

Self-consistent inventory control

Elvira Marie B. Aske,^{†,‡} and Sigurd Skogestad^{*,†}

*Department of Chemical Engineering, Norwegian University of Science and Technology,
Trondheim, Norway*

E-mail: skoge@ntnu.no

Abstract

Inventory or material balance control is an important part of process control. A requirement is that the inventory control system is consistent meaning that the steady-state mass balances (total, component and phase) for the individual units and the overall plant are satisfied. In addition, self-consistency is usually required, meaning that the mass balances are satisfied locally with local inventory loops only. In practice, if a control structure is inconsistent, then at least one control valve will become fully open (or in rare cases closed) and cannot attain its set point. The main result of this paper is a self-consistency rule for evaluating the consistency of inventory control systems.

Introduction

One of the more elusive parts of process control education is inventory or material balance control. An engineer with some experience can usually immediately say if a proposed inventory control system is workable. However, for a student or newcomer to the field it is not obvious, and even for an experienced engineer there may be cases where experience and intuition are not sufficient. The

[†]Norwegian University of Science and Technology

[‡]Current address: StatoilHydro Research Center, Trondheim, Norway

objective of this paper is to present concise results on inventory control, relate to previous work, tie up loose ends, and to provide some good illustrative examples. The main result (self-consistency rule) can be regarded as obvious, but nevertheless we have not seen it presented in this way before.

The main result is a simple rule to check whether an inventory control system is *consistent*. Here, consistency means that the mass balances for the entire plant are satisfied.¹ In addition, we usually want the inventory control system to be *self-consistent*. Self-consistency means that, in addition to plantwide consistency, the mass balance for each unit is satisfied by itself (locally), without the need to rely on control loops outside the unit. Consistency is a required property, because the mass balances must be satisfied in a plant, whereas self-consistency is a desired property of an inventory control system. In practice, an inconsistent control structure will lead to a situation with a fully open or closed control valve and the associated control loop cannot fulfill or attain the control set point.

In most plants, we want the inventory control system to use simple PID controllers and be part of the basic (regulatory) control layer. This is because it is generally desirable to separate the tasks of regulatory (stabilizing) control and supervisory (economic) control. From this it follows that the structure of the inventory control system is usually difficult to change later.

The importance of consistency of inventory control structures is often overlooked. Our work is partly inspired by the many examples of Kida, who has given industrial courses in Japan on control structures for many years. In a personal communication² he states that “*most process engineers, and even academic people, do not understand the serious problem of consistency of plantwide control configurations. When writing a paper, you have to clearly explain this point and make them convinced at the very outset. Otherwise they will not listen to or read through your detailed statements, but skip them all*”.

A very good early reference on inventory control in a plantwide setting is Buckley.³ He states that material balance control must be in the direction of flow downstream a given flow and opposite the direction of flow upstream a given flow. Price and Georgakis extended this and state that the inventory control must “radiate” outwards from the point of a given flow (throughput manipula-

tor).^{1,4} As shown in this paper, all these statements are a consequence of requiring the inventory control system to be self-consistent.

Downs⁵ provides a very good discussion of material balance control in a plantwide control environment, with many clarifying examples. However, it is somewhat difficult for the reader to find a general rule or method that can be applied to new cases.

Luyben et al.⁶ propose a mainly heuristic design procedure for plantwide control. The procedure consist of, among others, “*Step 6. Control inventories (pressures and levels) and fix a flow in every recycle loop*”. Possible limitations of this guideline are discussed in the present paper. Another guideline of Luyben et al. is to “*ensure that the overall component balances for each chemical species can be satisfied either through reaction or exit streams by accounting for the component’s composition or inventory at some point in the process*”. As discussed later, this guideline is a bit limited because entrance (feed) streams is not considered.

Specific guidelines for designing inventory control structures are presented by Georgakis and coauthors.^{1,4} They propose a set of heuristic guidelines for inventory control design in a plantwide environment and also discuss consistency. The authors also state the importance of a self-consistent inventory control structure: “*Self-consistency appears to be the single most important characteristic governing the impact of the inventory control structure on system performance*”.

As already mentioned, Fujio Kida from JGC Corporation in Japan has developed a lot of teaching material² and written several papers on inventory control, e.g.⁷ Unfortunately, the work is published in Japanese only, but nevertheless it is clear that there are many detailed rules and some require detailed calculations. Our objective is to derive, if possible, a single rule for evaluating the consistency of inventory control system that applies to all cases and that only requires structural information.

The objective of this paper is to present a simple rule, self-consistency rule, for evaluating the consistency of inventory control systems. Note that the present paper focuses on analysis of a given control structure. The design of the inventory control system, which in particular is related to the placement of the throughput manipulator, is discussed in more detail in Aske.⁸

Remark on notation: In this paper, when a flow is left unused or with a flow controller (FC), then this indicates that this flow is *not* used for inventory control. Instead the flow is either (1) used as a throughput manipulator (TPM), (2) given by another part of the plant (disturbance for our part), (3) fixed or (4) left as a degree of freedom for other control tasks. Also note that the general term used in this paper for an inventory controller is IC. This usually involves a level controller (LC) (liquid) or a pressure controller (PC) (gas).

Definition of self-consistent inventory control

The dynamic mass balance for total or component mass in any unit or process section can be written:⁵

$$\text{Inflow} + \text{Generation} - \text{Outflow} - \text{Consumption} = \text{Change in inventory} \quad (1)$$

To keep the inventory within bounds, the change in inventory must be within bounds, and over a long time (at steady-state) the change in inventory must be zero. Thus, there must be a balance between the In-terms (inflow + generation) and Out-terms (outflow + consumption). However, without control this is not necessarily satisfied. The main objective of the inventory control system is to “stabilize” or provide “self-regulation” of all inventories such that the mass balances are satisfied. This leads to the self-consistency rule, which is the main result in this paper, but let us first defines some terms.

Definition 1. Consistency. *An inventory control system is said to be **consistent** if the steady-state mass balances (total, components and phases) are satisfied for any part of the process, including the individual units and the overall plant.*

Remark. The use of mass balances for a phase may seem odd, and is discussed in more detail in the next section.

Since the mass balance must be satisfied for the overall plant, it follows that a consistent inventory control system must be “able to propagate a production rate change throughout the process

and in particular if such a change produces changes in the flow rates of major feed and product streams”.¹

Note that the above definition of consistency allows for “long loops” (not local loops) where, for example, the feed rate controls the inventory at the other end of the process (as illustrated in Figure Figure 4). This is often undesirable and self-consistency is when the steady-state mass balances are satisfied also locally. More precisely, we propose the following definition:

Definition 2. Self-consistency. *A consistent inventory control system is said to be **self-consistent** if there is local “self-regulation” of all inventories. This means that for each unit the local inventory control loops by themselves are sufficient to achieve steady-state mass balance consistency for that unit.*

Remark 1 “Self-regulation” here refers to the response of the process with its inventory control system in operation. If self-regulation is achieved without active control then this is referred to as “true” self-regulation.

Remark 2 The term “local inventory control loops” means that no control loops involving manipulated variables outside the unit are needed for inventory control of the unit (see Figure 4 for a system that does not satisfy this requirement).

Remark 3 The definitions require that the “steady-state mass balances” are satisfied. We are here referring to the *desired* steady-state, because an inconsistent inventory control system may give a steady-state but it is not the desired one. For example, a component with no specified exit will eventually have to exit somewhere but this may not be a desired operation point.

Example 1. Self-regulation. *“Self-regulation” may or may not require “active” control, as mentioned in Remark 1. As an example, consider regulation of liquid inventory (m) in a tank; see Figure 1(a). The outflow is given by a valve equation*

$$\dot{m}_{out} = C_v f(z) \sqrt{\Delta p \cdot \rho} \quad [\text{kg/s}] \quad (2)$$

where z is valve position. The pressure drop over the valve is

$$\Delta p = p_1 - p_2 + \rho gh \quad (3)$$

where h is the liquid level, which is proportional to the mass inventory, e.g., $m = h\rho A$ for a tank with constant cross section area A . If the pressure drop Δp depends mainly on the liquid level h , then the inventory m is self-regulated. This is the case in Figure 1(a) where $p_1 = p_2$ so $\Delta p = \rho gh$ and the entire pressure drop over the valve is caused by the liquid level. Thus, $\dot{m}_{out} \sim \sqrt{h}$, which means that without control a doubling of the flow \dot{m}_{out} will result in a four times larger liquid level (h). If this change is acceptable, then we have self-regulation. In other cases, it may be necessary to use “active” control to get sufficient self-regulation of the inventory. Specifically: In Figure 1(b), $p_1 - p_2 = 99$ bar so the relative pressure contribution from the liquid level (ρgh) is much too small to provide acceptable self-regulation. For example, for a large tank of water with $h = 10$ m, the contribution from the level is only about 1 % ($\rho gh \approx 1000 \text{ kg/m}^3 \cdot 10 \text{ kg m/s}^2 \cdot 10 \text{ m} = 10^5 \text{ N/m}^2 = 1 \text{ bar}$). In this case “active” control is required, where the level controller (LC) adjusts the valve position z , see Figure 1(b).

