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Near-Optimal Output Feedback Control of Dynamic Processes

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Optimal operation

- Two paradigms for implementation
  1. on-line optimization using measurements for model or state estimate updates (RTO, MPC)
  2. inherent optimal operation by exploiting off-line information in control structure design

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Off-line optimization

- Lower computational load
  - Still a significant problem for dynamic real-time optimization
- Less sensitive to modeling errors

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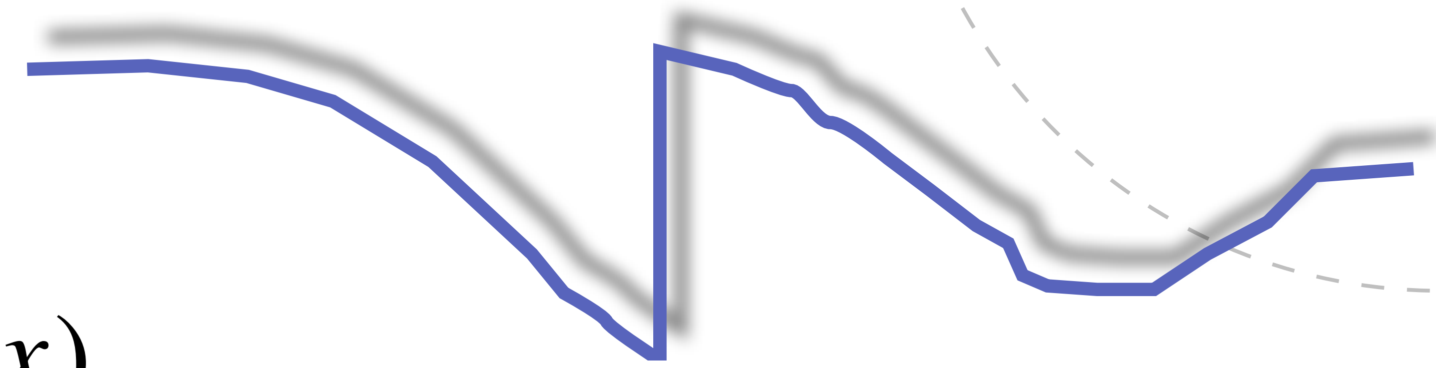
Dynamic optimization



$$\min_{u(t),t_f} J(x(t_f))$$

$$\dot{x} = f(x,u) \quad y = h(x)$$

$$c(x,u) \leq 0$$

$$u(t) \in U[0,T]$$

$$x(t_f) \in X$$

$$H(t) = \lambda^{\mathrm{T}} f(x,u) + \mu^{T} c(x,u)$$

$$\dot{\lambda}_i = -\frac{\partial H}{\partial x_i}$$

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Hamiltonian

- takes minimum along optimal path
- has a constant value along the optimal path for problems not depending explicitly on time
- can be used to define a loss in sub-optimal operation:

$$L(t) = H(t) - H_{\text{opt}}(t)$$

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Loss related to outputs

$$L(t) = \underbrace{H_u \delta u}_{0} + \frac{1}{2} \delta u^T H_{uu} \delta u$$

Need to relate variations in inputs to variations in outputs

$$\delta y = \frac{\partial h}{\partial x^T} \frac{\partial x}{\partial u^T} \delta u = G \delta u$$

$$L(t) = \frac{1}{2} \delta y^T G^{-T} H_{uu} G^{-1} \delta y$$

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Maximum gain rule for dynamic optimization

- Assume the model is scaled such that $\delta y_{max} \approx 1$

Select y's to minimize the following expression along the nominal trajectory

$$\min \left(\int_{t_0}^{t_f} \left\| G^{-T} H_{uu} G^{-1} \right\|_2^2 dt \right)^{1/2}$$

