Self-consistent inventory control

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Abstract

Inventory or material balance control is an important part of process control. In the literature, many rules have been proposed to held in designing such systems, but their justification is often unclear. The main contribution of this paper is to proposed the more general self-consistency rule for evaluating inventory control systems. Consistency means that the steady-state mass balances (total, component and phase) for the individual units and the overall plant are satisfied. In addition, self-consistency is a desired property, meaning that the mass balances are satisfied locally with local inventory loops only.

Introduction

One of the more elusive parts of process control education is inventory or material balance control. An engineer with some experience can usually immediately say if a proposed inventory control system is workable. However, for a student or newcomer to the field it is not obvious, and even for an experienced engineer there may be cases where experience and intuition are not sufficient. The objective of this paper is to present concise results on inventory control, relate to previous work, tie

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up loose ends, and to provide some good illustrative examples. The main result (self-consistency rule) can be regarded as obvious, but nevertheless we have not seen it presented in this way before.

The main result is a simple rule to check whether an inventory control system is *consistent*. Here, consistency means that the mass balances for the entire plant and units are satisfied.[1] In addition, we usually want the inventory control system to be *self-consistent*. Self-consistency means that consistency is satisfied locally, without the need to rely on control loops outside the unit. Consistency is a required property, because the mass balances must be satisfied in a plant, whereas self-consistency is a desired property of an inventory control system. In practice, an inconsistent control structure will lead to a situation with a fully open or closed control valve and the associated control loop cannot fulfill or attain the control set point.[2]

In most plants, we want the inventory control system to use simple PI controllers and be part of the basic (regulatory) control layer. This is because it is generally desirable to separate the tasks of regulatory (stabilizing) control and supervisory (economic) control. From this it follows that the structure of the inventory control system is usually difficult to change later.

The importance of consistency of inventory control structures is often overlooked. Our work is partly inspired by the many examples of Kida, who has given industrial courses in Japan on control structures for many years. In a personal communication[2] he states that *"most process engineers, and even academic people, do not understand the serious problem of inconsistency of plantwide control configurations. When writing a paper, you have to clearly explain this point and make them convinced at the very outset. Otherwise they will not listen to or read through your detailed statements, but skip them all"*.

A very good early reference on inventory control in a plantwide setting is Buckley.[3] He states that material balance control must be in the direction of flow downstream a given flow and opposite the direction of flow upstream a given flow. Price et. al[1,4] extended this and state that the inventory control must "radiate" outwards from the point of a given flow (throughput manipulator). As shown in this paper, these statements are a consequence of requiring the inventory control system to be self-consistent.

Downs [5] provides a very good discussion of material balance control in a plantwide control environment, with many clarifying examples. However, it is somewhat difficult for the reader to find a general rule or method that can be applied to new cases.

Luyben et al. [6] propose a mainly heuristic design procedure for plantwide control. Luyben's procedure consist of, among others, "*Step 6. Control inventories (pressures and levels) and fix a flow in every recycle loop*". Possible limitations of this guideline are discussed in the present paper. Another guideline of Luyben et al. [6] is to "*ensure that the overall component balances for each chemical species can be satisfied either through reaction or exit streams by accounting for the component's composition or inventory at some point in the process*". This guideline is a bit limited because entrance (feed) streams is not considered.

Specific guidelines for designing inventory control structures are presented by Georgakis and coauthors. [1,4] They propose a set of heuristic guidelines for inventory control design in a plantwide environment and also discuss consistency. The authors also state the importance of a self-consistent inventory control structure: "*Self-consistency appears to be the single most important characteristic governing the impact of the inventory control structure on system performance*".

As already mentioned, Fujio Kida from JGC Corporation in Japan has developed a lot of teaching material [2] and written several papers on inventory control. [7] Unfortunately, the work is published in Japanese only, but nevertheless it is clear that there are many detailed rules and some require detailed calculations.

In summary, it is clear that the present literature provides a number of specific rules of designing inventory control systems, but the justification and limitations of these rules are often unclear. The main result of this paper is to present the simple self-consistency rule for evaluating the consistency of inventory control systems, which applies to all cases and only requires structural information.

The outline of the paper is as follows. First the terms consistency and self-consistency is defined, then we present and derive the self-consistency rule. We then apply the rule to some flow networks, such as units in series and recycle system. This is followed by a derivation of some more specific rules before we end up with some more complex examples, including distillation

and recycle reactors. Note that the present paper focuses on analysis of a given control structure. The design of the inventory control system, which in particular is related to the placement of the throughput manipulator, is discussed in more detail in Aske.[8]

Remark on notation: In this paper, when a flow (valve) is left unused or with a flow controller (FC), then this indicates that this is a *given* flow. By the term "given flow" we mean that the flow is *not* used for inventory control but rather given by conditions outside the inventory control system. For example, a "given flow" can be

1. a throughput manipulator (TPM),

2. a flow that comes from another part of the plant (disturbance for our part),

3. a fixed flow

4. a flow that is used for other control tasks (eg., control of composition or temperature).

Definition of self-consistent inventory control

The dynamic mass balance for total or component mass in any unit or process section can be written:[5]

$$\frac{dI}{dt} = \text{Rate of change in inventory} = \text{Inflow + Generation - Outflow - Consumption} \qquad (1)$$

During operation we must have "inventory regulation", meaning that the inventories of total, component and phase mass are kept within acceptable bounds. To achieve this we need a "consistent" and preferably "self-consistent" inventory control system.

Definition 1. *Consistency. An inventory control system is consistent if it can achieve inventory regulation for any part of the process, including the individual units and the overall plant.*

More precisely, by "it can achieve inventory regulation" we mean that there exists controller

settings such that one can keep the inventories of total, component and phase mass within acceptable bounds.

Remark 1 The use of mass balances for a phase may seem odd, and is discussed in more detail in the next section.

Remark 2 Consistency requires that the steady-state mass balances (total, components and phases) are "satisfied", meaning at steady-state there is a balance between In-terms (inflow + generation) and Out-terms (outflow + consumption) such that $\frac{dI}{dt} = 0$. In addition, we must require for consistency that the inventories can be kept within acceptable bounds ($I_{\min} \leq I \leq I_{\max}$). For example, if we have a process where a component has no specified exit, then it will eventually have to exit somewhere (at steady-state), but the value of its steady-state inventory I (and composition) may not be acceptable, so we may still not have consistency.

Remark 3 Since the mass balance must be satisfied for the overall plant, it follows that a consistent inventory control system must be *"able to propagate a production rate change throughout the process and in particular if such a change produces changes in the flow rates of major feed and product streams"*.[1]

In most cases, we want the inventory control system also to be "self-consistent", meaning that the local control by "itself" gives consistency:

Definition 2. *Self-consistency. A consistent inventory control system (see Def. 1) is self-consistent if only local inventory loops are used.*

More precisely, by "only local inventory loops are used" we mean that the inventory control system of each unit uses only its in- or outflows (with no manipulated variables outside the unit).

For a process consisting of just one unit (like in Figure 2 where the unit is the black dot), self-consistency and consistency are the same, but not in general. To understand the difference between consistency and self-consistency consider the serial process in Figure 4, which is dissuced in more detail later in Example 3. Here, control of inventory in the last unit involves the main feed flow which is outside the unit, so we do not have self-consistency. The main problem with not having self-consistency in this case is that the last unit in Figure 4 cannot be operated by itself

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(independently of the other units). Another problem is that the "long" inventory control loop (with a large effective time delay from the feed valve to the last unit) may result in poor control of the inventory in the last unit.

Two related terms are "regulation" and "self-regulation". Here, regulation is associated with consistency (as already used in Definition 1) and self-regulation (local) is associated with self-consistency. Note that in this paper, we use the term "self-regulation (local)" which allows for the use of local control, so it is wider than "true" self-regulation where there is no "active" control at all. This is clear from the following definitions:

Definition 3. *Self-regulation (**true**) is when regulation (acceptable variation in the output variable when disturbances occur) is achieved by the process "itself" (with no "active" control).*

Definition 4. *Self-regulation (**local**) is when regulation (acceptable variation in the output variable when disturbances occur) is achieved by the local control system "itself" (with no control loops outside the unit).*

Self-regulation (true) always implies self-regulation (local) but not the other way around. In this paper, when we just write "self-regulation", then we mean "self-regulation (local)".

Example 1. *Self-regulation.* *"Self-regulation" (local) may or may not require "active" control. As an example, consider regulation of liquid inventory (m) in a tank; see Figure 1(a). The outflow is given by a valve equation*

$$\dot{m}_{out} = C_v f(z) \sqrt{\Delta p \cdot \rho} \quad [kg/s] \tag{2}$$

where z is valve position. The pressure drop over the valve is

$$\Delta p = p_1 - p_2 + \rho g h \tag{3}$$

where h is the liquid level, which is proportional to the mass inventory, e.g., $m = h\rho A$ for a tank with constant cross section area A. If the pressure drop Δp depends mainly on the liquid level h, then the inventory m is self-regulated (true). This is the case in Figure 1(a) where $p_1 = p_2$

so $\Delta p = \rho g h$ and the entire pressure drop over the valve is caused by the liquid level. Thus, $\dot{m}_{out} \sim \sqrt{h}$, which means that without active control a doubling of the flow $\dot{m}_{out}$ will result an a four times larger liquid level (h). If this change is acceptable, then we have self-regulation (true and local). In other cases, it may be necessary to use "active" control to get self-regulation (local). Specifically, in Figure 1(b), $p_1 - p_2 = 99$ bar so the relative pressure contribution from the liquid level ($\rho g h$) is much too small to provide true self-regulation. For example, for a large tank of water with $h = 10$ m, the contribution from the level is only about 1 % ($\rho g h \approx 1000$ kg/m^3· 10 m/s^2 · 10 m = 10^5 N/m^2 = 1 bar). In this case "active" control is required, where the level controller (LC) adjusts the valve position z, see Figure 1(b).

