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Quantitative methods for optimal regulatory layer selection

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Abstract: Controlled variables selection based on economic objectives using self optimizing concepts are developed. In this paper, we extend the self optimizing control ideas to find optimal controlled variables in the regulatory layer. The regulatory layer is designed to facilitate stable operation, to regulate and to keep the operation in the linear operating range and its performance is here quantified using the state drift criterion. Quantitative methods are proposed and evaluated on a distillation column case study with 41 stages that minimize state drift in composition states to obtain optimal regulatory layer with $1, 2$ and more closed loops.

1. INTRODUCTION

The plantwide control system for the overall plant is in most cases organized in a hierarchical structure (Fig. 1), based on the time scale separation between the layers. As

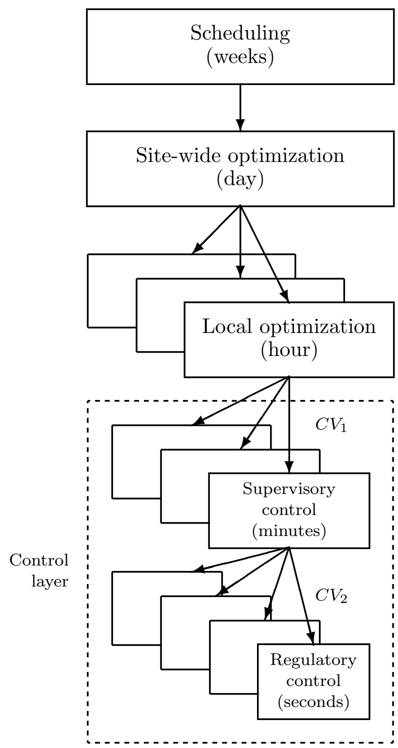

Fig. 1. Control system hierarchy for plantwide control in chemical plants Skogestad and Postlethwaite [1996]

shown in Fig. 1, the control layer is usually divided in two parts. The main task of the upper slower "supervisory" layer is to keep the "economic" or primary controlled variables CV_1 close to their economic optimal set points.

$$J_1 = \|CV_1 - CV_{1s}\|^2 \tag{1}$$

On the other hand, the task of the lower faster "regulatory" layer is to avoid the process drifts too far away from its desired steady state. More specifically, it should stabilize any unstable modes, provide for local (fast) disturbance rejection and keep the operation in the linear operating range.

In this paper, we focus on the selection of controlled variables CV_2 for regulatory control layer, and we will quantify the regulatory objectives in terms of a scalar function J_2, which is the weighted state drift away from the desired nominal point,

$$J_2 = \|\mathbf{Wx}\|^2 \tag{2}$$

where $\mathbf{W}$ is a weighting matrix, $\mathbf{x}$ is the drift in the states from nominal operating point. Many norms may be used, but we will consider the 2-norm of the state drift. The question is then what should we control (CV_2) in order to minimize J_2.

Another objective is that the regulatory layer should be "simple", and we will quantify this by the number of loops that need to be closed, that is, by the number of physical degrees of freedom (usually values) that are used by the regulatory layer. This can be related to the idea of partial control [Shinnar, 1981, Arbel et al., 1996, Kothare et al., 2000]. In partial control system, a few inputs are kept constant and the remaining inputs are used to control the process.

One may question the division of the control layer into a supervisory (economic) and regulatory (stabilizing) layer, but this paradigm is widely used and is the basis for this paper. The main justification is that the two tasks of stabilization and optimal operation are fundamentally different and that trying to do both at the same time is much more complex. This is captured, for example, by the common idiom "you need to learn to walk before you can run". Specifically, "learn to walk" means that the process (the child) first needs to be stabilized before attempting more high-level tasks like running.

This paper focuses on controlled variables (CV_2) selection in the regulatory layer. Another important decision in regulatory layer is the pairing of selected outputs (CV_2) with available inputs (valves). However, we do not address this issue by making a simplifying assumption of perfect control of selected controlled variables (CV_2).

Generally, the regulatory layer decisions are taken based on heuristic methods or on the intuition of process engineers (e.g.Luyben [1996] and references therein). These heuristic based methods may not be optimal and are also difficult to compare various proposals. Hence, systematic and good methods are needed to arrive at optimal regulatory layer that minimizes the state drift in the presence of disturbances. To arrive at controlled variables (CV) that are easy to control, various methods have been proposed for desirable control system controllability, achievable performance as criteria [van de Wal and de Jager, 2001]. But these methods are not directly applicable for controlled variables selection in regulatory layer that minimizes the state drift.

The rest of the paper is organized as follows: Section 2 describes the problem for optimal regulatory layer selection. Section 3 extends the SOC concepts to state drift. Section 4 discusses the regulatory layer selection. Section 5 presents evaluation on a distillation column case study with 41 stages to find optimal regulatory layer. The conclusions are given in Section 6.