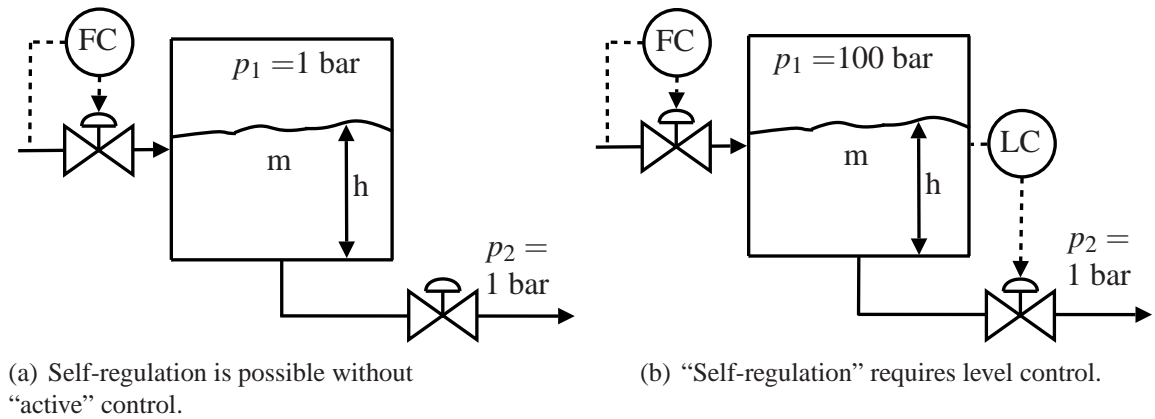


Figure 1: Self-regulation of inventory in a tank with a given feed rate.

Self-consistency rule

As a direct consequence (implication) of the statements in the section above, we propose the following rule to check if an inventory control system is self-consistent.

Rule 1. “Self-consistency rule”: *Self-consistency (local “self-regulation” of all inventories) requires that*

1. *The total inventory (mass) of any part of the process (unit) must be “self-regulated” by its in- or outflows, which implies that at least one flow in or out of any part of the process (unit) must depend on the inventory inside that part of the process (unit).*
2. *For systems with several components, the inventory of each component of any part of the process must be “self-regulated” by its in- or outflows or by chemical reaction.*
3. *For systems with several phases, the inventory of each phase of any part of the process must be “self-regulated” by its in- or outflows or by phase transition.*

Remark 1 The above requirement must be satisfied for “any part of the process”. In practice, it is sufficient to consider the individual units plus the overall process.

Remark 2 A flow that depends on the inventory inside a part of the process, is often said to be on “inventory control”. Inventory control usually involves a level controller (LC) (liquid) or pressure controller (PC) (gas and in some cases liquid), but it may also be a temperature controller (TC), composition controller (CC) or even no control (“true” self-regulation, e.g. with a constant valve opening). Obviously, a flow controller (FC) can not be used for inventory control because flow is not a measure of inventory.

Remark 3 It is possible to extend the “self-regulation” rule to energy inventory, but this is not done here. We also doubt if such an extension is very useful, because in most cases the energy balance will maintain itself by “true” self-regulation (without control), for example because a warmer inflow in a tank leads to a warmer outflow.

Proof of self-consistency rule.

1. A boundary (control volume) may be defined for any part of the process. Let m [kg] denote the inventory inside the control volume and let \dot{m}_{in} and \dot{m}_{out} [kg/s] denote in- and outflows. Then the (total) mass balance is

$$\frac{dm}{dt} = \sum \dot{m}_{\text{in}} - \sum \dot{m}_{\text{out}} \quad [\text{kg/s}] \quad (4)$$

If \dot{m}_{in} and \dot{m}_{out} are independent (or weakly dependent) of the inventory m then this is an integrating (or close to integrating) process where m will not return to its desired steady-state (it will drift to an undesirable steady-state). To stabilize the inventory we must have “self-regulation” where \dot{m}_{in} or \dot{m}_{out} depends on the inventory (m), such that m is kept within given bounds in spite of disturbances. More precisely, \dot{m}_{in} must decrease when m increases or \dot{m}_{out} must increase when m increases, such that m is kept within given bounds in spite of disturbances.

2. Similarly, let n_A [mol A] denote the inventory of component A inside the control volume and let $\dot{n}_{A,\text{in}}$ and $\dot{n}_{A,\text{out}}$ [mol A/s] denote the in- and outflows. The mass balance for component A is

$$\frac{dn_A}{dt} = \sum \dot{n}_{A,\text{in}} - \sum \dot{n}_{A,\text{out}} + G_A \quad [\text{mol A/s}] \quad (5)$$

where G_A is the net amount generated by chemical reaction. Again, if $\dot{n}_{A,\text{in}}$, $\dot{n}_{A,\text{out}}$ and G_A are independent (or weakly dependent) of the inventory n_A then this is an integrating (or close to integrating) process where n_A will not return to its desired steady-state. To stabilize the inventory we must have “self-regulation” where $\dot{n}_{A,\text{in}}$, $\dot{n}_{A,\text{out}}$ or G_A depend on n_A such that n_A is kept within given bounds in spite of disturbances.

An example where the inventory n_A is self-regulated because of the reaction term G_A is the irreversible reaction $A + B \rightarrow P$, where B is in excess and A is the limiting reactant. In this case, an increase in inflow of A ($\dot{n}_{A,\text{in}}$) will be consumed by the chemical reaction.

3. The rule for the individual phase follows by simply defining the control volume as the parts of the process that contain a given phase P and applying the mass balance to this control volume. Let m^P [kg] denote the inventory of the given phase inside the control volume and let \dot{m}_{in}^P and \dot{m}_{out}^P [kg/s] denote

the in- and outflows. The mass balance for a given phase is then

$$\frac{dm^P}{dt} = \sum \dot{m}_{in}^P - \sum \dot{m}_{out}^P + G^P \quad [\text{kg/s}] \quad (6)$$

where G^P is the net phase transition over the phase boundary. If \dot{m}_{in}^P , \dot{m}_{out}^P and G^P are independent (or weakly dependent) of the inventory then this is an integrating (or close to integrating) process where m^P will not return to its desired steady-state. To stabilize the inventory we must have “self-regulation” where \dot{m}_{in}^P , \dot{m}_{out}^P or G^P depends on the inventory (m^P) such that m^P is kept within given bounds in spite of disturbances.

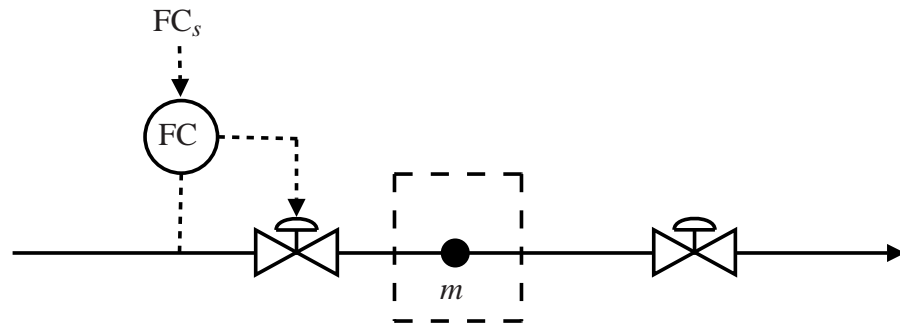
An example where we need to consider individual phases is a flash tank where a two-phase feed is separated into gas and liquid.

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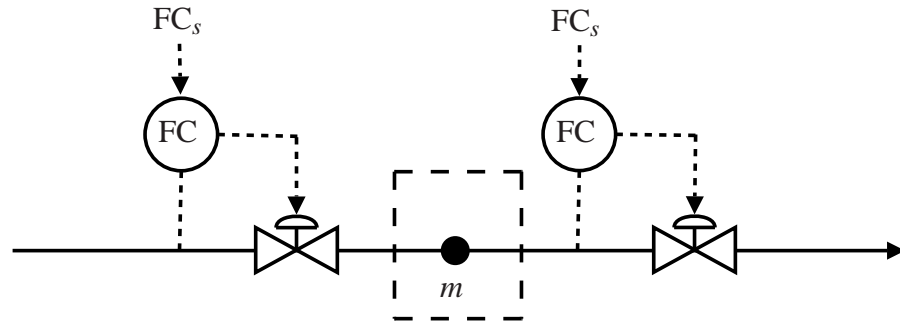
Example 2. Stream with two valves. *To demonstrate the self-consistency rule on a very simple example, consider a single stream with two valves; see Figure 2(a). There is only a single (small) hold-up m in this simple process (illustrated by the big dot), so consistency and self-consistency are here the same. The pressure p depends directly on the inventory m (for a liquid the dependency is very strong; for an ideal gas it is $p = \frac{mRT}{V}$). Thus, self-regulation of inventory is the same as self-regulation of pressure. To apply the self-consistency rule, we define a control volume (dotted box) as shown in Figure 2 and note that the inflow is om flow control in all four cases, that is, the inflow is independent of the inventory m . Thus, according to Rule 1, to have consistency (self-regulation), the outflow must depend on the pressure p (inventory m) and more specifically the outflow must increase when p increases.*

Four different control structures are displayed in Figure 2. According to Rule 1, the structure in Figure 2(a) is consistent since the outflow increases when the pressure p (inventory m) increases. Thus, we have “true” self-regulation with no need for active control.

