solid: bottom composition estimate

Table 1. Different filters and the final transfer functions

Filter matrix	Transfer function from u to ŷ
$\mathbf{H}_{F1} = \begin{bmatrix} \frac{1}{s+1} & 0 \\ 0 & \frac{1}{3s+1} \end{bmatrix}$	$\mathbf{G}_1 = \mathbf{H}\mathbf{H}_{F1}\mathbf{G}_x = \frac{1}{(3s+1)(s+1)}$
$\mathbf{H}_{F2} = \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{3s+1} \end{bmatrix}$	$\mathbf{G}_2 = \mathbf{H}\mathbf{H}_{F2}\mathbf{G}_x = \frac{2s+1}{(3s+1)(s+1)}$
$\mathbf{H}_{F3} = \begin{bmatrix} 1 & 0 \\ 0 & \frac{s+1}{3s+1} \end{bmatrix}$	$\mathbf{G}_3 = \mathbf{H}\mathbf{H}_{F3}\mathbf{G}_x = \frac{1}{3s+1}$
$\mathbf{H}_{F4} = \begin{bmatrix} \frac{3s+1}{s+1} & 0 \\ 0 & 1 \end{bmatrix}$	$\mathbf{G}_4 = \mathbf{H}\mathbf{H}_{F4}\mathbf{G}_x = \frac{1}{s+1}$

Figure 6 show the open-loop response of the primary variables (solid line) and the estimated values (dashed line) to a change in boilup (V). An inverse response is seen in the estimate of the top composition. To check why this is happening, the contribution of the temperatures to the final estimate is studied. Figures 7-8 show the temperature changes as each of the degrees of freedom is perturbed, and the contribution of each of the trays to the estimate of the top composition. Figure 8 is actually obtained by multiplying **H** to each of the time-series vectors of the measurements. The perturbation is small enough so that the results in the negative and positive directions are similar.

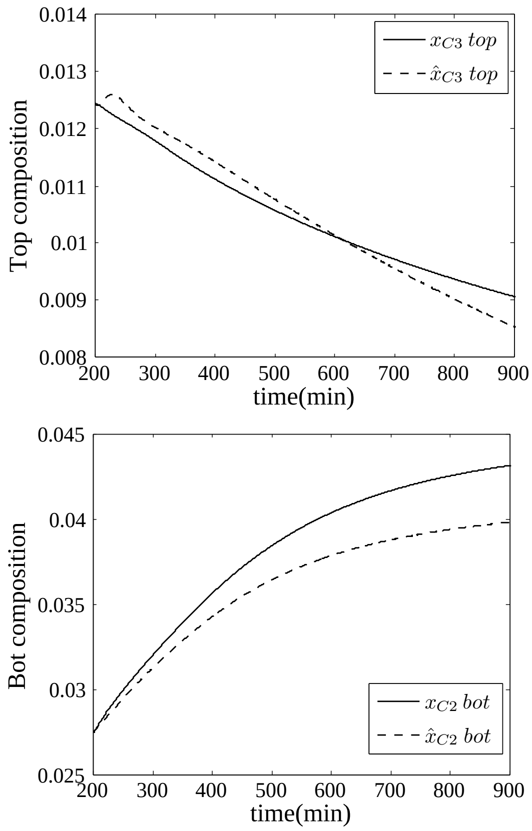

Fig. 6. Top and bottom estimates with -1% change in boilup

As a simple trial, by adding first-order filters on the 6th, 16th and 17th stages which display a fast response to change in boilup (see the first 100 minutes in Figure 8), we see in Figure 9 that the inverse response is removed.

5. OPTIMIZATION OF THE FILTERS

The filter time constants can be optimized to give the best performance. The objective function can be defined as minimizing the H_∞ norm

$$\min_{\mathbf{H_F}} \|\mathbf{G}_{ref} - \mathbf{H}\mathbf{H}_F\mathbf{G}_x\|_\infty \qquad (4)$$

where $\mathbf{G}_{ref}$ is the desired transfer function from input to the estimate. For monitoring purpose, the best performance means the closest response to the actual compositions. So, in this case $\mathbf{G}_{ref}$ would be the transfer function

