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Dynamic online optimization of a house heating system in a fluctuating energy price scenario

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Abstract: We consider dynamic optimization of the energy consumption in a building with energy storage capabilities. The goal is to find optimal policies which minimize the cost of heating and respect operational constraints. A main complication in this problem is the time-varying nature of the main disturbances, which are the energy price and outdoor temperature. To find the optimal operable policies, we solve a moving horizon optimal control problem assuming known disturbances. Next, we proposed simple implementation based on feedback control, which gives near-optimal operation for a range of disturbances. The methods were successfully tested using simulation, which show that there is a great economical gain in using dynamic optimization for the case of variable energy price.

1. INTRODUCTION

Due to increasing energy consumption and prices and greater concerns about greenhouse gases emissions, more efficient electric power production and usage is sought. Recently, great attention has been given to renewable generation sources like windturbine and photovoltaic parks. Although efficiency-wise attractive, these alternative energy sources suffer a major drawback due to their sharply varying energy production caused by wide-ranging weather conditions. This is an important limitation since the energy production should cover the demand at any given time.

One possible approach to overcome this, is demand side load management where the large fluctuations in the load are tackled by peak shaving and by shifting load to more beneficial periods (Molderink et al., 2009). Field tests in the USA have demonstrated that optimization of domestic energy consumption with variables prices can significantly reduce load peaks (Hammerstrom, 2007). This can be achieved by manipulating the energy price according to demand information and weather forecasts. The dynamic energy pricing for demand load management is in itself a non-trivial problem, and it is currently an active research area. The interested reader is invited to check the references Mardavij Roozbehani and Mitter (2010) and Goudarzi et al. (2011) for more information. This problem is outside the scope of this work.

In such a scenario, the adaptation of the energy consumption by the final consumer is essential to the success of the approach. Thus, in this article we focus on the local building heating system optimization where the goal is the minimization of energy costs.

The case studied here consists of a single room comprised of a floor heating device, a radiator and a ventilation system with adjustable flow. We consider bounds on the floor temperature, the room temperature (air) and the CO_2 levels. The floor heat capacity is assumed to be large enough so that we can store a considerable amount of energy in it, hence, giving us an extra degree of freedom for optimization. Other hardware configurations could also have been employed. For example, one could use a insulated tank filled with water.

The main complicating factor for this problem is the time-varying nature of the disturbances in the outdoor temperature and energy price. We assume that predictions of the temperature and price variation are available, but they are not necessarily correct. Thus, a dynamic real time optimization (DRTO) scheme is proposed to compensate this variations while minimizing the energy cost. In this scheme, a dynamic optimization problem is solved at each sample time with new states and disturbance measurements.

A drawback of the DRTO is the fact that the system operates in open-loop in between two consecutive optimizations. This may yield sub-optimal or even infeasible solutions in case of large disturbances. To deal with this problem, we propose simple solutions solely based on feedback and offline analysis, where near-optimal control inputs are generated at low computational and maintenance costs. This extends the self-optimizing control idea (Skogestad, 2000) to dynamic optimization problems. We show that near-optimal solutions can be obtained by tracking *optimally invariant trajectories*, which we defined here as being the function of the measurements whose optimal profile does not change with disturbances.

The paper is organized as follows: Section 2 details the derivation of the dynamic. Section 3 shows the formulation of the dynamic optimization problem and describes the solution method used. In Section 4, the implementation of the optimal control solution is discussed and various com-

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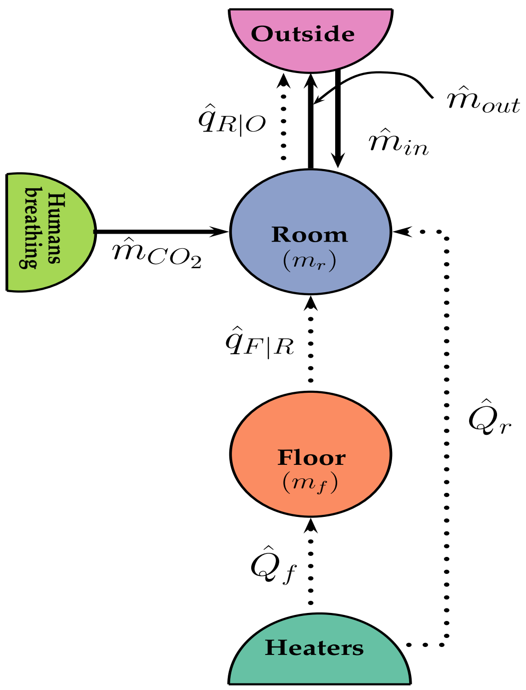

Fig. 1. The system topology

parative results presented. Section 5 gives the concluding remarks of the article.

