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Sensitivity of Optimal Operation of an Activated Sludge Process Model

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***Abstract*—This paper describes a systematic sensitivity analysis of optimal operation conducted on an activated sludge process model based on the test-bed Benchmark Simulation Model No. 1 (BSM1). The objective is to search for an operational structure that leads to optimal economic operation, while promptly rejecting disturbances at lower layers in the control hierarchy avoiding thus violation of the more important regulation constraints on effluent discharge. We start by optimizing a steady-state nonlinear model of the process. The resulting active constraints must be chosen as economic controlled variables. These are the effluent ammonia from the bioreactors and the final effluent total suspended solids at their respective upper limits, as well as the internal recycle flow rate at its lower bound. The remaining degrees of freedom need to be fulfilled, and we use several local (linear) sensitivity methods to find a set of unconstrained controlled variables that minimizes the loss between actual and optimal operation; particularly we choose to control linear combinations of readily available measurements so to minimize the effect of disturbances and implementation errors.**

I. Introduction

Much to the authors' surprise, optimization of wastewater treatment plants has not received much attention in the WWTP research community given the small number of contributions found in the literature. Only few articles discuss the subject from a heuristic economic point of view[1], [2], [3] to formal optimization using an explicit mathematical model of the process[4], [5], [6] for optimal design and operation. However, none of the publications define an optimal operation policy from a systematic prism. Araujo *et al.*[7] applied a systematic procedure for control structure design of an activated sludge process in which optimization for various operational conditions were carried out on a mathematical model of the process.

In this communication a systematic sensitivity analysis of optimal operation of an activated sludge process model based on the Benchmark Simulation Model No. 1 (BSM1) [8] is conducted. It must be clear that all analysis, and hence all conclusions, from this work are based on the underlying mathematical model of the process, and should not be considered as definite guidelines for actual plant operation since the mathematical model may not be able to reproduce many operational situations. However, the results can be used in practice as general rules-of-thumb to be tested in actual wastewater treatment plants of the kind discussed here.

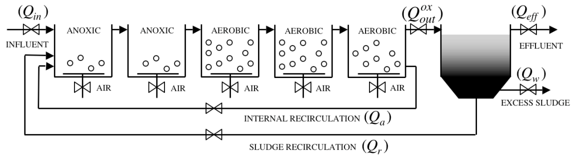

Fig. 1. Schematic representation of the BSM1 activated sludge process.

II. Process description

The BSM1 [8] represents a fully defined protocol that characterizes the process including the plant layout, influent loads, modeling and test procedures as well as evaluation criteria. Figure 1 shows a schematic of the process consisting of a bioreaction section divided into five compartments, which can be anoxic or aerobic, and a secondary settling device. In order to maintain the microbiological population, sludge from the settler is re-circulated into the reaction section (returned activated sludge, Q_r). Also, part of the mixed liquor leaving the last reactor can be recycled to the inlet of the bioreactor (internal recycle, Q_a) to enhance nitrogen removal. Moreover, excess sludge at a rate Q_w is continuously withdrawn from the settler underflow.

From a modeling point of view, the original BSM1 is based on two widespread accepted process models: the celebrated Activated Sludge Model No.1 (ASM1) [9] used to model the biological process, and a non-reactive Takacs one dimensional layer model for the settling process [10], [11]. The full model equations as well as kinetic and stoichiometric parameters are given within the benchmark description [8]. In addition, influent data are provided in terms of flow rates and ASM1 state variables over a period of 14 days with 15 minutes sampling time.

Each reactor is modeled as a perfectly-mixed, constant-volume tank within which complex biological reactions give

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rise to component mass balance equations, generating a system of (coupled) ordinary differential equations. The ASM1 is a well establish and reliable model widely used among WWT modelers, and further discussion on its known capabilities of reproducing with fidelity the behavior of the reaction section of an activated sludge process can be found in the vast wastewater literature.

