Effective implementation of optimal operation using off-line computations

Sridharakumar Narasimhan, Sigurd Skogestad

Department of Chemical Engineering, NTNU, Trondheim, Norway

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NTNU
Innovation and Creativity

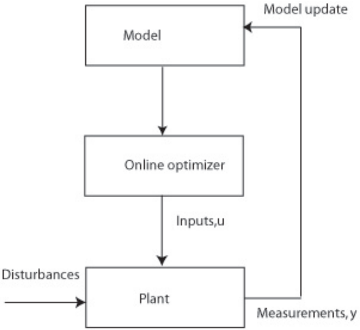
Optimal operation

- A typical dynamic optimization problem

$$\begin{aligned} &\min_{u} J(x, u, d) \\ \text{s.t. } &\dot{x} = f(x, u, d), \\ &h(x, u, d) = 0, \\ &g(x, u, d) \leq 0. \end{aligned}$$

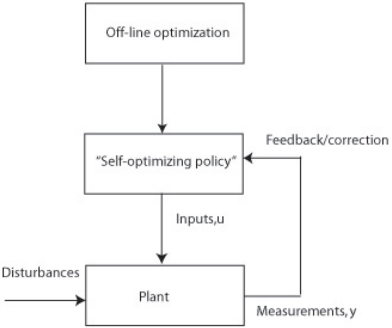
- “Open-loop" solutions not robust to disturbances or model uncertainty.
- Introduce feedback.

Introducing feedback: Paradigm 1



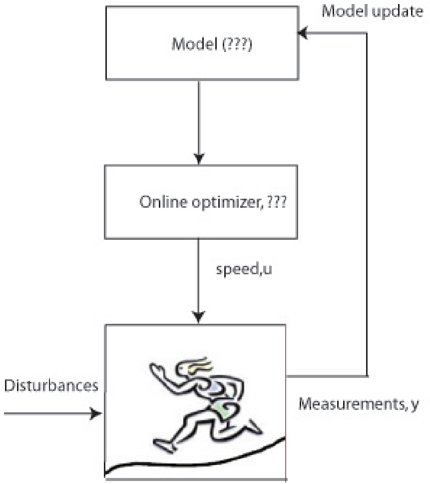
- Paradigm 1: Online optimizing control where measurements are primarily used to update the model.
- With the arrival of new measurements, the optimization problem is resolved online for the inputs.
- Also referred to as explicit schemes (Srinivasan and Bonvin, 2007)

Introducing feedback: Paradigm 2



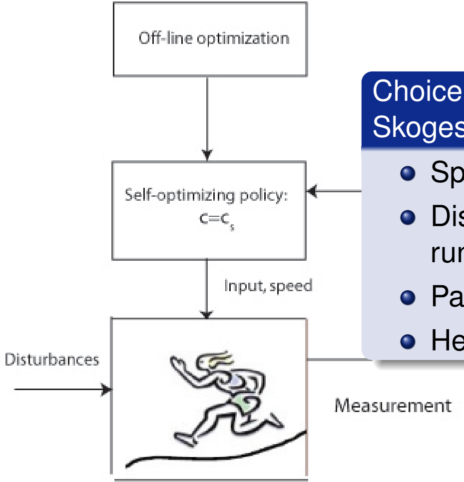
- Paradigm 2 : Use self-optimizing policy based on off-line analysis.
- Measurements are used to (indirectly) update the inputs using feedback control schemes.
- No online optimization.
- Also referred to as implicit schemes (Srinivasan and Bonvin, 2007)

Paradigm 1: Marathon runner



Clearly impractical!

Paradigm 2: Marathon runner

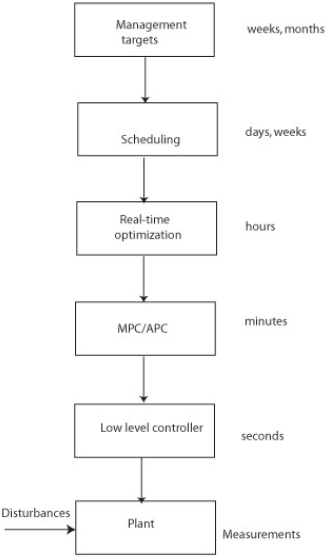


Choice of measurement, *c*, Skogestad, 2004

- Speed
- Distance to nearest runner
- Pain level
- Heart rate

Optimal operation of a typical chemical process

- Hierarchial decomposition based on time scale separation.
- Economics largely decided by slow time scale.