2. MODELING

In this section, we develop a dynamic model based on energy and mass balances. The model describes a single $25m^2$ room comprised of a floor heating device, a radiator and a ventilation system with adjustable flow. It is assumed that all the heat lost by the floor is transferred to the air in the room whereas the heat in the air can be lost both through the walls and through the ventilation. The air entering is assumed to be at outdoor temperature and behaves as an ideal gas. The CO_2 accumulation due to breathing is modelled as a constant feed and the consumption of O_2 is neglected. To help visualizing the energy and mass flows in the system it is useful to use system topology graph as shown in Fig. 1. All state, manipulated and disturbance variables are described in Table 1. Other constant parameters are summarized in Table A.1.

Table 1. Variables description

State variables	Description	Unit
T_f	Floor temperature	K
T_r	Room temperature	K
m_r	Mass of air	kg
w	CO_2 mass fraction	-
Manipulated variables	**Description**	**Unit**
Q_f	Floor heat input	kW
Q_r	Room heat input	kW
$\hat{m}_{in}$	Air inflow	kg/s
Disturbance variables	**Description**	**Unit**
T_o	Outdoor temperature	K
p	Energy price	\$/kW

The energy balance for the floor is simply

$$\frac{dE_f}{dt} = Q_f - q_{f,r} \tag{1}$$

where the energy transfer to the room (air) $q_{f,r}$ is given by

$$q_{f,r} = UA_{f,r}(T_f - T_r). \tag{2}$$

Since the floor mass is constant we get

$$\frac{dT_f}{dt} = \frac{Q_f}{m_f c_{p,f}} - \frac{UA_{f,r}}{m_f c_{p,f}}(T_f - T_r) \tag{3}$$

The energy balance for the room is

$$\frac{dE_r}{dt} = Q_r + q_{f,r} + q_{o,r} + q_{wall} - q_{r,o} \tag{4}$$

The mass of air in the room is not constant, therefore we get

$$\frac{dE_r}{dt} = c_{p,r}T_r\frac{dm_r}{dt} + c_{p,r}m_r\frac{dT_r}{dt} \tag{5}$$

Using the mass balance we have

$$\frac{dm_r}{dt} = \hat{m}_{in} - \hat{m}_{out} \tag{6}$$

where

$$\hat{m}_{out} = k(P_r - P_o) \tag{7}$$

is the out flow and $P_r = \frac{m_r RT_r}{M_r V_r}$ is the pressure inside the room. Combining Eq. 6 and (5) with (4) and using

$$q_{o,r} = \hat{m}_{in}c_{p,r}T_o$$
$$q_{r,o} = \hat{m}_{out}c_{p,r}T_r$$
$$q_{wall} = UA_{r,o}(T_o - T_r)$$

we obtain

$$\frac{dT_r}{dt} = \frac{Q_r}{m_r c_{p,r}} + \frac{\hat{m}_{in}}{m_r}(T_o - T_r) \tag{8}$$
$$+ \frac{UA_{fr}}{m_r c_{p,r}}(T_f - T_r) + \frac{UA_{r,o}}{m_r c_{p,r}}(T_o - T_r)$$

Finally, the component mass balance of CO_2 is given by

$$\frac{d(wm_r)}{dt} = \hat{m}_{in}w_{in} - \hat{m}_{out}w + B \tag{9}$$

using the product rule for differentiation we have

$$\frac{d(wm_r)}{dt} = m_r\frac{dw}{dt} + w\frac{dm_r}{dt} \tag{10}$$

and using the total mass balance in Eq.(6) yields

$$\frac{dw}{dt} = \frac{\hat{m}_{in}}{m_r}(w_{in} - w) + \frac{B}{m_r} \tag{11}$$

For sake of simplicity in the notation, we define the control inputs $u^T = [Q_f, Q_r, \hat{m}_{in}]$, the state vector $x^T = [T_f, T_r, m_r, w]$ and the disturbances $d^T = [T_o, p]$. Hence, we can pack the dynamics into the vector function f such that $\frac{dx}{dt} = f(x, u, d)$. In the next section we describe how to use this model to find optimal heating polices.

3. DYNAMIC OPTIMIZATION

This section presents the dynamic optimization problem and the approach used to solve it. It starts off by presenting the continuous time optimal control problem we would like to solve and evolves in a stepwise manner presenting modifications that helps the solution. Finally, we present the full discretization method based on orthogonal collocation as well as the formulation of the nonlinear program. The implementation is discussed in the subsequent section.

