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Control Structure Design for Optimal Operation of Heat Exchanger Networks

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When only single bypasses and utility duties are used as manipulations, optimal operation of heat exchanger networks (HENs) can be categorized as an active constraint control problem. This work suggests a simple split-range control scheme to implement the optimal operation. For a given HEN and information about the disturbances, the corresponding control structure can be found by solving an integer-linear programming (ILP) problem with two objective functions providing optimal split-range pairs (for tracking active constraints during the operation) and appropriate control pairings (for fast control action). A HEN case study is used to demonstrate the application of the proposed design technique. Dynamic simulation shows the ability to provide the optimal operation of the obtained control structure. © 2007 American Institute of Chemical Engineers *AIChE J,* 54: 150–162, 2008
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Introduction

We are looking for simple ways of implementing (economic) optimal operation. In general, we first control the active constraints, and for the remaining unconstraints we look for good "self-optimizing" variables.[1] For some problems, including the heat exchanger network problem considered in this paper, there are no optimally unconstrained degrees of freedom, that is, all degrees of freedom should be used to satisfy active constraints. For heat exchanger networks, the active constraints are typically given target temperatures and zero or maximum heat exchanger duties. The issue in terms of implementing optimal operation is then to identify the active constraints and change the control policy accordingly. A naive (or at least rather complex) approach is to use online optimization. In this paper, *the approach is to use offline optimization to identify all possible regions with different set of active constraints and then attempt to find a simple operation policy for switching between regions*. The approach taken here is to use *split-range control,* which probably is the simplest way of dealing with changes in active input constraints. In a previous paper,[2] we used a physically-based approach using structural information and an arithmetic sign for how the heat is transferred but this works only in simple cases. In this paper, we solve the problem by integer linear programming (ILP), which gives a solution in terms of split-range control if a feasible solution exists.

Heat exchanger networks (HENs) are widely used in chemical industries to reduce the utility consumption by energy interchange of hot and cold streams. However, without a good control strategy, the reduction may not be achieved in practice. Marselle et al.[3] proposed a method for control structure design based on graph theory and developed a policy to adjust flow distributions in the HEN to meet target temperatures with minimum utility consumption.

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Calandranis and Stephanopoulos[4] used the structural characteristics of HENs to develop an expert controller for allocating loads to available sinks. A method based on structural information using an arithmetic sign (directional effect between a manipulation and a controlled variable) to generate an optimal control policy was studied by Mathisen et al.,[5] Glemmestad et al.,[6] and Lersbamrungsuk et al.[2] Online and periodic optimizations for the operation of HENs were studied by Aguilera and Marchetti,[7] Glemmestad et al.,[8] and González et al.[9]

If only single bypasses and utility duties are used as manipulations, then the steady-state optimal operation of HENs can be formulated as a linear programming (LP) problem.[2,7] This assumes constant heat capacity for the streams and constant heat transfer coefficients, which is a reasonable assumption for many problems. The important implication in terms of operation is that the optimal solution is always at constraints. In most cases, the resulting active constraint solution can be implemented using a split-range control scheme.[2,10] In Lersbamrungsuk et al.,[2] the split-range control structure was found using the directional effect between a manipulation and a controlled variable,[5] which may be unclear in some cases (e.g. the sign is [±] for a HEN with loops). Instead in this work we use an integer-linear programming (ILP) formulation to suggest an optimal control structure based on a split-range control scheme.

The paper is divided into seven sections. In the following section, a LP problem for the optimal operation of HENs is formulated. This results in an active constraints control problem. Next, an idea for switching between active constraint regions and an ILP for finding an optimal split-range control structure are described. The fourth section illustrates the application of the proposed ILP to a HEN case study. Dynamic simulations to demonstrate the ability for tracking active constraints of the obtained control structure are presented in the fifth section. Further discussion is shown in the sixth section. The last section is the conclusions.

Optimal Operation of HENs

Consider heat exchanger networks where the objective is to maintain optimal operation in spite of the variations in the inlet temperature. Assume

- Constant heat capacity flowrate (mC_P) for all streams
- Constant heat transfer coefficients (UA) for all heat exchangers.

Further assume that the available degrees of freedom for control (operation) are

- Single bypasses (duties of individual exchangers, Q)
- Utility duties (Q_h, Q_c)

Under these assumptions, Aguilera and Marchetti[7] and Lersbamrungsuk et al.[2] show that the corresponding steady-state optimal operation of simple HENs can be formulated as a linear programming (LP) problem:

$$\min c^T x \tag{1a}$$

Subject to

$$Ax \le b \tag{1b}$$

$$A_\mathrm{eq} x = b_\mathrm{eq} \tag{1c}$$

The vector x consists of the inlet and outlet temperatures on the hot side ($T_i^\mathrm{hot,in}$ and $T_i^\mathrm{hot,out}$) and cold side ($T_i^\mathrm{cold,in}$ and $T_i^\mathrm{cold,out}$) of all the exchangers, as well as the duty of all exchangers (Q_i-process exchanger, $Q_{\mathrm{c}i}$-cold utility exchanger, and $Q_{\mathrm{h}i}$-hot utility exchangers). The equality constraints include the process models, the internal connection, and given supply temperatures T_i^s and target temperatures T_i^t. The inequality constraints include the lower and upper bounds on the duty of the heat exchangers. The objective function (1a) allows for many problem formulations including maximum temperature problem. In this paper, the objective is to minimize the utility cost in which all elements of the cost vector c are zero except the elements related to the duty of utility exchangers. The LP problem formulation for optimal operation of HENs is given in Eqs. 2a–2m:

Objective function:

$$\sum_i c_i Q_{ci} + \sum_j c_j Q_{hj} \quad i \in \mathrm{CU}, j \in \mathrm{HU} \tag{2a}$$

Subject to

Equality Constraints:

(a) Process models (energy balances): Assuming constant heat capacity flowrates (mC_P)

For process exchanger i:

$$Q_i - (mC_\mathrm{P})_i^\mathrm{cold}\left(T_i^\mathrm{cold,out} - T_i^\mathrm{cold,in}\right) = 0 \quad i \in \mathrm{PHX} \tag{2b}$$

$$Q_i - (mC_\mathrm{P})_i^\mathrm{hot}\left(T_i^\mathrm{hot,in} - T_i^\mathrm{hot,out}\right) = 0 \quad i \in \mathrm{PHX} \tag{2c}$$

For cooler i:

$$Q_{\mathrm{c}i} - (mC_\mathrm{P})_i^\mathrm{hot}\left(T_i^\mathrm{hot,in} - T_i^\mathrm{hot,out}\right) = 0 \quad i \in \mathrm{CU} \tag{2d}$$

For heater i:

$$Q_{\mathrm{h}i} - (mC_\mathrm{P})_i^\mathrm{cold}\left(T_i^\mathrm{cold,out} - T_i^\mathrm{cold,in}\right) = 0 \quad i \in \mathrm{HU} \tag{2e}$$

(b) Connecting equations

Supply connection:

$$T_i^\mathrm{hot,in} = T_i^\mathrm{s} \quad i \in \mathrm{HXHS} \tag{2f}$$

$$T_i^\mathrm{cold,in} = T_i^\mathrm{s} \quad i \in \mathrm{HXCS} \tag{2g}$$

Internal connection:

$$T_i^\mathrm{hot,out} - T_j^\mathrm{hot,in} = 0 \quad i \in \mathrm{HXHO}, j \in \mathrm{HXHI} \tag{2h}$$

$$T_i^\mathrm{cold,out} - T_j^\mathrm{cold,in} = 0 \quad i \in \mathrm{HXCO}, j \in \mathrm{HXCI} \tag{2i}$$