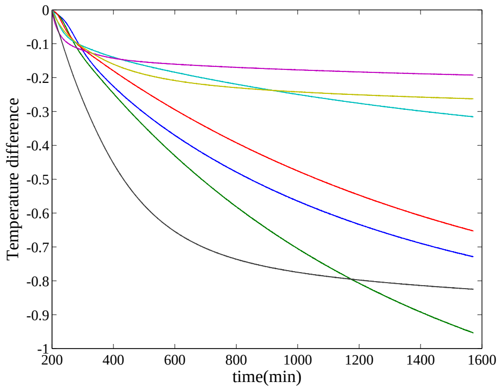

Fig. 7. Temperature changes in the column with -1% change in boilup and constant Reflux ratio

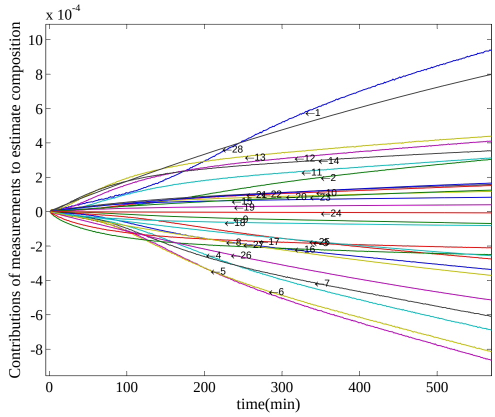

Fig. 8. Contributions to the top composition estimate with -1% change in boilup and constant Reflux ratio

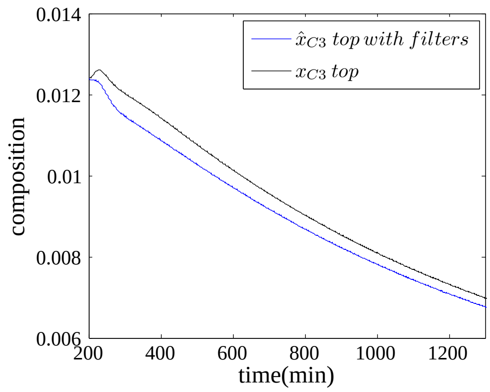

Fig. 9. Estimated composition in the top and the filtered estimate. Filters are on 6th, 16th and 17th measurements

from the inputs to the real primary variables. Figure 10 shows the optimized filtered top estimate together with the real primary value and the unfiltered estimate which is obtained from the steady-state calculations. We have focused on the first 100 min, since we don't want to let the steady-state offset value be part of the objective function value. As it is seen in Figure 10, the filtered estimate matches perfectly the real primary variable value in the first 100 minutes and it diverges to get to the steady state value of the unfiltered estimate (note that these are LP filters with no change in SS). Table 2 shows the values of the filter time constants for this optimization.

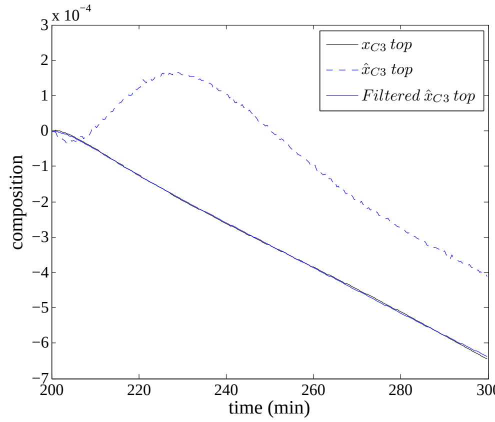

Fig. 10. Estimated composition with optimized filters

Table 2. The time constants of the filters from optimization

Tray no.	H for top comp.	τ_F
5	-0.0043	1039.8
6	-0.0013	1338.3
7	0.0012	33.6
8	0.0028	514.1
9	0.0037	1209.1
10	0.0036	55.0
11	0.0029	1211.4
12	0.0018	1589.6
13	0.0004	554.6
14	-0.0010	1976.2
15	-0.0021	909.4
16	-0.0030	1424.3
17	-0.0032	466.6
18	-0.0025	1640.2
19	-0.0006	278.2
20	0.0023	19.8
21	0.0015	8.3
22	0.0005	1577.0
23	-0.0003	484.9
24	-0.0008	1158.8
25	-0.0010	1026.5
26	-0.0009	925.0
27	-0.0005	860.4
28	0.0000	1992.7
29	0.0006	868.7
30	0.0009	1404.7
31	0.0005	831.4
32	-0.0009	477.4

5.1 Explicit solution for the optimization problem

In this section, we want to solve the optimization problem by considering it as a model-matching problem and solving Nehari problem obtained from it. The problem is to compute an upper bound γ and then compute a $\mathbf{Q}$ such that

$$\|\mathbf{T}_1 - \mathbf{T}_2\mathbf{Q}\mathbf{T}_3\|_\infty \leq \gamma$$

An optimal $\mathbf{Q}$ exists if the ranks of the two matrices $\mathbf{T}_2(j\omega)$ and $\mathbf{T}_3(j\omega)$ are constant for all $0 < \omega < \infty$ Francis (1987). Our reason to use this method is shown by the motivating example. We saw that we can not be sure about the structure of the filters, i.e. being lead-lag or LP, etc. The following Theorem is obtained from Francis (1987), based on which an algorithm to find an optimal $\mathbf{Q}$ is proposed Francis (1983).