However, the same cannot be said about the settler model, though, for the reason that these units display very complex mechanisms that are not still fully understood [12]. Nevertheless, much progress has been made towards building a physically sound model for the secondary clarifier based on the theory of partial differential equation applied to conservation law with discontinuous fluxes [13], [14], [15], [16]. While these more meaningfully grounded mathematical models satisfying fundamental physical properties [17] still have not found widespread application in the WWT field, it is a commonplace to resort to approximate models of the settler, and the one due to Takacs [10], [11] is the most widely used representation of the secondary settler in published studies and commercial software environments. Some authors [18], [19], [20], however, pointed out many setbacks related to this model, among which is the fact that the number of discretization layers is not in agreement with numerical convergence and without distinguishing model formulation and numerical solution, but instead it is used solely as a model parameter in order to match experimental observations [21]. Numerical simulations have showed [17] the failure of Takacs' model to represent the complex behavior of secondary settlers under certain conditions, and this has led researchers to switch to more reliable physically meaningful sedimentation models. One such development is described by Diehl [13] who formulated and analyzed dynamically the settler model based on the one-dimensional scalar mass conservation law (1)

$$\frac{\partial X(z,t)}{\partial t} + \frac{\partial}{\partial z}(F(X(z,t),z)) = s(t)\delta(z) \tag{1}$$

where X is the flocculated solids concentration, δ is the Dirac measure, s is a source, and F is a flux function, which is discontinuous at three points in the space coordinate z, namely at the inlet and the two outlets. Details are fully given in the cited references. We here are interested in the sensitivity of the static optimum of the settler coupled with the biological reaction section, and the steady-state solutions of the above equation provide the basis for our analysis.

A simple analysis of the model described in [15], shows that the steady-state model of the settler that must hold for optimization purposes is given by (2)

$$\begin{aligned} X_e &= \frac{s - f(X_M)}{q_e} \\ X_u &= \frac{f(X_M)}{q_u} \\ X_M &= M(q_u) \\ X_f &\in (X_m, X_M) \end{aligned} \tag{2}$$

where X_e and X_u are the solids concentration in the effluent and in the wastage streams, respectively; s is the feed flux given by $s = q_f X_f$, where q_f is the feed flux and X_f is the solids concentration in the feed; $f(X)$ is a flux function given by $f(X) = Xv_s(X) + q_u X$, where $v_s(X)$ is the settling velocity law given by the double exponential equation [10]; X_M is a minimizer of $f(X)$; X_m is a value strictly less than X_M satisfying $f(X_m) = f(X_M)$; q_e and q_u are the effluent and wastage fluxes, respectively; M is a function that computes the local minimizer of $f(X_M)$. In addition, we can also calculate the steady-state concentration of suspended solids in the clarification (X_{cl}) and thickening zones (X_{th}) as in (3) [15]

$$\begin{aligned} g(X_{cl}) + s &= f(X_M) \\ X_{th} &= X_M \end{aligned} \tag{3}$$

where $g(X_{cl}) = X_{cl}v_s(X_{cl}) - q_e X_{cl}$.

Note that, although in this communication a nonreactive settler is considered, we here follow [14] and treat the dissolved oxygen in the settler in a special way. We assume that the oxygen is consumed within the settler and, consequently, the oxygen concentration at the settler's outlets is set to zero, which is indeed a realist assumption. This results in a more conservative computation of the oxygen demand in the reaction section.

III. Systematic sensitivity analysis methodology

The methodology is mainly based on the first 4 steps, known as "top-down analysis", of the more general procedure described in [22], where economic variable selection is the key issue. The analysis conducted is of local nature, i.e., we use linearized models of the process to develop the methodology.

In this communication we use optimal measurement combinations [23] for unconstrained variable selection, i.e. the ones left after choosing the active constraints as "primary" economic variables. The basic idea is to select combinations c of the measurements y such that $c = Hy$, where H is a (static) selection matrix. To determine H, two approaches are developed based on a linearized model of the process and a second-order Taylor series expansion of the cost function used for optimization; two sources of uncertainty are assumed which are represented by (1) external disturbances (d) and (2) implementation (measurement) errors (n). The first of two approaches combines these uncertainties in one single scaled vector to minimize the worst case economic loss (L), defined as the difference between actual operation (with a given control structure in place) and operation under optimal control. The second approach is to first minimize the loss with respect to external disturbances, and then, if there are still available measurements, minimize the loss with respect to implementation (measurement) errors.

