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Near-optimal control of batch processes

- by tracking of approximated sufficient conditions of optimality

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What are we trying to do?

- Seek linear combination of measurements: $c = Sy$
- Select the best matrix S using systematic procedure
- Allow separation of problem into:
  - Economic optimization
  - Control structure design via selecting S
  - Implementation using for example PI controllers

Implementation strategies

- Online methods
  - Measurements used to update model (or state estimate)
  - Requires online optimization
  - High computational load
- Offline methods
  - Measurements used directly in output feedback structure
  - Requires offline analysis
  - Low computational load

Outline

- Goal: systematic control structure design
- Offline school of thought
- Remaining talk has three main parts:

a) Method development: $c(t) = S(t)y(t)$
b) Linear example with stable plant
c) Nonlinear batch distillation example

Mayer problem

$$\min_{u(t)} J(x(t_f))$$

$$\dot{x} = f(x,u,d), \quad x(0) = x_0.$$

$$y = h(x)$$

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Necessary and sufficient conditions for optimal control

$$H(t) = \lambda^T f(x,u,d), \quad \dot{\lambda} = -\partial H / \partial x$$

$$H_u = 0$$

$$\lambda(t_f) = \partial J / \partial x\left(t_f\right)$$

$$\delta^2 J = \int_0^{t_f} \begin{bmatrix} \delta x^T & \delta u^T \end{bmatrix} \begin{bmatrix} H_{xx} & H_{xu} \\ H_{ux} & H_{uu} \end{bmatrix} \begin{bmatrix} \delta x \\ \delta u \end{bmatrix} dt.$$

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Matching the Hamiltonian Matrix

$$\delta^2 J = \int_0^{t_f} \begin{bmatrix} \delta x^T & \delta u^T \end{bmatrix} \begin{bmatrix} H_{xx} & H_{xu} \\ H_{ux} & H_{uu} \end{bmatrix} \begin{bmatrix} \delta x \\ \delta u \end{bmatrix} dt.$$

- Form linear combinations: c = Sy = SCx
- New approximation:

$$\delta^2 J \approx \frac{1}{2} \int_0^{t_f} \left(S(t)C(t)\delta x(t)\right)^T Q_c \left(S(t)C(t)\delta x(t)\right) + \delta u^T H_{uu}\, \delta u \, dt$$

- Obviously, we must select $R = H_{uu}$
- Also

$$\left(SC(t)\right)^T Q_C(t) \left(SC(t)\right) = H_{xx}(t).$$

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Scaling matrix Q_c

- Variables "c" not known a priori
- Assume c's independent
- Assume worst-case change in c to occur for all c's at same d
- Scaling suggested:

$$Q_C(t) = \left(\operatorname{diag}\left(S Q_y^{-1}(t) e\right)\right)^{-1}.$$

$$\Rightarrow (SC(t))^T \left(\operatorname{diag}(S Q_y^{-1} e)\right)^{-1} (SC(t)) = H_{xx}(t).$$

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A linear-quadratic problem

$$J = \frac{1}{2}\int_0^{t_f} \|x\|_2^2 + \|u\|_2^2 dt$$

$$\dot{x} = Ax + Bu,$$
$$y = Cx$$

$$A = \begin{bmatrix} -3 & 1 & 1 \\ 0 & -2 & 1 \\ 0 & 0 & -3 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}, C_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