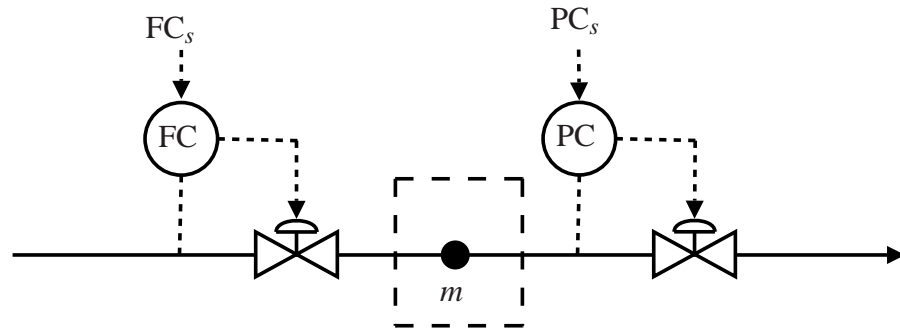
The control structure in Figure 2(b) is not consistent because the outflow is independent on the inventory m . Even if the set points for the two flow controllers were set equal, any error in the actual flow would lead to an imbalance, which would lead to accumulation or depletion of mass



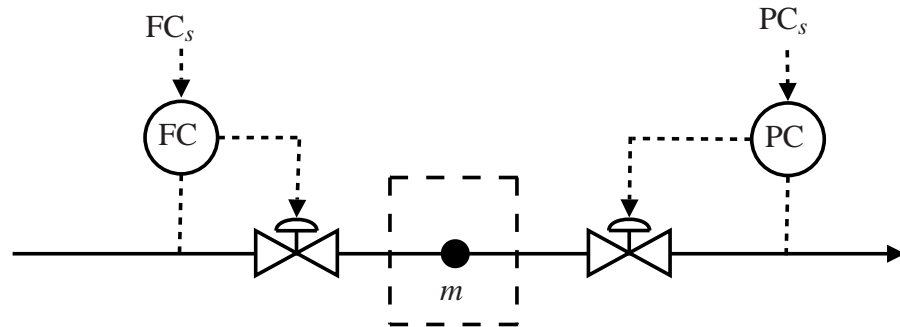
(a) OK (consistent control structure since outflow depends on inventory m).



(b) Not consistent control structure since outflow is fixed.



(c) OK (consistent control structure since outflow depends on inventory m).



(d) Not consistent control structure since outflow does not depend correctly on inventory m .

Figure 2: Four different control structures with two valves and fixed inflow. Note: For the flow controllers (FC) it does matter whether the valve is downstream (as shown above) or upstream of the flow measurement.

and the inventory would not be self-regulated.

The structure in Figure 2(c) is consistent because the outflow increases when the pressure (inventory m) increases.

Finally, the control structure in Figure 2(d) is not consistent because the outflow depends on the inventory m (and pressure) in the wrong (opposite) manner. To understand this, consider a decrease in inflow, which will lead to a decreased pressure in the control volume. A lower differential pressure over the pressure-controlled valve leads to a smaller flow through the valve and the pressure at the downstream measuring point will decrease, leading the pressure controller to open the valve. The result a further pressure decrease in the control volume, so the pressure controller is actually working in the wrong direction. The opening of the pressure-controlled calve will also affect the flow-controlled valve and, depending on the set point of the controllers, either the flow-controlled valve or the pressure-controlled valve will move to fully open. The other pressure-controlled valve or flow-controlled valve will continue to control pressure or flow. It should also be noted that the pressure control loop is in the direction opposite to flow, which is not correct when the inflow is given (see further discussion in next section).

This is confirmed by dynamic simulations of the simple configuration in Figure 2(d) using the flowsheet simulator Aspen HYSYS®(see Figure 3):

10% increase in FC set point: *The FC saturates at fully open and the PC maintains its set point (Figures 3(a) and 3(b)).*

10% decrease in FC set point: *The FC maintains its set point and the PC saturates at fully open (Figures 3(c) and 3(d)).*

5% increase in PC set point: *The FC maintains its set point and the PC saturates at fully open (Figures 3(e) and 3(f)).*

5% decrease in PC set point: *The FC saturates at fully open and the PC maintains its set point (Figures 3(g) and 3(h)).*

In all cases the system is assumed to be at steady-state initially.

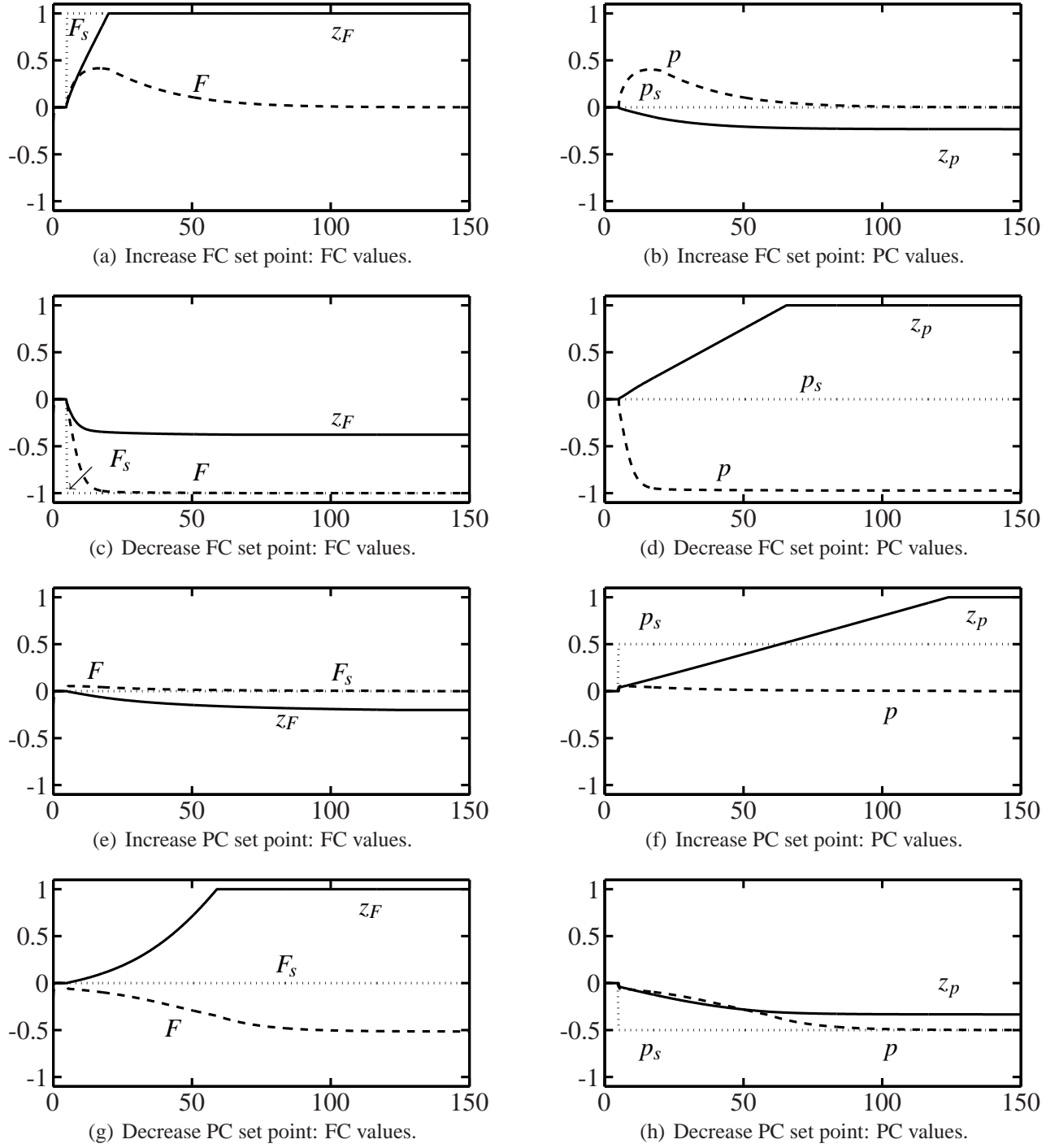


Figure 3: Dynamic simulations of the simple configuration in Figure 2(d). Left column: Flow controller. Right column: Pressure controller. In all cases, one of the valves move to fully open.

A remark about the sign of the controllers: Overall, the controller and the plant should give a negative feedback loop:

1. **Flow control.** Opening a valve always increases the flow (positive gain), so a flow controller is always “reverse acting” (with a negative feedback sign).
2. **Level and pressure control.** The controller sign depends on the location of the valve relative to the inventory (level or pressure). If control is in the direction of flow (with the inventory measurement for level or pressure upstream the valve) then the controller must be “direct acting” (positive feedback sign), if control is in opposite direction of flow then it must be “reverse acting”.

These rules were used when tuning the controllers in Figure 3.

Example 3. Units in series. To understand the difference between the terms consistency (Definition 1) and self-consistency (Definition 2), consider inventory control of the series process in Figure 4. The control structure is **consistent** and is able to propagate a production rate change to a change in the feed rate. However, the in- and outflows for the last unit (dashed box) do not depend directly on the inventory inside the unit and the control volume is therefore **not self-consistent** according to the “self-consistency rule” (Rule 1). This can also be seen because the inventory controllers are not in the direction opposite to flow as they should be for a self-consistent process with a given product rate (see also next section). To make the structure consistent we have in Figure 4 introduced a “long loop” where the inflow to the first unit is used to control the inventory in the last unit.

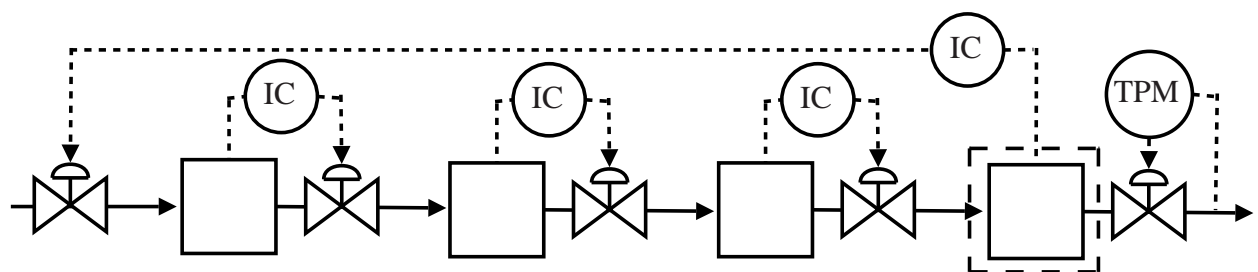
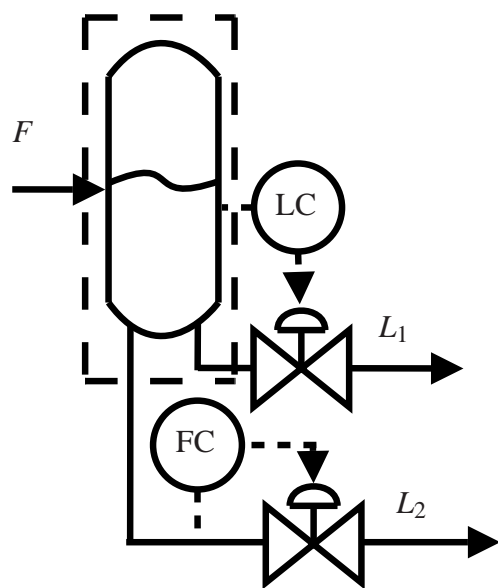


Figure 4: Consistent, but not self-consistent inventory control structure.