2. MINIMIZATION OF STATE DRIFT (PROBLEM DEFINITION)

In this section, we give a brief overview of the problem.
Classification of variables:
$\mathbf{u}$: Set of n_u independent variables (inputs). It does not really matter what these variables are as long as they form an independent set e.g. one may close loops and instead introduce the set points as the variables $\mathbf{u}$. In the frequency domain there is no causality and closing loops will not change the problem.
$\mathbf{u}_0$: Set of physical degrees of freedom (inputs) which we may want to keep constant in the regulatory layer (often $\mathbf{u}_0 = \mathbf{u}$, but this is not a requirement).
$\mathbf{y}_m$: Set of (additional) measurements
$\mathbf{y} = [\mathbf{y}_m \quad \mathbf{u}_0]$: Combined set of measurements and physical inputs that we consider as choice for CV_2.
$CV_2 = \mathbf{c} = \mathbf{H}\mathbf{y}$: Selected set of $n_c = n_u$ controlled variables in the regulatory layer
$\mathbf{H}$ is here assumed to be a constant real matrix (not frequency dependent).

Linear model is assumed at a nominal operating point and the models at each frequency ω are described as

$$\mathbf{x} = \mathbf{G}^x(j\omega)\mathbf{u} + \mathbf{G}_d^x(j\omega)\mathbf{W}_d\mathbf{d} \tag{3a}$$

$$\mathbf{y} = \mathbf{G}^y(j\omega)\mathbf{u} + \mathbf{G}_d^y(j\omega)\mathbf{W}_d\mathbf{d} + \mathbf{W}_n\mathbf{n}^y \tag{3b}$$

where $\mathbf{G}^x(j\omega)$,$\mathbf{G}_d^x(j\omega)$ and $\mathbf{G}^y(j\omega)$,$\mathbf{G}_d^y(j\omega)$ are the frequency based gain functions between $\mathbf{x}$ to $\mathbf{u}$, $\mathbf{x}$ to $\mathbf{d}$ and $\mathbf{y}$ to $\mathbf{u}$, $\mathbf{y}$ to $\mathbf{d}$, respectively. $\mathbf{W}_d$ and $\mathbf{W}_n$ are the magnitudes of disturbances and implementation errors.

Note that the controlled variables can be either individual measurements or combinations of measurements.

$$\mathbf{c} = \mathbf{H}\mathbf{y} \tag{4}$$

where $\mathbf{c}$ are controlled variables, $\mathbf{y}$ are candidate measurements and $\mathbf{H}$ is a real valued measurements combination matrix. We include inputs $\mathbf{u}_0$ also as candidate measurements.

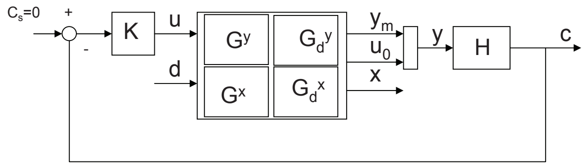

Fig. 2. General approach

The objective is to find what to control ($\mathbf{c} = \mathbf{H}\mathbf{y}$) in the stabilizing layer, given that we want to minimize the state drift (2) for the expected disturbances ($\mathbf{d}$) and implementation error (measurement noise, $\mathbf{n}^y$), and that we want to close k loops, $\forall k = 1, 2, \ldots, n_u$.

In order to avoid the need to design controller for each loop in the regulatory layer we assume that the selected $\mathbf{c}$ is perfectly controlled, i.e. $\mathbf{c} = 0$ as a function of frequency. In frequency domain, we may assume perfect control, but closed loop stability needs to be addressed separately.

Problem: In the frequency domain, the problem can be stated as follows: Assuming perfect control of the selected $\mathbf{c} = \mathbf{H}\mathbf{y}$ at a given frequency i.e. $\mathbf{c}(j\omega) = 0$ (Fig. 2), we want to find optimal $\mathbf{H}$ that minimizes the state drift $J_2(j\omega)$ when there are disturbances. In our case, minimizing the loss $L = J_2(\mathbf{c}, \mathbf{d}) - J_{2,opt}(\mathbf{d})$ on an average basis, e.g. using the frobenius norm, is the same as minimizing the cost J_2. Hence, we use the self-optimizing control concepts [Skogestad, 2000] and we consider minimization of loss rather than cost as the loss can be formulated as convex optimization problem in $\mathbf{H}$ [Alstad et al., 2009].

$$\begin{aligned} \mathbf{H} &= [\mathbf{H}_y \quad \mathbf{H}_u] \\ \mathbf{c} = \mathbf{H}\mathbf{y} &= \mathbf{H}_y\mathbf{y}_m + \mathbf{H}_u\mathbf{u}_0 \end{aligned} \tag{5}$$

We want to find the best controlled variables for each of the possibilities for closing loops
Close 0 loops: in the set $\mathbf{c}$, select n_c variables from the set $\mathbf{u}_0$ ($\mathbf{H}_y = 0, \mathbf{H}_u = I$)
Close 1 loop: in the set $\mathbf{c}$, select $n_c - 1$ variables from the set $\mathbf{u}_0$ (one row in $\mathbf{H}_y$ is nonzero, the rest are zero)
Close 2 loops: in the set $\mathbf{c}$, select $n_c - 2$ variables from the set $\mathbf{u}_0$ (two rows in $\mathbf{H}_y$ are nonzero, the rest are zero)
Close k loops: in the set $\mathbf{c}$, select $n_u - k$ variables from the set $\mathbf{u}_0$
Close n_c loops: in the set $\mathbf{c}$, select 0 variables from the set $\mathbf{u}_0$

In addition, we can have restrictions on the set $\mathbf{c}$ such as selecting only single measurements (each row in $\mathbf{H}$ containing one 1 and the rest 0's).