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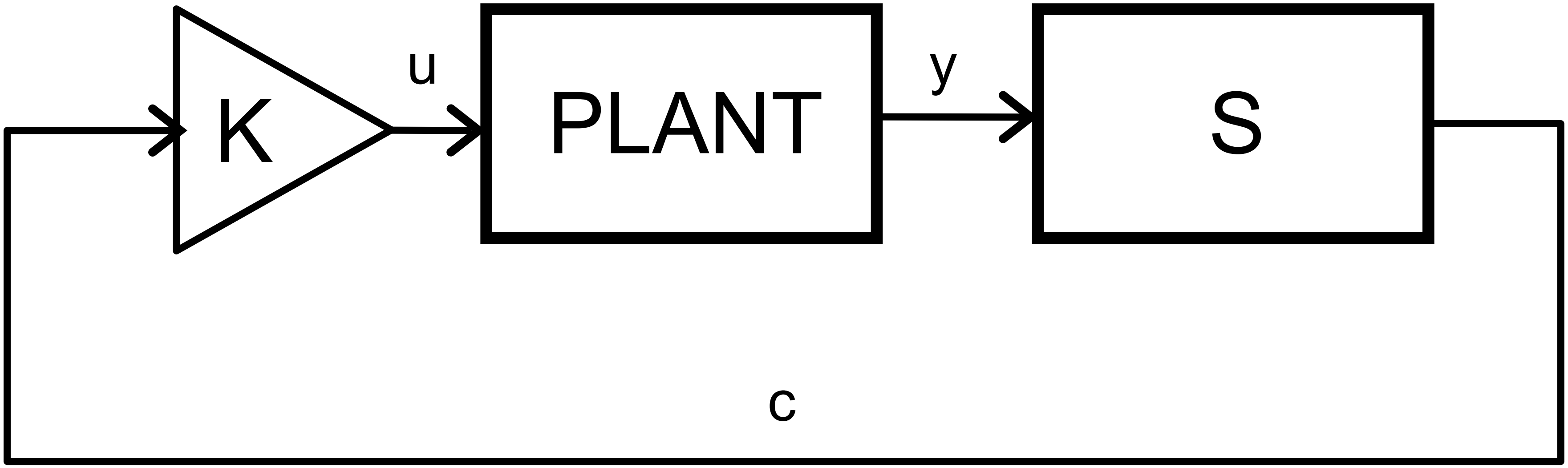
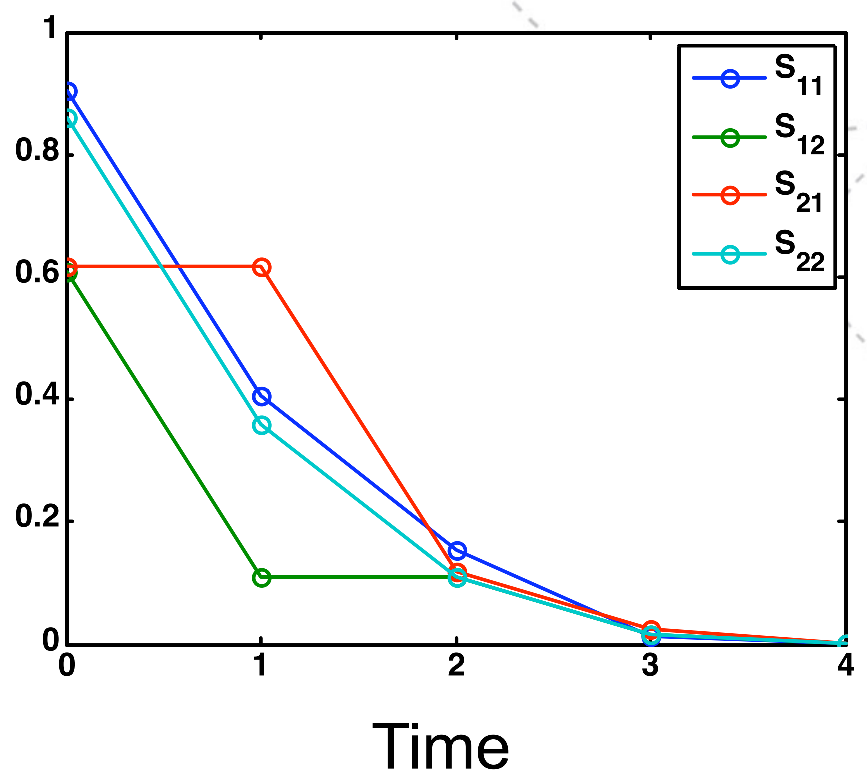
Block diagram: output feedback