Target connection:

$$T_i^\mathrm{hot,out} = T_i^\mathrm{t} \quad i \in \mathrm{HXHT} \cup \mathrm{CUT} \tag{2j}$$

$$T_i^\mathrm{cold,out} = T_i^\mathrm{t} \quad i \in \mathrm{HXCT} \cup \mathrm{HUT} \tag{2k}$$

Inequality Constraints

Lower bound:

$$-Q_i \leq 0 \quad i \in \mathrm{PHX} \cup \mathrm{CU} \cup \mathrm{HU} \tag{2l}$$

Upper bound: assuming constant thermal efficiency ($P_{\mathrm{h},i}$) and heat capacity flowrate (mC_{P})

$$Q_i \leq P_{h,i}(mC_{\mathrm{P}})_i^{\mathrm{hot}}(T_i^{\mathrm{hot,in}} - T_i^{\mathrm{cold,in}}) \quad i \in \mathrm{PHX} \cup \mathrm{CU} \cup \mathrm{HU} \tag{2m}$$

where

PHX: set of all process–process heat exchangers

CU: set of cold utility exchangers

HU: set of hot utility exchangers

HXHT: subset of PHX with hot side outlet is a controlled target

HXCT: subset of PHX with cold side outlet is a controlled target

CUT: subset of CU with outlet is a controlled target

HUT: subset of HU with outlet is a controlled target

HXHO: subset of PHX with hot side outlet entering a hot side inlet of the adjacent exchanger

HXCO: subset of PHX with cold side outlet entering a cold side inlet of the adjacent exchanger

HXHI: subset of PHX with hot side inlet coming from a hot side outlet of the adjacent exchanger

HXCI: subset of PHX with cold side inlet coming from a cold side outlet of the adjacent exchanger

HXHS: subset of PHX with hot side inlet directly coming from a hot supply

HXCS: subset of PHX with cold side inlet directly coming from a cold supply.

$P_{\mathrm{h},i}$: thermal efficiency of exchanger i,

$$P_{\mathrm{h},i} = \frac{\mathrm{NTU}_{\mathrm{h},i}(1 - e^{(\mathrm{NTU}_{\mathrm{c},i} - \mathrm{NTU}_{\mathrm{h},i})})}{\mathrm{NTU}_{\mathrm{h},i} - \mathrm{NTU}_{\mathrm{c},i}e^{(\mathrm{NTU}_{\mathrm{c},i} - \mathrm{NTU}_{\mathrm{h},i})}}$$

$$\mathrm{NTU}_{\mathrm{h},i} = \frac{(\mathrm{UA})_i}{(mC_{\mathrm{P}})_i^{\mathrm{hot}}} \quad \mathrm{NTU}_{\mathrm{c},i} = \frac{(UA)_i}{(mC_{\mathrm{P}})_i^{\mathrm{cold}}}$$

$(mC_{\mathrm{p}})_i^{\mathrm{cold}}$ and $(mC_{\mathrm{p}})_i^{\mathrm{hot}}$: Heat capacity flowrate on cold and hot side (kW/°C) of exchanger i.

$(\mathrm{UA})_i$: Product of heat transfer coefficient and heat transfer area of exchanger i (kW/°C).

As shown in the formulation, one process exchanger generates five variables in the vector x (inlet and outlet temperatures of hot and cold side, and heat duty, see Eqs. 2b–2c), while one utility exchanger generates three variables (inlet and outlet temperatures and heat duty, see Eqs. 2d and 2e). Therefore, for a HEN containing $N_{\mathrm{h}x}$ process exchangers, N_{cu} coolers and N_{hu} heaters, the number of variables (N_{var}) in the vector x becomes:

$$N_{\mathrm{var}} = 5N_{\mathrm{h}x} + 3N_{\mathrm{cu}} + 3N_{\mathrm{hu}} \tag{3}$$

In terms of equality constraints, one process exchanger generates two equality constraints [removed heat on hot side and received heat on cold side (Eqs. 2b and 2c), while one utility exchanger generates one equality constraint (removed heat to a cooler or received heat from a heater (see Eqs. 2d and 2e)]. The number of connecting equations is the sum of the number of supply specification (N_{s}, Eqs. 2f and 2g), number of internal variable connection between the adjacent heat exchangers ($N_{\mathrm{int,connect}}$, Eqs. 2h and 2i), and number of target specification (N_{t}, Eqs. 2j and 2k). Therefore, the number of equality constraints (N_{eq}) is:

$$N_{\mathrm{eq}} = 2N_{\mathrm{h}x} + N_{\mathrm{cu}} + N_{\mathrm{hu}} + N_{\mathrm{s}} + N_{\mathrm{t}} + N_{\mathrm{int,connect}} \tag{4}$$

Each process exchanger and utility exchanger generates two inequality constraints (see Eqs. 2l and 2m) and hence the number of inequality constraints (N_{ineq}) is:

$$N_{\mathrm{ineq}} = 2(N_{\mathrm{h}x} + N_{\mathrm{cu}} + N_{\mathrm{hu}}) \tag{5}$$

Theorem 1. *The optimal operation problem of a simple HEN* is a LP problem.*

Proof: Equations 2a–2m

*A simple HEN in this context refers to a HEN with (1) only single bypasses (duties on individual process heat exchangers) and utility duties as degrees of freedom (manipulations), (2) given heat capacity flowrates, and (3) given UA values for the heat exchangers. Note that the process stream flowrates and stream-splits are not considered as degrees of freedom. □

The main "trick" used above to show that the optimal operation of a HEN is a LP problem is to introduce the thermal efficiency $P_{\mathrm{h},i}$ which avoids introducing the logarithmic mean temperature difference (LMTD) in the model for the heat transfer. The efficiency factors are constant under the assumption of constant heat capacity flowrates (mC_{P}) and constant heat transfer coefficients (UA).

Corollary 1.1. *The optimal operation of a simple HEN lies always at constraints.*

Proof: Property of a LP problem. □

An important property of a LP problem is that one optimal solution is always in a "corner." This implies that after satisfying in equality constraints (i.e., target temperatures), it is optimal to use all remaining degrees of freedom to satisfy active constraints (i.e., fully closing or opening of some bypasses or utility duties). From this follow Theorem 1 and Corollary 1.1.

The above LP problem may have multiple solutions (but not always) if there are some free degrees of freedom that may not affect the utility cost. This occurs when the HEN contains some loops. An idea to handle multiple solutions of the LP is discussed in the Discussion section.

Note that it is possible to extend the LP formulation to include, for example, inequality constraints on temperatures (rather than targets) and other objective functions, for example, maximum temperature. However, the results in this paper are based on the above formulation by use of split-range control.

The LP formulation implies that the optimal solutions are always at a constraint (vertex). The inequality constraints in the above formulation (see Eqs. 2l and 2m) imply active constraints on manipulations (i.e., duties of individual process and utility exchangers). This means that after the necessary degrees of freedom (manipulations) are used for control of the target temperatures (equality constraints), it is optimal to keep all remaining manipulations at constraints. However, under the variation of operating conditions, the optimal vertex (set of active constraints) may change. For a given operating window, we may have several optimal vertices for active constraint regions. Hence, if one can track the right active constraints during the operation, optimality can be obtained. One solution is to use an online optimization technique (e.g., Arkun and Stephanopoulos[11]). Alternatively, one may try to avoid an online optimization task by using some logic to determine switching between active constraint regions and combine this with decentralized control. A particular implementation of the latter using common split-range control is the focus of this paper.

Switching Between Active Constraints

Preliminaries

In this section we describe methods for possible implementation of the optimal policy by tracking the changing set of active constraints. We make the following assumptions:

A1: Target temperatures are feasible for the given disturbance window (output constraints do not change).