In terms of finding the optimal $\mathbf{H}$ we consider two approaches.
1. General approach: To avoid numerical problems with poles on the jw-axis (including integrators), one may introduce P-controllers to shift (stabilize) these modes, but then the corresponding set points (e.g, level set points) should replace the corresponding MV in the set $\mathbf{u}$ (Fig. 3). In Fig. 3, general approach $\mathbf{u} = \{\mathbf{u}_1 \cup \mathbf{u}_2 \cup L_s\}$.
2. Steady-state approach. To reduce the dimension of $\mathbf{u}$ (and of $\mathbf{H}$), we remove variables with no steady-state effect, which in most cases are liquid levels. In practice, this may be done by choosing an independent base set for

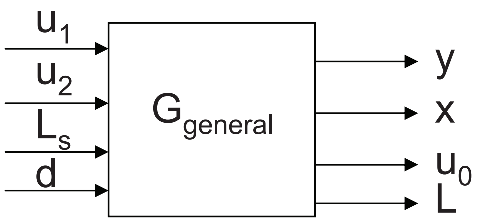

Fig. 3. General approach

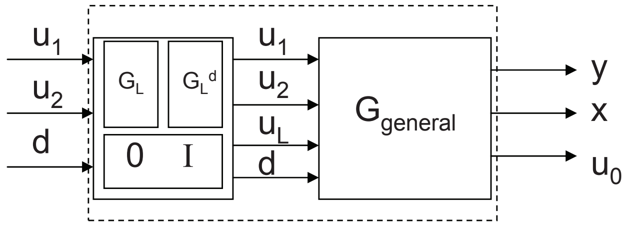

Fig. 4. Steady state approach

$\mathbf{u} = \{\mathbf{u}_1 \cup \mathbf{u}_2\}$, and then using the steady-state model to obtain the linearized effect of $\mathbf{u}$ and the $\mathbf{d}$ on the flows $\mathbf{u}_L$ (to be included in $\mathbf{u}_0$) which are used for level control,

$$\mathbf{u}_L = \mathbf{G}_L \mathbf{u} + \mathbf{G}_L^d \mathbf{d} \tag{6}$$

This approach reduces the available steady state degrees of freedom to n_u.

3. STATE DRIFT FOR A GIVEN FREQUENCY

By the term "state drift" we refer to the dynamic drift which is not taken care of by the actions of the slower supervisory control layer. Since disturbances often are sinusoidal with varying frequency it may be useful to consider the state drift as a function of frequency

$$J_2(w) = \|\mathbf{W}(j\omega)\mathbf{x}(j\omega)\|^2 \tag{7}$$

We have here included the possibility for a frequency-dependent weight matrix $\mathbf{W}(j\omega)$, but in most cases we will assume it is constant diagonal matrix.

3.1 Previous work

Skogestad and Postlethwaite [1996] use minimization of weighted state drift from nominal point at steady state and suggest controlled variables, $\mathbf{c} = \mathbf{H}\mathbf{y}$, with $\mathbf{H} = (\mathbf{W}\mathbf{G}^x)^T [\mathbf{W}\mathbf{G}^x \quad \mathbf{W}\mathbf{G}_d^x] [\mathbf{G}^y \quad \mathbf{G}_d^y]^\dagger$, where $\dagger$ represents pseudo inverse of the matrix, when the number of measurements $n_y \geq n_u + n_d$.

3.2 Loss as a function of $\mathbf{d}$,$\mathbf{n}^y$ *and control policy* $\mathbf{H}$

We extend the methods of self-optimizing control [Skogestad, 2000, Kariwala et al., 2008] for state drift criterion J_2 as a functional of $\mathbf{u}$ and $\mathbf{d}$ for a given frequency, where $J_2 = \|\mathbf{W}\mathbf{x}\|^2$, $\mathbf{x}$ is the deviation of states from the desired operating point, $\mathbf{W}$ is the square diagonal state weighting matrix. The selection of appropriate weighting matrix $\mathbf{W}$ allows the user to easily study the state drift in certain states only.