Lemma 2. Let $\mathbf{U}$ be an inner matrix and define

$$\mathbf{E} = \begin{bmatrix} \mathbf{U}^{\sim} \\ \mathbf{I} - \mathbf{U}\mathbf{U}^{\sim} \end{bmatrix}$$

Then, $\|\mathbf{E}\mathbf{G}\|_\infty = \|\mathbf{G}\|_\infty$

Proof. It suffices to show that $\mathbf{E}^{\sim}\mathbf{E} = \mathbf{I}$

Lemma 3. Suppose $\mathbf{F}$ and $\mathbf{G}$ are matrices with no poles on imaginary axis with equal number of columns. If

$$\| \begin{bmatrix} \mathbf{F} \\ \mathbf{G} \end{bmatrix} \|_\infty < \gamma \tag{5}$$

then

$$\|\mathbf{G}\|_\infty < \gamma \tag{6}$$

and

$$\|\mathbf{F}\mathbf{G}_o^{-1}\|_\infty < 1 \tag{7}$$

Proof. Equation (6) follows immediately from (5), because

$$\|\mathbf{G}\|_\infty \le \| \begin{bmatrix} \mathbf{F} \\ \mathbf{G} \end{bmatrix} \|_\infty$$

$\gamma^2 - \mathbf{G}^{\sim}\mathbf{G}$ has a spectral factorization:

$$\gamma^2 - \mathbf{G}^{\sim}\mathbf{G} = \mathbf{G}_o^{\sim}\mathbf{G}_o \tag{8}$$

where $\mathbf{G}_o$, $\mathbf{G}_o^{-1} \in \mathrm{RH}_\infty$ Define

$$\epsilon := \gamma - \| \begin{bmatrix} \mathbf{F} \\ \mathbf{G} \end{bmatrix} \|_\infty \tag{9}$$

and

$$g := \mathbf{G}_o^{-1} f \tag{10}$$

where f is a Lb_2-vector of unit norm. Starting from (9), we have

$$\| \begin{bmatrix} \mathbf{F} \\ \mathbf{G} \end{bmatrix} g\|_2 \le (\gamma - \epsilon)\|g\|_2^2$$

$$< \begin{bmatrix} \mathbf{F} \\ \mathbf{G} \end{bmatrix} g, \begin{bmatrix} \mathbf{F} \\ \mathbf{G} \end{bmatrix}, g > \le (\gamma - \epsilon)^2 < g, g >$$

$$< g, \mathbf{F}^{\sim}\mathbf{F}g > \le \gamma^2 < g, g > - \epsilon(2\gamma - \epsilon)\|g\|_2^2$$

$$< g, \mathbf{F}^{\sim}\mathbf{F}g > \le < g, (\gamma^2 - \mathbf{G}^{\sim}\mathbf{G})g > -\epsilon(2\gamma - \epsilon)\|\mathbf{G}_o\|_\infty^{-2}$$

The last step used the inequality

$$1 = \|\mathbf{G}_o g\| \le \|\mathbf{G}_o\|_\infty \|g\|_2$$

Now using (8) we get

$$\|\mathbf{F}g\|_2^2 \le \|\mathbf{G}_o g\|_2^2 - \epsilon(2\gamma - \epsilon)\|\mathbf{G}_o\|_\infty^{-2}$$

Hence,

$$\|\mathbf{F}\mathbf{G}_o^{-1} f\|_2^2 \le 1 - \epsilon(2\gamma - \epsilon)\|\mathbf{G}_o\|_\infty^{-2}$$

Since f was arbitrary, we find that

$$\|\mathbf{F}\mathbf{G}_o^{-1}\|_2^2 \le 1 - \epsilon(2\gamma - \epsilon)\|\mathbf{G}_o\|_\infty^{-2}$$