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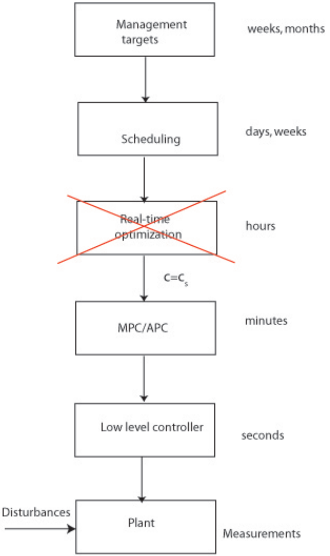
Self-optimizing control...

is when acceptable operation (=acceptable loss) can be achieved using constant set points (c_s) for the controlled variables c (without the need for re-optimizing when disturbances occur) at the faster time scale.



Goal of current research

- Extend the idea of self-optimizing control to more general systems.
- Find analytical or pre-computed solutions suitable for on-line implementation.
- Determine the structure of the optimal solution. Typically, this involves identifying regions where different sets of constraints are active.
- Determine optimal values (or trajectories) for the unconstrained variables.
- Find good self-optimizing controlled variables, c associated with the unconstrained degrees of freedom.
- Determine a switching policy between different regions.
- Ensure simplicity in implementation.

Pre-computed solutions: LQ regulator

Consider the system:

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx, \end{aligned}$$

and the cost (to be minimized)

$$J = \int_0^\infty (x'Qx + u'Ru)dt,$$

the optimal solution is the of the form: (Bryson, 1999)

$$u(t) = -Kx(t)$$

MPC: Paradigm 1

Consider the system:

$$\begin{aligned} x_{t+1} &= Ax_t + Bu_t \\ y_t &= Cx_t, \end{aligned}$$

find $U = [u_{t+1}, u_{t+2}, \ldots, u_{t+N_u-1}]'$, that minimizes:

$$J = x'_{t+N_y|t} P x_{t+N_y|t} + \sum_{k=0}^{N_y-1} x'_{t+k|t} Q x_{t+k|t} + u'_{t+k} R u_{t+k}$$

subject to:

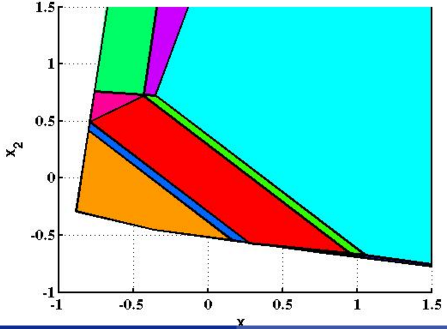
$$x_t \in X_c, y_t \in Y_c, u_t \in U_c.$$

Implement u_t at time t and re-solve the problem at time $t+1$.

Explicit MPC: Paradigm 2

The optimal solution $U^*(x)$ is a Piece-Wise Affine function of the current state x_t: (Bemporad et al., 2002)

$$U^*(x) = \begin{cases} K_1 x + g_1, & if, \ x \in X_1 \\ K_2 x + g_2, & if \ x \in X_2 \\ \vdots \\ K_n x + g_n, & if \ x \in X_n \end{cases}$$



Controlling the necessary conditions of optimality

- Static optimization: KKT conditions (Arkun and Stephanopolous, 1980)
  - Active constraints can be controlled.
  - Gradient of the Lagrangian is zero. However, it is usually not measured.
  - Self-optimizing control (Skogestad, 2000) or indirect gradient control using measurements (Cao, 2006).
- Dynamic optimization:
  - Sensitivities are zero, however, unmeasured.
  - Constraints in the future.
  - Some constraints implicitly defined.
  - Use measurements. (Bonvin and coworkers)