The steps to be followed are:

1. Define operational objectives: We first quantify the operational objectives in terms of a scalar cost function (here denoted J) as given in (4) that should be minimized

$$J = \textit{cost of feed} + \textit{cost of utilities (energy)} - \textit{revenue from valuable products} \quad (4)$$

Constraints can then be added to the process as inequality equations of the form $g \leq 0$.

2. Determine the steady-state optimal operation: Using a steady-state model of the process, identify degrees of freedom and expected disturbances, and perform optimizations to assess sensitivity for the expected disturbances.

Usually, the economics of the plant are primarily determined by the (pseudo) steady-state behavior [24], so the steady-state degrees of freedom (u_0) are usually the same as the economic degrees of freedom. Which variables to include in the set u_0 is immaterial, as long as they make up an independent set. The important disturbances (d) and their expected range for future operation must then be identified. These are generally related to feed rate and feed composition, as well as external variables such as temperature and pressure of the surroundings. We should also include as disturbances possible changes in specifications and active constraints (such as product specifications or capacity constraints) and changes in parameters (such as equilibrium constants, rate constants and efficiencies). Finally, we need to include as disturbances the expected changes in prices of products, feeds and energy.

In order to achieve near optimal operation without the need to re-optimize the process when disturbances occur, one needs to minimize the loss in (5)

$$L = J_0(c, d) - J_0(c^{opt}(d), d) \geq 0 \quad (5)$$

where $J_0(c, d)$ is the value of the cost for a chosen set of constant setpoint variables c that fulfill all remaining degrees of freedom and $J_0(c^{opt}(d), d)$ is the value of the cost after re-optimization. Clearly, the loss in (5) depends on the objective function as well as on the measurements through c, since c is a function of the available y. We then need to learn about the sensitivity to disturbances not only of the cost function, but also of the measurements.

At last, the steady-state optimization problem can be formulated as in (6)

$$\min_{u_0} J_0(x, u_0, d) \quad (6)$$

subject to

Model equations: $f(x, u_0, d) = 0$

Operational constraints: $g(x, u_0, d) \leq 0$

where x are internal variables (states). In $f(x, u_0, d) = 0$ possible operational equality constraints (like a given feed flow) is also included. The main objective is to determine the optimal nominal operating condition to be used in the variable selection step.

3. Select "economic" (primary) controlled variables: In this step, the issue is the implementation of the optimal operation point found in the previous step in a robust and, most importantly, simple manner. We need to identify as many economic controlled variables (c) as there are economic degrees of freedom (u_0). For economic optimal operation, active constraints must be selected [25], which in turn consumes part of the degrees of freedom (u'). For the remaining degrees of freedom u (with $n_u = n_{u_0} - n_{u'}$), we select variables for which close-to-optimal operation is achieved with constant nominal setpoints, even when there are disturbances [26]. Because our considerations in this communication are of local nature, we assume that the set of active constraints does not change with changing disturbances, and we consider the problem in reduced space in terms of the remaining unconstrained degrees of freedom u, which can be expressed as in (7) [27]

$$\min_{u} J_0(x, u, d) \quad (7)$$

subject to

Model equations: $f(x, u, d) = 0$

Active constraints: $g_{active}(x, u, d) = 0$

where we consider as active constraints a subset $g_{active}(x, u, d)$ of $g(x, u_0, d)$ for which optimal values are always at bounds for all disturbances. By eliminating the states using the equality constraints in (7), the unconstrained optimization problem can be expressed simply as in (8)

$$\min_{u} J(u, d) \quad (8)$$

Ensuring active constraint operation consumes part of the degrees of freedom for optimization. The remaining degrees of freedom need to be fulfilled, and we select variables such that when kept at optimal setpoints leads to near optimal economic operation despite of disturbances, i.e., the deviation (loss L in (5)) from re-optimization as a function of disturbances should be small. The optimal setpoints of c are then determined from the optimization at the nominal operating point. This is the celebrated "self-optimizing" control technology [26]. It can be shown that the loss in (5) can be expressed in its worst case form (L_{wc}) as in (9) [23]

$$L_{wc} = \max_{\left\| \begin{bmatrix} d' \\ n^{y\prime} \end{bmatrix} \right\|_2 \leq 1} L = \frac{1}{2}\bar{\sigma}^2(M) \quad (9)$$