(a) Self-regulation (true) possible without "active" control.
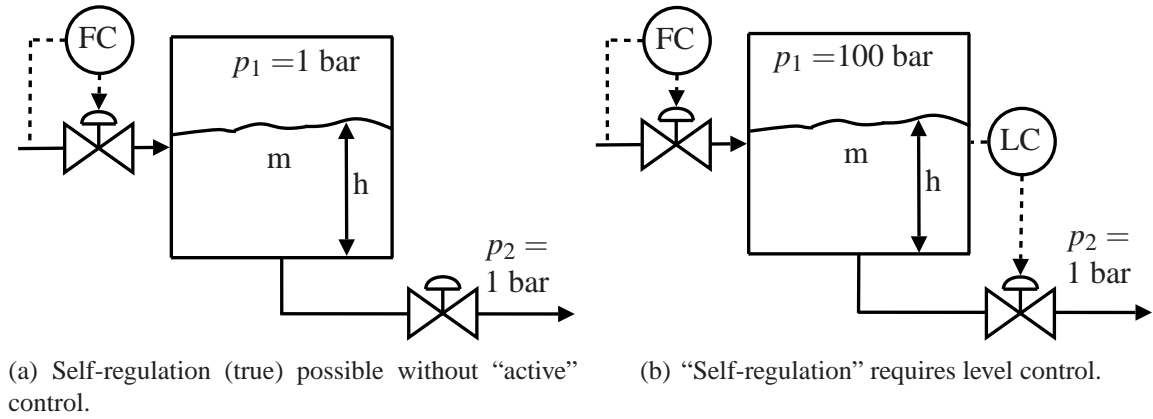
(b) "Self-regulation" requires level control.

Figure 1: Self-regulation of inventory in a tank with a given feed rate.

Self-consistency rule

From the above definitions it follows that self-consistency is equivalent to requiring self-regulation (local) of all inventories, and we can derive the following rule.

Rule 1. *"Self-consistency rule": Self-consistency (local "self-regulation" of all inventories) requires that*

1. *The total inventory (mass) of any part of the process (unit) must be "self-regulated" by its in- or outflows, which implies that at least one flow in or out of any part of the process (unit) must depend on the inventory inside that part of the process (unit).*

2. *For systems with several components, the inventory of each component of any part of the process must be "self-regulated" by its in- or outflows or by chemical reaction.*

3. *For systems with several phases, the inventory of each phase of any part of the process must be "self-regulated" by its in- or outflows or by phase transition.*

Remark 1 The above requirement must be satisfied for "any part of the process". In theory, since the local balances of the individual units sum up to the balances around any group of units, it is sufficient to apply the above rule to the individual units only. However, in practice, it is recommended to consider at least the individual units plus the overall process.

Remark 2 A flow that depends on the inventory inside a part of the process, is often said to be on "inventory control". Inventory control usually involves a level controller (LC) (liquid) or pressure controller (PC) (gas and in some cases liquid), but it may also be a temperature controller (TC), composition controller (CC) or even no control ("true" self-regulation, e.g. with a constant valve opening as in Figure 1(a)). On the other hand, a flow controller (FC) can not be used for inventory control because flow is not a measure of inventory.

Remark 3 It is possible to extend the "self-regulation" rule to energy inventory, but this is not done here. We also doubt if such an extension is very useful, because in most cases the energy balance will maintain itself by "true" self-regulation (without control), for example because a warmer inflow in a tank leads to a warmer outflow.

The above rule may seem obvious. Nevertheless, a more formal proof is useful and may clarify some issues.

Proof of self-consistency rule.

1. A boundary (control volume) may be defined for any part of the process. Let m [kg] denote the inventory inside the control volume and let $\dot{m}_{in}$ and $\dot{m}_{out}$ [kg/s] denote in- and outflows. Then the (total) mass balance is

$$\frac{dm}{dt} = \sum \dot{m}_{in} - \sum \dot{m}_{out} \quad [\text{kg/s}] \tag{4}$$

If all terms are independent of the inventory m, then this is an integrating process where m will drift out of bounds ($\frac{dm}{dt} \neq 0$ at steady-state) when there is a disturbance in one of the terms (e.g. $\dot{m}_{\text{in}}$, $\dot{m}_{\text{out}}$). To stabilize the inventory we must have "self-regulation" where $\dot{m}_{\text{in}}$ or $\dot{m}_{\text{out}}$ depends on the inventory (m), such that m is kept within given bounds in spite of disturbances. More precisely, $\dot{m}_{\text{in}}$ must decrease when m increases or $\dot{m}_{\text{out}}$ must increase when m increases, such that m is kept within given bounds in spite of disturbances.

2. Similarly, let n_A [mol A] denote the inventory of component A inside the control volume and let $\dot{n}_{A,\text{in}}$ and $\dot{n}_{A,\text{out}}$ [mol A/s] denote the in- and outflows. The mass balance for component A is

$$\frac{dn_A}{dt} = \sum \dot{n}_{A,\text{in}} - \sum \dot{n}_{A,\text{out}} + G_A \quad \text{[mol A/s]} \tag{5}$$

where G_A is the net amount generated by chemical reaction. To stabilize the inventory we must have "self-regulation" where $\dot{n}_{A,\text{in}}$, $\dot{n}_{A,\text{out}}$ or G_A depend on n_A such that n_A is kept within given bounds in spite of disturbances.

An example where the inventory n_A may be self-regulated because of the reaction term G_A is the irreversible reaction $A + B \rightarrow P$, where B is in excess and A is the limiting reactant. In this case, an increase in inflow of A ($\dot{n}_{A,\text{in}}$) will be partly consumed by the chemical reaction.

3. The rule for the individual phase follows by simply defining the control volume as the parts of the process that contain a given phase P and applying the mass balance to this control volume. Let m^P [kg] denote the inventory of the given phase inside the control volume and let $\dot{m}^P_{\text{in}}$ and $\dot{m}^P_{\text{out}}$ [kg/s] denote the in- and outflows. The mass balance for a given phase is then

$$\frac{dm^P}{dt} = \sum \dot{m}^P_{\text{in}} - \sum \dot{m}^P_{\text{out}} + G^P \quad \text{[kg/s]} \tag{6}$$

where G^P is the net phase transition over the phase boundary. To stabilize the inventory we must have "self-regulation" where $\dot{m}^P_{\text{in}}$, $\dot{m}^P_{\text{out}}$ or G^P depends on the inventory (m^P) such that m^P is kept within given bounds in spite of disturbances.

An example where we need to consider individual phases is a flash tank where a two-phase feed is separated into gas and liquid, see Figure 5(b).

□

Example 2. *Stream with two valves.* *To demonstrate the self-consistency rule on a very simple example, consider a single stream with two valves; see Figure 2(a). There is only a single (small) inventory (hold-up) m in this simple process (illustrated by the big dot), so consistency and self-consistency are here the same. In this case, the pressure p is a direct measure of inventory m (for a liquid the dependency is very strong; for an ideal gas it is $p = \frac{mRT}{V}$). Thus, self-regulation of pressure is the same as self-regulation of inventory. To apply the self-consistency rule, we define a control volume (dotted box) as shown in Figure 2 and note that the inflow is on flow control in all four cases, that is, the inflow is independent of the inventory m. Thus, according to Rule 1, to have consistency (self-regulation), the outflow must depend on the inventory m (pressure p) and more specifically the outflow must increase when m (p) increases.*

Four different control structures are displayed in Figure 2. According to Rule 1, the structure in Figure 2(a) is consistent since the outflow increases when the inventory m (pressure p) increases. Thus, we have "true" self-regulation with no need for active control.
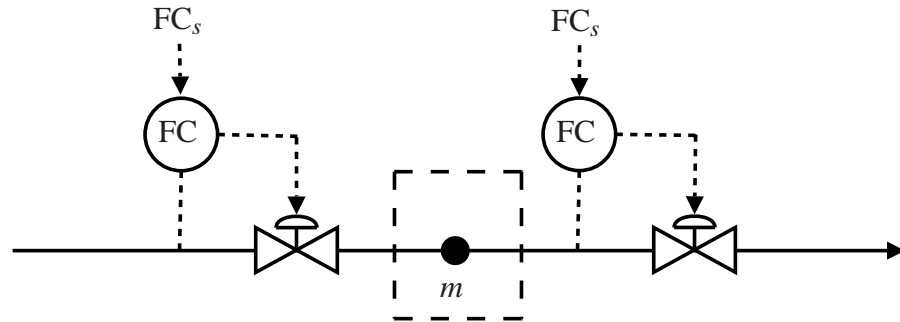
The control structure in Figure 2(b) is not consistent because the outflow is independent on the inventory m. Even if the set points for the two flow controllers were set equal, any error in the actual flow would lead to an imbalance, which would lead to accumulation or depletion of mass and the inventory would not be self-regulated.

The structure in Figure 2(c) is consistent because the outflow increases when the inventory m (pressure p) increases.