A2: The output constraints do not change and are always active. The optimal point is a vertex, i.e., at the intersection of constraints and hence, a certain number of inputs are at the constraints.

Under these assumptions, the optimal solution has the following properties:

1. The set of active constraints remains constant in a certain region of the disturbance space. The largest region in the disturbance space where the set of active constrains remains the same is known as critical region. Critical regions are polyhedral in shape for a LP and can be determined using offline optimization or parametric programming tools.[12]

2. If there are two or more critical regions in the given disturbance window, from the definition of critical region, it follows that the set of constraints are different. Since the output constraints do not change, it follows that the set of input constraints are different in each critical region. At the interface between two neighboring critical regions, constraints corresponding to both critical regions are active (which is a degenerate LP solution). However, since this constitutes a set of measure zero (i.e., the probability of being exactly on the boundary is zero), it does not affect the controllability properties of the network on the whole.

Using these properties of the optimal solution, it is possible to operate the HEN optimally using the following procedure:

1. In a given critical region R_o, it is possible to operate the HEN optimally using a decentralized control structure where some manipulations are used to control the output constraints using SISO control loops with zero steady state error, for example, PI controllers. The remaining manipulations are maintained at the constraints.

2. If the disturbances are such that we have moved from R_o to a different region R_1, it is possible to implement the optimal policy in R_1 by tracking the transition or change in active constraints.

Table 1. Set of Active Constraints for Example Process

Region	MV_1	MV_2	MV_3
1	S	U	U
2	U	U	S
3	U	S	U

U, Unsaturated manipulation (inactive constraint) to be used for control of target temperatures; S, Saturated manipulation (active constraint).

For example, suppose we have a system with three manipulations and two controlled variables (target temperatures, T1 and T2). Clearly, we need two manipulations for control. Furthermore, since one optimal solution is always at input constraints, the remaining manipulation may be at constraints (saturated). For a given operating window, active constraint regions can be found using parametric programming and the results can be summarized as shown in Table 1.

Thus, in Region 1, it is optimal to use MV_2 and MV_3 to control the outputs T1 and T2 respectively using SISO PI control loops and keep MV_1 at constraint. When moving into Region 2, MV_3 saturates and so, the optimal policy is to keep MV_3 at the constraint and instead use MV_1 as a manipulation for control. Thus MV_1 and MV_2 are used for control in Region 2. Likewise, in Region 3, the optimal policy is to control T1 and T2 using MV_1 and MV_3 and keep MV_2 at constraint. It is possible to keep track of the regions by tracking the changes in active constraints. When the new region is determined, the optimal policy corresponding to the new region is implemented. We discuss two ways of implementing this policy.

Implementation 1: Using Switching Logic. In this method, a switching logic based on the current state and change in some set of active constraints is used to determine the corresponding control law. The switching logic can be represented as:

Switching between Regions 1 and 2

- MV_2 is inactive constraint, MV_3 becomes active constraint in Region 2
- MV_2 is inactive constraint, MV_1 becomes active constraint in Region 1

Switching between regions 1 and 3

- MV_3 is inactive constraint, MV_2 becomes active constraint in Region 3
- MV_3 is inactive constraint, MV_1 becomes active constraint in Region 1

Switching between regions 2 and 3

- MV_1 is inactive constraint, MV_2 becomes active constraint in Region 3
- MV_1 is inactive constraint, MV_3 becomes active constraint in Region 2

This switching logic with the three sets of decentralized controllers (corresponding to Regions 1–3) can be used to implement the operating policy and is optimal in the presence of disturbances without the need to directly measure the disturbances and reoptimize the plant. The logic can be extended to more general situations using finite state machines.

However, in general, the switching logic can become very complicated. In some circumstances, a simpler implementation is possible using a split range controller. In the remainder of this work, we focus on implementation of the optimal solution using a split range controller.

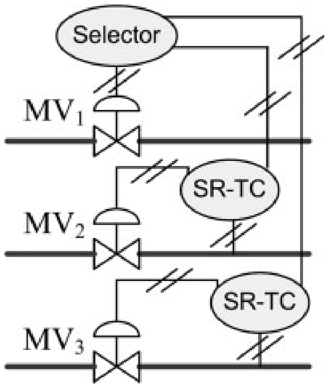


Figure 1. Control system of the example process.

(SR-TC, split-range temperature controller).

Implementation 2: Using Split Range Control. Split range controllers are commonly used to control two or more manipulations using a single controller. A technique using structural information (sign matrix) to find a control structure for optimal operation of HENs was proposed by Mathisen et al.[5] and Glemmestad et al.[6] They commented that in most case the obtained control structure can be implemented in a split-range control manner. Depending on the directional effect of the manipulations, different control configurations are possible.[2] When two manipulations are used, we refer to one of them as primary and the other as secondary. The primary manipulation can be thought of as the manipulation that is used to control a target under nominal conditions. However, the final choice of primary and secondary can be based on other considerations also. This flexibility will be exploited in the final control structure design.

In addition to the Assumptions A1 and A2, in order to obtain a decentralized control structure using split range control, further assumptions made are:

A3: One split-range combination contains only two manipulations. Hence, each primary manipulation can have only one secondary manipulation. Note that this does not rule out the possibility that a variable that is treated as a secondary manipulation can be used in two or more split range controllers.

A4: Only one saturation (upper or lower bounds) is allowed for each manipulation.

A simple illustration will be provided for the above example. Assume that Region 1 is the "primary" region. Then MV_2 and MV_3 are the "primary" manipulations used for control of the target temperatures. For optimality, the active constraint should be switched to MV_3 when operation moves into Region 2, and to MV_2 in Region 3. In terms of control, when moving to Region 2, MV_1 needs to take over the task of saturated MV_3 ("MV_1 is used as a secondary manipulation for MV_3"), and when moving to Region 3 MV_1 needs to take over the task of saturated MV_2 ("MV_1 is used as a secondary manipulation for MV_2"). Hence, we should combine MV_2 & MV_1 and MV_3 & MV_1 as split-range pairs using MV_1 as the secondary manipulation. This control system can be shown in Figure 1.

In the above example, the choices of the secondary manipulations for the primary manipulations could be determined by inspection. In general problems, with a large number of manipulations and regions of active constraints, this is not a trivial task. Hence, a systematic method of determining this pairing is necessary. Lersbamrungsuk et al.[2] showed how the optimal split-range control structure can be found by using the information of directional effect (sign element). However, when the sign is unclear, the control structure cannot guarantee optimality. We here present an optimization formulation that determines an optimal split range control structure.

ILP formulation to determine split-range control structure

Assuming that A1–A4 hold and the set of active constraints in the critical regions is known, an integer linear programming (ILP) formulation for the design of an optimal split-range control structure is as follows (Table 2).

Objective Function I: Minimizing the number of "inter-connection" or "complexity" of control structure

Table 2. Definitions and Notation for the ILP Formulation

Definition 1: Set of controlled and manipulated variables
- CV: set of controlled variables, $CV = \{CV_1, CV_2, \ldots, CV_{N_{cv-1}}, CV_{N_{cv}}\}$
- MV: set of manipulations, $MV = \{MV_1, MV_2, \ldots MV_{N_{m-1}}, MV_{N_m}\}$
- MVAAT: subset of MV with manipulations which are always active constraints (saturated at upper or lower bounds)
- MVINAT: subset of MV with manipulations which are always inactive constraints (never saturated)
- MVAT: subset of MV with manipulations which change between being active and inactive constraints

Definition 2: Primary and secondary manipulations
- Primary manipulation: A manipulation that is used for controlling an output (target), except when it is saturated.
- Secondary manipulation: A manipulation that is used to take over the task of a saturated primary manipulation.