where d' and $n^{y\prime}$ are the scaled disturbance and measurement error variables related by $d = W_d d'$ and $n^y = W_{n^y} n^{y\prime}$ (W_d and W_{n^y} are scaling matrices), and $M = -M_n H \tilde{F}$, where $\tilde{F} = [F W_d W_{n^y}]$ being $F = \frac{\partial y^{opt}}{\partial d}$ the optimal measurement (y^{opt}) sensitivity with respect to the disturbances and M_n any nonsingular $n_u \times n_u$ matrix. In other words, we need to find H that minimizes $\bar{\sigma}(M)$, i.e., $H = arg \min_H \bar{\sigma}(M)$.There are basically two approaches to solve for this minimization problem. The first one solves the minimization at once by combining disturbances and measurement errors, and an explicit formula for H is given by (10) [23]

$$H^T = (\tilde{F}\tilde{F}^T)^{-1}G^y(G^{yT}(\tilde{F}\tilde{F}^T)^{-1}G^y)^{-1}J_{uu}^{1/2} \quad (10)$$

where G^y is the static model of the process from the unconstrained inputs u to the measurements and $J_{uu} = \left(\frac{\partial^2 J}{\partial u^2}\right)_{u^{opt}}$ is the Hessian of J with respect to u evaluated at u^{opt} (u^{opt} is the optimal value of the manipulated variables).

The second approach, called the extended nullspace method [23], solves the problem by first minimizing the loss with respect to disturbances, and then, if there are still enough measurements left, minimize the loss with respect to measurement errors. It can be shown that the explicit expression for H in this case is given in (11)

$$H = M_n^{-1}\tilde{J}(W_{n^y}^{-1}\tilde{G^y})^{\dagger}W_{n^y}^{-1} \quad (11)$$

where $\tilde{J} = [J_{uu}^{1/2} \quad J_{uu}^{1/2}J_{uu}^{-1}J_{ud}]$ ($J_{ud} = \frac{\partial^2 J}{\partial u \partial d}$) and $\tilde{G^y} = [G^y \quad G_d^y]$ (G_d^y is the static model from disturbances d to y).

There are four cases where (11) can be applied:

3a. "Just-enough" measurements, i.e., $n_y = n_u + n_d$. Here, the expression for H becomes (12)

$$H = M_n^{-1}\tilde{J}(\tilde{G^y})^{-1} \quad (12)$$

which is the same as having H in the left null space of F, i.e., $H \in N(F^T)$.

3b. Extra measurements (select just enough measurements), i.e., $n_y > n_u + n_d$, and we want to select a subset of the measurements y such that $n_y = n_u + n_d$. The solution is to find such a subset that maximizes $\underline{\sigma}(\tilde{G^y})$ using, e.g. existing efficient branch-and-bound algorithms [28]. The resulting $\tilde{G^y}$ is then used to compute H in (12).

3c. Extra measurements (use all available measurements), i.e., $n_y > n_u + n_d$. H is calculated using (11), where $\dagger$ denotes the left inverse, calculated as $A^{\dagger} = (A^TA)^{-1}A^T$ for any given matrix A.

3d. "Too few" measurements, i.e., $n_y < n_u + n_d$. In this case, the optimal H in (11) is not affected by the noise weight and therefore becomes

$$H = M_n^{-1}\tilde{J}(\tilde{G^y})^{\dagger} \quad (13)$$

where $\dagger$ denotes the right inverse, calculated as $A^{\dagger} = A^T(AA^T)^{-1}$.

The above procedure boils down to selecting suitable candidate measurements, i.e. identify n_y vis-a-vis $n_u + n_d$, and find that linear combination (matrix H) of all, or a given subset, which results in the smallest loss among all possible solutions. One big hurdle to be surmounted is the numerical calculation of J_{uu} and J_{ud}. For some ill-posed problems, it may become an intractable task, and one solution is to compute F numerically instead, since $F = \frac{dy^{opt}}{dd}$. Particularly, the extended nullspace general formula (11) can, after some matrix algebra, be reformulated as in (14)

$$H = M_s(G^y)^{\dagger}[G^y \quad (G_d^y - F)](W_{n^y}^{-1}\tilde{G^y})^{\dagger}W_{n^y}^{-1} \quad (14)$$

where $M_s = (J_{uu}^{-1/2}M_n)$ can be any non-singular $n_u \times n_u$ matrix. In this case, we could select $M_n = J_{uu}^{1/2}$ so that (10) and (11) are independent of Hessian information.