Finally, the control structure in Figure 2(d) is not consistent because the outflow depends on the inventory m (and pressure) in the wrong (opposite) manner. To understand this, consider a decrease in inflow, which will lead to a decreased pressure in the control volume. A lower differential pressure over the pressure-controlled valve leads to a smaller flow through the valve and the pressure at the downstream measuring point will decrease, leading the pressure controller to open the valve. The result is a further pressure decrease in the control volume, so the pressure controller is actually working in the wrong direction. The opening of the pressure-controlled valve will
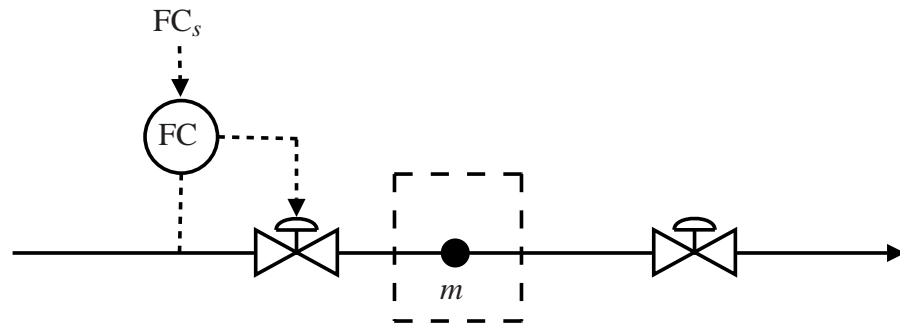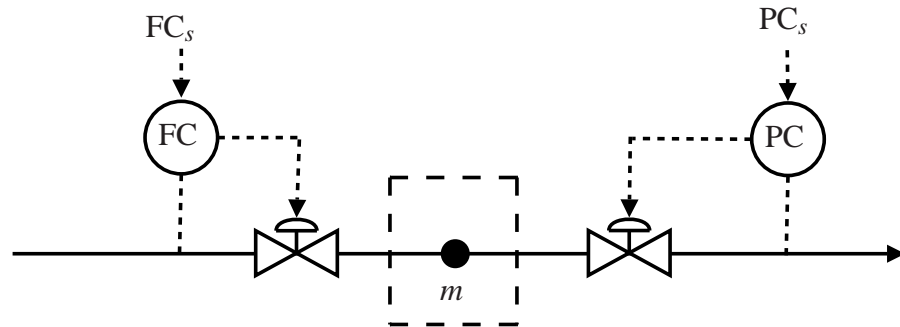
(a) OK (consistent control structure since outflow depends on inventory m).

(b) Not consistent control structure since outflow is set.
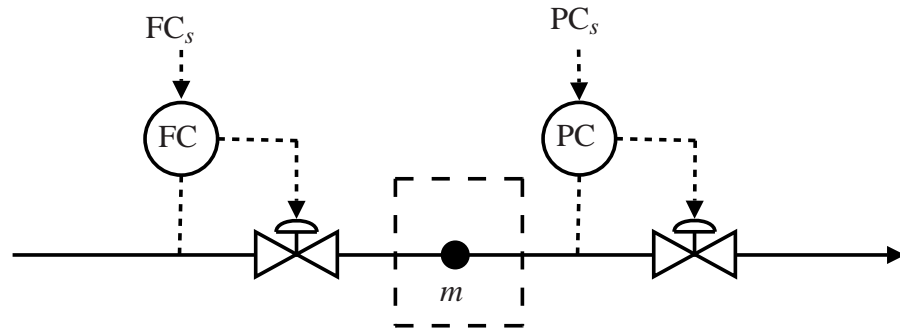
(c) OK (consistent control structure since outflow depends on inventory m).

(d) Not consistent control structure since outflow does not depend correctly on inventory m.

Figure 2: Four different control structures with two valves and set inflow. Note: For the flow controllers (FC) it does not matter whether the valve is downstream (as shown above) or upstream of the flow measurement.

also affect the flow-controlled valve and, depending on the set point of the controllers, either the flow-controlled valve or the pressure-controlled valve will move to fully open. The other pressure-controlled valve or flow-controlled valve will continue to control pressure or flow. It should also be noted that the pressure control loop is in the direction opposite *to flow, which is not correct when the inflow is given (see further discussion in next section).*

This is confirmed by dynamic simulations of the simple configuration in Figure 2(d) using the flowsheet simulator Aspen HYSYS® (see Figure 3):

(i). **10% increase in FC set point***: The FC valve saturates at fully open and the PC maintains its set point*

(ii). **10% decrease in FC set point***: The FC maintains its set point and the PC valve saturates at fully open*

(iii). **5% increase in PC set point***: The FC maintains its set point and the PC valve saturates at fully open*

(iv). **5% decrease in PC set point***: The FC valve saturates at fully open and the PC maintains its set point*

In all cases the system is assumed to be at steady-state initially.

Remarks about sign of controllers: Overall, for closed-loop stability the controller and the plant should give a negative feedback loop:

1. **Flow control**. Opening a valve always increases the flow (positive gain), so a flow controller is always "reverse acting" (with a negative feedback sign).

2. **Level and pressure control**. The controller sign depends on the location of the valve relative to the inventory (level or pressure). If control is in the direction of flow (with the inventory measurement for level or pressure upstream the valve) then the controller must be "direct acting" (positive feedback sign), if control is in opposite direction of flow then it must be "reverse acting".
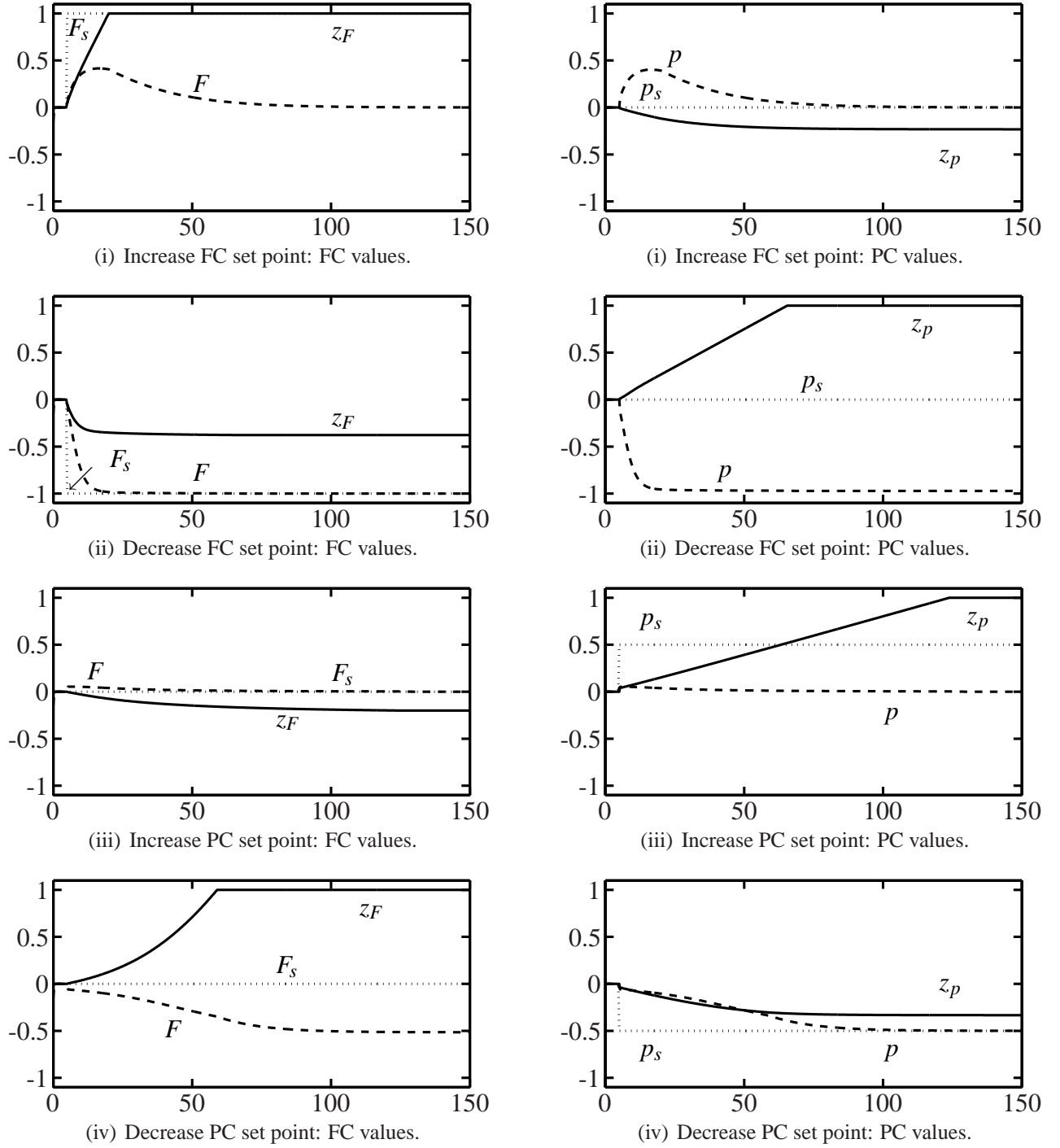
Figure 3: Dynamic simulations of the simple configuration in Figure 2(d). Left column: Flow controller. Right column: Pressure controller. In all cases, one of the valves moves to fully open. (z_F: inlet valve opening, z_P: outlet valve opening).

These remarks were used when choosing the sign for controller gains for the dynamic simulations in Figure 3.

Example 3. *Units in series. To understand the difference between the terms consistency (Definition 1) and self-consistency (Definition 2), consider inventory control of the series process in Figure 4. The control structure is **consistent** and is able to propagate a production rate change to a change in the feed rate. However, the in- and outflows for the last unit (dashed box) do not depend directly on the inventory inside the unit and the control volume is therefore **not self-consistent** according to the "self-consistency rule"(Rule 1). Also, the inventory controllers are not in the opposite direction to flow as they should be for a self-consistent process with a given product rate (see also next section). To make the structure consistent we have in Figure 4 introduced a "long loop" where the inflow to the first unit is used to control the inventory in the last unit.*
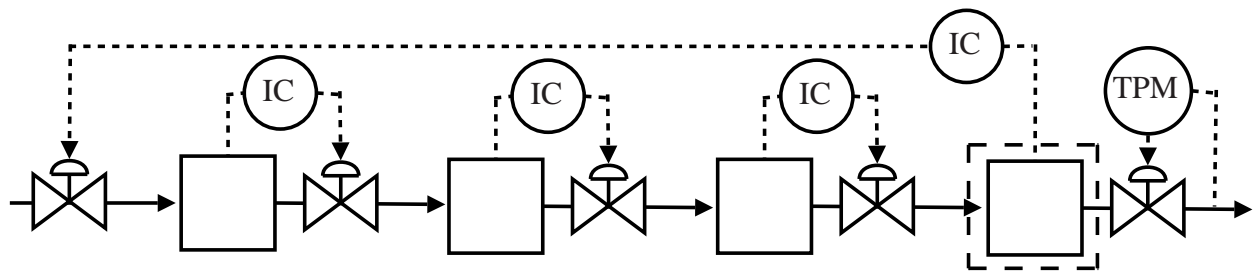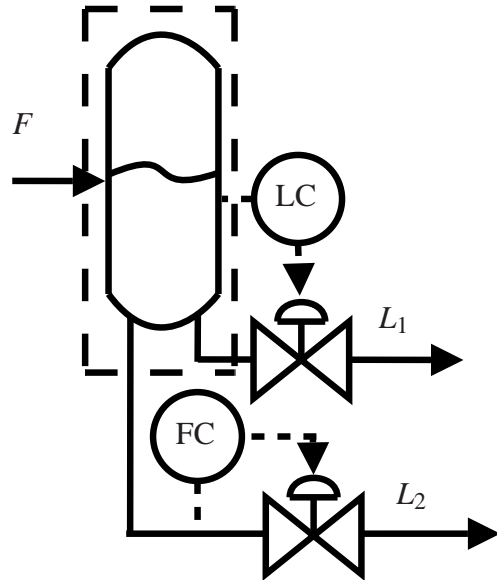

Figure 4: Consistent, but not self-consistent inventory control structure.