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3.1 Problem definition

The optimization objective is to minimize the energy costs over an infinite horizon. A solution method is to use a moving horizon approach where we solve an optimal control problem within the fixed interval $[t_0, t_0+h]$ where the horizon h is large enough to capture important trends in the system. At each time point t_0 a different optimization problem (12) is solved with different initial condition x_0 that is unknown in advance. We formulate our moving horizon problem in the Lagrangian form as:

$$\min_u \int_{t_0}^{t_0+h} p(t)(Q_f + Q_r)\, dt \tag{12}$$

subject to

$$\dot{x} = f(x, u, d), \quad x(t_0) = x_0 \tag{13}$$
$$T_r \geq T_{min} \tag{14}$$
$$T_f \leq T_{max} \tag{15}$$
$$w \leq w_{max} \tag{16}$$
$$Q_f \leq Q_{max} \tag{17}$$
$$Q_r \leq Q_{max} \tag{18}$$
$$Q_f, Q_r \geq 0 \tag{19}$$

3.2 Singular optimal control problem

Define the Hamiltonian functional of the optimal control problem (12).

$$\mathcal{H} = L(x, u, d, t) + \lambda^T f(x, u, d) + \nu^T g_I(x, u) \tag{20}$$

where λ and ν are the multipliers, g_I denotes the set of inequality constraints $g_I \leq 0$ and $L(x, u, d, t) = p(t)(Q_f + Q_r)$. The necessary conditions of optimality for this problem include (Kirk, 1970):

$$\frac{\partial \mathcal{H}}{\partial u} = 0 \tag{21}$$

Its clear that, for this problem, u enters linearly in $\mathcal{H}$ and $\frac{\partial \mathcal{H}}{\partial u}$ does not depend explicitly on u. Hence, the necessary conditions of optimally do not determine directly the minimizing u and the control problem is singular. It is known that a singular optimal control problem may create troubles when direct numerical methods are used if accurate control profile is sought. It can be shown that the Hessian matrix becomes very ill-conditioned as the time step size decreases (Biegler, 2010). To avoid convergence problems, we modify the cost function by adding a quadratic term:

$$\min_u \int_{t_0}^{t_0+h} p(t)[\beta(Q_f + Q_r)^2 + (Q_f + Q_r)]dt \tag{22}$$

where the weighting factor β is adjusted such that the linear term dominates the expression. In a constant price scenario the two formulations are equivalent since it would be optimal to simply minimize the input usage.

3.3 Disturbance modelling

The main disturbances are the outdoor temperature $T_o(t)$ and energy price $p(t)$. For simplicity, we assume that $p(t)$ is periodic and follows

$$p(t) = p_0 + A_p \text{sign}[\sin(\omega_p t + \phi_p)] \tag{23}$$

where parameters the A_p and ϕ_p are uncertain. More general dynamic pricing polices can also be treated in this framework in a straightforward manner. We assume the weather predictions are available numerically from weather models such that we can interpolate the predictions using polynomials. Therefore, we assume we have the predictions $\hat{T}_o(t) = P(t)$ where P is a polynomial fitted using the weather model data. For this case study we have used weather prediction data from (yr.no, 2012). It would not be realistic to embed a weather forecast model in the optimization loop due to its highly complex nature.

3.4 Softening constraints

During operation is possible that a disturbance brings the system outside the feasible region. The formulation based on hard constrains (14)-(19) would then fail to produce a reasonable solution since the initial state would already be infeasible. This problem can be overcome by softening the output constraints (14)-(16). It would not make sense to soften the input constrains as they represent real physical limitations.

Firstly, we rewrite the output constraints in a vector form such that we have $h_o(x, u) \geq 0$. Next, we introduce a vector of slack variables ε and define the following constraints in the optimization problem:

$$h_o(x, u) \geq 0 - \varepsilon \tag{24}$$
$$\varepsilon \geq 0 \tag{25}$$

Finally, the cost function is modified by adding penalties for the violation of the constraints

$$\min_u \int_{t_0}^{t_0+h} \{p(t)[\beta(Q_f + Q_r)^2 + (Q_f + Q_r)] + \mu \cdot \varepsilon\}dt \tag{26}$$

The linear penalty function was chosen because it is exact in the sense that minimizing (26) also minimizes the original cost function (22) provided that μ is large enough (Nocedal and Wright, 2006).