IV. Sensitivity analysis results

A. Step 1. Operational objectives

The operational costs in a wastewater treatment plant depend on the wastewater system itself and can be divided into manpower, energy, maintenance, chemicals usage, chemical sludge treatment, and disposal costs. However, in this work, the objective is to reduce the cost of energy and sludge disposal as much as possible. Therefore, the following costs are considered:

- Required pumping energy (E_P expressed in kWh/d);
- Required aeration energy (E_A expressed in kWh/d);
- Required mixing energy when the aeration is too low (E_M expressed in kWh/d);
- Sludge disposal (C_D expressed in \$ $/d$).

The mathematical expressions for all these quantities can be found in [8], and by assuming a constant energy price of $k_E = 0.09$ \$ $/kWh$ and a sludge disposal price of $k_D = 80$ \$ $/ton$, the total cost in \$$/d$ can be calculated as:

$$Cost = k_E(E_P + E_A + E_M) + k_DC_D \quad [\$/d] \quad (15)$$

The overall cost function in (15) is then minimized subject to environment regulations for the effluent and some constraints related to process operability, as listed in Table I.

TABLE I
CONSTRAINTS TO THE PROCESS.

Constraint	Unit	Status
$COD^{(eff)} \leq 100$	$gCOD/m^3$	Regulation constraint
$TSS^{(eff)} \leq 30$	gSS/m^3	Regulation constraint
$TN^{(eff)} \leq 18$	gN/m^3	Regulation constraint
$BOD_5^{(eff)} \leq 10$	$gBOD/m^3$	Regulation constraint
$S_{NH}^{(eff)} \leq 4$	gN/m^3	Regulation constraint
$Q_w \leq 1845$	m^3/d	Manipulation constraint
$Q_r \leq 36892$	m^3/d	Manipulation constraint
$Q_a \leq 92230$	m^3/d	Manipulation constraint
$K_La^{(1-5)} \leq 360$	d^{-1}	Manipulation constraint

B. Step 2. Steady-state optimal operation

There are 8 manipulated variables (last four entries in Table I), which correspond to 8 steady-state degrees of freedom (u). The liquid levels in the reactor tanks are assumed constant at maximum capacity due to the overflow layout considered for the plant.

Compared to other process industries, a wastewater treatment plant is subject to very distinct operation modes because of daily, weekly and seasonal variation in the incoming wastewater. In this paper we consider the influent load data as given by the IWA Task Group in the benchmark website. The data are presented in terms of ASM1 state variables and influent flow rates. In general, these data reflect expected diurnal trend variations in weekdays which are typical for normal load behavior at a municipality treatment facility. Table II summarizes the given disturbances in terms of influent flow rate and load. The average composition and flow rate and the average values for the process inputs are taken from the various weather data.

TABLE II
WEATHER PROFILES EVENTS.

	Q_0 $[m^3/d]$	$COD^{(in)}$ $[gCOD/m^3]$	$TSS^{(in)}$ $[gSS/m^3]$	$TN^{(in)}$ $[gN/m^3]$	T $[^oC]$
e_0	18446	381	211	54	15
e_1	21320	333	183	48	15
e_2	40817	204	116	28	15
e_3	19746	353	195	50	15
e_4	34286	281	101	37	15
e_5	20850	347	199	41	15
$e_{5,min}$	20850	347	199	41	9
$e_{5,max}$	20850	347	199	41	21

C. Step 3. Variable selection

The results of the optimization shows that three constraints are active, namely, $TSS^{(eff)}$ (upper limit), $S_{NH}^{(eff)}$ (upper limit), and Q_a (lower limit). As expected, $TSS^{(eff)}$ is at its maximum to make Q_w small. In general, the reason why free ammonia ($S_{NH}^{(eff)}$) is active at its upper bound is that, as nitrification is an oxygen demanding process and because the transfer efficiency of oxygen from gas to liquid is relatively low so that only a small amount of oxygen supplied is used by the microorganisms, the aeration demand (E_A), which is the major cost contributor in a wastewater treatment plant, is high. One interesting fact is that the process is optimally operated aerobically, that is to say, with no anaerobic zone. The possible reason is due to the attempt to minimize the high aeration costs and to the fact that the effluent total nitrogen and ammonia constraints are quite easily reached for the given influent loads.