Example 4. *Phase inventories. In some cases, phase inventories needs to be considered for self-consistency. Consider Figure 5 where the inflow F is given. Thus, according to Rule 1, to have consistency the outflow must depend on the inventory in the tank.*

In Figure 5(a), the inflow is a single phase (liquid) and the outflow from the single-phase tank is split in two liquid streams (L_1 and L_2). There is one inventory, so for self-consistency, one of the outflows must be on inventory control whereas the other outflow can be flow controlled (adjustable split).

In Figure 5(b) the inflow is two-phase (liquid and vapor) and there are two inventories (liquid and vapor) that needs to be regulated. To have a consistent inventory control structure, both the

outflows (vapor and liquid) must be used for inventory control. In Figure 5(b) this is illustrated by the LC (liquid inventory) and PC (vapor inventory). In this case, the split is not adjustable in practice because the split is indirectly determined by the feed quality (fraction of vapor).

(a) Single-phase tank: Adjustable split.
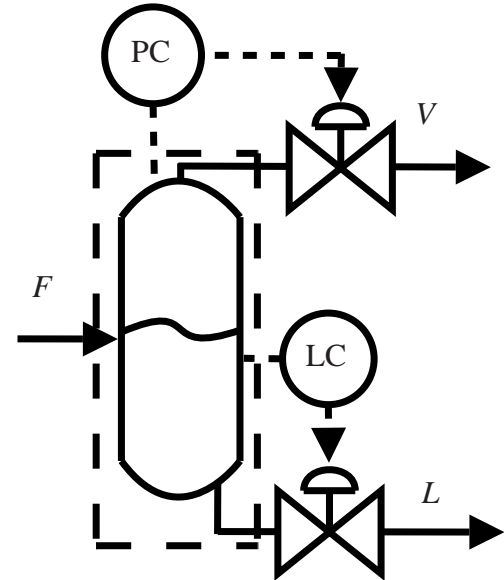
(b) Two-phase tank: Inventory control determines split.

Figure 5: Self-consistent inventory control of split with one and two phases.

Self-consistency of flow networks

Throughput manipulator.   In a flow network there is at least one degree of freedom, called the throughput manipulator (TPM), which sets the network flow. More generally, *a TPM is a degree of freedom that affects the network flow and which is not directly or indirectly determined by the control of the individual units, including their inventory control.*[9] Typically, a given flow (e.g. flow controller with an adjustable set point) is a TPM. As discussed in more detail below, the location of the TPM is very important. In particular, if the flow network has no splits or junctions, then for a given placement of the TPM, there is only one *self-consistent* inventory control system.

Flow split. In most cases a flow split is adjustable and thus introduces an extra degree of freedom for control of flow and inventory at the network level.[2] However, a flow split does not always introduce a degree of freedom for network flow as illustrated by the flash tank in Figure 5(b) where the two outflows are indirectly determined by the feed enthalpy (phase distribution). Another example, where a split does not give an extra degree of freedom for control of network flow, is a distillation column where the outflows (distillate flow D and bottoms flow B) are indirectly determined by the feed composition.

For an adjustable split and junction (e.g., multiple feeds), which introduces an extra degrees of freedom for control flow and inventory at the network level, a common choice is to use the largest flow for inventory control.[6] For example, with a given feed, the largest product stream may be used for inventory control with the flow rates of the smaller product streams used for quality control. Similarly, with a given production rate, the largest feed rate is often used for inventory control and the smaller feed flows are set in ratio relative to this, with the ratio set point possibly used for quality control.

The objective is now to apply the self-consistency rule to analyze inventory control structures for real processes (flow networks). We consider three network classes:

1. Units in series

2. Recycle systems

3. Closed systems

A series network may have splits, provided the flow is still in the same direction. Note that each single-phase split introduces one extra degree of freedom (the split ratio; see Figure 5). A recycle system contains one or more splits that are (partly) fed back to the system. A closed system has total recycle with no feeds or products.

Units in series ("radiating rule")

As mentioned above, if there are no splits or junctions, the location of the throughput manipulator determines the self-consistent inventory control structure. Specifically, a direct consequence of the self-consistency rule is

- *Inventory control must be in direction of flow downstream the location of a given flow (TPM).*

- *Inventory control must be in direction opposite to flow upstream the location of a given flow (TPM).*

More generally, we have:

Rule 2. *Radiation rule:[1] A self-consistent inventory control structure must be radiating around the location of a given flow (TPM).*

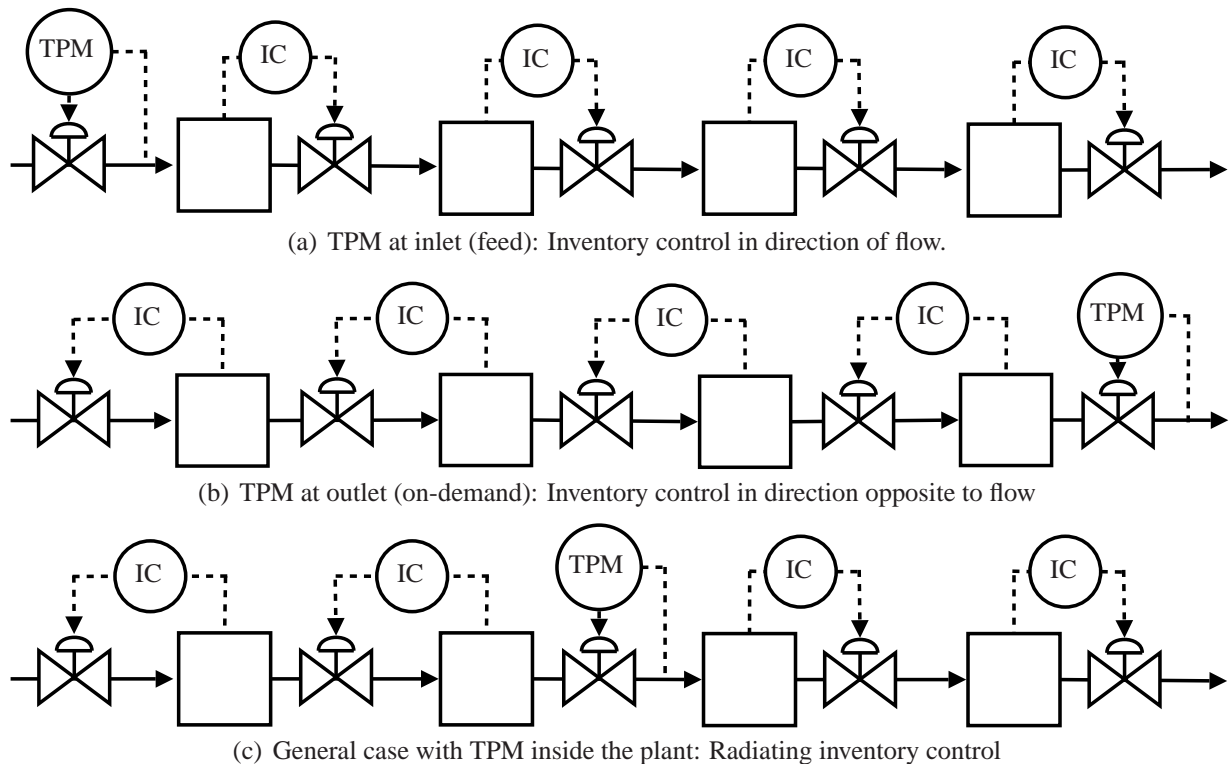These rules are further illustrated in Figure 6.

(a) TPM at inlet (feed): Inventory control in direction of flow.

(b) TPM at outlet (on-demand): Inventory control in direction opposite to flow

(c) General case with TPM inside the plant: Radiating inventory control

Figure 6: Self-consistency requires a radiating inventory control around a given flow (TPM).

Recycle systems

A recycle system has a flow split that usually introduces an extra degree of freedom for control of network flow and inventory. A simple recycle flow network with a given feed flow is considered in Figure 7 (there is a pump or compressor in the recycle loop which is not shown). The flow split introduces a degree of freedom, which means that we may fix one of the three remaining flows, and four alternative inventory control structures are shown. Figures 7(a) and 7(b) have consistent inventory control structures, because the outflows from units 1 and 2 depend on the inventory inside each unit. In both cases one flow in the recycle loop is set (flow controlled with an adjustable set point that may be used for other purposes than inventory control). Note that *the inventory control in the recycle loop can be either in direction of flow* (Figure 7(a)) or *direction opposite to flow* (Figure 7(b)), because the flow rate can be set at any location in the recycle loop.

In Figure 7(c) the inventory loops for units 1 and 2 are paired opposite. This structure is consistent (as the material balances are satisfied), but not self-consistent because the inventory of unit 2 is not "self-regulated by its in- or outflows" and thus violates Rule 1.