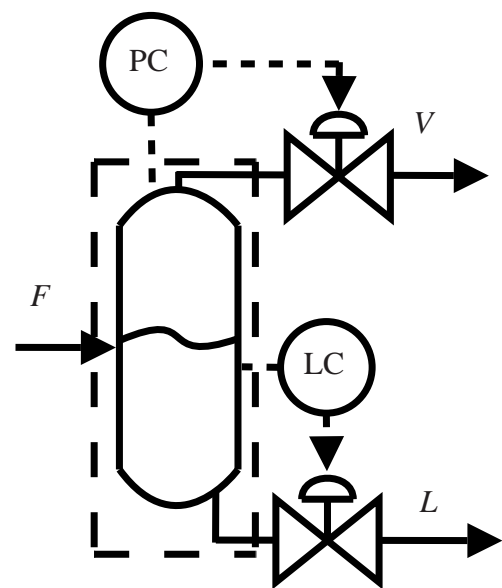
Example 4. Phase transition. In some cases, phase transition needs to be considered for self-consistency. Consider Figure 5 where the inflow F is given. Thus, according to Rule 1, to have consistency the outflow must depend on the inventory in the tank.

In Figure 5(a), the inflow is a single phase (liquid) and the outflow from the single-phase tank is split in two liquid streams (L_1 and L_2). There is one inventory, so for self-consistency, one of the outflows must be on inventory control whereas the other outflow can be flow controlled. This follows because the adjustable split introduces an extra degree of freedom, but the number of inventories that need to be controlled is unchanged.

In Figure 5(b) the inflow is two-phase (liquid and vapor) and there are two inventories (liquid and vapor) that needs to be regulated. To have a consistent inventory control structure, both the outflows (vapor and liquid) must be used for inventory control. In Figure 5(b) this is illustrated by the LC (liquid inventory) and PC (vapor inventory). In this case, the split does not actually give an extra degree of freedom because the split is indirectly determined by the feed quality (fraction of vapor).



(a) Single-phase tank: Adjustable split.



(b) Two-phase tank: Split indirectly fixed by inventory control.

Figure 5: Self-consistent inventory control of split with one and two phases.

Specific rules and consistency of flow networks

In a flow network there is at least one degree of freedom, called the throughput manipulator (TPM), which sets the network flow. More generally, *a TPM is a degree of freedom that affects the network flow and which is not directly or indirectly determined by the control of the individual units, including their inventory control.*⁹ Typically, a fixed flow (e.g. flow controller with an adjustable set point) is a TPM. As discussed in more detail below, the location of the TPM is very important. In particular, if the flow network has no splits or junctions, then for a given placement of the TPM, there is only one *self-consistent* inventory control system.

However, at splits (e.g. multiple products) or junctions (e.g. multiple feeds), there are several possibilities. At a split or junction, a common choice is to use the largest flow for inventory control.⁶ For example, with a given feed, the largest product stream may be used for inventory control with the flow rates of the smaller product streams used for quality control. Similarly, with a given production rate, the largest feed rate is often used for inventory control and the smaller feed flows are set in ratio relative to this, with the ratio set point possibly used for quality control.

The objective is now to apply the self-consistency rule to analyze inventory control structures for real processes (flow networks). We consider three network classes:

1. Units in series
2. Recycle systems
3. Closed systems

A series network may have splits, provided the flow is still in the same direction. Note that each single-phase split introduces one extra degree of freedom (the split ratio; see Figure 5). A recycle system contains one or more splits that are (partly) fed back to the system. A closed system has total recycle with no feeds or products.

Units in series (“radiating rule”)

As mentioned above, if there are no splits or junctions, the location of the throughput manipulator determines the self-consistent inventory control structure. Specifically, a direct consequence of the self-consistency rule is

- *Inventory control must be in direction of flow downstream the location of a fixed flow (TPM).*
- *Inventory control must be in direction opposite to flow upstream the location of a fixed flow (TPM).*

More generally, we have:

Rule 2. Radiation rule Price and Georgakis:¹ *A self-consistent inventory control structure must be radiating around the location of a fixed flow (TPM).*

These rules are further illustrated in Figure 6.

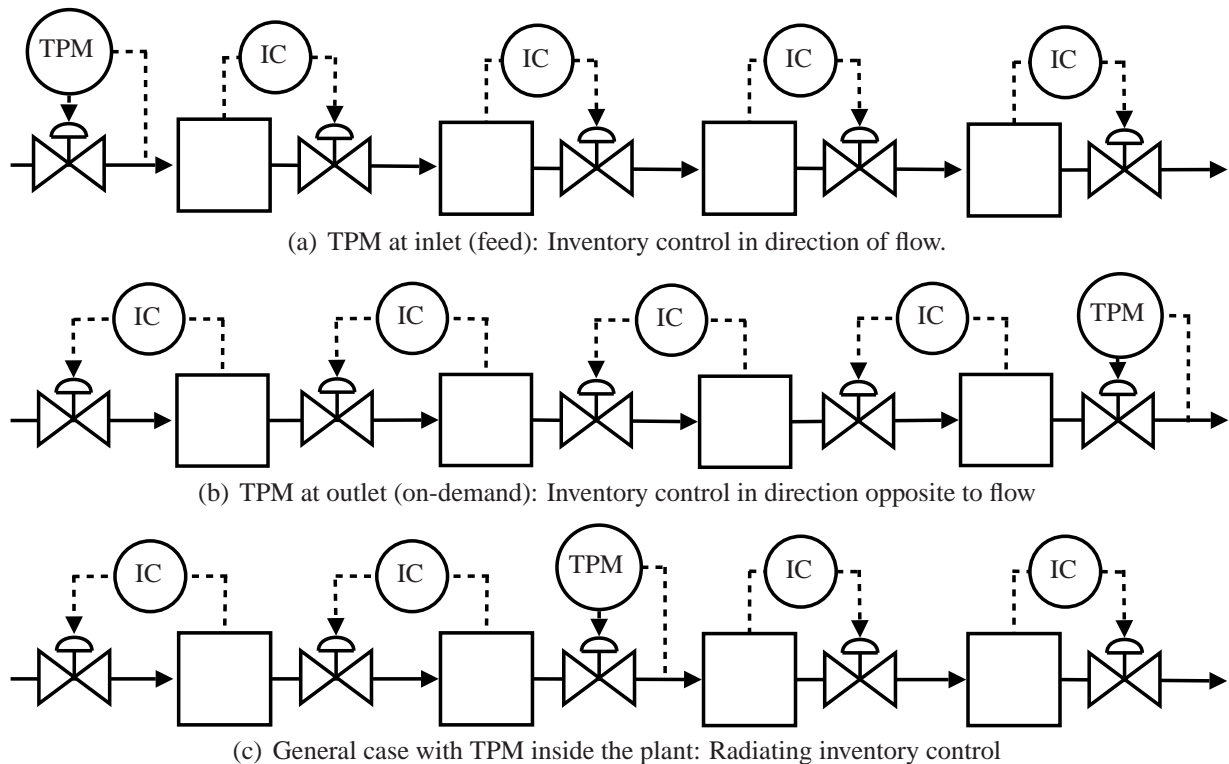


Figure 6: Self-consistency requires a radiating inventory control around a fixed flow (TPM).

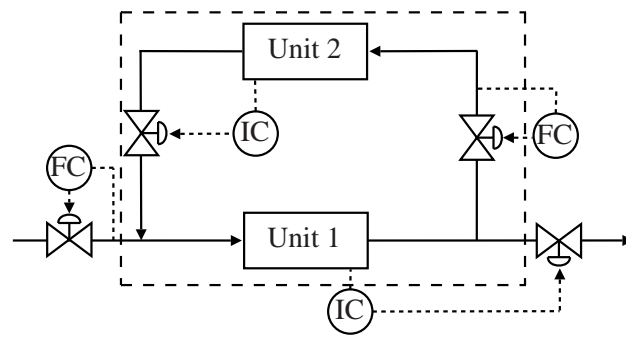
Recycle systems

A recycle system usually has an adjustable split, which (but not always) “*introduces an extra degree of freedom for control of the network flow*”.² On the other hand, the requirement of self-consistency imposes limitations. As an example, consider the simple single-phase recycle example with a fixed feed and an adjustable split in Figure 7 (there is a pump or compressor in the recycle loop which is not shown). Figures 7(a) and 7(b) have consistent inventory control structures, because the outflows from units 1 and 2 depend on the inventory inside each unit. In both cases one flow in the recycle loop is fixed (flow controlled with an adjustable set point that may be used for other purposes than inventory control). Note that *the inventory control in the recycle loop can be either in direction of flow* (Figure 7(a)) or *direction opposite to flow* (Figure 7(b)), because the flow rate can be fixed at any location in the recycle loop.