Definition 3: Relationship between primary and secondary manipulations
- Let $x_{i,j}$ (where $i,j \in MV$) be a binary variable which represents the relationship between manipulation MV_i and manipulation MV_j
  - for $i = j$, $x_{i,i} = 1$ implies manipulation MV_i is a primary manipulation
  - $x_{i,i} = 0$ implies manipulation MV_i is a secondary manipulation or unused
  - for $i \neq j$, $x_{i,j} = 1$ implies manipulation MV_j is a secondary manipulation for MV_i
  - $x_{i,j} = 0$ implies manipulation MV_j is not a secondary manipulation for MV_i

Definition 4: Relative order between manipulations and controlled variables
- Let $r_{k,j}$ be a relative order between controlled variable CV_k and manipulation MV_j. Relative order is a structural measure of how direct an effect an input has on an output (i.e. physical closeness).[13] However, for simplicity we here assume $r_{k,j}$ as a number of exchanger units between controlled variable CV_k and manipulation MV_j

Definition 5: Relationship between controlled variables and manipulations
- Let $z_{k,j}$ (where $k \in CV$, $j \in MV$) be a binary variable that represents the relationship between controlled variable CV_k and manipulation MV_j
  - $z_{k,j} = 1$ implies controlled variable CV_k is paired with manipulation MV_j
  - $z_{k,j} = 0$ implies controlled variable CV_k is not paired with manipulation MV_j

(unnecessary relationships between primary and secondary manipulations).

$$\min J_{\mathrm{I}} = \sum_{i}\sum_{j\neq i} x_{i,j} \quad i,j \in \mathrm{MV} \tag{6}$$

Constraint 1: Assign one primary manipulation to each control objective.

Number of primary manipulation is equal to number of controlled variables (N_{CV})

$$\sum_{i} x_{i,i} = N_{\mathrm{CV}} \quad i \in \mathrm{MV} \tag{7}$$

Constraint 2: A manipulation MV_i that always is an active constraint should not be used for other purposes

Manipulation MV_i is not used for control

$$x_{i,i} = 0 \quad \mathrm{i} \in \mathrm{MVAAT} \tag{8}$$

Manipulation MV_i has no need of a secondary manipulation

$$\sum_{j\neq i} x_{i,j} = 0 \quad i \in \mathrm{MVAAT}, j \in \mathrm{MV} \tag{9}$$

Manipulation MV_i is not used as a secondary manipulation

$$\sum_{j\neq i} x_{j,i} = 0 \quad i \in \mathrm{MVAAT}, j \in \mathrm{MV} \tag{10}$$

Constraint 3: A manipulation MV_i that is never an active constraint is used as a primary manipulation with no need of a secondary manipulation.

Manipulation MV_i is a primary manipulation

$$x_{i,i} = 1 \quad i \in \mathrm{MVINAT} \tag{11}$$

Manipulation MV_i has no need of a secondary manipulation

$$\sum_{j\neq i} x_{i,j} = 0 \quad i \in \mathrm{MVINAT}, j = \mathrm{MV} \tag{12}$$

Manipulation MV_i is not used as a secondary manipulation

$$\sum_{j\neq i} x_{j,i} = 0 \quad i \in \mathrm{MVINAT}, j = \mathrm{MV} \tag{13}$$

Constraint 4: A manipulation MV_i that changes between being an active and inactive constraint may be a primary or secondary manipulation.

1. MV_i may have a need or no need of a secondary manipulation if
   - MV_i is chosen to be a primary manipulation that can be saturated (active constraint), then a secondary manipulation is needed

$$\text{if } x_{i,i} = 1 \quad \text{then} \quad \sum_{j\neq i} x_{i,j} = 1 \quad i,j \in \mathrm{MVAT}$$

   - MV_i is not chosen to be a primary manipulation, then it has no need of a secondary manipulation

$$\text{if } x_{i,i} = 0 \quad \text{then} \quad \sum_{j\neq i} x_{i,j} = 0 \quad i,j \in \mathrm{MVAT}$$

The above two statements can be written

$$-x_{i,i} + \sum_{j\neq i} x_{i,j} = 0 \quad i,j \in \mathrm{MVAT} \tag{14}$$

2. MV_i can be or cannot be used as a secondary manipulation if
   - MV_j is chosen to be a primary manipulation, then it is not used as a secondary manipulation for the other manipulations

$$\text{if } x_{j,j} = 1 \quad \text{then} \quad \sum_{i\neq j} x_{i,j} = 0 \quad i,j \in \mathrm{MVAT}$$

   - MV_j is chosen to be a secondary manipulation, then it is used for at least one primary manipulation

$$\text{if } x_{j,j} = 0 \quad \text{then} \quad \sum_{i\neq j} x_{i,j} \geq 1 \quad i,j \in \mathrm{MVAT}$$

The above two statements can be written

$$x_{j,j} + \sum_{i\neq j} x_{i,j} \geq 1 \quad i,j \in \mathrm{MVAT} \tag{15}$$

and

$$M(x_{j,j} - 1) + \sum_{i\neq j} x_{i,j} \leq 0 \quad i,j \in \mathrm{MVAT} \tag{16}$$

where M is a positive integer which is greater than the number of members in **MVAT**.

Constraint 5: Possible and impossible split-range combination of manipulations (these constraints are obtained from the information of active constraint regions).

Constraint 5A: Impossible split-range combination of manipulations

"Impossible pair: Two manipulations which are active constraints (saturated) at the same time cannot be combined as a split-range pair"

For an active constraint Region R, we have

$$\sum_{i}\sum_{j\neq i} x_{i,j} = 0 \quad i,j \in \mathrm{MVAT}^{\mathrm{A,R}} \tag{17}$$

where $\mathrm{MVAT}^{\mathrm{A,R}}$ is the subset of MVAT with manipulations being active constraints in Region R.

Constraint 5B: Possible split-range combination of manipulations

"Possible pair: two manipulations which are not active (inactive) constraint at the same time may be combined as a split-range pair"

For an active constraint Region R, we have

$$x_{j,j} + \sum_{i\neq j} x_{i,j} \geq 1 \quad i \in \mathrm{MVAT}^{\mathrm{I,R}}, j \in \mathrm{MVAT}^{\mathrm{A,R}} \tag{18}$$

$$x_{i,i} + \sum_{j\neq i} x_{j,i} \geq 1 \quad \mathrm{i} \in \mathrm{MVAT}^{\mathrm{I,R}}, \quad j \in \mathrm{MVAT}^{\mathrm{A,R}} \tag{19}$$

where $\text{MVAT}^{\text{I,R}}$ is the subset of MVAT with manipulations being inactive constraints in Region R.

See an illustration of Constraint 5 in Appendix.

Combining Objective Function I and Constraints 1–5, Problem P1 can be written,

Problem P1.

$$\min J_I = \sum_i \sum_{j \neq i} x_{i,j} \quad i,j \in \text{MV}$$

Subject to Eqs. 7–19.

By solving the Problem P1, one obtains split-range pairs that can provide optimal switching between active constraint regions. However, the solution of Problem P1 may be nonunique. Hence, relative orders are introduced as an additional criterion for screening the set of poorly controllable structure solutions. The additional objective function and constraints are as follows:

Objective Function II: Minimizing the sum of relative order of the control pairs.

$$\min J_{II} = \sum_k \sum_j r_{k,j} z_{k,j} \quad k \in \text{CV}, j \in \text{MV} \tag{20}$$

Constraint 6: Assign one manipulation to each control objective

$$\sum_j z_{k,j} = 1 \quad k \in \text{CV}, j \in \text{MV} \tag{21}$$

Constraint 7: Only primary manipulations are paired with controlled variables.