Finally, Figure 7(d) is obviously not consistent since both the feed rate and the product rate are given. In particular, the inflow and outflow to the dotted box do not depend on the inventory inside this part of the process, which violates Rule 1.
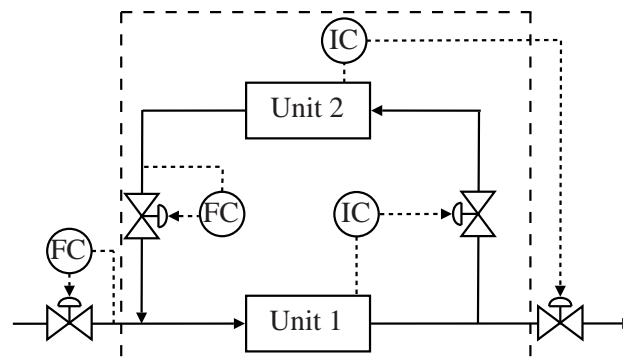
Closed systems

Closed systems require particular attention. First, the total inventory is constant. Second, since there are no in- or out streams our previously derived rule (Rule 1) does not really apply to the overall system. As an example, consider a closed system with two inventories. In Figure 8(a) we follow Rule 1 and attempt to control both inventories, but the two loops will "fight each other" and will drift to a solution with either a fully open or fully closed valve. For example, a (feasible) solution is to have zero flow in the cycle. The problem is that the flow is not set anywhere in the loop. To get a consistent inventory control structure, *one must let one of the inventories be uncontrolled*, as shown in Figures 8(b) and 8(c). *The corresponding unused degree of freedom*
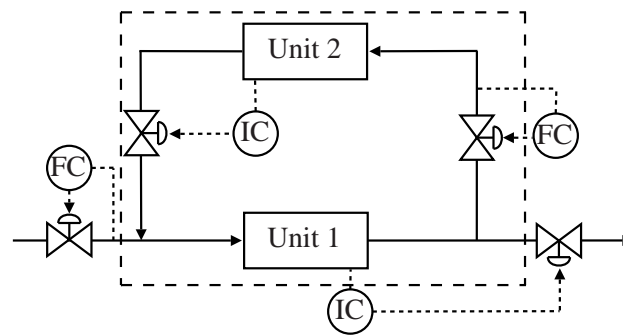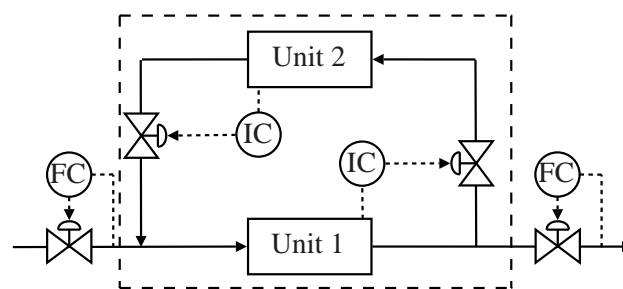
(a) Self-consistent inventory control.
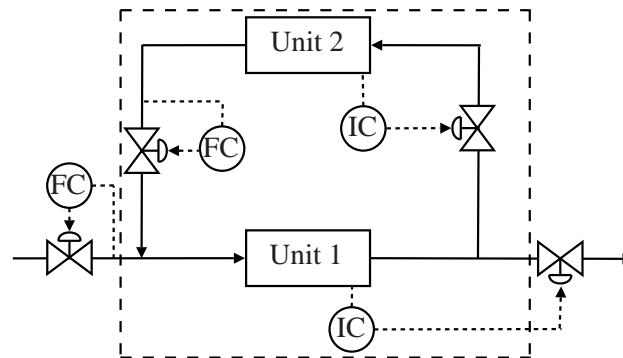
(b) Self-consistent inventory control.

(c) Consistent, but not self-consistent inventory control.

(d) Not consistent inventory control.

Figure 7: Inventory control of simple recycle process with given feed.

(flow) sets the flow rate ("load", throughput) of the closed system.

To be able to use our self-consistency rule (Rule 1) for closed systems there are two alternative "fixes":

1. Let the total inventory be uncontrolled (*not* self-regulated), which is how such systems are usually operated in practice. Typically the largest single inventory is uncontrolled. However, the remaining inventories must be self-regulated, as usual, to have self-consistency of the inventory control system.

2. Introduce a "dummy" (small) stream that keeps the total inventory constant. This corresponds to allowing for filling (charging) or emptying the system. In practice, this stream may be a make-up stream line that refills or empties the largest inventory, e.g. on a daily or monthly basis.

Both approaches allow for disturbances, such as leaks or supply. The inventory control system can then be analyzed using the normal self-consistency rule (Rule 1). Figure 8(a) is clearly not allowed by Fix 1 as the total inventory is not left uncontrolled. Figure 8(a) is also not consistent by Fix 2, since for self-consistency the dummy inlet stream must be used for inventory control instead of one of the two flows in the recycle loop.

Example 5. *Absorber-regenerator example. In this example, the consistency rule (Rule 1) is used for an individual phase (liquid), which forms a closed system. Consider the absorber and regenerator example in Figure 9[2] where a component (e.g. CO_2) is removed from a gas by absorption. The inlet gas flow (feed) is indirectly given because there is a pressure control in the direction of flow at the inlet. The gas outlet flows are on pressure control in the direction of flow and thus depend on the gas holdup in the plant. Therefore the gas-phase inventory control is consistent. However, the liquid flows between the absorber and regenerator make up a "closed system" (expect for minor losses). There is a flow controller for the recycled liquid, but its set point is set by the inventory in the regenerator, hence all inventories in the closed system are on inventory control, which violates the rule just derived. To get a consistent inventory control structure, we must break the level-flow*
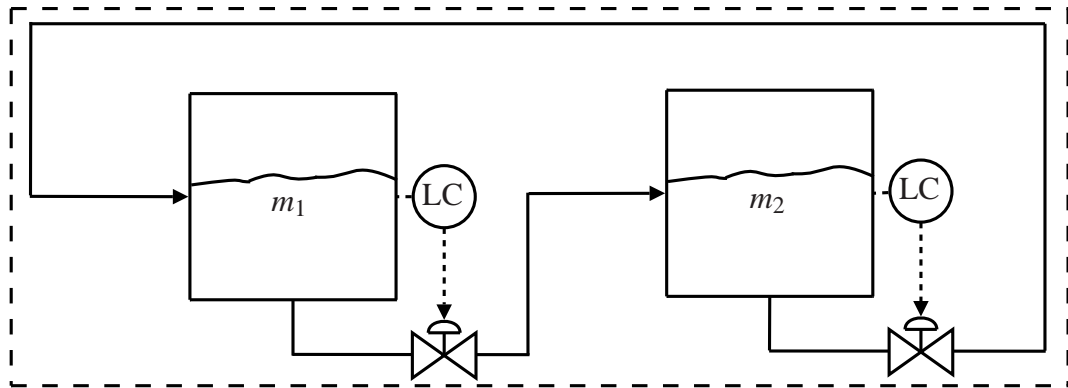
(a) Not consistent (because there is no uncontrolled inventory).
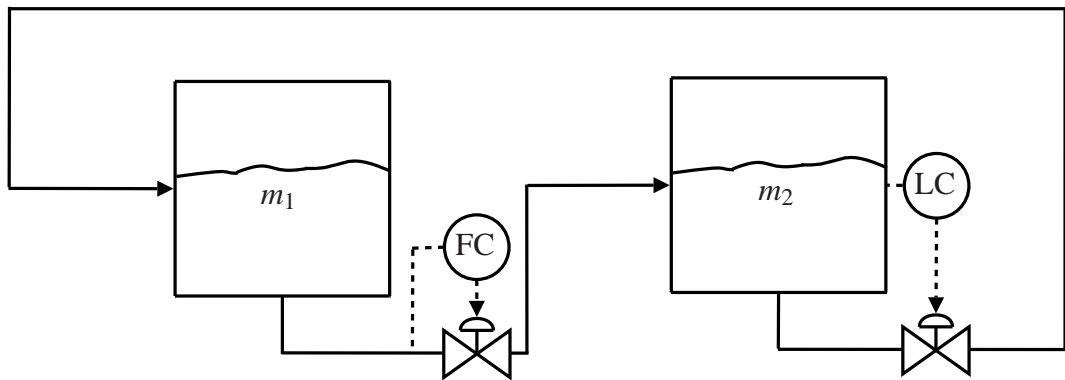
(b) Self-consistent (inventory m_1 is uncontrolled).
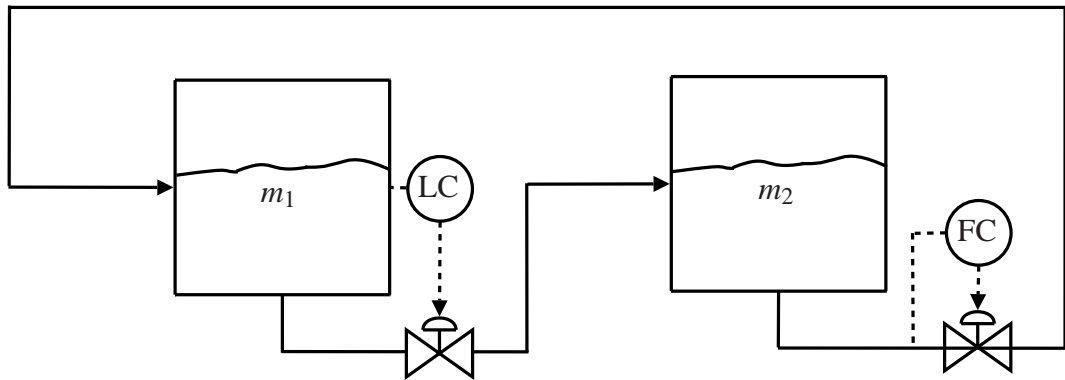
(c) Self-consistent (inventory m_2 is uncontrolled).