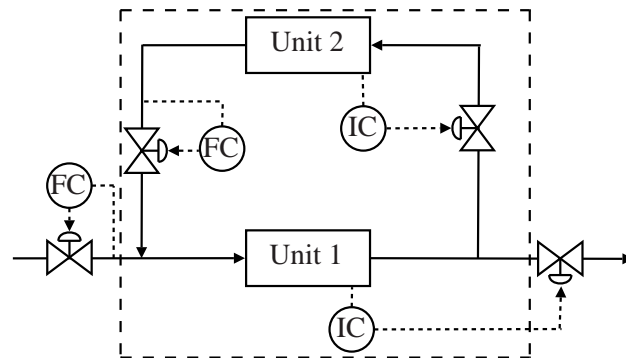
In Figure 7(c) the inventory loops for units 1 and 2 are paired opposite. This structure is not self-consistent because the inventory of unit 2 is not “self-regulated by its in- or outflows” and thus violates Rule 1. In addition, the inventory control of unit 2 requires that the other inventory loop is closed, and thus violates Definition 2.

Finally, Figure 7(d) is obviously not consistent since both the feed rate and the product rate are fixed. In particular, the inflow and outflow to the dotted box do not depend on the inventory inside this part of the process, which violates Rule 1.

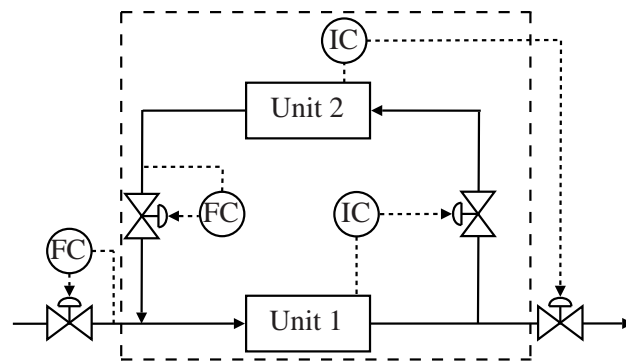
Remark. This simple example seems to prove the rule that “*one flow rate somewhere in the recycle loop should be flow controlled*”.¹⁰ This rule follows because there is an extra degree of freedom introduced by the split, but the number of inventories that need to be controlled are unchanged. However, first one should note that the set point of the flow controller is a degree of freedom which may be used for other purposes, for example, control of composition. Second, a “counter-example” is provided by the self-consistent reactor-separator-recycle process in Figure 11(a). In this case, the split is not actually an extra degree of freedom because the split is indirectly determined by the feed composition to the separator (distillation column), as discussed in Example 4.



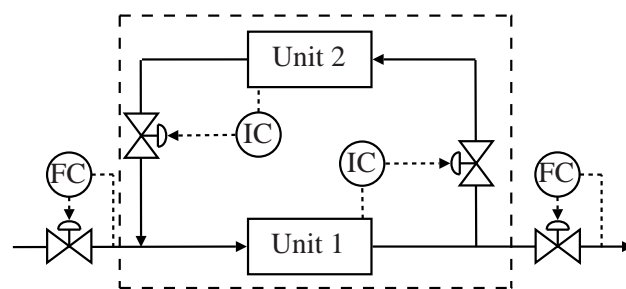
(a) Self-consistent inventory control.



(b) Self-consistent inventory control.



(c) Consistent, but not self-consistent inventory control.



(d) Not consistent inventory control.

Figure 7: Inventory control of simple recycle process with given feed.

Closed systems

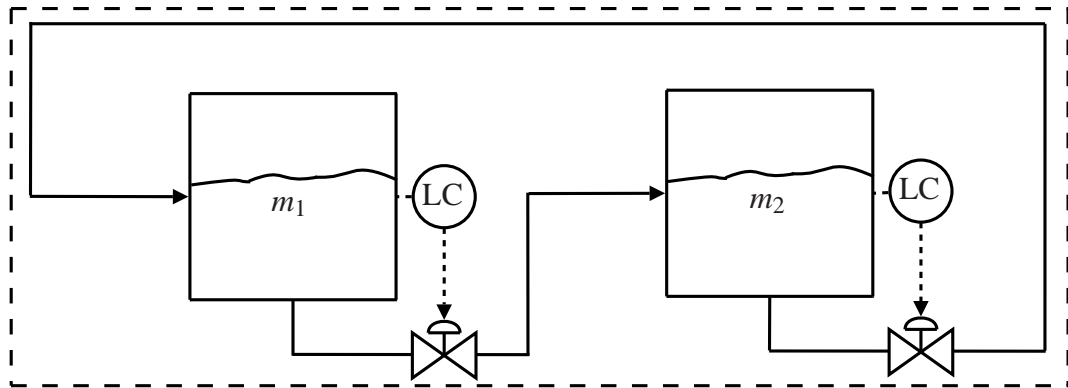
Closed systems require particular attention. It is clear from the total mass balance that the total inventory of a closed system cannot be self-regulated since there are no in- or out streams. Thus, our previously derived rule (Rule 1) does not really apply. As an example, consider a closed system with two inventories. In Figure 8(a) we attempt to control both inventories, but the two loops will “fight each other” and will drift to a solution with either a fully open or fully closed valve. For example, a (feasible) solution is to have zero flow in the cycle. The problem is that the flow is not set anywhere in the loop. To get a consistent inventory control structure, *one must let one of the inventories be uncontrolled*, as shown in Figures 8(b) and 8(c). *The corresponding unused degree of freedom (flow) sets the flow rate (“load”, throughput) of the closed system.*

To be able to use our self-consistency rule (Rule 1) for closed systems there are two alternative “fixes”:

1. Let the total inventory be uncontrolled (*not* self-regulated), which is how such systems are usually operated in practice. Typically the largest single inventory is uncontrolled. However, the remaining inventories must be self-regulated, as usual, to have self-consistency of the inventory control system.
2. Introduce a “dummy” stream that keeps the total inventory constant. This corresponds to allowing for filling (charging) or emptying the system. In practice, this stream may be a make-up stream line that refills or empties the largest inventory, e.g. on a daily or monthly basis.

Both approaches allow for disturbances, such as leaks or supply. The inventory control system can then be analyzed using the normal self-consistency rule (Rule 1). Figure 8(a) is clearly not allowed by Fix 1 as the total inventory is not left uncontrolled. Figure 8(a) is also not consistent by Fix 2, since for self-consistency the dummy stream must be used for inventory control instead of one of the two flows in the recycle loop.

Example 5. Absorber-regenerator example. *In this example, the consistency rule (Rule 1) is used*



(a) Not consistent (because there is no uncontrolled inventory).

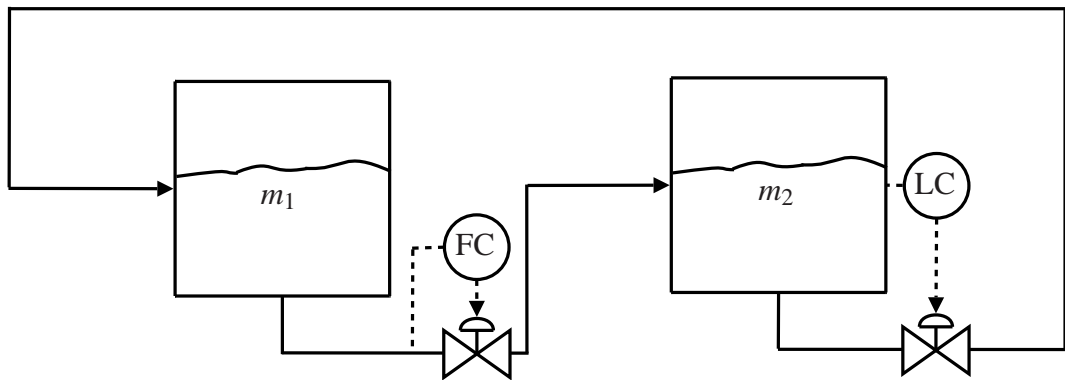
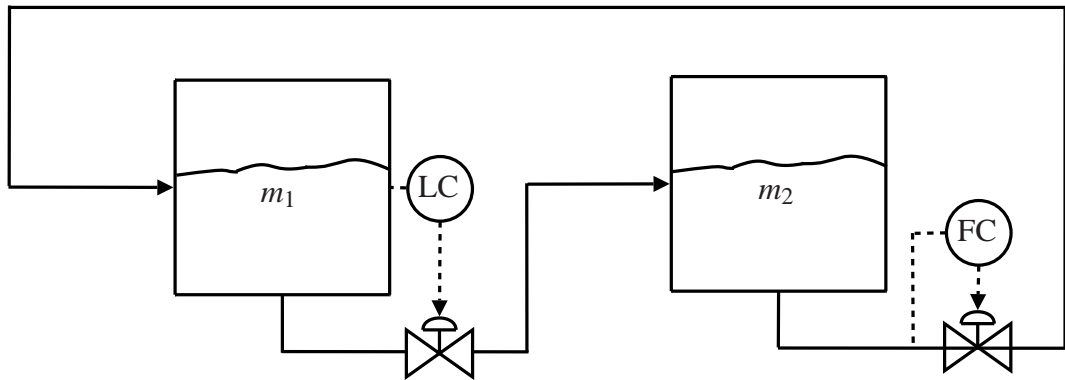
(b) Self-consistent (inventory m_1 is uncontrolled).(c) Self-consistent (inventory m_2 is uncontrolled).

Figure 8: Inventory control for closed system.

for an individual phase (liquid), which forms a closed system. Consider the absorber and regenerator example in Figure 9² where a component (e.g. CO_2) is removed from a gas by absorption. The inlet gas flow (feed) is indirectly given because there is a pressure control in the direction of flow at the inlet. The gas outlet flows are on pressure control in the direction of flow and thus depend on the gas holdup in the plant. Therefore the gas-phase inventory control is consistent. However, the liquid flows between the absorber and regenerator make up a “closed system” (except for minor losses). There is a flow controller for the recycled liquid, but its set point is set by the inventory in the regenerator, hence all inventories in the closed system are on inventory control, which violates the rule just derived. To get a consistent inventory control structure, we must break the level-flow cascade loop and let the inventory in the bottom of the regenerator remain uncontrolled.