If MV_j is a primary manipulation, it must be paired with a controlled variable

$$\text{If } x_{j,j} = 1 \quad \text{then} \quad \sum_k z_{k,j} = 1 \quad k \in \text{CV}, j \in \text{MV}$$

If MV_j is not a primary manipulation, it must not be paired

$$\text{If } x_{j,j} = 0 \quad \text{then} \quad \sum_k z_{k,j} = 0 \quad k \in \text{CV}, j \in \text{MV}$$

Therefore,

$$-x_{j,j} + \sum_k z_{k,j} = 0 \quad k \in \text{CV}, j \in \text{MV} \tag{22}$$

The ILP problem now concerns with two objective functions that can be solved using lexicographic optimization. In lexicographic optimization, the objectives are arranged in a decreasing order of preference and objectives with a higher preference are considered to be infinitely more important than those with lower orders. Among solutions that are optimal with respect to the first objective, solutions that are optimal with respect to the second objective are chosen.

Using the idea of lexicographic optimization, we first solve P1:

$$J_I^* = \min_x J_I(x) \quad x \in S$$

where S is the feasible set and then solve an associated Problem P1′:

$$\min_x J_{II}(x) \quad x \in S, J_I = J_I^*(x)$$

which ensures that among minimized J_{I} solutions, the minimized J_{II} solutions are chosen. In principle, we need to solve two optimization problems in sequence. However, it is possible to solve P1 and P1′ as a single optimization problem by minimizing a weighted objective function $wJ_{\text{I}} + J_{\text{II}}$, where w is a sufficiently large positive number chosen appropriately. Suggestions for choice of w are given in Sherali,[14] and Sherali and Soyster.[15] Hence, we solve the following problem P2:

Problem P2.

$$J = \min(wJ_I + J_{II}) \quad i,j \in \text{MV}, k \in \text{CV}$$

$$J_I = \sum_i \sum_{j \neq i} x_{i,j}, \quad J_{II} = \sum_k \sum_j r_{k,j} z_{k,j}$$

Subject to Eqs. 7–19, 21, and 22.

It can be seen that Constraints 6 and 7 (Eqs. 21 and 22) do not alter the feasible set for P1. The ILP problem P2 consists of two objective functions with a weighting factor (w) between the two. The first objective is used to minimize complexity when changing between active constraints whereas the second objective (controllability) is used to select the most controllable control structure. A large value of w will imply that the second objective (controllability) will only be considered when there are multiple solutions.

A HEN Case Study

The HEN in Figure 2 is from the work of Aguilera and Marchetti[7] but we have modified it to use only single bypasses. The HEN contains two hot and two cold streams with four target outlet temperatures of stream H1, H2, C1 and C2 ($T_{\text{H1}}^{\text{out}}$, $T_{\text{H2}}^{\text{out}}$, $T_{\text{C1}}^{\text{out}}$, and $T_{\text{C2}}^{\text{out}}$). The utility prices are 0.05 \$/kWh for hot utility H, 0.02 \$/kWh for cold utility C1 and 0.01 \$/kWh for cold utility C2.

There are six degrees of freedom (heat duties of all exchangers, Q) and four equality constraints on the target outlet temperatures. This leaves two degrees of freedom. Assume that the disturbances are the inlet temperature of each stream, with the expected variation $\pm 10°\text{C}$ for streams

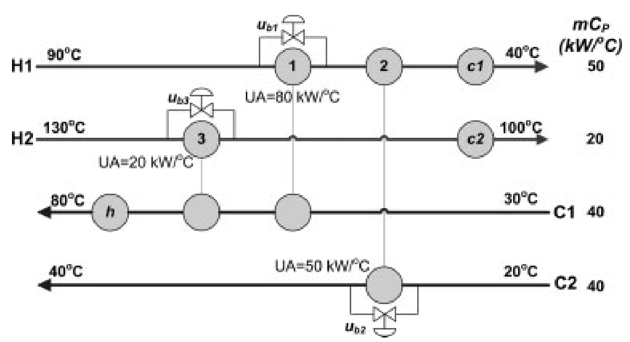


Figure 2. A HEN case study.

Table 3. Set of Active Constraints in the Case Study

Region	Manipulations					
	Q_{C1}	Q_{C2}	Q_h	u_{b1}	u_{b2}	u_{b3}
1	S_L	U	S_L	U	U	U
2	S_L	S_L	U	U	U	U
3	U	S_L	U	S_L	U	U
4	U	U	S_L	S_L	U	U
5	U	U	S_L	U	U	S_U

U, unsaturated manipulation (inactive constraint); S_L, saturated manipulation (active constraint) at the lower bound; S_U, saturated manipulation (active constraint) at the upper bound.

H1, H2, and C1 and ±5°C for stream C2. This results in feasible LP optimal solutions and the resulting five active constraint regions can be obtained using parametric programming[12] as shown in Table 3. As expected there are two active constraints (saturated manipulations) in each region. Note that the setpoints for the target temperatures are assumed constant, that is, these are not included as disturbances.

Table 3 demonstrates that manipulations Q_{C1}, Q_{C2}, Q_h, and u_{b1} can become active constraints at the lower bounds (i.e., zero utility duties or fully close of bypasses) while manipulation u_{b3} can become an active constraint at the upper bound (i.e., fully open of bypasses). The manipulation u_{b2} is never an active constraint (never saturated) and hence it should be used as a primary manipulation with no need of a secondary manipulation.

The software "GAMs" with the solver "CPLEX" was used to solve the ILP. The solution to Problem P1 (minimizing complexity of optimal split-range pairs) in Table 4 shows that Q_{C1}, Q_{C2}, u_{b1}, and u_{b2} are chosen to be primary manipulations (see diagonal elements with $x_{i,i} = 1$) while Q_h and u_{b3} are chosen to be secondary manipulations (see diagonal elements with $x_{i,i} = 0$). Q_h is the secondary manipulation for Q_{C2} ($x_{2,3} = 1$) and u_{b3} is the secondary manipulation for Q_{C1} and u_{b1} ($x_{1,6} = 1$ and $x_{4,6} = 1$). However, the solution obtained from the Problem P1 may not be unique. For example, by including a constraint $x_{3,3} = 1$ (i.e., set Q_h as a primary manipulation) in Problem P1, a different solution with the same value of objective function I ($J_I = 3$) is obtained as shown in Table 5.

To handle the multiple solutions of Problem P1, the second objective J_{II} (controllability purpose in terms of minimizing the sum of relative orders) is introduced and included in problem P2 for selecting the most controllable control structure. The additional information of relative orders is shown in Table 6. The values of binary variables $x_{i,j}$ and $z_{k,j}$

Table 4. The Values of $x_{i,j}$ After Solving the ILP Problem P1 ($J_I = 3$)*

Pri MV	Sec MV					
	Q_{C1}	Q_{C2}	Q_h	u_{b1}	u_{b2}	u_{b3}
Q_{C1}	1					1
Q_{C2}		1	1			
Q_h						
u_{b1}				1		1
u_{b2}					1	
u_{b3}						

*Cells without entries indicate zeros.

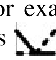
Table 5. The values of $x_{i,j}$ After Solving the ILP Problem P1 with Setting $x_{3,3} = 1$ ($J_I = 3$)*

Pri MV	Sec MV					
	Q_{C1}	Q_{C2}	Q_h	u_{b1}	u_{b2}	u_{b3}
Q_{C1}	1					1
Q_{C2}						
Q_h		1	1			
u_{b1}				1		1
u_{b2}					1	
u_{b3}						

*Cells without entries indicate zeros.

from solving the problem P2 are shown in Tables 7 and 8, respectively.