Figure 8: Inventory control for closed system.

cascade loop and let the inventory in the bottom of the regenerator remain uncontrolled. Alterna-
tively, let the absorber liquid inventory be uncontrolled and break the level-flow cascade loop and
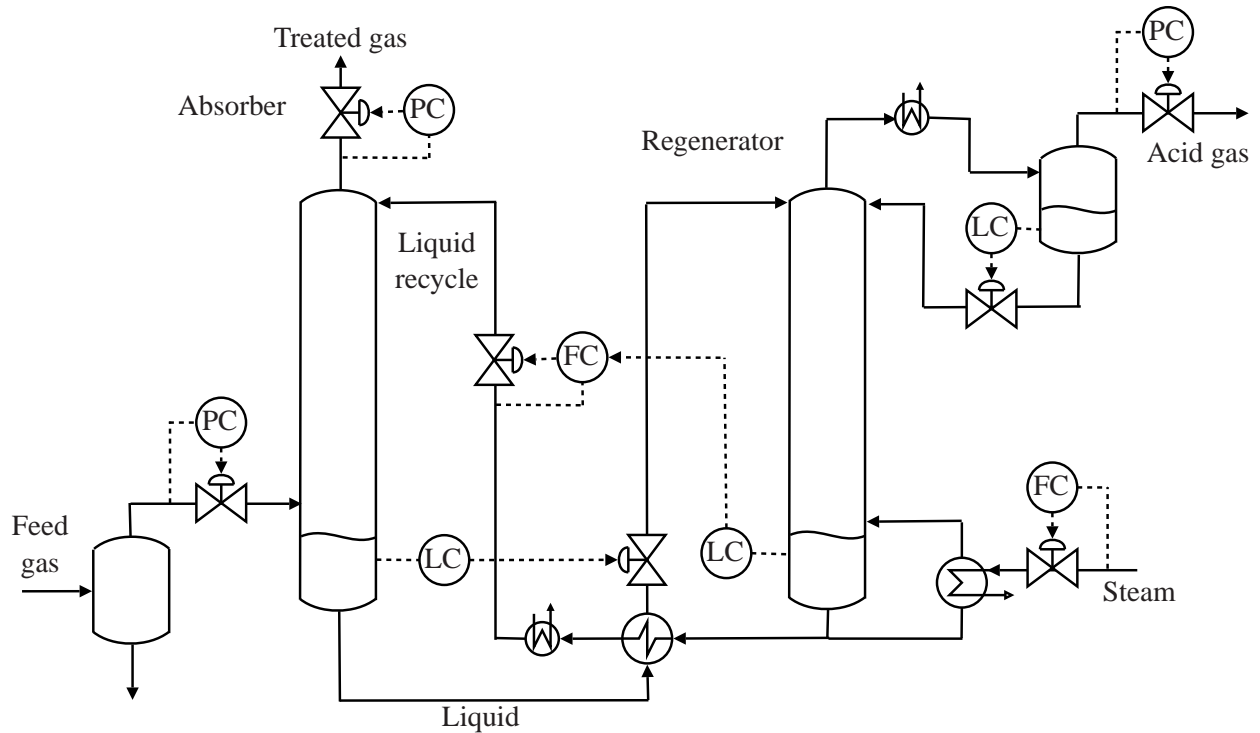*let the feed into the regenerator control the regenerator liquid inventory.*

Figure 9: Absorber and regenerator example: Not consistent liquid inventory control.

Summary and discussion of specific rules

In the literature there are many rules that deal with the inventory control structure. In addition to
the radiating rule (Rule 2), some useful rules that can be developed from the self-consistency rule
(Rule 1) are:

1. *All systems must have at least one given flow.*

   Proof. Assume there is no given flow such that all flows are on inventory control. This will not result
   in a unique solution, for example, zero flow will be an allowed solution. □

2. *Component balance rule Downs [5], p. 414: Each component, whether important or insignificant, must have its inventory controlled within each unit operation and within the whole process.* This is also referred as "Downs drill" in Luyben et al. [10], p. 56.

   Proof. This comes from the requirement of component self-consistency (Rule 1). □

3. *A stream cannot be flow controlled more than once, that is, a structure with two flow controllers on the same stream is not consistent.*

   Proof. Make a control volume with the two flow-controlled streams as in- and outflows. Then neither the inflow nor the outflow depends on the control volume and the inventory is not self-regulated. This is demonstrated in Figure 2(b). □

4. *Price and Georgakis [1], p.2699: If a change in the throughput manipulator does not result in a change in the main feed flow, then the control structure is inconsistent.*

   Justification. This follows from the total steady-state mass balances. □

5. *Generalized from Price and Georgakis [1], p.2699: A self-consistent inventory control structure must use the feed or the product (or both) for inventory control.*

   Justification. This follows from the total steady-state mass balance. This is also discussed in Section "Units in series" and a clear illustration of this statement is found in Figure 6. □

6. *For closed systems: One inventory must be left uncontrolled and one flow in the closed system must be used to set the load.*

   Justification. This follows from that all systems must have at least one given flow to be unique. To be able to set the load for a closed system, one inventory must be uncontrolled. □

The rules are summarized by the proposed procedure for inventory control system design in Table 1, which is inspired by the inventory control guidelines of Price et al. [4].

Table 1: Proposed guidelines for design of self-consistent inventory control system. In case of doubt consult the general self-consistency rule (Rule 1).

1	Choose the location of the throughput manipulator
2	Identify inventories that need to be controlled including: a) Total mass b) Components c) Individual phases
3	Identify manipulators suitable for adjusting each inventory
4	Design a self-consistent radiation inventory control system that controls all the identified inventories. This means: a) Inventory control in direction of flow downstream the throughput manipulator b) Inventory control in direction opposite to flow upstream the throughput manipulator
5	At junctions or splits a decision has to be made on which flow to use for inventory control. Typically, the largest flow is used, or both streams are changed such that their ratio is held constant (the ratio itself is often set by a slower outer composition loop).
6	Recycles require special consideration. Make a block (control volume) around the entire section and make sure that there is self-consistency for total mass, (individual) components and phases (if relevant).
7	Assign control loops for any process external flow that remain uncontrolled. Typically, "extra" feed rates are put on ratio control with the ratio set point being set by an outer composition loop.

Remark. Luyben [11] provides the rule to "fix a flow in each recycle". If we interpret the term "fix a flow" to mean "do not use a flow for inventory control", then this rule follows from the requirement of self-consistency provided the recycle loop contains a split that introduces an extra degree of freedom. However, if no degree of freedom is introduced by the recycle, as is in the case if we have a separator or flash where the split is (indirectly) fixed by the feed properties (e.g. see Figure 5(b)), then this rule is *not* a requirement, (e.g. see the self consistent control structure in Figure 11(a), where all the flows in the recycle loop are on inventory control).

Examples

In this section we apply the self-consistency rule to some more difficult examples from the academic literature where self-consistency of component inventory is an issue.

Example 6: Distillation column with DB-configuration

An example of a recycle system is a distillation column. As seen from Figure 10, a distillation column has one split in the condenser (V_T splits into L and D) and one split in the reboiler (L_B splits into B and V). In both cases one of the streams is recycled to the column (L and V, respectively). The two splits introduce two degrees of freedom and this gives rise to many possible inventory control structures ("configurations"), as has been discussed widely in the literature (see Skogestad [12] for a summary of this discussion).

Figure 10 displays the DB-configuration, which uses reflux L and boilup V for inventory control (condenser and reboiler level control), such that the flows of D and B remain as degrees of freedom for other purposes. The DB-configuration has earlier been labeled "impossible", "unacceptable" or "infeasible" by distillation experts.[13,14] This inventory control system also violates Luybens rule of "fixing a flow in the recycle loop" and it is indeed true that this inventory control system is not self-consistent. To see this, consider the dashed box in Figure 10 where we note that none of the flows in or out of the column (F, D and B) depend on the inventory inside the column. However, an inconsistent inventory control system can usually be made consistent by adding control loops outside the local units (which here are the tanks with M_D and M_B) and the DB-configuration is workable (and consistent) provided one closes at least one extra loop, for example by using D to control a temperature inside the column.[15,16] Thus, labeling the DB-configuration as "impossible" is wrong. In summary, the DB-configuration is not self-consistent, but it can be made consistent by adding a temperature (or composition) control loop.

Remark 1 An example of a self-consistent inventory control structure for distillation is the common LV-configuration, where the two level loops have been interchanged such that D and B are used for level control and L and V remain as degrees of freedom (e.g. on flow control). In the LV-configuration, inventory is controlled in the direction of flow, as expected since the feed is given.

Remark 2 An additional inventory issue for distillation columns is related to the split between light and heavy components (component inventory). One may regard the column as a "tank" with light component in the upper part and heavy in the lower part. Thus, one is not really free to set the split between D and B and
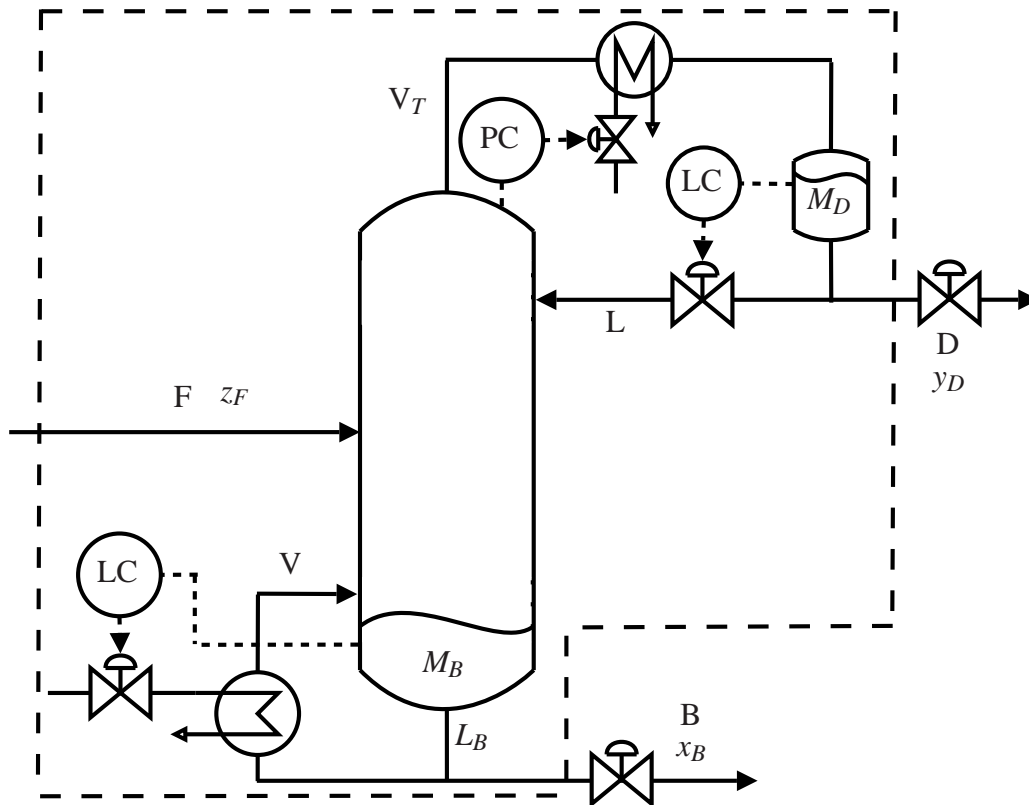
Figure 10: Example of inconsistent inventory control: Distillation column with DB-configuration.