As those 3 active constraints must be implemented to ensure optimal operation [25], we are left with 5 degrees of freedom, and we use the local methods described in step 3 of the procedure to decide for the best (optimal) set of unconstrained self-optimizing control variables to fulfill the available degrees of freedom. We consider $n_y = 28$ measurements (the list is not shown here for the sake of compactness), $n_u = 5$ manipulated variables, and $n_d = 5$ disturbances, and clearly with $n_y > n_u + n_d$ one can expect to substantially reduce the loss for disturbances and measurement errors. As there are as many measurements as there are manipulations and disturbances, one can compute various H matrices and their respective local losses. The methods considered in this communication are

1. The combined disturbances and measurements errors using all available measurements, where H is computed by (10). In this case, H_1 is a 5×28 combination matrix.
2. The extended nullspace using all measurements, with H computed by (14). In this case, H_2 is also a 5×28 combination matrix.
3. The extended nullspace using just enough measurements, where $\tilde{G}^y$ in (14) is found by a branch and bound algorithm [28]. In this case, H_3 is a 5×10 combination matrix.

For the sake of compactness only the resulting local losses calculated using (9) are reported, and they are $L_{wc}^{H_1} = 0.1184$, $L_{wc}^{H_2} = 0.1291$, and $L_{wc}^{H_3} = 0.0761$, and one should expect to have actual (nonlinear) losses of the same magnitude for any of the calculated H matrices. In the last case, where the loss is expected to be the smallest, the variables chosen by the branch and bound algorithm that maximized the minimum singular value of $\tilde{G}^y$ were $S_O^{(3)}$, $S_O^{(4)}$, $S_{NO}^{(4)}$, $MLSS$, $K_La^{(1)}$, $K_La^{(2)}$, $K_La^{(3)}$, $K_La^{(4)}$, $COD^{(in)}$ and $T^{(in)}$.

V. Discussion

The nominal optimization results showed that it is economically optimal to keep effluent suspended solids and ammonia concentrations at their respective upper bounds, and that no internal recirculation of sludge should be used, at least under the steady-state assumption. This fact is surprising but quite realistic. Indeed, the main purpose of the internal recirculation is to provide enough nitrate to enhance denitrification in the bioreactor anoxic zones and from an economical point of view this can be efficiently achieved by the return sludge (Qr) only which brings back sufficient nitrate for denitrification reducing the pumping costs due to Q_a. However, when operating the process dynamically, one may consider using Q_a to control some internal variable so as to improve the disturbance rejection capability of the process.

If these variables are controlled at their respective optimal setpoints (active constraint control), a choice had to be made on the selection of the remaining 5 degrees of freedom, and we use the sensitivity analysis based on a plantwide procedure to decide on which 5 variables to fix/control at their respective nominal optimum values. The exact local (linear) method and the extended nullspace method based on the concept of self-optimizing control were used to systematically select those variables such that the cumbersome combinatorial curse of choosing and testing 5 out of 28 possible variable combinations is avoided. The resulting combination matrices H were easily computed using elementary matrix algebra, as described by the formulas (10), (11), and (12). The only burden with those calculations lies on the computation of the optimal matrices J_{uu}, J_{ud}, and F. Since accuracy of second order information found numerically is known to be difficult to guarantee, in addition to assuring positive definiteness of J_{uu}, calculation of F might become more attractive, and a replacement formula for (11) was derived as in (14). M_n in this equation can be freely selected, as long as it is a non-singular matrix, and we chose $M_n = J_{uu}^{1/2}$ so to avoid the need to compute J_{uu}. Moreover, since the solution for H in (10) is not unique [23], we can also find a non-singular $n_u \times n_u$ D matrix such that $H_{new} = DH$ is another yet solution, and we can select D as a function of $J_{uu}^{1/2}$; in this paper we assumed $D = J_{uu}^{-1/2}$.