3.5 Simultaneous approach

The dynamic optimization stated so far is infinite dimensioned and in order to solve it numerically, a discretization method is needed. We have decided to discretize the problem using orthogonal collocation methods. In this approach, both the states and manipulated variables profiles are approximated by orthogonal polynomials and their coefficients become the decision variables. The polynomial approximation of the states is required to respect the model equations only at the solution of the optimization problem. This formulation yields a large-scale sparse nonlinear program (NLP) and is known as the simultaneous approach (Biegler, 2010).

For simplicity, we first transform the problem to the Mayer form by expanding the state vector with $\dot{J} = p(t)[\beta(Q_f + Q_r)^2 + (Q_f + Q_r)] + \mu \cdot \varepsilon$ such that we have $z^T = [x, J]$ and $\dot{z} = \hat{f}(z, u, d)$. The equivalent dynamic optimization problem is

$$\min_u J(t_0 + h) \tag{27}$$

subject to the constraints (17)-(19) and the model $\dot{z} = \hat{f}(z, u, d)$.

Proceeding to the discretization, we first divide the time interval into N time periods. Within each time period i the control inputs are represented by Lagrange interpolation

$$u(t) = \sum_{j=1}^{K} \bar{l}_j(\tau) u_{ij} \tag{28}$$

where

$$\bar{l}_j(\tau) = \prod_{k=1,\neq j}^{K} \frac{\tau - \tau_k}{\tau_j - \tau_k} \tag{29}$$

The collocation equations for the differential equations can be written as

$$\sum_{j=0}^{K} \dot{l}_j(\tau_k) z_{ij} - h_i \hat{f}(u_{ik}, z_{ik}, d_{ik}) = 0 \tag{30}$$

where $i \in [1, \ldots, N]$, $k \in [1, \ldots, K]$, $\dot{l}_l(\tau) = \frac{dl_j}{d\tau}$ and K is the degree of the polynomials. The length of the time intervals h_i are considered fixed and are not decision variables for the optimization problem. In fact, for this case we have chosen $N = 1$ which leads to a *pseudospectral method*. This class of methods can give very accurate solutions for dynamic optimization problems with smooth profiles (Biegler, 2010). Finally, the collocation points τ_k are chosen as the roots of the Gauss-Legendre orthogonal polynomials. The resulting NLP is as follows:

$$\min J(t_0 + h) \tag{31}$$

$$\text{s.t.} \sum_{j=0}^{K} \dot{l}_j(\tau_k) z_j - h \hat{f}(u_k, z_k, d_k) = 0 \tag{32}$$

$$h_o(x_k, u_k, d_k) \geq -\varepsilon_k, \quad \varepsilon_k \geq 0 \tag{33}$$

$$k \in [1, \ldots, K] \tag{34}$$

The above problem is formulated in Matlab and solved using the sparse NLP solver SNOPT. This solver employs a sparse SQP algorithm with quasi-Newton approximations to the Hessian. Gradient information is obtained using automatic differentiation approach. The interface between Matlab and SNOPT is handled by the optimization environment TOMLAB.

4. IMPLEMENTATION APPROACHES

We propose the implementation of a dynamic real time optimization where the optimal control problem is solved in a moving horizon fashion. At each time sample, t_0, a dynamic optimization problem is solved with a new initial state and disturbance measurements. We specified a horizon length $h = 24$h so that all the important dynamics are captured. However, only the first portion of the optimal profile corresponding to $t \in [t_0 + t_s]$ is implemented, where $t_s < h$ is the time between successive optimizations. In this paper we assume limited computation power so that we need to have $t_s = 2$h. During this period the optimal inputs are extracted by using the Lagrange interpolation shown in (28).

In order to improve the accuracy of the solution and improve the convergence, the NLP is solved with successively larger number of collocation points, where the solution to the previous lower dimensioned problem is used as an initial guess for the next one. Here, we solve the NLP with $K = 25$ and then $K = 45$ collocation points. Another important point is the warm start of the NLP solver. This is done in two steps: first, the control inputs from previous solutions are shifted to to the next time window by assuming the inputs remain constant in the final time period. Then, the shifted inputs are used to simulate the model and the states are extracted. The shifted inputs and the simulated states are the initial guess to the next optimization problem. The overall algorithm is summarized as follows:

Algorithm 1 Simple moving horizon optimal control

```
Initialize: x_0, h, t_s, initial guess x_g and u_g
while t ≤ t_f do
    Solve the NLP (31)-(34)
    Implement solution for t ∈ [t_0, t_0 + t_s]
    Measure or estimate x(t_0 + t_s)
    Set x_0 ← x(t_0 + t_s)
    Shift previous solution u_g ← u_opt(t) with t ∈ [t_0 + t_s, t_0 + t_s + h]
    Use u_g to simulate the model from x_0 and obtain x_g
    Set t ← t_0 + t_s
end while
```

4.1 Nominal optimal solution

Assuming perfect predictions, the solution for a whole day obtained with Algorithm 1 is shown. Figure 2 depicts the nominal price variations, the outdoor temperature variation and the accumulated energy cost. This temperature profile corresponds to the temperature measured in Trondheim, Norway on 03 January 2012 provided by the Norwegian Meteorological Institute which made the data freely available in (yr.no, 2012).