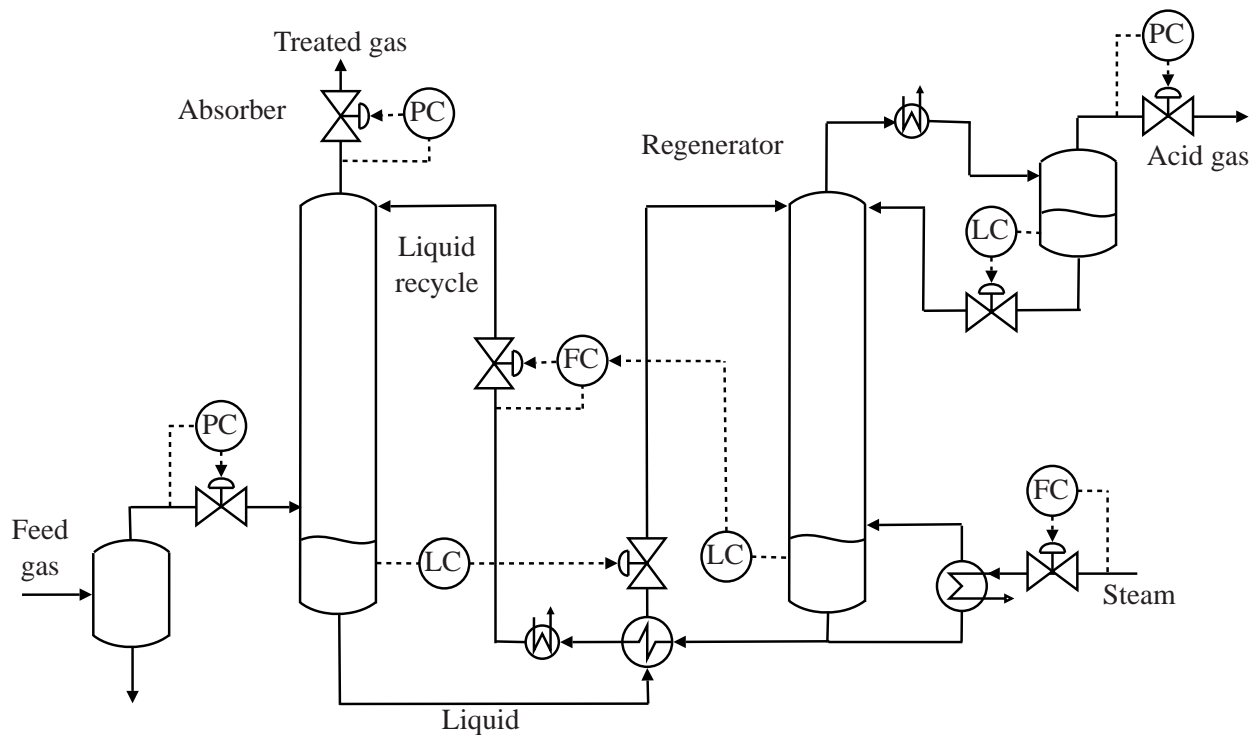


Figure 9: Absorber and regenerator example: Not consistent liquid inventory control.

Summary of specific rules

In the literature there are many rules that deal with the inventory control structure. In addition to the radiating rule, some useful rules that can be developed from the self-consistency rule (Rule 1) are:

1. *All systems must have at least one given flow (throughput manipulator).*

Proof. Assume there is no throughput manipulator. Then all flows must be on inventory control, which will not result in a unique solution. For example, zero flow will be an allowed solution. \square

2. *Component balance rule:¹¹ Each component, whether important or insignificant, must have its inventory controlled within each unit operation and within the whole process. This is also referred as “Downs drill”.¹³*

Proof. This comes from the requirement of component self-consistency (Rule 1). \square

3. *A stream cannot be flow controlled more than once, that is, a structure with two flow controllers on the same stream is not consistent.*

Proof. Make a control volume with the two flow-controlled streams as in- and outflows. Then neither the inflow nor the outflow depends on the control volume and the inventory is not self-regulated. This is demonstrated in Figure 2(b). \square

4. *Price and Georgakis:¹⁴ If a change in the throughput manipulator does not result in a change in the main feed flow, then the control structure is inconsistent.*

Proof. This follows from the requirement of satisfying the steady-state mass balances. \square

5. *Generalized from:¹⁵ A self-consistent inventory control structure must use the feed or the product (or both) for inventory control.*

Proof. This follows from the steady-state mass balance. This is also discussed in Section “Units in series” and a clear illustration of this statement is found in Figure 6. \square

6. *For closed systems: One inventory must be left uncontrolled and one flow in the closed system must be used to set the load.*

Proof. This follows from that all systems must have at least one given flow to be unique. To be able to fix the load for a closed system, one inventory must be uncontrolled. \square

The rules are summarized by the proposed procedure for inventory control system design in Table 1, which is inspired by the inventory control guidelines in Price et al..

Table 1: Proposed guidelines for design of self-consistent inventory control system. In case of doubt consult the general self-consistency rule (Rule 1).

1	Choose the location of the throughput manipulator
2	Identify inventories that need to be controlled including: a) Total mass b) Components c) Individual phases
3	Identify manipulators suitable for adjusting each inventory
4	Design a self-consistent radiation inventory control system that controls all the identified inventories. This means: a) Inventory control in direction of flow downstream the throughput manipulator b) Inventory control in direction opposite to flow upstream the throughput manipulator
5	At junctions or splits a decision has to be made on which flow to use for inventory control. Typically, the largest flow is used, or both streams are changed such that their ratio is held constant (often the ratio is set by a slower outer composition loop).
6	Recycles require special consideration. Make a block (control volume) around the entire section and make sure that there is self-consistency for total mass, (individual) components and phases (if relevant).
7	Assign control loops for any process external flow that remain uncontrolled. Typically, “extra” feed rates are put on ratio control with the ratio set point being set by an outer composition loop.

Examples

In this section we apply the self-consistency rule to some examples from the academic literature.

Distillation column with DB-configuration

An example of a recycle system is a distillation column. As seen from Figure 10, a distillation column has one split in the condenser (V_T splits into L and D) and one split in the reboiler (L_B splits into B and V). In both cases one of the streams is recycled to the column (L and V , respectively). The two splits introduce two degrees of freedom and this gives rise to many possible inventory control structures (“configurations”), as has been discussed widely in the literature (see¹⁶ for a summary of this discussion).

Figure 10 displays the DB-configuration, which uses reflux L and boilup V for inventory control (condenser and reboiler level control), such that the flows of D and B remain as degrees of freedom for other purposes. The DB-configuration has earlier been labeled “impossible”, “unacceptable” or “infeasible” by distillation experts^{18, 20} This inventory control system also violates Luybens rule of “fixing a flow in the recycle loop” and it is indeed true that this inventory control system is not self-consistent. To see this, consider the dashed box in Figure 10 where we note that none of the flows in or out of the column (F , D and B) depend on the inventory inside the column. However, an inconsistent inventory control system can usually be made consistent by adding control loops and the DB-configuration is workable (and consistent) provided one closes at least one extra loop, for example by using D to control a temperature inside the column.^{21, 22} Thus, labeling the DB-configuration as “impossible” is wrong. In summary, the DB-configuration is not self-consistent, but it can be made consistent by adding a temperature (or composition) control loop.

Remark 1 An example of a self-consistent inventory control structure for distillation is the common LV-configuration, where the two level loops have been interchanged such that D and B are used for level control and L and V remain as degrees of freedom (e.g. on flow control). In the LV-configuration, inventory is controlled in the direction of flow, as expected since the feed is given.

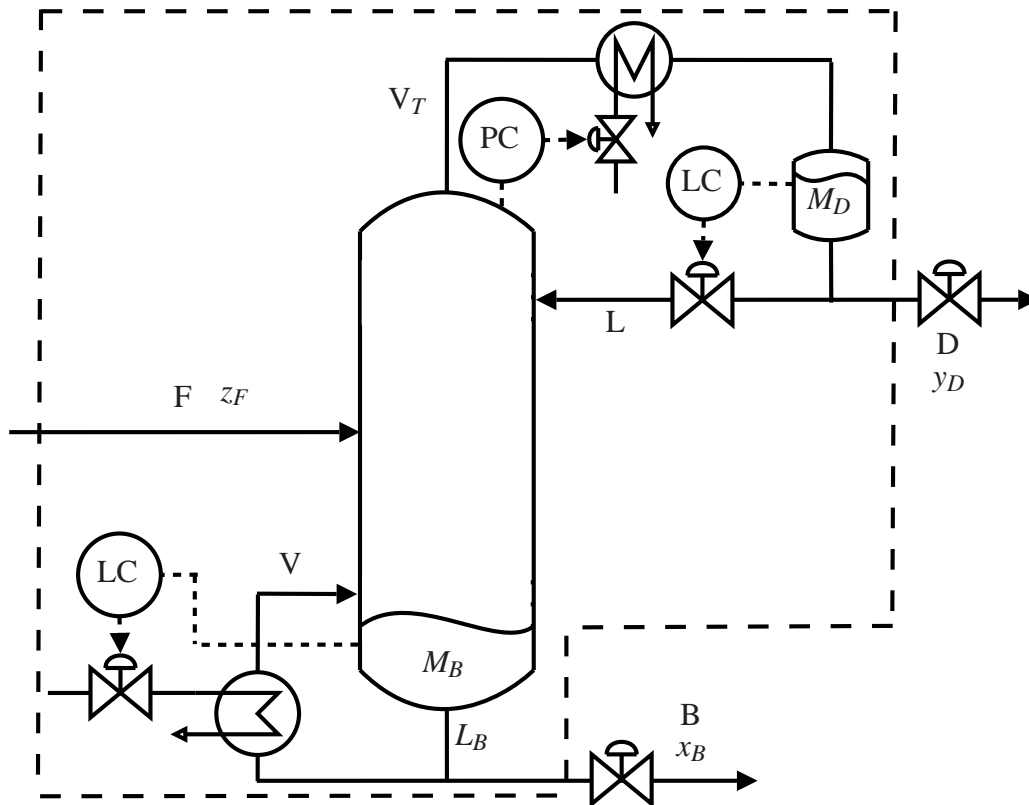


Figure 10: Example of inconsistent inventory control at recycle process: Distillation column with DB-configuration.