The linear model (3) of the process around the nominal operating point for the states and output as a function of frequency w is assumed. For a given frequency the gain matrices from $\mathbf{u}$ to $\mathbf{x}$, $\mathbf{d}$ to $\mathbf{x}$ are $\mathbf{G}^x, \mathbf{G}_d^x$ and $\mathbf{u}$ to $\mathbf{y}$, $\mathbf{d}$ to $\mathbf{y}$ are $\mathbf{G}^y, \mathbf{G}_d^y$, respectively. Then the second order derivatives can easily be found analytically based on the linear model obtained at the nominal operating point.

$$\begin{aligned} \mathbf{J}_{2_{uu}} &\triangleq \frac{\partial^2 J_2}{\partial \mathbf{u}^2} = 2\mathbf{G}^{xT}\mathbf{W}^T\mathbf{W}\mathbf{G}^x \\ \mathbf{J}_{2_{ud}} &\triangleq \frac{\partial^2 J_2}{\partial \mathbf{u}\partial \mathbf{d}} = 2\mathbf{G}^{xT}\mathbf{W}^T\mathbf{W}\mathbf{G}_d^x \end{aligned} \tag{8}$$

For any disturbance $\mathbf{d}$, the input must be changed to $\mathbf{u}_{\text{opt}}(\mathbf{d})$ to have optimal state drift. Any input $\mathbf{u}$ that is different from $\mathbf{u}_{\text{opt}}(\mathbf{d})$ will result in the loss in state drift (i.e. the deviation from the optimum state drift), that can be defined as

$$L = J_2(\mathbf{u}, \mathbf{d}) - J_{2,opt}(\mathbf{u}_{\text{opt}}(\mathbf{d}), \mathbf{d}) \tag{9}$$

The loss with self-optimizing control by keeping (4) at constant set point is denoted by L_c. The worst case loss, average loss $L_{c,wc}, L_{c,av}$ in terms of state drift are obtained by considering various realizations of $\mathbf{d} \in \mathscr{D}$ and $\mathbf{n}^c \in \mathscr{E}$.

$$L_{c,wc}(\mathbf{d}, \mathbf{n}^c) = \max_{\mathbf{d}\in\mathscr{D}, \mathbf{n}^c\in\mathscr{E}} L_c(\mathbf{d}, \mathbf{n}^c) \tag{10}$$

$$L_{c,av}(\mathbf{d}, \mathbf{n}^c) = \mathbb{E}\left[L_c(\mathbf{d}, \mathbf{n}^c)\right] \quad \forall \mathbf{d} \in \mathscr{D}, \forall \mathbf{n}^c \in \mathscr{E} \tag{11}$$

Subsequently the worst case loss in state drift L_{wc} for the bounded set $\| \begin{bmatrix} \mathbf{d}' \\ \mathbf{n}^{y'} \end{bmatrix} \| \leq 1$ [Halvorsen et al., 2003], average loss in state drift L_{av} for normally distributed random variables $\mathbf{d}', \mathbf{n}^{y'}$ with unit variance in the local methods are obtained [Kariwala et al., 2008] as

$$L_{c,wc} = \frac{1}{2}\|\mathbf{J}_{2_{uu}}^{1/2}(\mathbf{H}\mathbf{G}^y)^{-1}\mathbf{H}\mathbf{Y}_x\|_2^2 \tag{12}$$

$$L_{c,av} = \mathbb{E}(L) = \frac{1}{2}\|\mathbf{J}_{2_{uu}}^{1/2}(\mathbf{H}\mathbf{G}^y)^{-1}\mathbf{H}\mathbf{Y}_x\|_F^2 \tag{13}$$

where $\mathbf{Y}_x = [\mathbf{F}_x\mathbf{W}_d \quad \mathbf{W}_n]$;$\mathbf{F}_x = \frac{\partial \mathbf{y}_{\text{opt}}}{\partial \mathbf{d}} = \mathbf{G}^y J_{2_{uu}}^{-1} J_{2_{ud}} - \mathbf{G}_d^y$, $\mathbf{J}_{2_{uu}} \triangleq \frac{\partial^2 J_2}{\partial \mathbf{u}^2}$, $\mathbf{J}_{2_{ud}} \triangleq \frac{\partial^2 J_2}{\partial \mathbf{u}\partial \mathbf{d}}$. We use only the average loss in state drift (13) involving the frobenius norm (F).

4. REGULATORY LAYER SELECTION

The regulatory layer with $1, 2$ and more closed loops can be viewed as partially controlled systems and optimal regulatory layer selection as the selection of the best partially controlled system with $1, 2, \ldots, n_u$ closed loops that minimize the state drift loss in (13). For each partial controlled system we solve (13) with an MIQP to find best CVs [Yelchuru et al., 2010]. Based on the loss from the partially controlled systems and the acceptable steady state drift loss defined by the user, the minimum regulatory layer can be obtained.

4.1 Optimal $\mathbf{H}$ *with* CV_2 *as individual measurements*

The optimal $\mathbf{H}$ with CV_2 as individual measurements, e.g. $\mathbf{H} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$, gives an MIQP that requires us to solve a convex QP at each node. For a process with i closed loops require to solve (13) as an MIQP [Yelchuru et al., 2010] that gives a globally optimal $\mathbf{H}$ to obtain i CV_2. Overall we need to solve $n_u + 1$ MIQP problems to find optimal regulatory layer with $0, 1, \ldots, n_u$ closed loops. The computational requirement of these methods increase as the number of MIQP problems increase with n_u, but these are tractable as these are offline methods.