For the sake of comparison, we also implemented the most trivial solution to the problem where the room temperature is kept at minimum allowed value by varying the heat input Q_r using a PI controller. To get a fair comparison, the optimal air inflow was used. The second heat input,Q_f, was left unused. Note that keeping the room temperature at minimum allowed value is, in fact, the optimal policy if we would like to minimize the energy consumption instead of the economical cost.

A comparison between the optimal profiles and the simple strategy is given in Figures 3 and 4. Some interesting conclusions can be drawn from this results. First, notice that it is optimal to overheat the room and floor above the minimum constraint when the price is low. In this case, when the energy is cheap we will store enough heat in order to meet the temperature constraints until the next low price valley. We also confirmed (not shown here for brevity) that the air inflow is increased just enough to meet the CO_2 level constraint. This is trivial since over-ventilation would unnecessarily cool the room down and it would require extra energy to keep the temperature constraint.

The optimal energy cost for one day was \$12.45, whereas the simple temperature controller gave a cost of \$21.62, which is considerably higher than the optimal. The energy usage is 12.5kWh and 10.9kWh, respectively. It is clear that this difference in the cost is proportional to the ratio between high and low energy price. Notice that, in a constant price scenario, the optimal is to keep the temperatures at the minimum allowed value.

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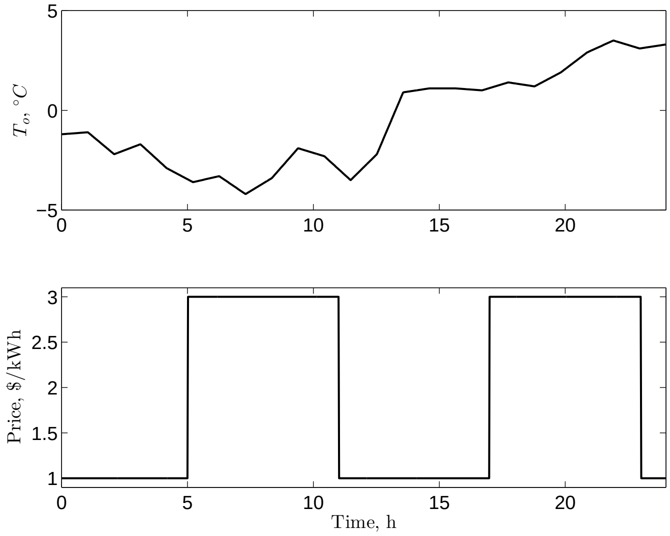

Fig. 2. Disturbances - energy price and outdoor temperature.

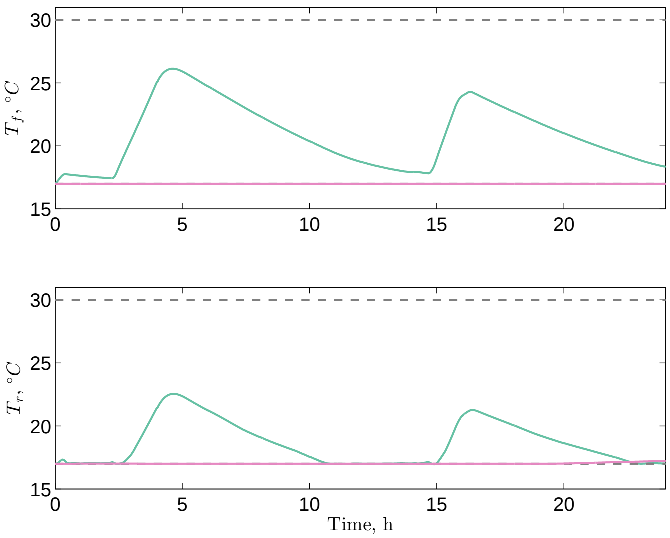

Fig. 3. Temperatures - green lines: optimal solution; magenta lines: simple temperature controller with constant setpoint.