Remark 2 An additional inventory issue for distillation columns is related to the split between light and heavy components (component inventory). One may regard the column as a “tank” with light component in the upper part and heavy in the lower part. Thus, one is not really free to set the split between D and B and to avoid a “drifting” composition profile (with possible “breakthrough” of light component in the bottom or of heavy component in the top), one must in practice close a quality (e.g., temperature or pressure) loop to achieve component self-consistency.¹⁶ For example, for the LV-configuration one may use the boilup V to control a temperature inside the column. This consideration about controlling the column profile also applies to the DB-configuration. Thus, in practice, the DB-configuration requires closing *two* quality loops to maintain mass and component balances. This means that both D and B must be used for quality control for the DB-configuration, rather than only one (L or V) for the LV-configuration.

Reactor-separator-recycle example with one reactant

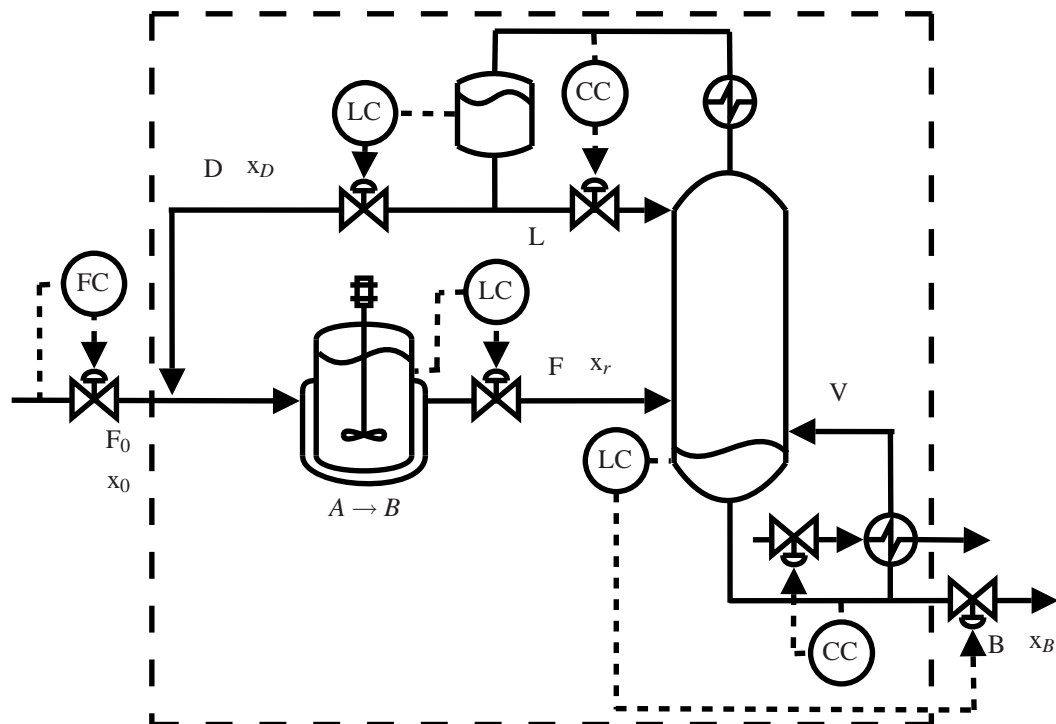
A common recycle example from the academic literature is the reactor-separator-recycle system in Figure 11. The system has a continuous stirred-tank reactor (CSTR) with an irreversible, isothermal, first order reaction $A \rightarrow B$, followed by separation (distillation) and recycle of the unreacted feed component back to the reactor (e.g.^{1,23–25}).

The feed (F_0) is pure reactant A and the component mass balances become

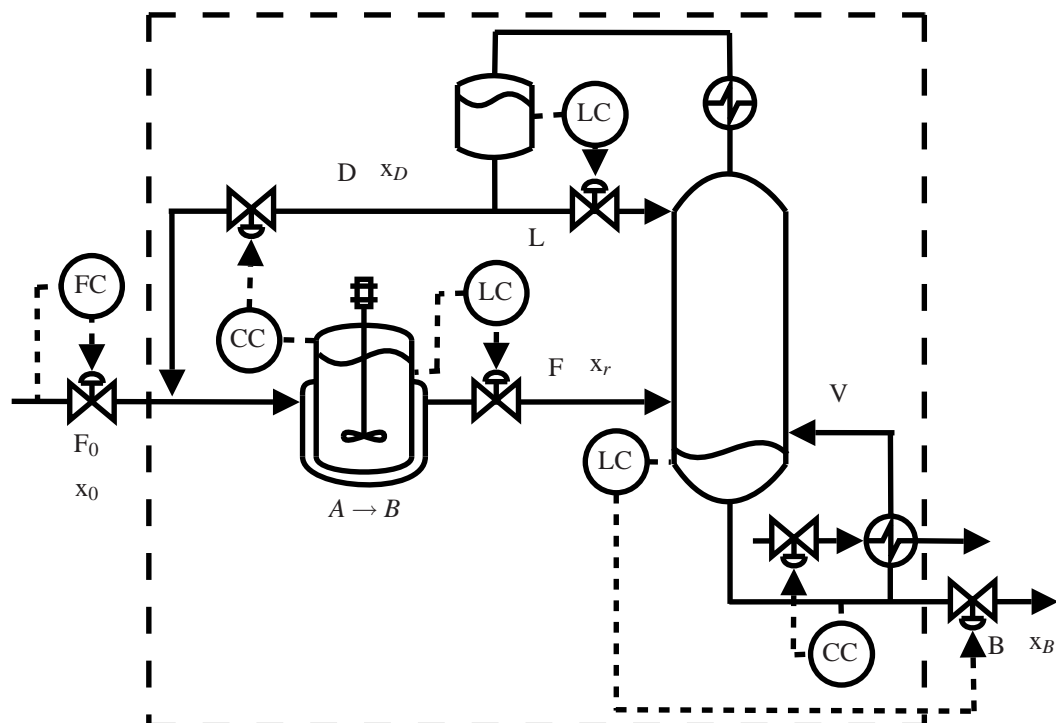
$$\begin{aligned} \text{Component A: } F_0 &= \underbrace{k(T) \cdot x_{r,A} \cdot V + B \cdot x_{B,A}}_{-G_A=G_B} \\ \text{Component B: } \underbrace{k(T) \cdot x_{r,A}}_{G_B} &= B \cdot x_{B,B} \end{aligned}$$

where x is the mole fraction, V is the reactor volume and $k(T)$ is the reaction rate constant. Note that $B = F_0$ [mol/s] at steady-state. Component A enters the process in the feed stream and its consumption in the reactor increases with the amount of A . The inventory of component A is therefore expected to be self-regulated by the reaction. Component B is produced in the reactor (G_B) and exits the process in stream B . Component B is not self-regulated by the reaction (because the reaction rate is independent of the amount of B) and thus requires a controller to adjust its inventory.

Two different control structures for the reactor-separator-recycle process are displayed in Figure 11. Both have fixed feed (F_0) and inventory control is the direction of flow. Thus, both of them are self-consistent in total mass, because the outflow B from the process depends on the inventory inside the process (indicated by the dashed control volume) (Rule 1). Since the outflow B mainly consist of component B , this implies that both structures are also consistent (self-regulated) with respect to the inventory of component B . The difference between the two structures is related to the control of component A . The “conventional” structure in Figure 11(a) uses the LV-configuration for the distillation column where the reflux (L) controls the composition in the recycle (distillate) D . The structure in Figure 11(b) uses the DV-configuration for the column where the reactor com-



(a) Conventional: Self-consistent inventory control structure.



(b) Composition control of reactor composition: Not consistent for component A.

Figure 11: Reactor-separator-recycle process with one reactant (A).

position $x_{r,A}$ is controlled instead of the recycle (distillate) composition.

As already mentioned, the inventory of component A is expected to be self-regulated by the reaction $A \rightarrow B$, so one would expect both structures to be consistent with respect to component A . In fact, both structures would be consistent if one *removed* the composition loop in the recycle loop (thus, fixing reflux L in Figure 11(a) and fixing recycle D in Figure 11(b)). With the composition loop closed, the “conventional” structure in Figure 11(a) remains consistent, but not the structure with control of reactor composition in Figure 11(b). The reason for the inconsistency is that control of reactor composition eliminated the self-regulation by reaction: The amount of A that reacts is given by $-G_A = G_B = k(T)x_{r,A}V$ and with fixed $x_{r,A}$ (because of the controller), T and V there is no self-regulation. The inconsistency of this control structure is pointed out by e.g.^{5,26}

Remark 1 The control structures in Figure 11 would both be self-consistent without closing the composition loop (CC) in the recycle part of the process, that is, with (a) L fixed or (b) D fixed. The reason for closing these composition loops is therefore not for consistent inventory control but rather for other (economic) reasons.²⁵ The interesting point to note, is that closing an extra loop can in some cases make the system inconsistent (Figure 11(b)).