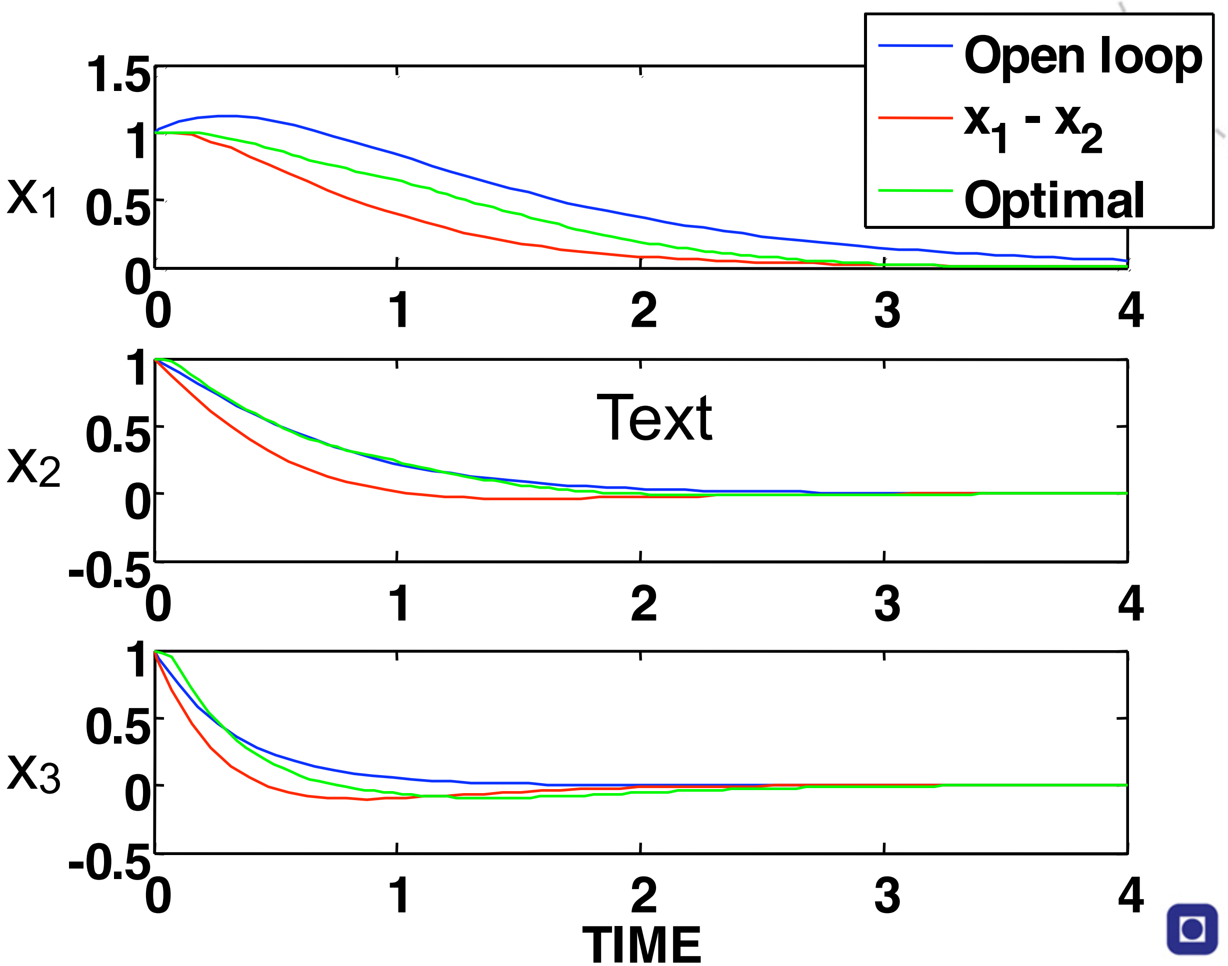


Control strategy

- S computed for times {0,1,2,3,4}
- Linear interpolation used between known points
- Static feedback with pure gain
- Tuned for x0=[1,1,1]
- Look at responses for [1,1,1]