4.2 Optimal $\mathbf{H}$ with CV_2 as measurement combination

We consider partial control problem where we allow for measurement combination for the controlled variables CV_2. This can be viewed as solving (13) with a particular structure in $\mathbf{H}$, which is generally a non-convex problem. For example, a partially controlled system with 3 process measurements and 2 inputs, resulting in 5 candidate measurements in $\mathbf{y}$, is

$$\mathbf{H} = \begin{bmatrix} h_{11} & h_{12} & h_{13} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \text{ or } \mathbf{H} = \begin{bmatrix} h_{11} & h_{12} & h_{13} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \quad (14)$$

To solve the (13) with this particular structure (14), we propose a two step approach which may not be optimal but which is convex. The first step is to partition the system inputs to 2 sets $\mathbf{u}_1$ and $\mathbf{u}_2$ ($\mathbf{u} = \{\mathbf{u}_1 \cup \mathbf{u}_2\}$) as we keep the inputs in input set $\mathbf{u}_2 \in \mathbf{u}_0$ as constants. The matrix for such a partial control system $\mathbf{G}^{y,partial} \in \mathbb{R}^{n_y \times n_{u_1}}$ is obtained by picking the columns associated to input set $\mathbf{u}_1$ and $\mathbf{J}_{uu,x}^{partial} \in \mathbb{R}^{n_{u_1} \times n_{u_1}}$,$\mathbf{J}_{ud,x}^{partial} \in \mathbb{R}^{n_{u_1} \times n_d}$ has elements associated to the inputs in the input set $\mathbf{u}_1$. The disturbance gain matrix $\mathbf{G}_d^y \in \mathbb{R}^{n_y \times n_d}$, disturbance magnitude matrix $\mathbf{W}_d \in \mathbb{R}^{n_d \times n_d}$ and measurement noise magnitude matrix $\mathbf{W}_n \in \mathbb{R}^{n_y \times n_y}$ will remain the same. The second step is to solve (13) with the matrices obtained in the first step as a convex optimization problem [Yelchuru et al., 2010] to obtain $\mathbf{H}^{partial}$ as a full matrix for the partially controlled system.

As $\mathbf{u}_2 \in \mathbf{u}_0$ varies in each partial controlled system, we cannot directly compare losses obtained from different partial control systems. Hence, in order to compare the losses on an equivalent basis, the loss value is calculated for the full system with the optimal controlled variables $CV_2^{partial}$ obtained for the partially controlled system and the constant inputs in $\mathbf{u}_2$ as the other CV_2. The number of partial controlled systems and the number of MIQP problems that need to be solved are $2^{n_u} - 2$. Thus based on the acceptable steady state drift loss defined by the user, the minimum regulatory layer with CV_2 as combination of n measurements can be obtained.

5. DISTILLATION COLUMN CASE STUDY

The proposed methods are evaluated on binary distillation column case study with 41 stages in LV-configuration [Skogestad, 1997, Hori and Skogestad, 2008], where distillate flow (D) and bottoms flow (B) are used to control the integrating hold ups and reflux (L) and boil up (V) are the remaining steady-state degrees of freedom ($\mathbf{u}$) (Fig. 5). The main disturbances are in feed flow rate (F), feed composition (z_F) and vapor fraction (q_F), which can vary between $1 \pm 0.2, 0.5 \pm 0.1$ and 1 ± 0.1, respectively. As the composition measurements assumed not to be available online, we use the tray temperatures to control the compositions indirectly. The boiling points difference between light key component (L) and heavy key component (H) is 13.5 ^oC. Then temperature $T_i(^oC)$ on each stage i is calculated as a linear function of the liquid composition x_i on each stage [Skogestad, 1997]

$$T_i = 0x_i + 13.5(1 - x_i) \quad (15)$$

To be consistent with the notation $\mathbf{u}_1 = L, \mathbf{u}_2 = V$ and $\mathbf{u}_L = \{D, B\}$. As we have integrating modes (condenser and reboiler levels), we use steady state approach with

$$\mathbf{u}_0 = \begin{pmatrix} L \\ V \\ D \\ B \end{pmatrix}, \quad \mathbf{u} = \begin{pmatrix} L \\ V \end{pmatrix}$$

Note that we have steady state degrees of freedom $n_u = 2$. For this distillation column in LV-configuration [Häggblom and Waller, 1990], the base variables, $\mathbf{u} = \begin{pmatrix} L \\ V \end{pmatrix}$ and $\mathbf{u}_L = \begin{pmatrix} D \\ B \end{pmatrix}$ are used for level control and we need to obtain from mass balances for the steady-state flow relationships (6) with $\mathbf{u}$ and $\mathbf{d} = \begin{pmatrix} F \\ z_F \\ q_F \end{pmatrix}$. Mass balances assuming perfect level control

$$\frac{dM_d}{dt} = 0 = V_{top} - L - D$$
$$\frac{dM_b}{dt} = 0 = L_{btm} - V - B$$