4.2 Near-optimal solution by tracking optimally invariant trajectories

In this section, we propose a simple control implementation that gives near-optimal solutions without the need for re-optimization online. The main idea is to find a function of the measurements whose trajectory is optimally invariant to disturbances and then track the trajectory using standard feedback controllers. The structure is shown in Fig. 5 where $c_r(t)$ is the optimally invariant reference trajectory that we wish to track. In the sequel, we will derive a procedure to obtain such trajectories.

We define $y \in \mathcal{R}^{n_y}$ as the vector of known variables (measurements), which may include states, disturbances and control inputs. The disturbance model of price and outdoor temperature is parametrized by a vector of constants d_0. However, the *real* (unknown) parameters are denoted by d, and we may have deviations $\Delta d = d - d_0$.

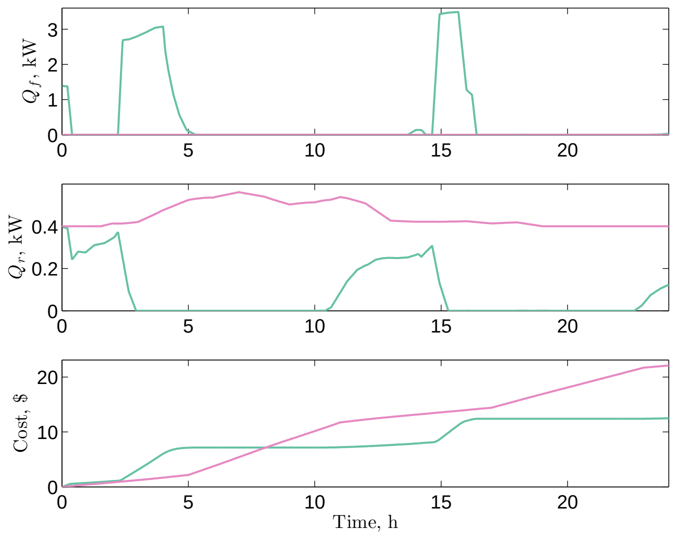

Fig. 4. Inputs and energy cost - green lines: optimal solution; magenta lines: simple temperature controller with constant setpoint.

The nominal optimal measurement trajectory is referred to as $y_0(t, d_0)$.

It can be shown that if the cost function J is twice continuously differentiable in a neighbourhood of the nominal solution and the linear independence constraint qualifications and the sufficient second-order conditions hold, then the optimal sensitivity matrix F is well defined:

$$F(t) = \frac{\partial y_{opt}(t, d)}{\partial d} \tag{35}$$

and, a first order, local approximation of the optimal solution in the neighbourhood can be obtained from

$$y_{opt}(t, d) \approx y_0(t, d_0) + F(t)\Delta d \tag{36}$$

Here, we are after a function of measurements $c(y(t), d)$ whose optimal value is independent of d, i.e., we want $c_{opt}(y(t), d) = c_0(y(t), d_0)$ for any d sufficiently small. A simple choice is a linear combination of the measurements:

$$c(t) \equiv H(t)y(t) \tag{37}$$

where $H(t)$ is a $n_u \times n_y$ matrix, and $c(t)$ is a $n_u \times 1$ vector. This way we can write

$$c_{opt}(t, d) = H(t)[y_0(t, d_0) + F(t)\Delta d] \tag{38}$$

and we define the nominal combination of measurements:

$$c_0(t, d_0) = H(t)y_0(t, d_0) \tag{39}$$

By subtracting (39) from (38) we obtain:

$$c_{opt}(t, d) - c_0(t, d_0) = H(t)F(t)\Delta d \tag{40}$$

Therefore, the optimal combination $c_{opt}(t, d)$ equals the nominal $c_0(t, d_0)$ for any d if we select $H(t)$ such that $H(t)F(t) = 0$. This is always true if $H(t)$ lies in the left null space of $F(t)$. Using this approach we obtain a trajectory $c_{opt}(t, d)$ that is optimally invariant due to disturbance. We can transform the problem of implementing $u(t)$ in a 'open-loop' manner to a reference tracking problem with optimal setpoints $c_r(t,d) = c_{opt}(t,d)$ (see Fig. 5). By tracking c_r, a simple controller automatically generates inputs u that are optimal for any disturbance d sufficiently small and thus, the online optimization is avoided.

The whole procedure has offline and online steps which are summarized as follows:

Offline:

- Solve the dynamic optimization problem with d_0;
- Select appropriate measurements y;
- Compute the optimal sensitivities $F(t)$ and the combination $H(t)$;
- Compute the reference trajectories $c_r(t) = H(t)y_0(t)$.