State trajectories



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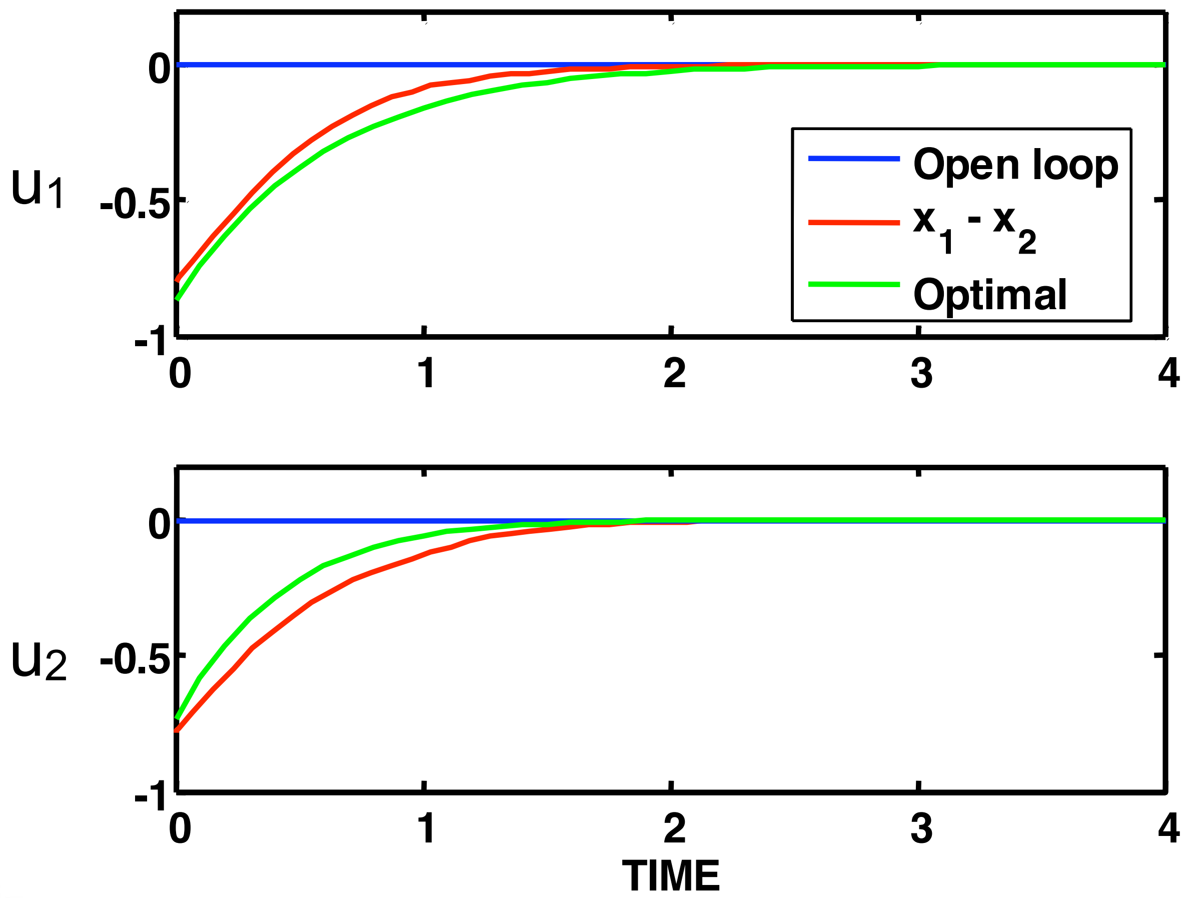
Input trajectories