Table 7 shows Q_{C1}, Q_{C2}, u_{b2}, and u_{b3} are chosen to be primary manipulations while Q_h and u_{b1} are chosen to be secondary manipulations. Table 8 shows the appropriate control pairing, $T_{H1}^{out} - Q_{C1}$, $T_{H2}^{out} - Q_{C2}$, $T_{C1}^{out} - u_{b3}$, and $T_{C2}^{out} - u_{b2}$ (see $z_{1,1} = z_{2,2} = z_{3,6} = z_{4,5} = 1$). The control structure for optimal operation of the HEN in this case study is shown in Figure 3.

From Figure 3, the SR-TC block represents a split-range controller with Port 1 to send a signal to a primary manipulation and Port 2 to send a signal to a secondary manipulation. The split-range signal of each split-range controller can be obtained by considering the information of active constraint regions (Table 3). For example, the split-range signal of the pair of Q_{C1} and u_{b1} is  (thick line, primary manipulation; dash line, secondary manipulation) because Q_{C1} and u_{b1} switch alternately to their lower constraints. Furthermore, because u_{b1} is chosen as the secondary manipulation for more than one primary manipulation (i.e. Q_{C1} and u_{b3}), a selective controller is used to select the secondary signal from control loops.

Dynamic Simulation

The HEN in the case study with the suggested control structure is tested by performing dynamic simulation on Aspen Dynamics v12.1. The information of disturbances and active constraints of the system at each period are shown in Table 9. Figure 4b shows the ability of the control structure to keep all target temperatures at the desired values even under the saturation of some manipulations (see Figures 4d, e). The input saturation problem is solved by switching ability to use a secondary manipulation when a primary manipulation is saturated. Furthermore, the optimality (in terms of utility cost) is also given as shown in Figure 4c that the graph of utility cost can track the optimal line. This conse-

Table 6. Relative Orders of the HEN in the Case Study

CV	MV					
	Q_{C1}	Q_{C2}	Q_h	u_{b1}	u_{b2}	u_{b3}
T_{H1}^{out}	1	∞	∞	3	2	∞
T_{H2}^{out}	∞	1	∞	3	∞	2
T_{C1}^{out}	∞	∞	1	3	∞	2
T_{C2}^{out}	∞	∞	∞	2	1	∞

Table 7. The Values of $x_{i,j}$ After Solving the ILP Problem P2 ($J_I = 3$)*

Pri MV	Sec MV					
	Q_{C1}	Q_{C2}	Q_h	u_{b1}	u_{b2}	u_{b3}
Q_{C1}	1			1		
Q_{C2}		1	1			
Q_h						
u_{b1}						
u_{b2}					1	
u_{b3}				1		1

*Cells without entries indicate zeros.

quence comes from the ability of the control structure to track the right active constraint during the operation (see Figures 4d, e, and set of active constraints in Table 9).

Note that the resulting control structure depends upon the specific disturbances (i.e., inlet temperatures in the HEN case study) considered in the design stage. If any other disturbances (e.g., setpoints, stream flowrates) come to the system, the optimality and feasibility of some target temperatures may not be obtained since the optimal active constraint regions may change. Consequently, in general case all expected disturbances should be included in the design phase of the control structure.

Discussion

Simple control structures versus online optimization

The heat exchanger network problem studied in this work is an example of a problem where, after satisfying the given target temperatures (equality CV constraints), there are extra degrees of freedom (inputs) left for optimization. Furthermore, assuming that the heat capacity flow rates (mC_P) and heat transfer coefficient (UA) are independent of temperature (which is reasonable) such that the coefficients in the model (2a–2m) are fixed, then this is a linear programming (LP) problem and it is always optimal to keep these extra degrees of freedom at constraints (max or min). There are three main approaches for implementing optimal operation in such a case:

1. Repeated online optimization (RTO) based on estimating the present state ("reconciliation") (e.g., Lid et al.[16]).
2. Use of offline optimization to generate all active constraint regions, and using information about the present state to switch between regions and then implement the precomputed optimal solution for the region.
3. Simpler implementation of Approach 2 (if possible) using (a) only information about the active constraints (and not the entire state vector) and (b) simple control structures such as split-range control and selectors.

Table 8. The Values of $z_{k,j}$ After Solving the ILP Problem P2 ($J_{II} = 5$)*

CV	MV					
	Q_{C1}	Q_{C2}	Q_h	u_{b1}	u_{b2}	u_{b3}
T_{H1}^{out}	1					
T_{H2}^{out}		1				
T_{C1}^{out}						1
T_{C2}^{out}					1	

*Cells without entries indicate zeros.

Approach 1 is the most general, and is always possible if a feasible solution exists (for the HEN problem, this means that the target temperatures must be feasible for all disturbances), but it requires a detailed steady-state model the online computation demand may be large. Approach 2 is always possible for cases like simple HENs, where the optimal solution is always at constraints. The online computation demand is reduced, but it still requires information about the present state, which may require online estimation of the present state ("data reconciliation"). The implementation of Approach 3 is much simpler, but is not always possible.

This work is concerned with Approach 3, so the main issue addressed is whether it is possible to identify a simple method of switching between the active constraints using something close to single-loop control where one unsaturated input is always paired with one output (target temperature). Note that the number of active constraints remains constant, so that if one input saturates and is no longer available for controlling a target temperature, then another input has to come out of saturation to take over this job (this is referred to as "protection" in the language of split range control used in this paper). It is simple online to identify that an input reaches a constraint, but it is not clear which input (manipulation) should come out of saturation and take over the job. There are two questions here. First (Q3a), is it possible, just from the information about the active constraints, to determine uniquely which input to take out of saturation? Second (Q3b), can this be implemented in a simple manner using, for example, only split-range control and selectors? If the answer to these questions (Q3a and Q3b) is positive, then optimal operation is always guaranteed with a simple control structure (Approach 3) without the need for online optimization (RTO; Approach 1).

Except for very simple cases, it is not easy to answer these questions by inspection. One systematic approach is the sign method[2] but this can only handle a subset of the cases where Approach 3 is feasible. The method used in this work for Approach 3 is to first perform offline steady-state optimization using the steady-state model to generate all the possible active constraint regions (e.g. Table 2), and then use integer

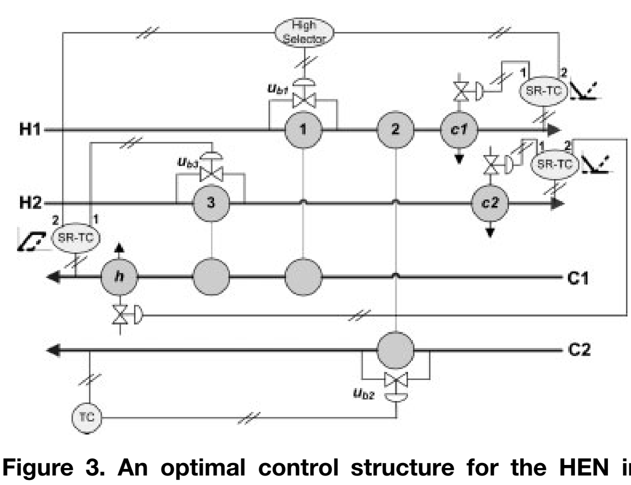


Figure 3. An optimal control structure for the HEN in the case study.

Table 9. Disturbances and Active Constraints for Each Period

Time (sec)	Disturbance				Active Constraint					
	ΔT_{H1}^{in}	ΔT_{H2}^{in}	ΔT_{C1}^{in}	ΔT_{C2}^{in}	Q_{C1}	Q_{C2}	Q_h	u_{b1}	u_{b2}	u_{b3}
<5	0	0	0	0	S_L		S_L			
5–15	−10	−10	−10	−5	S_L	S_L				
15–25	−10	−10	10	5		S_L		S_L		
25–35	10	10	−10	−5	S_L		S_L			
>35	10	10	10	5			S_L			S_U

S_L, Saturated manipulation (active constraint) at the lower bound; S_U, Saturated manipulation (active constraint) at the upper bound.

linear programming (ILP) to determine if a simple optimal control structure based on considering only the active constraints can be identified.