Online:

- Track the reference c_r by a feedback controller.

Remark: It is only possible to choose H in the left null space of F if the number of independent measurements respect the condition $n_y \geq n_u + n_d$ where n_d and n_u are the number of disturbances and inputs, respectively. See (Alstad and Skogestad, 2007) for proof.

Here, we assume the air inflow q_{in} will remain at nominal trajectory such that two manipulated variables are available. Thus, since we are considering two disturbances we will need at least $n_y = 2 + 2 = 4$ measurements and we seek two trajectories $c_1(t)$ and $c_2(t)$ to track. Defining the measurement vector $y = [T_f, T_r, m_r, p]^T$ we compute the optimal sensitivities $F(t)$ for the whole horizon and obtain $H(t)$ and the reference trajectory $c_r(t)$. As controllers, we use two decentralized P controllers. Note that the only way to adapt to price changes is by measuring it explicitly as the model of the physical process does not depend on price explicitly.

This idea was tested by considering a disturbance in the phase shift (ϕ_p) of the energy price as well as a mismatch between prediction and actual outdoor temperatures. Figure 6 compares the predictions with the measured disturbance values. We compare the proposed method with the moving horizon strategy given in Algorithm 1 and with the true optimal solution assuming perfect knowledge of the disturbances. Figures 7 and 8 depict the output and input trajectories for the three different cases. The economical comparison is shown in bottom Fig. 8. The proposed simple method works surprisingly well for this case, given a relative loss of optimality of only 0.3175%. The relative loss given by the moving horizon strategy with imperfect disturbance model was 24.4%, which is considerably higher.

One of the reasons for the success of the method is the fact that, in this range of disturbances, the dynamics are close to linear and, therefore, the linear approximation of the NLP ends up near the true solution. A drawback of this approach is that it cannot explicitly handle constraints. Therefore, for a realistic implementation the proposed method should be combined with a periodic solution of the dynamic optimization where a new reference solution is obtained, and new invariant trajectories $c(t)$ are computed. The idea is to recompute the optimal sensitivities $F(t)$ online after solving the current NLP and then apply the approach shown in Fig 5 in between two successive

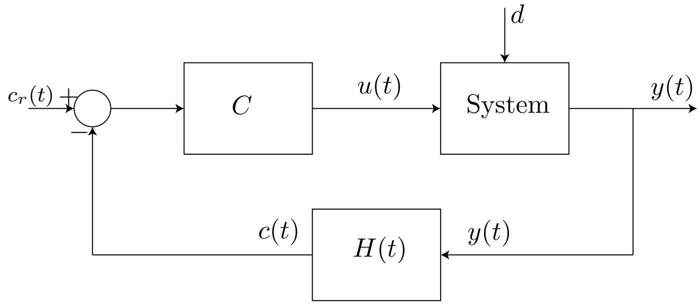

Fig. 5. Proposed implementation based on simple feedback

optimizations. This requires, however, fast online calculations of the sensitivities as those provided by the methods proposed by (H. Pirnay and Biegler., 2012). Similar idea has been published in (Würth et al., 2009) where the authors proposed to use sensitivity based neighbouring-extremal updates combined with real-time optimization. In this way, the frequency of optimizations can be greatly reduced.