to avoid a "drifting" composition profile (with possible "breakthrough" of light component in the bottom or of heavy component in the top), one must in practice close a quality (e.g., temperature or pressure) loop to achieve component self-consistency.[12] For example, for the LV-configuration one may use the boilup V to control a temperature inside the column. This consideration about controlling the column profile also applies to the DB-configuration. Thus, in practice, the DB-configuration requires closing *two* quality loops to maintain mass and component balances, otherwise the split D/F (or B/D) will be fixed and there is no adjustment to changes in feed composition. This means that both D and B must be used for quality control for the DB-configuration, rather than only one (L or V) for the LV-configuration.

Example 7: Reactor-separator-recycle example with one reactant

A common recycle example from the academic literature is the reactor-separator-recycle system in Figure 11. The system has a continuously stirred-tank reactor (CSTR) with an irreversible, isothermal, first order reaction $A \rightarrow B$, followed by separation (distillation) and recycle of the

unreacted feed component back to the reactor (e.g. [1,17–19]). Note that in this case the recycle does not introduce an extra degree of freedom for control of flow at the network level because the split in the distillation column is indirectly determined by the column feed composition.

The feed (F_0) is pure reactant A and the steady-state component mass balances give

$$\text{Component A:} \quad F_0 = \underbrace{k(T) \cdot x_{r,A} \cdot V}_{-G_A = G_B} + B \cdot x_{B,A}$$

$$\text{Component B:} \quad \underbrace{k(T) \cdot x_{r,A} \cdot V}_{G_B} = B \cdot x_{B,B}$$

where x is the mole fraction, V is the reactor volume and $k(T)$ is the reaction rate constant. Note that $B = F_0$ [mol/s] at steady-state. Component A enters the process in the feed stream and its conversion (consumption) in the reactor increases with the amount of A. The inventory of component A is therefore expected to be sufficiently self-regulated by the reaction. Component B is produced in the reactor (G_B) and exits the process in stream B. Component B is not self-regulated by the reaction (because the reaction rate is independent of the amount of B) and thus requires a controller to adjust its inventory.

Two different control structures for the reactor-separator-recycle process are displayed in Figure 11. Both have given feed (F_0) and inventory control is in the direction of flow. Thus, both of them are self-consistent in total mass, because the outflow B from the process depends on the inventory inside the process (indicated by the dashed control volume) (Rule 1). Since the outflow B mainly consists of component B, this implies that both structures are also consistent (self-regulated) with respect to the inventory of component B. The difference between the two structures is related to the control of component A. The "conventional" structure in Figure 11(a) uses the LV-configuration for the distillation column where the reflux (L) controls the composition in the recycle (distillate) D. The structure in Figure 11(b) uses the DV-configuration for the column where the reactor composition $x_{r,A}$ is controlled instead of the recycle (distillate) composition.

As already mentioned, the inventory of component A is expected to be self-regulated by the
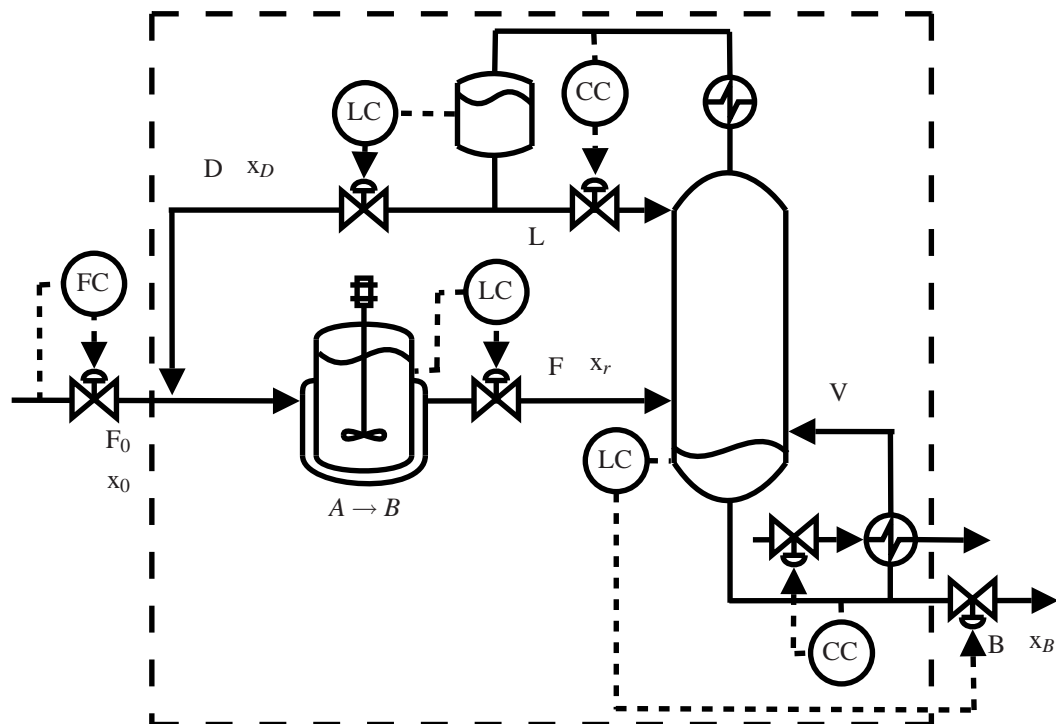
(a) Conventional: Self-consistent inventory control structure.
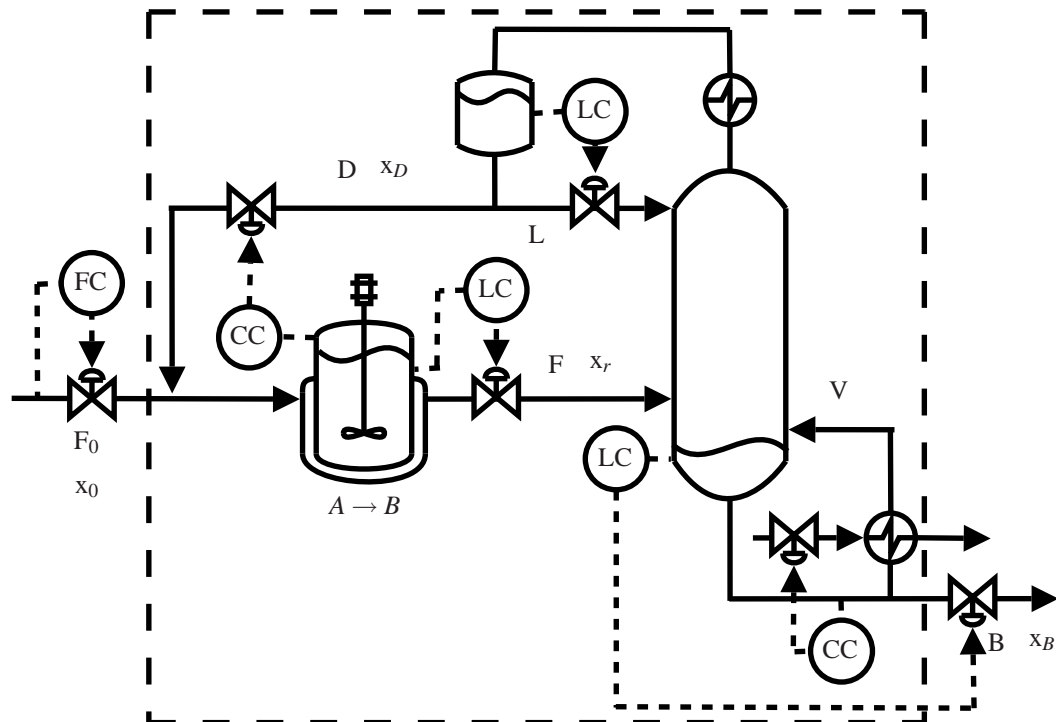
(b) Composition control of reactor composition: Not consistent for component A.

Figure 11: Reactor-separator-recycle process with one reactant (A).

reaction $A \rightarrow B$, so one would expect both structures to be consistent with respect to component A. In fact, both structures would be consistent if one *removed* the composition controller (CC) in the recycle loop (thus, setting reflux L in Figure 11(a) and setting recycle D in Figure 11(b)). With the composition loop closed, the "conventional" structure in Figure 11(a) remains consistent, but not the structure with control of reactor composition in Figure 11(b). The reason for the inconsistency is that control of reactor composition eliminated the self-regulation by reaction: The amount of A that reacts is given by $-G_A = G_B = k(T)x_{r,A}V$ and with given T, V and $x_{r,A}$ (because of the controller) and there is no self-regulation. The inconsistency of this control structure is pointed out by e.g.[5,20]

Remark 1 The control structures in Figure 11 would both be self-consistent without composition control (CC), that is, with (a) L given or (b) Dgiven. The reason for closing these composition loops is therefore not for consistent inventory control but rather for other (economic) reasons.[19] The interesting point to note, is that closing an extra loop can in some cases make the system inconsistent (Figure 11(b)).