Note: These two equations can now be used to eliminate D and B from the original $\mathbf{u}_0 = \begin{pmatrix} L \\ V \\ D \\ B \end{pmatrix}$, to get the reduced $\mathbf{u} = \begin{pmatrix} L \\ V \end{pmatrix}$. Here, at steady state and assuming constant molar flows

$$V_{top} = V + (1 - q_F)F$$
$$L_{btm} = L + q_F F$$

So we find at steady state

$$D = V - L + (1 - q_F)F$$
$$B = L + q_F F - V$$

Note that there is no effect of z_F in this case. We linearize (6) to get $\mathbf{G}_L$ and $\mathbf{G}_L^d$ as

$$\mathbf{G}_L = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}, \quad \mathbf{G}_L^d = \begin{bmatrix} (1 - q_F)^* & 0 & -F^* \\ q_F^* & 0 & F^* \end{bmatrix}$$

At a steady state operating point $L = 2.706$, $V = 3.206$, $F = 1$, $z_F = 0.5$, $q_F = 1$, $x_D = 0.99$, $x_B = 0.01$ based on steady state conservation laws

$$\mathbf{G}_L = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}, \quad \mathbf{G}_L^d = \begin{bmatrix} 0 & 0 & -1 \\ 1 & 0 & 1 \end{bmatrix}$$

The 41 stage temperatures and the manipulated input flows $\{L, V, D, B\}$ are taken as candidate measurements. The implementation error for temperatures is $\pm 0.5^oC$ and it is $\pm 10\%$ for the flows. The total state drift J_T for the compositions on each of the 41 trays is

$$J_2 = \|\mathbf{W}\mathbf{x}\|^2 \quad (16)$$

The $\mathbf{W} \in \mathbb{I}^{41 \times 41}$ (identity matrix) to have equal weights on compositions on each tray. where $\mathbf{x}$ denote the drift in composition states from nominal operating point.

To find the frequency where the state drift has a peak, we consider linear models around a nominal operating point

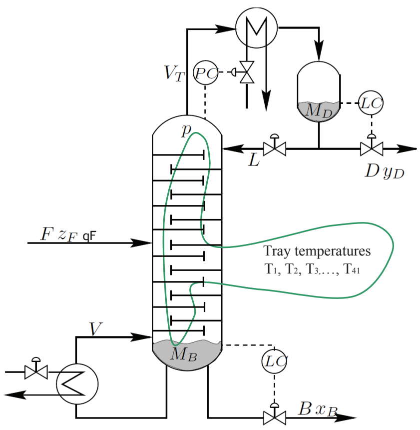

Fig. 5. Distillation column using LV-configuration

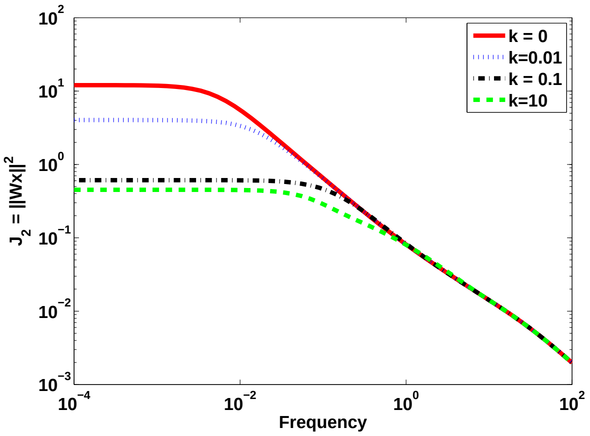

Fig. 6. State drift J_2 for various frequencies with 1 temperature loop closed with varying proportional control gains (in addition to level control with proportional control ($k_L = 10$)

for the effect of $\mathbf{u}$ and $\mathbf{d}$ on the states $\mathbf{x}$ and measurements $\mathbf{y}$ (3). For this case, we have $n_u = 2$ inputs and 2×2 system. In the presence of disturbances ($\mathbf{d}$), the state drift (i.e. $J_2(j\omega) = \|\mathbf{Wx}(j\omega\|^2)$ is derived as functions of $\mathbf{H}$ and $\mathbf{d}$ with their associated gains at each frequency. We assume proportional controllers for level control with $k_L = 10$, disturbances as normally distributed random variables and only single loop control for a chosen $\mathbf{H}$ with boilup (V) for varying proportional gains for various frequencies is shown in Fig. 6. In Fig. 6, proportional controller gain $k = 0$ correspond to an open loop policy and $k = 10$ correspond to perfect control. From Fig. 6, it is clear that for a chosen $\mathbf{H}$ the state drift and the loss (not shown) are almost constant over a frequency band [0.0001 0.02] for a single loop control. Hence performing a steady state analysis for state drift alone is sufficient and we focus only on the loss from optimal state drift (13) for the steady state for this case study.