If the ILP method fails to find a feasible solution for Approach 3, then one may allow for a more complex structure than the one used in this work by relaxing Assumptions A3 and A4, for example by allowing two inputs (and not only one) to protect another input. A simple example, is keeping constant room temperature with three available inputs (manipulations): a cooler (c_1), inexpensive heater (h_1), and expensive heater (h_2). For example, the expensive heater could be electricity, and the inexpensive one is hot water.

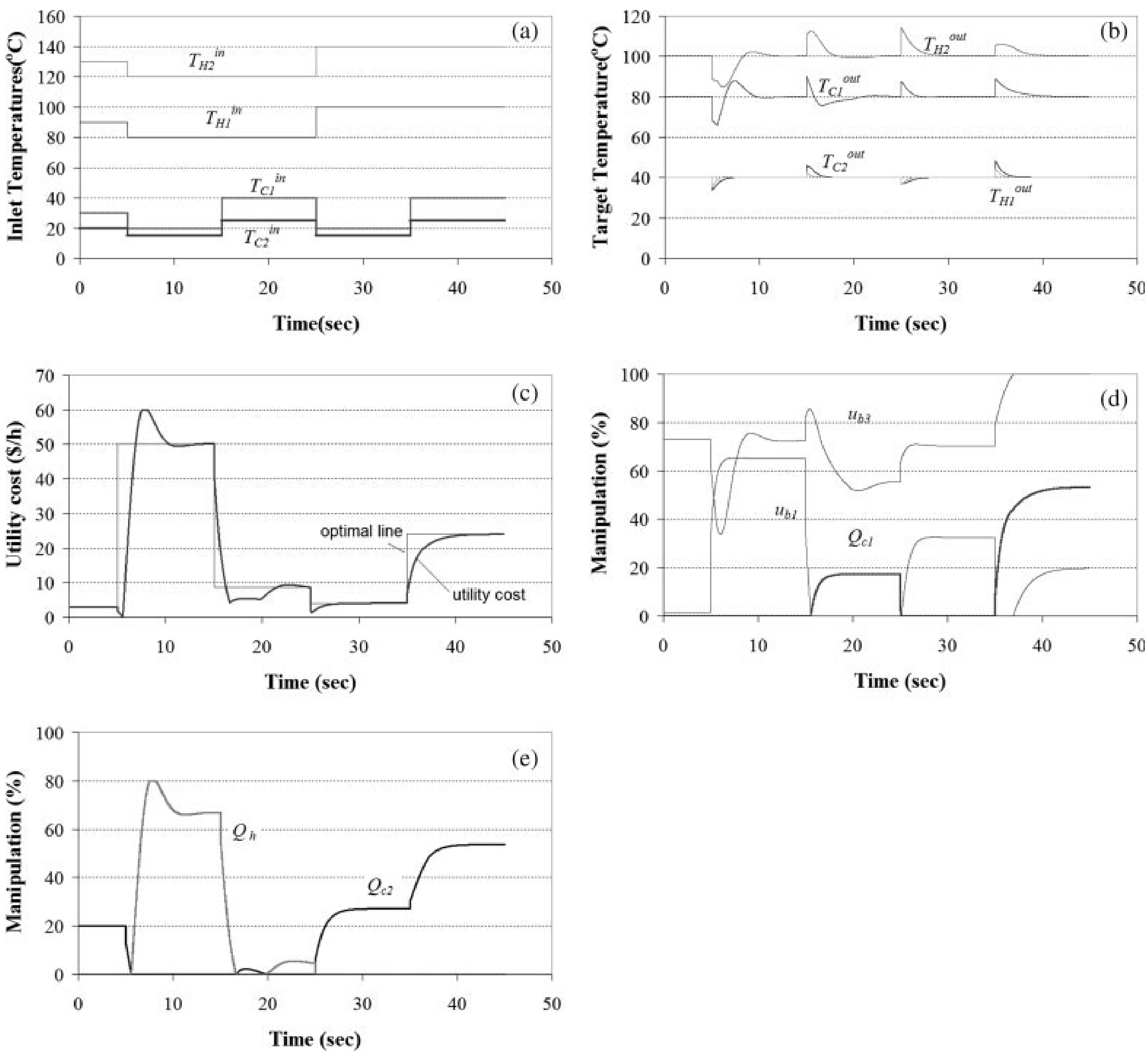


Figure 4. Dynamic simulation of the HEN in case study with the suggested control structure.

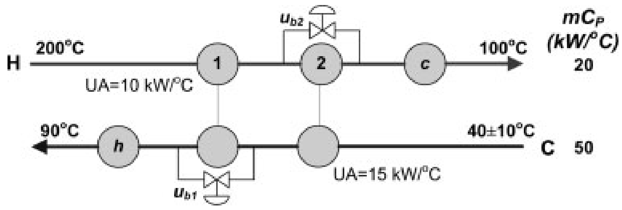


Figure 5. A trivial HEN with a loop.

This problem always has two active constraints (saturated inputs), and it is trivial to design an optimal split range control using all three inputs to control the room temperature at its setpoint; c_1 is used when it is hot, and one switches to h_1 when it gets cold and supplements with h_2 when it gets even colder. In this particular case the solution is simple, but in general such solutions can get very complicated, and at some points the structure may become so complex that it is difficult to maintain and understand. In this case, it is probably better to switch to online optimization (Approach 1).

In cases where Approach 3 fails because the answer to Q3a is negative, then to use still Approach 3 one may assume that the disturbance changes are gradual and introduce the concept of neighboring regions to decide where to switch. This is feasible in this case since the regions are convex, but the assumption of gradual changes may not hold. If detailed state information is available (essentially knowing the temperatures in and out of all heat exchangers), then Approach 2 is feasible, but the required logic may become very complicated.

In practice, Approach 1 (RTO) is therefore the preferred approach for cases where we fail to find a simple solution using Approach 3. With RTO one can also include nonlinear effects in the model, including varying flow rates (m) and heat capacities (C_P), and may include stream splits as manipulations. A successful case study for a crude oil preheating network is reported by Lid et al.[16]

In summary, online optimization (RTO) is to be recommended for more complex and general cases.

The above issues are discussed in more detail in a forthcoming article on the ILP approach.

Multiple solutions of the LP optimization

We have already seen that the split-range implementation (solution to Problem P1) of the LP optimal utility cost solution may be nonunique. This was the main reason for including the controllability in term of the relative order as a secondary objective (Problem P2). However, it is also possible that the LP optimal solution itself is nonunique. This may happen if there are loops in the network[17] because of the possibility to shift duty around loops without affecting the utility cost. Also in this case, same secondary objective may be added into the LP optimization, but one should be careful to avoid changing the optimal solution including the range of feasible solutions. The number of loops (N_{loops}) is given by[17]

$$N_{\text{loops}} = N_{\text{units}} - R - N_{\text{U}}$$

where N_{units} is the number of process exchangers and utility exchangers, R is the dimensional space spanned by the manipulations in the inner HEN to the outer HEN, and N_{U} is the number of utility types.

In the network of the case study, we have $N_{\text{units}} = 6$, $R = 3$, and $N_{\text{U}} = 3$, so $N_{\text{loops}} = 0$. Thus there are no loops and the LP optimal solution is unique. Note that if the two cold utilities have the same cost, then $N_{\text{U}} = 2$ and we would have $N_{\text{loops}} = 1$, and the optimal solution might in some cases be nonunique because of the possibility to have duty shift between the two coolers without affecting the utility cost. Next, we will consider a trivial example with one "internal" loop.