The above derivations are local since we assume a linear process and a second-order objective function in the inputs and the disturbances. Thus, the proposed controlled variables are only globally optimal for the case with a linear model and a quadratic objective. In this article, for a final validation, the actual losses are checked using the nonlinear model of the process. Table III shows that the losses are about the same order of magnitude for a given disturbance. Note also that feasibility is not always guaranteed for all alternatives, and indeed only the alternative where H was computed using the extended nullspace method with "just-enough" measurements is feasible for all disturbance spectrum.

TABLE III
NONLINEAR LOSS CALCULATION FOR VARIOUS DISTURBANCES.

	e_1	e_2	e_3	e_4	e_5	$e_{5,min}$	$e_{5,max}$
J^{opt}	426.78	490.09	420.56	599.36	419.83	491.28	357.96
J^{H_1}	427.08	507.18	420.62	602.53	420.36	494.37	Inf
$\%L^{H_1}$	0.07	3.49	0.014	0.53	0.13	0.63	Inf
J^{H_2}	426.97	495.86	420.59	608.95	420.37	492.82	358.71
$\%L^{H_2}$	0.04	1.18	0.01	1.60	0.13	0.31	0.21
J^{H_3}	427.07	Inf	420.60	Inf	419.94	507.74	359.40
$\%L^{H_3}$	0.07	Inf	0.01	Inf	0.03	3.35	0.40

VI. CONCLUSION

This paper discussed the application of a sensitivity procedure for optimal operation of a wastewater treatment plant. For the given modified mathematical model of the process, where the settler is modeled based upon the static one-dimension scalar mass conservation law with discontinues fluxes theory, keeping the active constraints ($S_{NH}^{(eff)}$, $TSS^{(eff)}$, and Q_a) at their optimal values and using linear combinations of the measurements as the five remaining unconstrained degrees of freedom can guarantee near-optimal operation with minimum loss when operating at the nominal optimal mode despite the severe disturbances that affect the process.

REFERENCES

[1] A. Stare, D. Vrecko, S. Hvala, and S. Strmcnik, "Comparison of control strategies for nitrogen removal in an activated sludge process in terms of operating costs: a simulation study," *Water Research*, vol. 41, pp. 2004–2014, 2007.

[2] P. Ingildsen, G. Olsson, and Y. Z., "A hedging point strategy - balancing effluent quality, economy and robustness in the control of wastewater treatment plants," *Wat. Sci. Tech.*, vol. 45, no. 4-5, pp. 317–324, 2002.

[3] P. Samuelsson, B. Halvarsson, and B. Carlsson, "Cost-efficient operation of a denitrifying activated sludge process," *Water Research*, vol. 41, pp. 2325–2332, 2007.

[4] E. Ayesa, B. Goya, A. Larrea, L. Larrea, and A. Rivas, "Selection of operational strategies in activated sludge processes based on optimization algorithms," *Wat. Sci. Tech.*, vol. 37, no. 2, pp. 327–334, 1998.

[5] A. Rivas, I. Irizar, and E. Ayesa, "Model-based optimisation of wastewater treatment plants design," *Environ. Model. Softw.*, vol. 23, pp. 435–450, 2008.

[6] B. Chachuat, N. Roche, and M. A. Latifi, "Dynamic optimisation of small size wastewater treatment plants including nitrification and denitrification processes," *Comp. Chem. Engn.*, vol. 25, pp. 585–593, 2001.

[7] A. C. B. Araujo, S. Gallani, M. Mulas, and G. Olsson, "Systematic approach to the design of operation and control policies in activated sludge systems," *Industrial and Engineering Chemistry Research*, vol. 50, no. 14, pp. 8542–8557, 2011.

[8] J. Alex, L. Benedetti, J. Copp, K. V. Gernaey, U. Jeppsson, I. Nopens, M. N. Pons, L. Rieger, C. Rosen, J. P. Steyer, P. Vanrolleghem, and S. Winkler, "Benchmark simulation model no. 1 (bsm1)," Dept. of Industrial Electrical Engineering and Automation - Lund University, Sweden, Technical Report, 2008.