Remark 2 Luyben has proposed to make the system in Figure 11(b) consistent by introducing an adjustable reactor volume, but this is not a good solution, because we always want to use the maximum reactor volume for economic reasons (energy saving).²⁵

Remark 3 The inventory of component A is expected to be self-regulated by the reaction $A \rightarrow B$. More precisely, the amount that reacts is $-G_A = kx_{r,A}V$ and the composition $x_{r,A}$ will “self-regulate” such that at steady-state $F_0 \approx -G_A$, that is, $x_{r,A} \approx F_0/(kV)$.

Remark 4 We already noted that fixing $x_{r,A}$ (Figure 11(b)) breaks this self-regulation and makes the system inconsistent. A related problem is when the reactor volume V is too small relative to the feed F_0 , such that the required $x_{r,A}$ exceeds 1, which is impossible. In practice, if we increase the feed rate F_0 and approach this situation, we will experience “snow-balling”¹⁰ where the recycle D becomes very large, and also the boilup V becomes very large. Eventually, V may reach its maximum value, and we lose composition control and

we will get “break-through” of A in the bottom product. Snow-balling is therefore a result of a too small reactor.

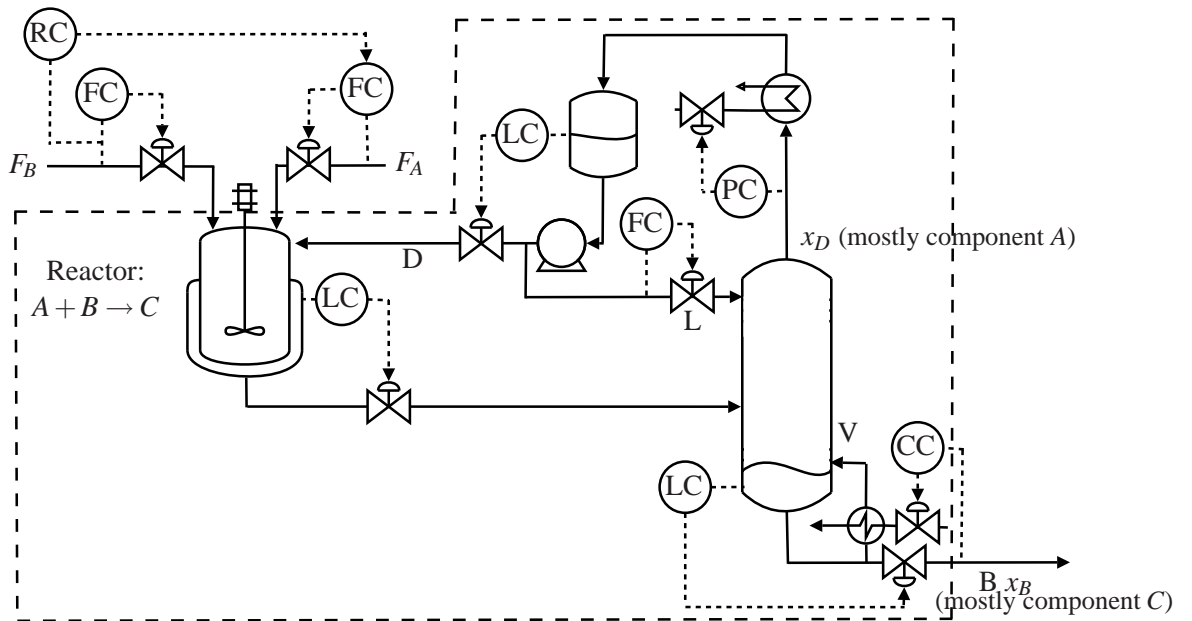
Remark 5 Consider the same process (Figure 11), but assume that the fresh feed (F_0) contains an inert component I in addition to the reactant A . If I is more volatile than component B , then component I will be recycled back to the reactor and will accumulate in the process. None of the inventory control systems in Figure 11 are consistent for the inert I . To make the system self-consistent for the inert, a purge stream must be introduced where part of stream D is taken out as a by-product.

Reactor-separator-recycle process with two reactants

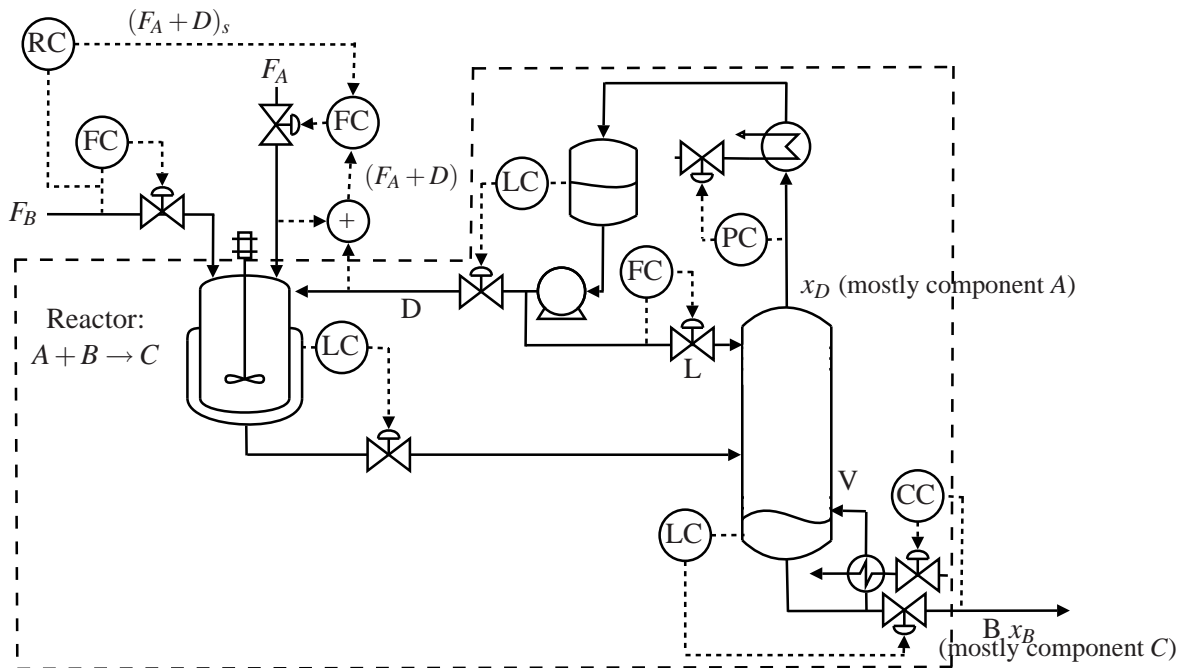
Another well studied recycle example is a reactor-separator-recycle process where two reactants A and B reacts according to the reaction $A + B \rightarrow C$ (e.g.²⁷). Component B is the limiting reactant as the recycle D contains mostly component A . Two different control structures are displayed in Figure 12. In both cases the distillate flow D (recycle of A) is used to control the condenser level (main inventory of A).

In Figure 12(a), both fresh reactant feeds (F_A and F_B) are flow controlled into the reactor, where reactant A is set in ratio to reactant B such that $F_A/F_B = 1$. This control strategy is not consistent because the two feeds is not independent and one of them needs to be dependent of the inventory inside since it not possible to feed exactly the stoichiometric ratio of the two reactants.²⁸ Any imbalance will over time lead to a situation where the recycle of A either goes towards zero or towards infinity.

To get a consistent inventory control structure, the first requirement is that one of the feed rates (F_A or F_B) must be dependent on what happens inside the process, such that we at steady-state can achieve $F_A = F_B$. One solution is to fix F_B (the limiting reactant) and adjust F_A such that the desired excess of A is achieved, resulting in the self-consistent control structure in Figure 12(b). Here F_A depends on the inventory of A as reflected by the recycle flow D by keeping the reactor feed ratio $(F_A + D)/F_B$ constant at a given value (larger than 1 to make B the limiting reactant). The structure is consistent for all components: C has an outlet in the bottom of the column; B is self-regulated



(a) Inconsistent structure with both reactant flows fixed



(b) Self-consistent structure where feed of reactant A depends on inventory of A (as reflected by D)

Figure 12: Reactor-recycle system with two reactants ($A + B$).

by reaction because it is the limiting reactant, and the feed of A depends on the inventory of A .

There exist also other consistent inventory control structures, e.g. see,²⁹ but these seem to be more complicated than the one proposed in Figure 12(b). For example, one could keep the recycle D constant and use F_A to control the condenser level (main inventory of A), but the dynamics for this “long level” loop are not favorable and this consistent structure is not self-consistent.

Conclusion

Consistency is a required property since the mass balances must be satisfied for the individual units and the overall plant. An inventory control system can be evaluated whether it is self-consistent (local “self-regulation” of all inventories) by using the self-consistency rule (Rule 1). The self-consistency rule follows from the mass balance that must be satisfied for the total mass, component and individual phases.

A direct consequence of the self-consistency rule is the “radiation rule”,¹ which states that the inventory control structure must be radiating around the location of a fixed flow. Other useful rules that can be developed from the self-consistency rule, is that all system must have at least one given flow (throughput manipulator). Thus, for closed systems, one inventory (preferable the largest) must be left uncontrolled.

Luyben provides the rule to “fix a flow in each recycle”. If we interpret the term “fix a flow” to mean “do not use a flow for inventory control”, then this rule follows from the requirement of self-consistency provided the recycle loop contains a split that introduced an extra degree of freedom. If no degree of freedom is introduced by the recycle, as is in the case if we have a separator or flash where the split is (indirectly) fixed by the feed properties, then this rule is not a requirement, e.g. see Figure 11(a), where all the flows in the recycle loop are on inventory control.

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