The distillation column case study has $n_u = 2$ number of steady state degrees of freedom with $n_y = 45$ number of candidate measurements, there are totally $n_u + 1 = 3$ MIQPs to be solved to find CV_2 as individual measurements. The loss with 0 loops closed, 1 loop closed (other input is kept constant), 2 loops closed are tabulated in Table 1. From Table 1, best 0 loops closed is to keep

Table 1. Distillation Column case study: The self optimizing variables $\mathbf{c}'s$ as individual measurements for the partially controlled systems

No. of loops closed†	$\mathbf{c}'s$	Loss
0	$[V \quad B]$	109.6695
1	$[T_{18} \quad L]$	0.1884
2	$[T_{15} \quad T_{27}]$	0.0265

† In addition to two closed level loops

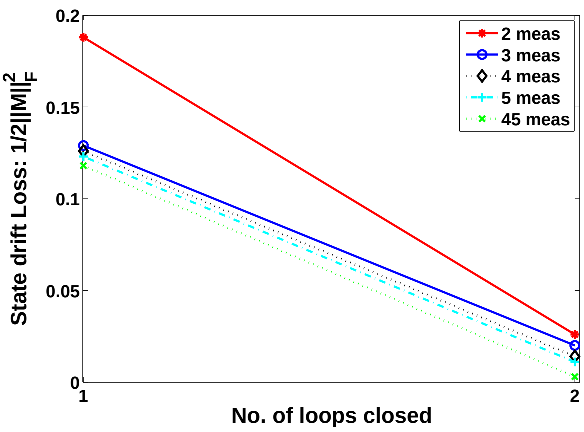

Fig. 7. Distillation case study: The optimal total state drift vs number of loops closed (in addition to level control)

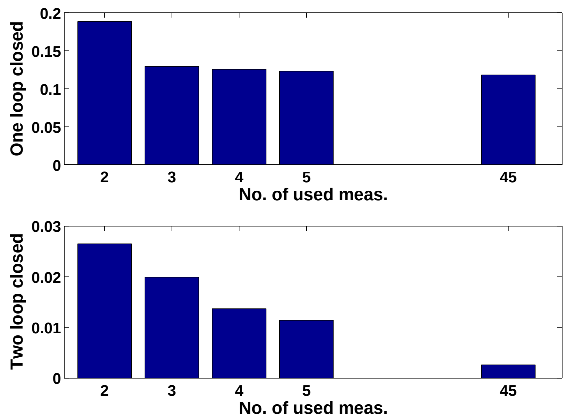

Fig. 8. Distillation case study: The reduction in loss in total state drift vs number of used measurements, Top: Loss with one loop closed, Bottom : Loss with two loops closed

$\{V, B\}$ constant, best single closed loop policy is to keep L constant and to control tray temperature T_{18}, best 2 loops closed policy is to control the tray temperatures T_{15} and T_{27} to minimize the compositions state drift. The loss reduction ratio by closing 1 loop and 2 loops increased at least by 2.7 and 3.6 orders of magnitude, respectively.

To find optimal $\mathbf{H}$ with CV_2 as measurement combination we used partial control ideas. The number of partial controlled systems and the number of MIQP problems that need to be solved for this case study with $n_u = 2$ are $2^{n_u} - 2 = 2$. The optimal CV_2 as measurement combination for the partially controlled systems with $3, 4, 5$ and 45

Table 2. Distillation Column case study: The self optimizing variables $\mathbf{c}'s$ as combinations of $3, 4, 5, 45$ measurements with their associated losses (total state drift) for the partially controlled systems

No. of loops closed †	No. of meas. used	Optimal meas.	$\mathbf{c}'s$	Loss
1	3	$[T_{15} \quad T_{26} \quad L]$	$\mathbf{c}_1 = L$ $\mathbf{c}_2 = 1.072T_{15} + T_{26}$	0.1294
2	3	$[T_{15} \quad T_{26} \quad T_{28}]$	$\mathbf{c}_1 = T_{15} - 0.1352T_{28}$ $\mathbf{c}_2 = T_{26} + 1.0008T_{28}$	0.0198
1	4	$[T_{15} \quad T_{16} \quad T_{27} \quad L]$	$\mathbf{c}_1 = L$ $\mathbf{c}_2 = 0.6441T_{15} + 0.6803T_{16} + T_{27}$	0.1256
2	4	$[T_{14} \quad T_{16} \quad T_{26} \quad T_{28}]$	$\mathbf{c}_1 = T_{14} - 6.1395T_{26} - 6.3356T_{28}$ $\mathbf{c}_2 = T_{16} + 6.2462T_{26} + 6.2744T_{28}$	0.0137
1	5	$[T_{15} \quad T_{16} \quad T_{26} \quad T_{27} \quad L]$	$\mathbf{c}_1 = L$ $\mathbf{c}_2 = 1.1926T_{15} + 1.1522T_{16} + 0.9836T_{26} + T_{27}$	0.1231
2	5	$[T_{14} \quad T_{16} \quad T_{26} \quad T_{27} \quad T_{28}]$	$\mathbf{c}_1 = T_{14} - 4.9975T_{26} - 5.0717T_{27} - 4.9813T_{28}$ $\mathbf{c}_2 = T_{16} + 5.1013T_{26} + 5.0847T_{27} + 4.9166T_{28}$	0.0114
1	45	$[T_1, T_2, \ldots, T_{41}, \quad L, V, D, B]$	$\mathbf{c}_1 = L$ $\mathbf{c}_2 = f(T_1, T_2, \ldots, T_{41}, L, V, D, B)$	0.1181
2	45	$[T_1, T_2, \ldots, T_{41}, \quad L, V, D, B]$	$\mathbf{c}_1 = f(T_1, T_2, \ldots, T_{41}, L, V, D, B)$ $\mathbf{c}_2 = f(T_1, T_2, \ldots, T_{41}, L, V, D, B)$	0.0026