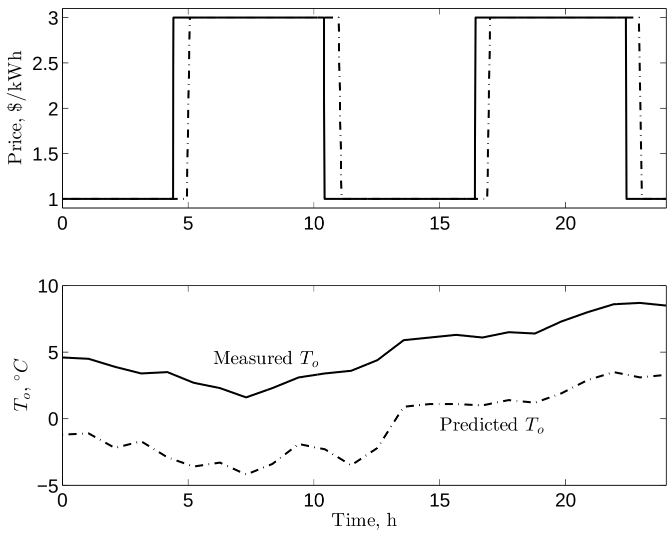

Fig. 6. Disturbances - solid lines: measured; dash-dotted lines: predicted

5. CONCLUSION

In this paper various solutions to the optimal heating of a room problem have been proposed. We proposed a moving horizon dynamic optimization method, which uses predictions to compute the optimal heating polices and ensure feasibility. We showed that, in a scenario where the energy price is time varying, the economical benefit of using a real time dynamic optimization scheme is substantial. Finally, simple solutions based on feedback control and offline was derived and successfully tested. The simulation exampled showed that very little loss of optimality could be obtained for relatively small disturbances. The benefit of this method is the negligible online computational cost and the simplicity of the implementation. The ideas discussed here could also be applied to any other problem with energy storage capabilities where the energy price changes, such as the dynamic optimization of supermarket refrigeration systems.

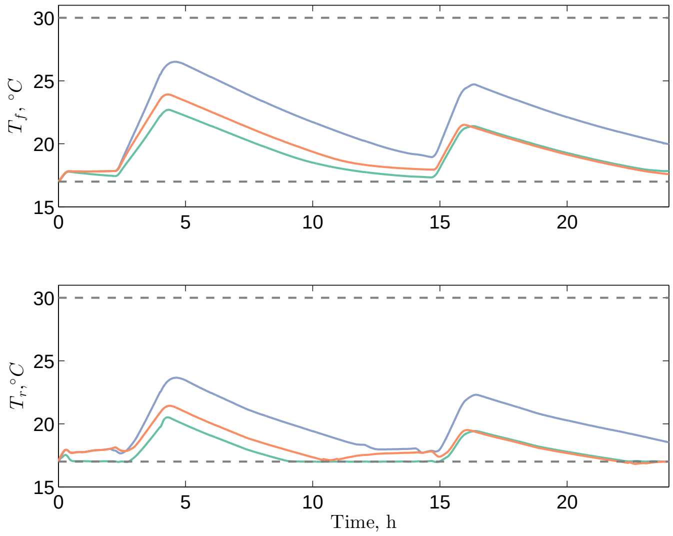

Fig. 7. Temperatures - blue lines: Algorithm 1 with imperfect predictions; orange lines: proposed implementation as shown in Fig. 5; green lines: optimal solution

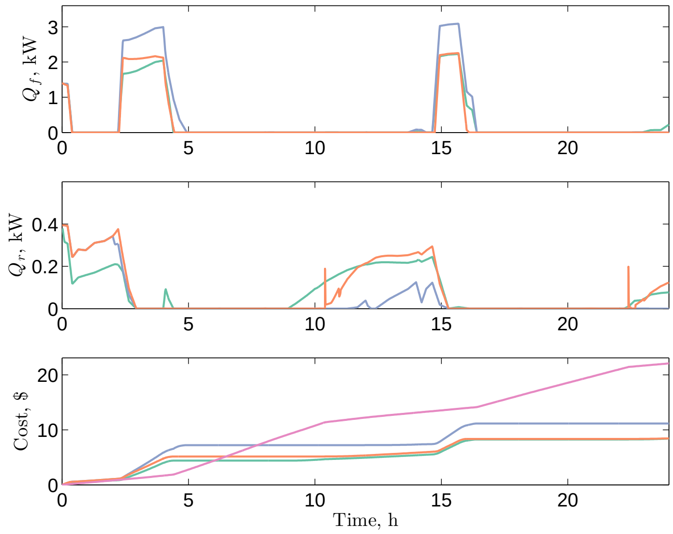

Fig. 8. Inputs and economical comparison - blue lines: Algorithm 1 with imperfect predictions; orange lines: proposed implementation as shown in Fig. 5; green lines: optimal solution; magenta line: cost of a simple temperature tracking controller

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Appendix A. MODEL PARAMETERS

Table A.1. Parameters description

Parameter	Description	Value	Unit
$UA_{f,r}$	Heat transfer coefficient floor	0.1801	$\mathrm{kJ/(s\cdot K)}$
$UA_{r,o}$	Heat transfer coefficient walls	0.0216	$\mathrm{kJ/(s\cdot K)}$
m_f	Mass of the floor	3000	Kg
$c_{p,f}$	Heat capacity of the floor	0.63	kg/kJ
$c_{p,r}$	Heat capacity of the air	1.005	kg/kJ
k	Valve constant	100	$\mathrm{kg/(bar\cdot s)}$
w_{in}	CO_2 fraction in flow	$6.16\cdot10^{-4}$	-
B	CO_2 generated by breathing	$9.02\cdot10^{-6}$	kg/s

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