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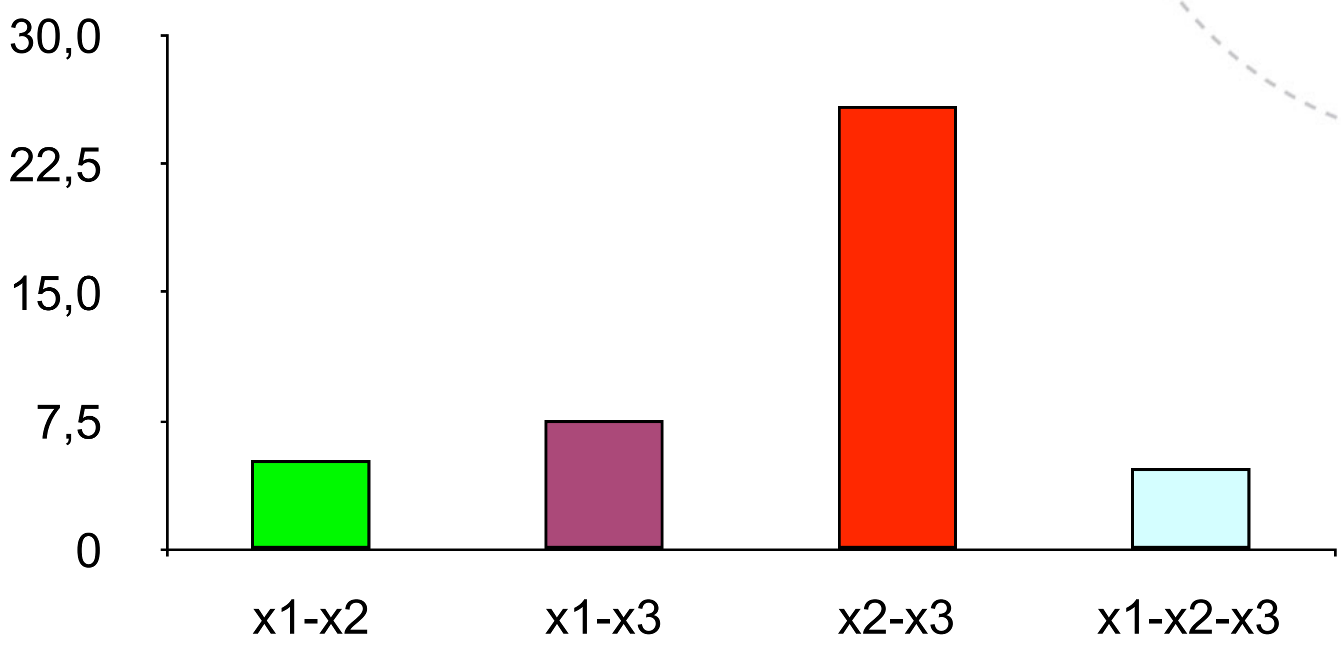
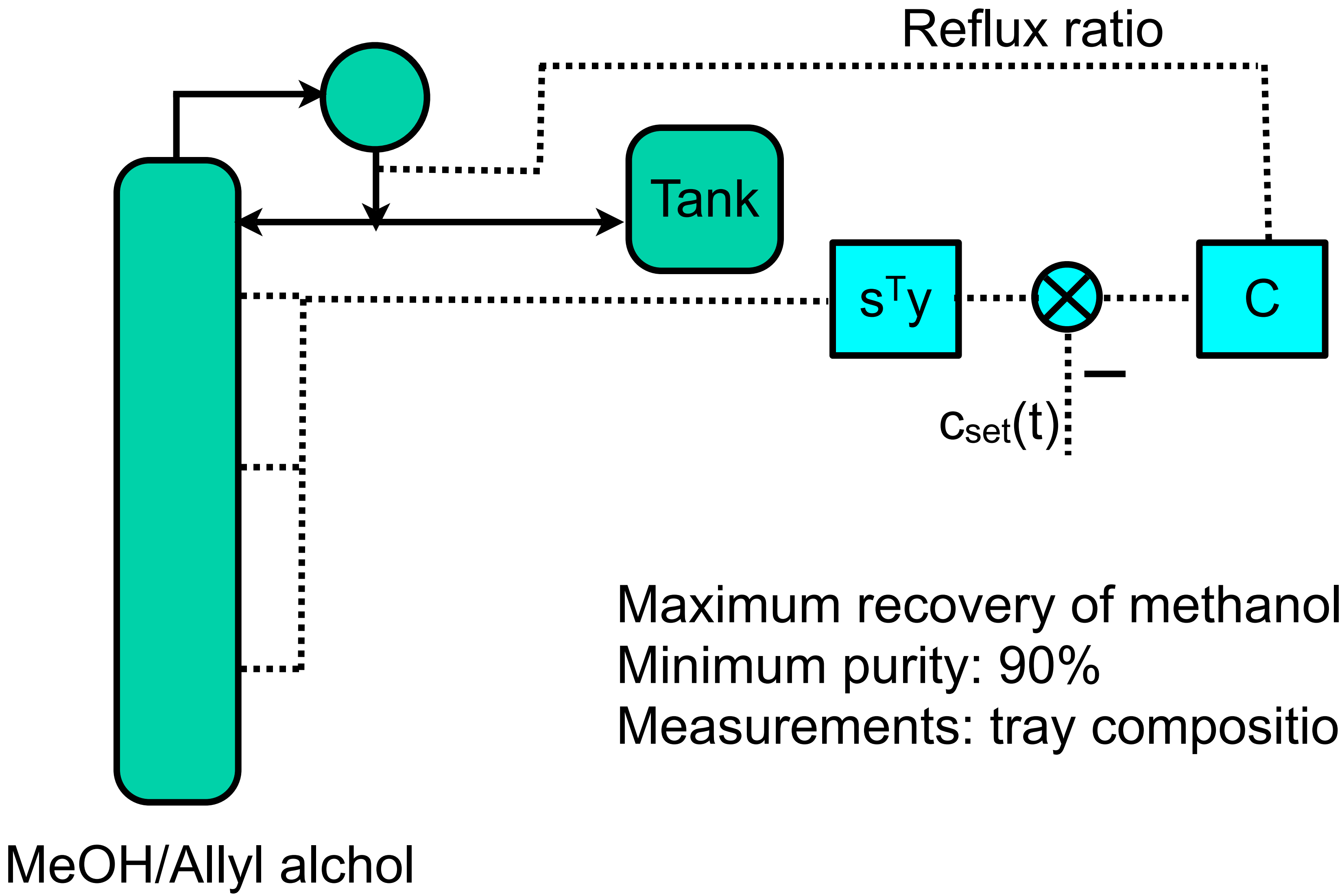
Average losses (%)