† In addition to two closed level loops

measurements while closing $0, 1, 2$ loops are shown in Fig. 7 and are also tabulated in Table 2. From Fig. 7, using more number of measurements reduce the loss in state drift. From Fig. 7, the state drift loss ratio increase as we use more number of measurements to obtain optimal CV_2. The reduction of loss with number of measurements, when one loop, two loops are closed is shown as a bar chart in Fig. 8. From Table 2, the best single loop control CV_2 with 3 measurements is to combine $1.072T_{15} + T_{26}$ and L constant. Thus based on the acceptable steady state drift loss from the optimal state drift defined by the user for the considered disturbances, the minimum regulatory layer can be obtained.

6. CONCLUSIONS

The self optimizing control concepts are extended to select optimal controlled variables that minimize the state drift in the presence of disturbances. We chose a frequency that has peak in state drift and we minimized the state drift for that frequency. The reduction in loss from optimal state drift by closing the loops in the presence of disturbances is quantified. We used self-optimizing control minimum loss method to minimize the state drift to arrive at regulatory layer with $1, 2$ and more closed loops. We described on how to use the proposed methods to find both optimal individual and combinations of measurements as controlled variables and are evaluated on a distillation column case study with 41 stages.

REFERENCES

V. Alstad, S. Skogestad, and E. Hori. Optimal measurement combinations as controlled variables. *Journal of Process Control*, 19(1):138–148, 2009.

Arnon Arbel, Irven H. Rinard, and Reuel Shinnar. Dynamics and control of fluidized catalytic crackers. 3. designing the control system: Choice of manipulated and measured variables for partial control. *Industrial & Engineering Chemistry Research*, 35(7):2215–2233, 1996. doi: 10.1021/ie9507080.

Kurt E. Häggblom and Kurt V. Waller. Control structures for disturbance rejection and decoupling of distillation. *AIChE Journal*, 36(7):1107–1113, 1990. ISSN 1547-5905. doi: 10.1002/aic.690360717.

Ivar J. Halvorsen, Sigurd Skogestad, John C. Morud, and Vidar Alstad. Optimal selection of controlled variables. *Ind. Eng. Chem. Res.*, 42, 2003.

Eduardo Shigueo Hori and Sigurd Skogestad. Selection of controlled variables: Maximum gain rule and combination of measurements. *Industrial & Engineering Chemistry Research*, 47(23):9465–9471, 2008. doi: 10.1021/ie0711978.

Vinay Kariwala, Yi Cao, and S. Janardhanan. Local self-optimizing control with average loss minimization. *Ind. Eng. Chem. Res.*, 47:1150–1158, 2008.

Mayuresh V. Kothare, Reuel Shinnar, Irven Rinard, and Manfred Morari. On defining the partial control problem: Concepts and examples. *AIChE Journal*, 46(12):2456–2474, 2000. ISSN 1547-5905. doi: 10.1002/aic.690461213.

W. L. Luyben. Simple regulatory control of the eastman process. *Industrial & Engineering Chemistry Research*, 35(10):3280–3289, 1996.

Reuel Shinnar. Chemical reactor modelling for purposes of controller design. *Chemical Engineering Communications*, 9(1-6):73–99, 1981. doi: 10.1080/00986448108911016.

Sigurd Skogestad. Dynamics and control of distillation columns: A tutorial introduction. *Chemical Engineering Research and Design*, 75(6):539 – 562, 1997. ISSN 0263-8762. Distillation.

Sigurd Skogestad. Plantwide control: The search for the self-optimizing control structure. *Journal of Process Control*, 10:487–507, 2000.

Sigurd Skogestad and Ian Postlethwaite. *Multivariable Feedback Control, 1. Edition*. Wiley, 1996.

M.M.J. van de Wal and A.G. de Jager. A review of methods for input/output selection. *Automatica*, 37(4): 487–510, 2001.

Ramprasad Yelchuru, Sigurd Skogestad, and Henrik Manum. MIQP formulation for controlled variable selection in self optimizing control. *DYCOPS, July 7-9, Brussels*, pages 61–66, 2010.