Example 1. A trivial HEN with a loop.

The HEN in Figure 5 contains one hot stream and one cold stream with two target temperatures (outlet temperatures of two streams). Each process exchanger has a single bypass. This network has $N_{\text{units}} = 4$, $R = 1$, $N_{\text{U}} = 2$, and $N_{\text{loops}} = 4 - 1 - 2 = 1$. This implies that there is one degree of freedom that may be used for some purposes without affecting the optimum of utility cost due to the duty shift between Exchangers 1 and 2.

Consider a disturbance of ± 10°C in the inlet temperature of the cold stream C (T_{C}^{in}). The optimization result and active constraint regions (see the columns 2–5 of Tables 10 and 11) from the LP utility cost optimization problem shows that the manipulation Q_{h} (duty of the heater) is never saturated and hence Q_{h} has no need of a secondary manipulation. For the control pairing, to get a direct effect, Q_{h} is used to control $T_{\text{C}}^{\text{out}}$ (the outlet temperature of the cold stream C) while Q_{c} (duty of cooler) is used to control $T_{\text{H}}^{\text{out}}$ (the outlet temperature of hot stream H). However, because Q_{c} can be saturated in some operating conditions, it requires a secondary manipulation which may be u_{b1} or u_{b2} (bypasses of Exchangers 1 or 2) or probably both. If the result in the columns 2–5 of Table 11 is considered, the choice of secondary manipulation is not quite clear because both u_{b1} and u_{b2} are in use. However, for the reason of direct effect, u_{b2} seems to be a better choice. In general, this solution can be found by performing a two-step optimization (i.e., lexicographic optimization) with first solving for the utility cost, and then maximizing

Table 10. Optimization Result of the HEN in Example 1

Disturbance	Minimize Utility Cost (Without Handling Multiple Solutions)				Minimize Utility Cost and Maximize Q_1 (with Handling Multiple Solutions)			
ΔT_C^{in}	u_{b1}	u_{b2}	Q_{C} (kW)	Q_{h} (kW)	u_{b1}	u_{b2}	Q_{C} (kW)	Q_{h} (kW)
0	0.3171*	0.1966*	0	500	0	0.2424	0	500
−10	0.6002*	0.3612*	0	1000	0	0.4635	0	1000
+10	0	0	48	48	0	0	48	48

*Multiple optimal solutions due to duty shift between exchangers in loops.

Table 11. Active Constraint Regions of the HEN in Example 1

Region	Minimize Utility Cost (Without Handling Multiple Solutions)				Minimize Utility Cost and Maximize Q_1 (with Handling Multiple Solutions)			
	u_{b1}	u_{b2}	Q_C (kW)	Q_h (kW)	u_{b1}	u_{b2}	Q_C (kW)	Q_h (kW)
1	U*	U*	S_L	U	S_L	U	S_L	U
2	S_L	S_L	U	U	S_L	S_L	U	U

*Multiple optimal solutions.
U, unsaturated manipulation (inactive constraint); S_L, saturated manipulation (active constraint) at the lower bound.

the duty of Exchanger 1 (Q_1) according to the optimal utility cost in the first step. This results in the LP solution with two active constraints as shown in the columns 6–9 of Tables 10 and 11.

Note that the information of active constraints needed in Constraint 5 of the ILP should be obtained from the solution in the columns 6–9 of Table 11. For example, if the information of active constraints in the columns 2–5 of Table 11 is used in the ILP, there will be no feasible solution.

The result of active constraints regions from the solution in the columns 6–9 of Table 11 shows that Q_c and u_{b2} switch alternately to be an active constraint and hence should be combined as a split-range pair. Moreover, because u_{b1} is always an active constraint, it should be assigned at the constraint for optimality. For this simple HEN, the optimal split-range control structure can be obviously found without the need of the ILP as shown in Figure 6.

Conclusions

When only single bypasses and utility duties are used as manipulations, optimal operation of HENs can be formulated as a linear programming implying the operation lies always at some input constraints. However, under the change of operating condition, the active constraints may change. This motivates the need of a control strategy with the ability to track active constraints under the change of operating condition. In this work, we focused on a decentralized control structure with the ability to provide appropriate switching between active constraints regions in a given operating window. This results in an optimal split-range control structure which can be found by solving an integer linear programming.

It is possible for some HENs to have no optimal split-range control structure (i.e., no feasible solution of the ILP). Hence, a study on a technique for switching between active constraint regions should be further investigated. We expect that this technique is not only able to be applied for constraint (vertex) optimal operation problem, but also for unconstraint (nonvertex) optimal operation problem (e.g., simplifying an online optimization task).

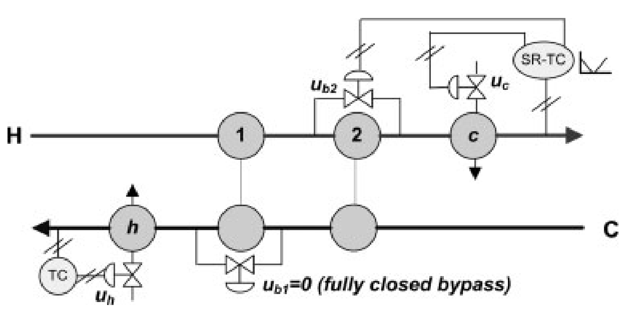


Figure 6. A trivial HEN with an optimal split-range control structure.

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Appendix

Example A1: A trivial example to illustrate Constraint 5

Suppose we have a system with six manipulations and three controlled variables. For optimality, the number of active manipulations during the operation is $6 - 3 = 3$. The information of set of active constraints within a given operating window is shown in Table A1. The results in Table A1 shows MV_5 is never an active constraint (satisfy Constraint 3) while MV_6 is always an active constraint (satisfy Constraint 2). Therefore, MVAT = {MV_1, MV_2, MV_3, MV_4}.

For Region 1, we have $MVAT^{A,R=1}$ = {MV_1, MV_3} and $MVAT^{I,R=1}$ = {MV_2, MV_4} and hence the constraints extracted from this region are

Impossible pair:

$$x_{1,3} + x_{3,1} = 0$$

Possible pair:

$$x_{1,1} + x_{2,1} + x_{4,1} \geq 1$$
$$x_{2,2} + x_{1,2} + x_{3,2} \geq 1$$
$$x_{3,3} + x_{2,3} + x_{4,3} \geq 1$$
$$x_{4,4} + x_{1,4} + x_{3,4} \geq 1$$

Table A1. Set of Active Constraints of the System in Example A1

Region	MV_1	MV_2	MV_3	MV_4	MV_5	MV_6
1	S_L	U	S_L	U	U	S_U
2	U	S_L	S_L	U	U	S_U
3	U	S_L	U	S_L	U	S_U

U, unsaturated manipulation (inactive constraint); S_L, saturated manipulation (active constraint) at the lower bound; S_U, saturated manipulation (active constraint) at the upper bound.

For Regions 2 and 3, we have

$$x_{2,3} + x_{3,2} = 0$$
$$x_{1,1} + x_{2,1} + x_{3,1} \geq 1$$
$$x_{2,2} + x_{1,2} + x_{4,2} \geq 1$$
$$x_{3,3} + x_{1,3} + x_{4,3} \geq 1$$
$$x_{4,4} + x_{2,4} + x_{3,4} \geq 1$$
$$x_{2,4} + x_{4,2} = 0$$
$$x_{1,1} + x_{2,1} + x_{4,1} \geq 1$$
$$x_{2,2} + x_{1,2} + x_{3,2} \geq 1$$
$$x_{3,3} + x_{2,3} + x_{4,3} \geq 1$$
$$x_{4,4} + x_{1,4} + x_{3,4} \geq 1$$

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