Can improve loss by better controller tuning

Batch distillation



Maximum recovery of methanol
Minimum purity: 90%
Measurements: tray compositions

Computations

- Optimize for 49 vol%, 50 vol% and 51 vol% methanol to find Q_y (www.gpops.org)
- Compute H_{xx}
- Solve

$$C^T S^T \mathrm{diag}^{-1}(S Q_y^{-1} e) S C = H_{xx}$$

using direct search optimization for every hour
- Verify by simulations in closed loop

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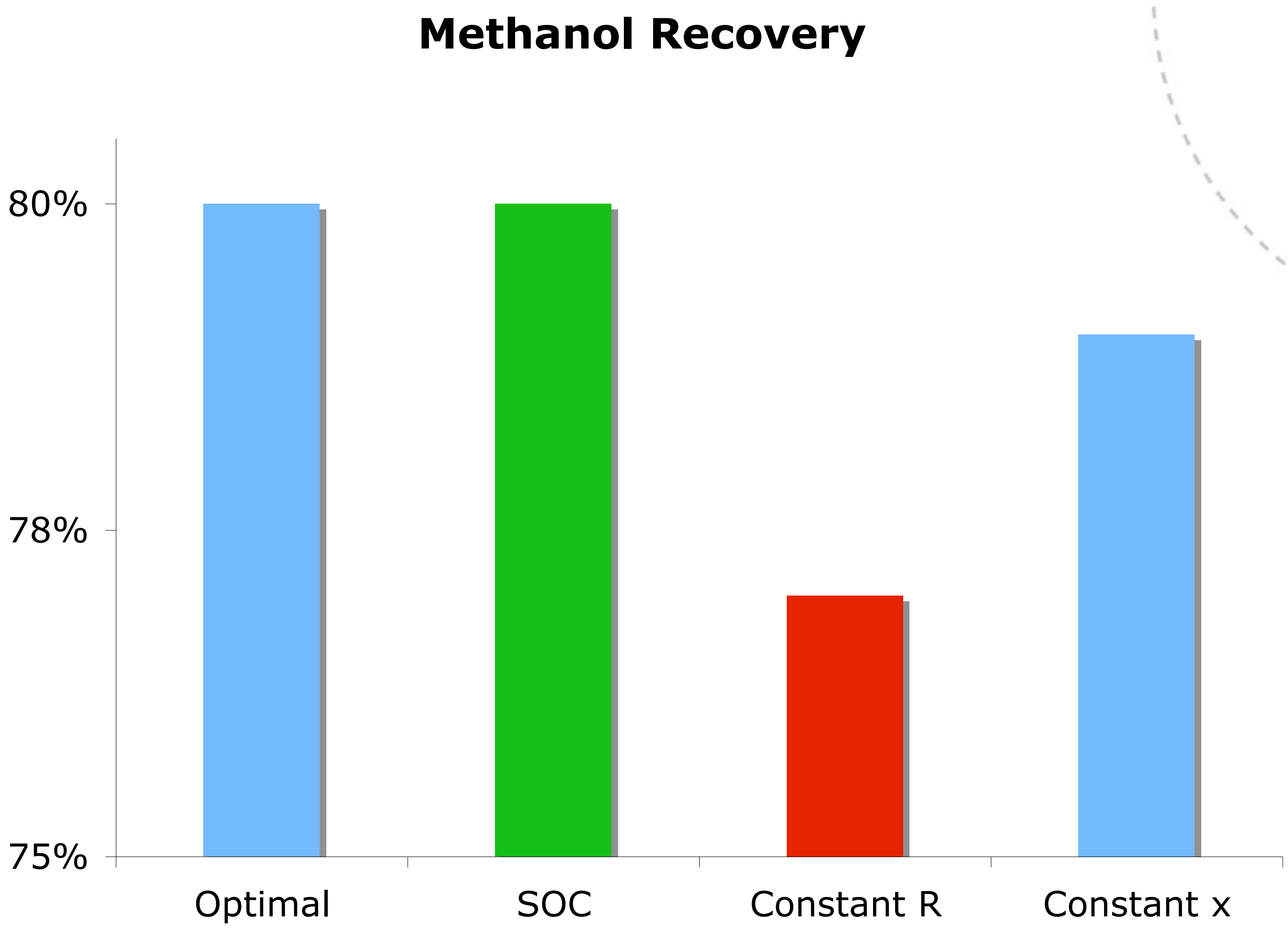

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- Algorithm for control structure design
- Applicable to both linear and nonlinear systems
- A step on the way to automatic control structure design

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