Since $\epsilon(2\gamma - \epsilon) > 0$, we arrive at 7

Theorem 4. (i) $\alpha = \inf\{\gamma : \|\mathbf{Y}\|_\infty < \gamma, \mathrm{dist}(R, \mathrm{RH}_\infty) < 1\}$
(ii) Suppose $\gamma > \alpha$, $\mathbf{G}, \mathbf{X} \in \mathrm{RH}_\infty$

$$\begin{aligned} \|R - \mathbf{X}\|_\infty &\le 1 \\ \mathbf{X} &= \mathbf{U}_0\mathbf{Q}\mathbf{Y}_o^{-1} \end{aligned} \tag{11}$$

Then $\|\mathbf{T}_1 - \mathbf{T}_2\mathbf{Q}\|_\infty \le \gamma$

Proof. (i) Let

$$\gamma_{inf} = \inf\{\gamma : \|\mathbf{Y}\|_\infty < \gamma, \mathrm{dist}(R, \mathrm{RH}_\infty) < 1\}$$

choose $\epsilon > 0$ and then choose γ such that $\alpha + \epsilon > \gamma > \alpha$. Then there exist $\mathbf{Q}$ in RH_∞ such that

$$\|\mathbf{T}_1 - \mathbf{T}_2\mathbf{Q}\|_\infty < \gamma$$

From Lemma 2 we have:

$$\| \begin{bmatrix} \mathbf{U}_i^{\sim} \\ \mathbf{I} - \mathbf{U}_i\mathbf{U}_i^{\sim} \end{bmatrix} (\mathbf{T}_1 - \mathbf{T}_2\mathbf{Q}) \|_\infty \le \gamma \tag{12}$$

This is equivalent to

$$\| \begin{bmatrix} \mathbf{U}_i^{\sim}\mathbf{T}_1 - \mathbf{U}_o\mathbf{Q} \\ \mathbf{Y} \end{bmatrix} \|_\infty < \gamma \tag{13}$$

This implies from Lemma 3 that

$$\|\mathbf{Y}\|_\infty < \gamma \tag{14}$$

$$\|\mathbf{U}_i^{\sim}\mathbf{T}_1\mathbf{Y}_o^{-1} - \mathbf{U}_o\mathbf{Q}\mathbf{Y}_o^{-1}\|_\infty < 1 \tag{15}$$

The latter inequality implies

$$\mathrm{dist}\left(R, \mathbf{U}_o\mathrm{RH}_\infty\mathbf{Y}_o^{-1}\right) < 1 \tag{16}$$

$\mathbf{U}_o$ is right-invertible in RH_∞ and $\mathbf{Y}_o$ is invertible in RH_∞. So, (17) gives

$$\mathrm{dist}(R, \mathrm{RH}_\infty) < 1 \tag{17}$$

Lemma 5. For R in RL_∞

$$\mathrm{dist}(R, \mathrm{RH}_\infty) = \mathrm{dist}(R, \mathrm{H}_\infty) = \|\Gamma_R\|$$

Proof. We have

$$\mathrm{dist}(R, \mathrm{RH}_\infty) \ge \mathrm{dist}(R, \mathrm{H}_\infty) = \|\Gamma_R\|$$

The latter is the Nehari's theorem. Choose $\epsilon > 0$ and set $\beta := \|\Gamma_R\|$. Then

$$\begin{aligned} \mathrm{dist}\left[(\beta + \epsilon)^{-1} R, \mathrm{H}_\infty\right] &= (\beta + \epsilon)^{-1} \|\Gamma_R\| \\ &= \beta / (\beta + \epsilon) \\ &< 1 \end{aligned}$$

This inequality implies that there exists $\mathbf{X}$ in RH_∞ such that

$$\|(\beta + \epsilon)^{-1} R - \mathbf{X}\|_\infty \le 1$$

Thus,

$$\begin{aligned} \mathrm{dist}(R, \mathrm{RH}_\infty) &\le \beta + \epsilon \\ &= \mathrm{dist}(R, \mathrm{H}_\infty) + \epsilon \end{aligned}$$