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How to obtain G

- How does variations in inputs map to the states?
- Neighboring optimal control gives u=Kx
- We estimate G by

$$G \approx \frac{\partial h}{\partial x^T} K^+$$

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Example

$X \xrightarrow[S]{O_2} \alpha X + \beta P$

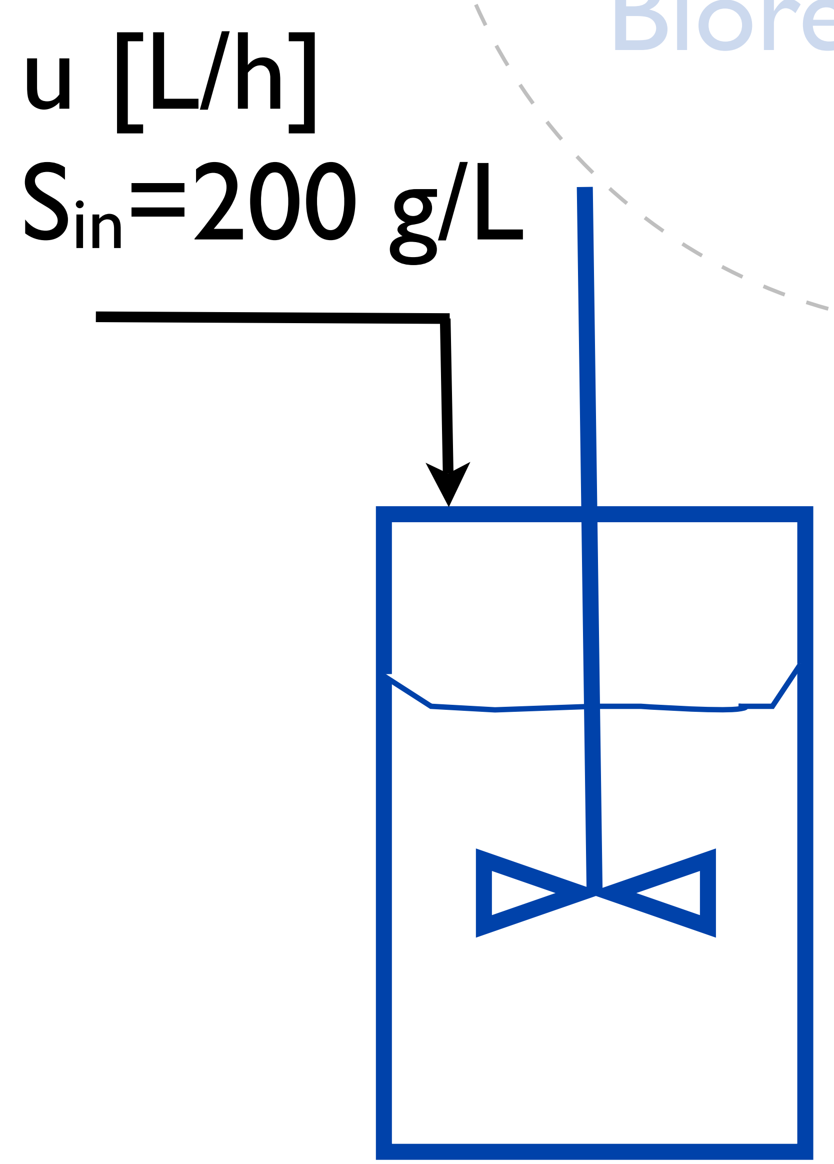
Bioreactor

- Maximize production of product P in a fed-batch bioreactor with fixed final time of 150 hours
- Reaction is driven by the presence of a substrate S, which is consumed in the biomass generation
- The biomass concentration is constrained

Srinivasan, B., D. Bonvin, et al. (2002). "Dynamic optimization of batch processes II. Role of measurements in handling uncertainty." Comp. chem. eng. **27**: 27-44.

Example

$X < 3.7$	Biomass concentration
S	Substrate concentration
P	Product concentration
V	Volume
$u < 1$	Substrate feedrate



Example

X < 3.7	Biomass concentration
S	Substrate concentration
P	Product concentration
V	Volume
u < 1	Substrate feedrate

