BILEVEL PROGRAMMING FOR ANALYSIS OF REDUCED MODELS FOR USE IN MODEL PREDICTIVE CONTROL

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Extended abstract

In this paper we use bilevel programming to find the maximum difference between a model predictive controller (MPC) using a full model and an MPC using a reduced model. The results apply to MPC with quadratic cost function and linear model with linear constraints.

Traditional method addresses the quality of a reduced model either in open or closed loop, but not in the presence of constraints. Our main contribution is therefore to asses the performance of reduced order models for use in MPC with constraints.

To achieve this we first set up a bilevel program on the form

$$\begin{aligned} &\max_{x\in X} d(u_{\text{red}},u) \\ &\text{subject to } u = \arg\min \ \{\text{MPC formulation with full model}\} \\ &\qquad\qquad u_{\text{red}} = \arg\min \ \{\text{MPC formulation with reduced model}\} \end{aligned} \tag{1}$$

We show that for a well-posed MPC problem, the optimization problem (1) can be reformulated as a mixed-integer linear program (MILP). This is done by writing up the Karush-Kuhn-Tucker (KKT) conditions for both MPC formulations and representing the complementarily conditions with binary variables.

To calculate the difference in the controllers we use the infinity norm of the input difference (through the B-matrix), i.e.

$$d(u_{\text{red}},u) = \left\| B(u-u_{\text{red}}) \right\|_\infty \tag{2}$$

Here we could also have used differences in outputs y using the framework presented in the paper. This measure renderes problem (1) non-convex, however it may be rendered into a MILP using standard techniques.

The method is demonstrated on a 16-state linear model, where we compare the full model with reduced models ranging from 1 to 15 states.

Planned further work in this project is to test the method on more realistic examples. In addition we will let the physical disturbances define the search-space in problem (1), rather than the intial states. Letting the initial states represent the disturbances can be very conservative as in the worst case we get combinations of disturbances that can never occur in practise.