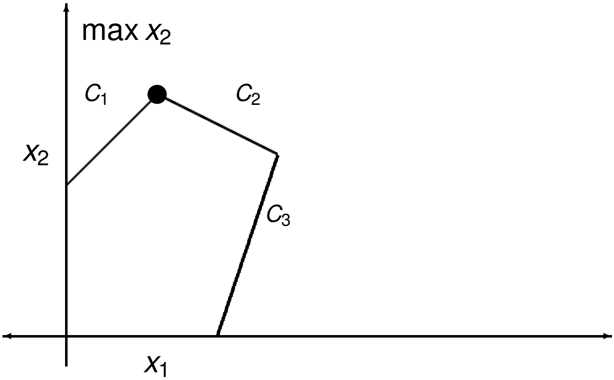
Exploiting the structure of optimal solution

Exploit known structure of the optimal solution to avoid online optimization.

- Identifying the active constraints
- Controlling the active constraints
- Selecting “self-optimizing variables" corresponding to the unconstrained degrees of freedom

Linear Program

- Optimal solution is at constraint vertex.
- Control the active constraints.
- No further degrees of freedom.



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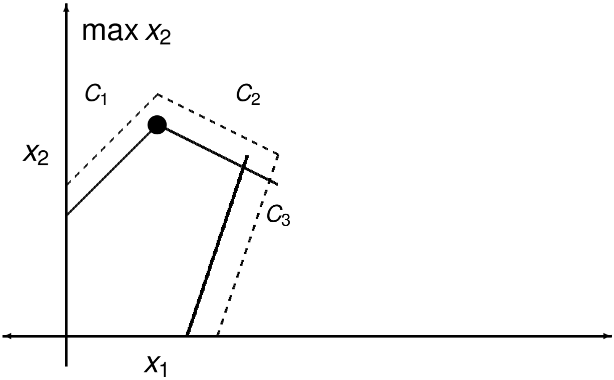

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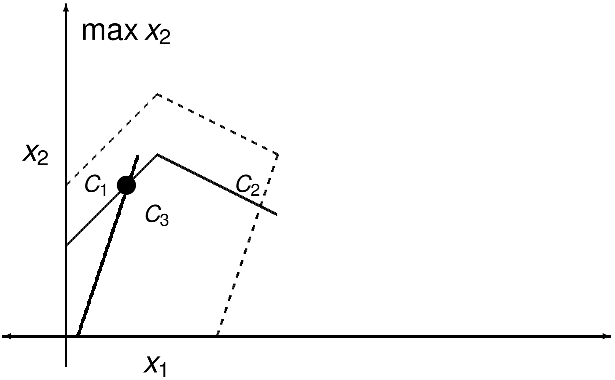

Switching policy

- Three manipulations u_1, u_2, u_3 and 2 outputs y_1, y_2.

Region	u_1	u_2	u_3
1	S	U	U
2	U	S	U

S: Saturated
U: Unsaturated

- Suggested pairing: Use u_3 to control y_2, and combine u_1 and u_2 in a split range pair to control y_1.

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Heat Exchanger Network

- Optimal operation (minimal utility consumption) of certain HENs can be reformulated as a L.P. Problem[1].

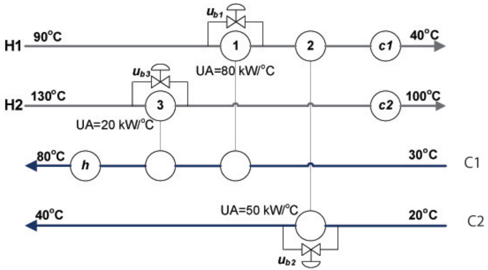

Figure: HEN example: Linear problem

- Control outlet temperatures at targets
- Inlet temperatures are unmeasured disturbances

[1] Aguilera and Marcheti, 1998, Lersbamrungsuk et al., 2006, 2007

Heat Exchanger Network: control structure

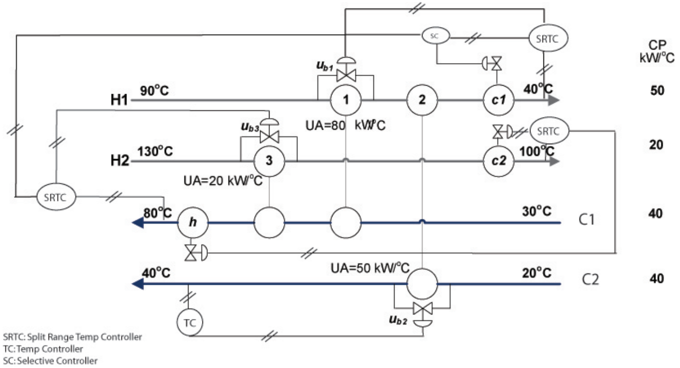

Figure: HEN example: Control structure for optimal operation (Lersbamrungsuk et al., 2007)

HEN: Using structure of the optimal solution.