Remark 2 Luyben[20] has proposed to make the system in Figure 11(b) consistent by introducing an adjustable reactor volume, but this is not generally a good solution, because we want to use the maximum reactor volume for economic reasons (including energy saving).[19]

Remark 3 The inventory of component A is expected to be self-regulated by the reaction $A \rightarrow B$. More precisely, the amount that reacts is $-G_A = kx_{r,A}V$ and the composition $x_{r,A}$ will "self-regulate" such that at steady-state (assuming $x_{B,A} \approx 0$): $F_0 \approx -G_A$, that is, $x_{r,A} \approx F_0/(kV)$.

Remark 4 We already noted that setting $x_{r,A}$ (Figure 11(b)) breaks this self-regulation and makes the system inconsistent. A related problem is when the reactor volume V is too small relative to the feed F_0, such that the required $x_{r,A}$ exceeds 1, which is impossible. In practice, if we increase the feed rate F_0 and approach this situation, we will experience "snow-balling"[11] where the recycle D becomes very large, and also the boilup V becomes very large. Eventually, V may reach its maximum value, and we loose composition control and we will get "break-through" of A in the bottom product. Snow-balling is therefore a result of a too small reactor.

Remark 5 Consider the same process (Figure 11), but assume that the fresh feed (F_0) contains an inert component I in addition to the reactant A. If I is more volatile than component B, then component I will be recycled back to the reactor and will accumulate in the process. None of the inventory control systems in Figure 11 are consistent for the inert I. To make the system self-consistent for the inert, a purge stream must be introduced where part of stream D is taken out as a by-product.
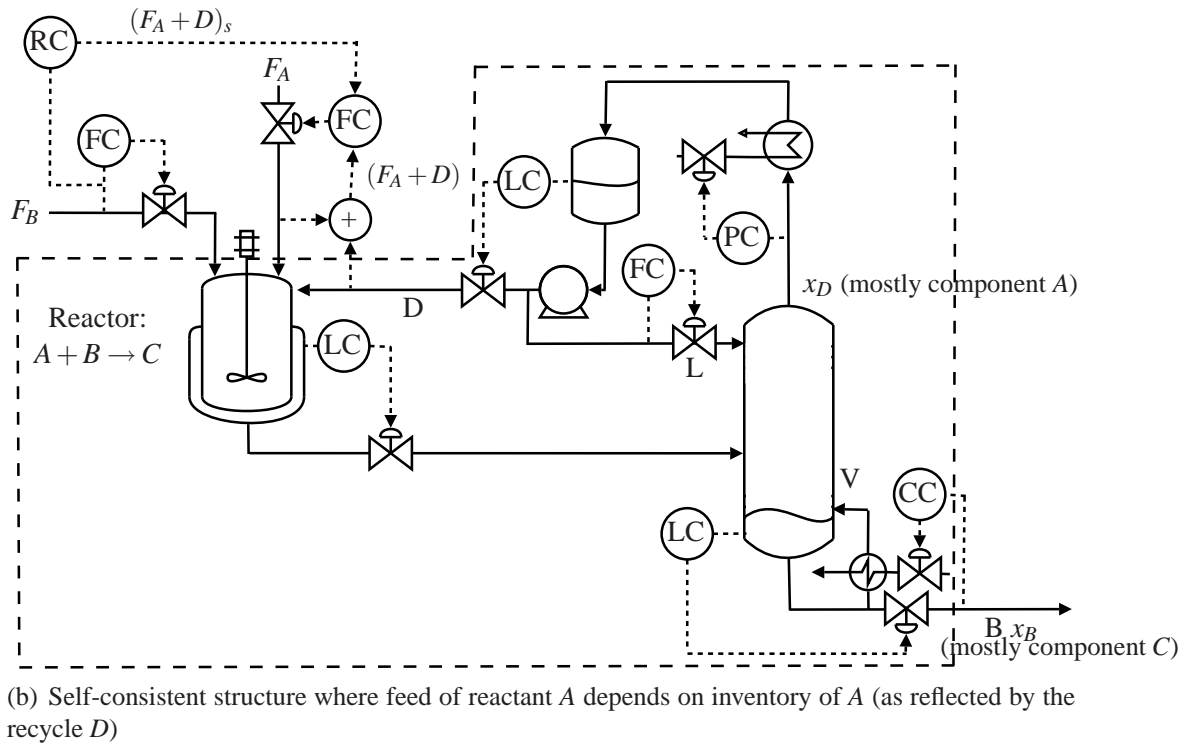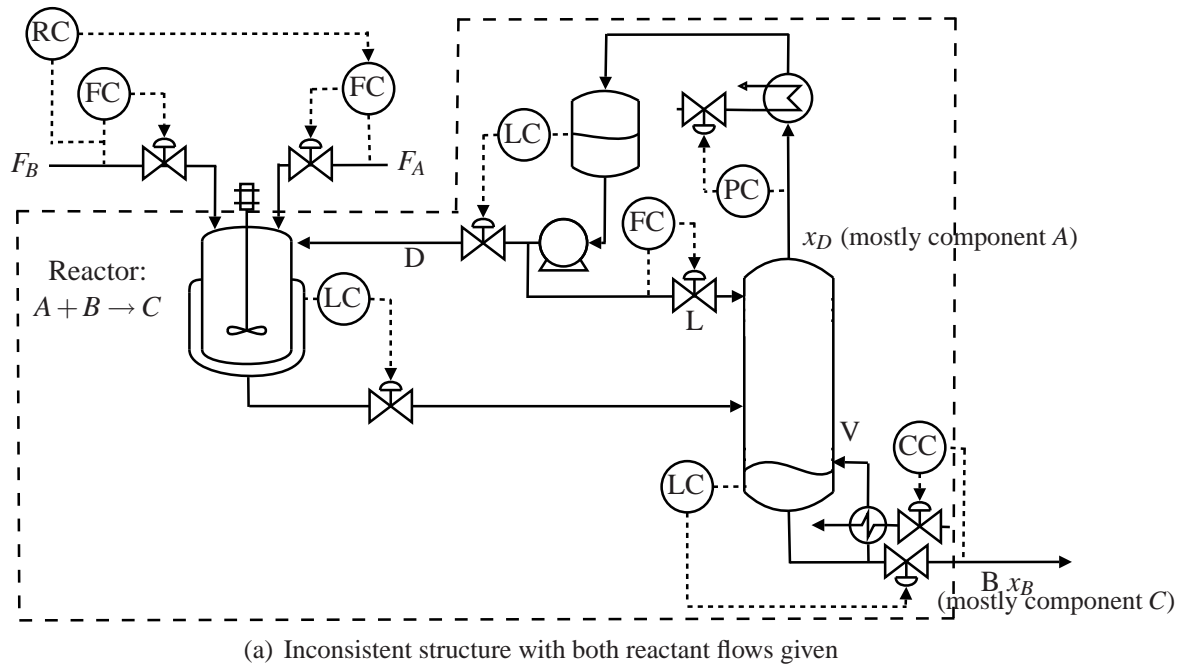
Example 8: Reactor-separator-recycle process with two reactants

Another well studied recycle example is a reactor-separator-recycle process where two reactants A and B reacts according to the reaction $A + B \rightarrow C$ (e.g.[21]). Component B is the limiting reactant as the recycle D contains mostly component A. Two different control structures are displayed in Figure 12. In both cases the distillate flow D (recycle of A) is used to control the condenser level (main inventory of A).

In Figure 12(a), both fresh reactant feeds (F_A and F_B) are flow controlled into the reactor, where reactant A is set in ratio to reactant B such that $F_A/F_B = 1$. This control strategy is not consistent because one of the two feeds depend on the inventory inside. This follows because it is not possible to feed exactly the stoichiometric ratio of the two reactants[10] and any imbalance will over time lead to a situation where the recycle of A either goes towards zero or towards infinity.

To get a consistent inventory control structure, the first requirement is that one of the feed rates (F_A or F_B) must depend on what happens inside the process, such that we at steady-state can achieve $F_A = F_B$. One solution is to set F_B (the limiting reactant) and adjust F_A such that the desired excess of A is achieved, resulting in the self-consistent control structure in Figure 12(b). Here F_A depends on the inventory of A as reflected by the recycle flow D by keeping the reactor feed ratio $(F_A + D)/F_B$ constant at a given value (larger than 1 to make B the limiting reactant). The structure is consistent for all components: C has an outlet in the bottom of the column; B is self-regulated by reaction because it it the limiting reactant, and the feed of A depends on the inventory of A.

There exist also other consistent inventory control structures, e.g. see Figure 2.11(b) in Luyben et al.[10], but these seem to be more complicated than the one proposed in Figure 12(b). For

(a) Inconsistent structure with both reactant flows given

(b) Self-consistent structure where feed of reactant A depends on inventory of A (as reflected by the recycle D)

Figure 12: Reactor-recycle system with two reactants ($A + B$).

example, one could keep the recycle D constant and use F_A to control the condenser level (main inventory of A), but the dynamics for this "long level" loop are not favorable and this consistent structure is not self-consistent.

Conclusion

Consistency is a required property since all inventories must be regulated (kept within bounds). A desired property is to have self-consistency where only local loops are used for regulation of inventory. Self-consistency of a given control system can be checked by using the proposed self-consistency rule (Rule 1). The self-consistency rule follows from mass balances for total mass, component and individual phases, and its use for control design is summarized in Table 1.

The self-consistency rule may be regarded as "obvious", but has nevertheless proven to be very useful in many applications, and is consistent with previously proposed rules. For example, a direct consequence of the self-consistency rule is the "radiation rule",[1] which states that the inventory control structure must be radiating around the location of a given flow. The self-consistency rule can also be applied to more complex cases where previously proposed rules fail.

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