[9] M. Henze, L. C. P. Grady, W. Gujer, G. V. R. Maris, and T. Matsuo, "Activated sludge model no. 1 (ASM1)," IAWQ, London, UK, Scientific and Technical Report no. 1, 1987.

[10] I. Takacs, G. G. Patry, and D. Nolasco, "A dynamic model of the clarification-thickening process," *Water Research*, vol. 29, no. 10, pp. 1263–1271, 1991.

[11] Z. Z. Vitasovic, "Continuous settler operation: A dynamic model," in *Dynamic Modeling and Expert Systems in Wastewater Engineering*, Lewis, Chelsea, Michigan, USA, 1989, pp. 59–81.

[12] B. G. Plosz, I. Nopens, J. DeClerq, L. Benedetti, and P. A. Vanrolleghem, "Shall we upgrade one-dimensional secondary settler models used in wwtp simulators? an assessment of model structure uncertainty and its propagation," *Water Science and Technology*, vol. 63, no. 8, pp. 1726–1738, 2011.

[13] S. Diehl, "A conservation law with point source and discontinuous flux function modelling continuous sedimentation," *SIAM Journal on Applied Mathematics*, vol. 56, no. 2, pp. 388–419, 1996.

[14] S. Diehl and U. Jeppsson, "A model of the settler coupled to the biological reactor," *Water Research*, vol. 32, no. 2, pp. 331–342, 1998.

[15] S. Diehl, "Operating charts for continuous sedimentation I - Control of steady states," *Journal of Engineering Mathematics*, vol. 41, pp. 117–144, 2001.

[16] S. Diehl, "The solids-flux theory - Confirmation and extension by using partial differential equations," *Water Research*, vol. 42, no. 20, pp. 4976–4988, 2008.

[17] R. Burger, S. Diehl, and I. Nopens, "A consistent modelling methodology for secondary settling tanks in wastewater treatment," *Water Research*, vol. 45, pp. 2247–2260, 2011.

[18] U. Jeppsson and S. Diehl, "An evaluation of a dynamic model of the secondary clarifier," *Water Science and Technology*, vol. 34, no. 5/6, pp. 19–26, 1996.

[19] D. Queinnec and D. Dochain, "Modelling and simulation of the steady-sate of secondary settlers in wastewater treatment plants," *Water Science and Technology*, vol. 43, no. 7, pp. 39–46, 2001.

[20] L. B. Verdickt and J. F. Van Impe, "Simulation analysis of a one-dimensional sedimentation model," in *Preprints of the 15th triennial IFAC World Congress (CDROM)*, Barcelona, Spain, 2002, p. 6.

[21] R. David, J. L. Vasel, and A. Vande Wouwer, "Settler dynamic modeling and matlab simulation of the activated sludge process," *Chemical Engineering Journal*, vol. 146, pp. 174–183, 2009.

[22] S. Skogestad, "Control structure design for complete chemical plants," *Computers and Chemical Engineering*, vol. 28, pp. 219–234, 2004.

[23] V. Alstad, S. Skogestad, and E. S. Hori, "Optimal measurement combinations as controlled variables," *Journal of Process Control*, pp. 138–148, 2009.

[24] M. Morari, G. Stephanopoulos, and Y. Arkun, "Studies in the synthesis of control structures for chemical processes, part I: formulation of the problem, process decomposition and the classification of the control task, analysis of the optimizing control structures," *AIChE Journal*, vol. 26, no. 2, pp. 220–232, 1980.

[25] A. Maarleveld and J. E. Rijnsdorp, "Constraint control on distillation columns," *Automatica*, vol. 6, pp. 51–58, 1970.

[26] S. Skogestad, "Plantwide control: The search for the self-optimizing control structure," *Journal of Process Control*, vol. 10, pp. 487–507, 2000.

[27] I. J. Halvorsen, S. Skogestad, J. C. Morud, and V. Alstad, "Optimal selection of controlled variables," *Ind. Eng. Chem. Res.*, vol. 42, pp. 3273–3284, 2003.

[28] V. Kariwala, Y. Cao, and Janardhanan, "Local self-optimizing control with average loss minimization," *Industrial and Engineering Chemistry Research*, vol. 47, pp. 150–1158, 2008.