Table: List of saturated manipulations[2]

Set of active constraints	Q_{c1}	Q_{c2}	Q_h	u_{b1}	u_{b2}	u_{b3}
1	S	U	S	U	U	U
2	S	S	U	U	U	U
3	U	S	U	S	U	U
4	U	U	S	S	U	U
5	U	U	S	U	U	S

- In general, pairings determined by solving an ILP[2], if feasible.

[2] Lersbamrungsuk et al., 2007

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HEN: Quadratic program

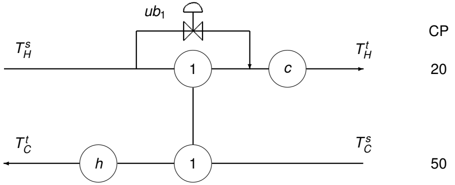
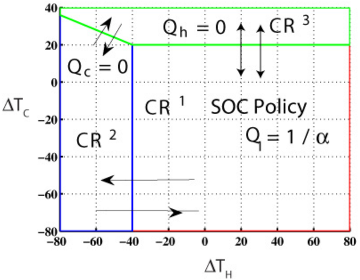


Figure: Simple HEN

- Minimize $Q_c + Q_h + \alpha Q_1^2$
- Can be formulated as a QP. [3]

[3] Manum et al., 2007, in preparation



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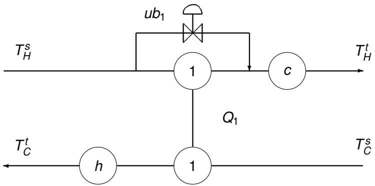

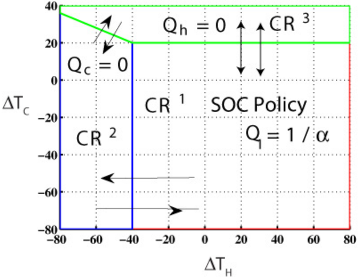

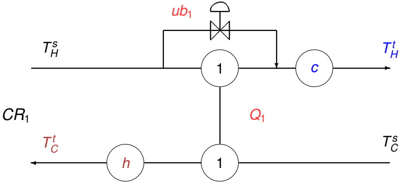

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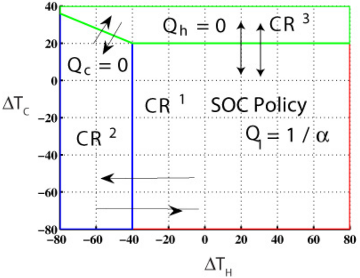
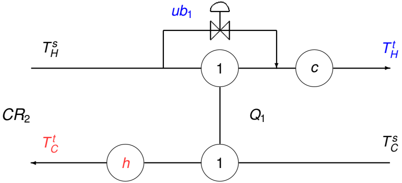
Optimal operation:Implementation





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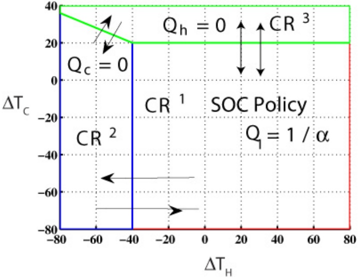

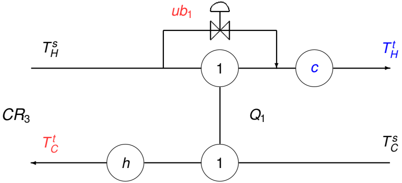

Manum et al., 2007, in preparation

Conclusions

- Paradigm 2: Using off-line analysis to replace online optimization.
- Search for self-optimizing policies.
- Use structure of the optimal solution for efficient implementation.
- Extensions to other classes of systems including dynamic systems.
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  - Henrik Manum
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