So,

$$\mathrm{dist}(R, \mathrm{RH}_\infty) \le \mathrm{dist}(R, \mathrm{H}_\infty)$$

The general algorithm to obtain $\mathbf{Q}$ is as follows

- Step 1 Compute $\mathbf{Y}$ and $\|\mathbf{Y}\|_\infty$
- Step 2 Find an upper bound α_1 for α ($\|\mathbf{T}_1\|_\infty$ is the simplest choice)
- Step 3 Select a trial value for γ in the interval $(\|\mathbf{Y}\|_\infty, \alpha_1]$
- Step 4 Compute $\mathbf{R}$ and $\|\Gamma_R\|$. Then $\|\Gamma_R\| < 1$ iff $\alpha < \gamma$. Change the value of γ correspondingly to meet this criteria
- Step 5 Find a matrix $\mathbf{X}$ such that $\|\mathbf{R} - \mathbf{X}\|_\infty \le 1$.
- Step 6 Solve $\mathbf{X} = \mathbf{U}_o\mathbf{Q}\mathbf{Y}_o^{-1}$ for $\mathbf{Q}$

In our case, $\mathbf{T}_1$ is the transfer function $\mathbf{G}_{ref}$ from input to the primary variables (the top composition estimate change by boilup flow perturbation), and $\mathbf{T}_2$ is the transfer function $\mathbf{G}_x^T$ from input to the measurements (a matrix of 28×1 transfer functions of the temperature change by boilup flow perturbation).

6. DISCUSSION

Specifying $\mathbf{G}_{ref}$ for control purpose is not that easy. When the system is not only stabilizable but also controllable, one can make the closed-loop eigenvalues arbitrarily fast Antsaklis and Michel (1997). We need to know what is the fastest response we can get. One idea is to specify a first-order transfer function with the smallest time constant in the process as the desired transfer function from inputs to the estimates. From Skogestad and Morari (1987) we know that the internal time constant are smaller than the external time constants. These can be found from changing the two inputs boilup and reflux rate at the same time such that the external flows remain constant. This is very difficult to do in practice. The responses to internal flow changes while the external flows are constant are shown in Figures 11 and 12.

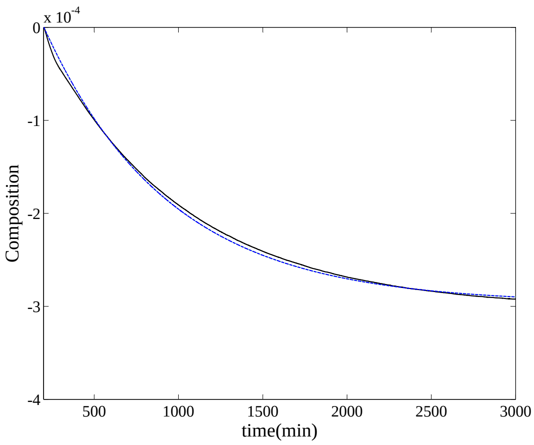

Fig. 11. Top composition, $\Delta L = \Delta V$, $\Delta D = \Delta B = 0$

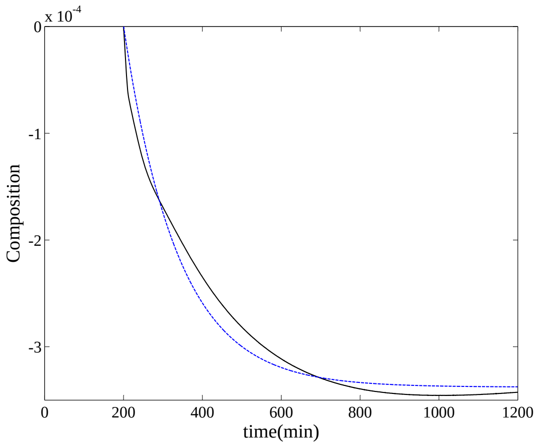

Fig. 12. Bot. composition, $\Delta L = \Delta V$, $\Delta D = \Delta B = 0$

The transfer functions of the compositions when the internal flows are changed so that the external flows remain constant are as below. The reason is that for high purity distillation, the product compositions are sensitive to changes in external flows and the effect of internal flows may not be seen since the gain is small.

$$\Delta y_D = (exp(-2s) \times \frac{-7.99e-5}{s+0.00135})\Delta V$$
$$\Delta x_B = (exp(-0.33s) \times \frac{-4.92e-4}{s+0.0073})\Delta V \qquad (18)$$

7. CONCLUSION

In this paper, we have discussed different methods to overcome the band-width limitations caused by combining different measurements with different dynamics to build the static estimators. We have shown that adding filters is the best option. By using filters, we will correct the dynamic behavior while keeping the optimal steady-state estimator untouched. We have done it with two approaches. First, we suggested filtering some of the measurements based on the insight from the process. Then, we use a more systematic way to construct a filter which is almost optimal.

ACKNOWLEDGEMENTS

We would like to thank Vinay Kariwala for the useful discussions.

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