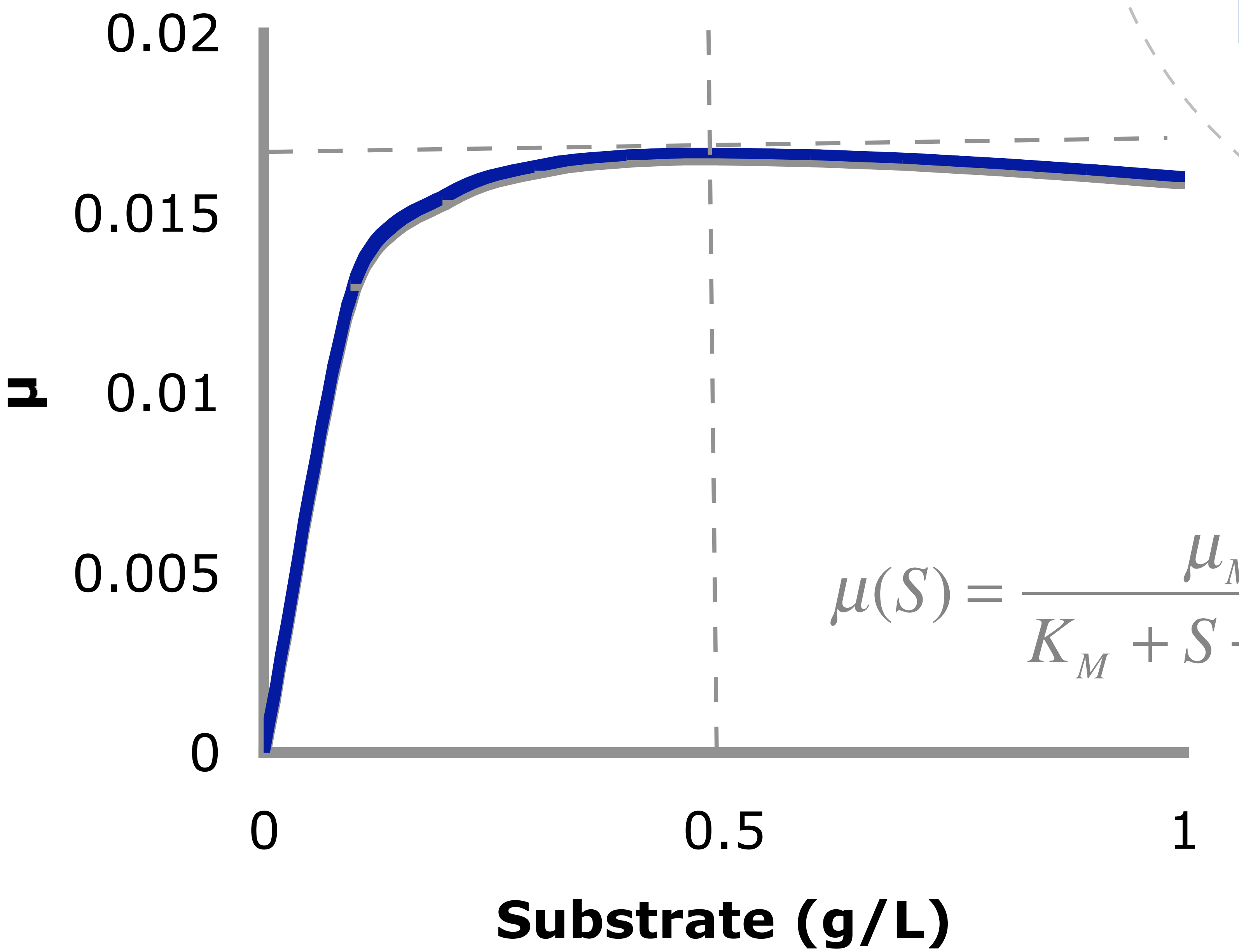
$$\dot{X} = \mu(S)X - \frac{u}{V}X$$

$$\dot{S} = -\frac{\mu(S)X}{Y_X} - \frac{\nu X}{Y_P} + \frac{u}{V}\left(S_{\text{in}} - S\right)$$

$$\dot{P} = \nu X - \frac{u}{V}P$$

$$\dot{V} = u$$

Biomass growth rate



$$\mu(S) = \frac{\mu_M S}{K_M + S + S^2 / K_i}$$

$X \underset{S}{\overset{O_2}{\rightleftharpoons}} \alpha X + \beta P$

Bioreactor

Consider gain magnitude

- Transformation of input: $\xi = \sqrt{u}$
- $H_{\xi\xi}$ is constant
- Minimizing of $1/K^2$ corresponds to maximizing K^2

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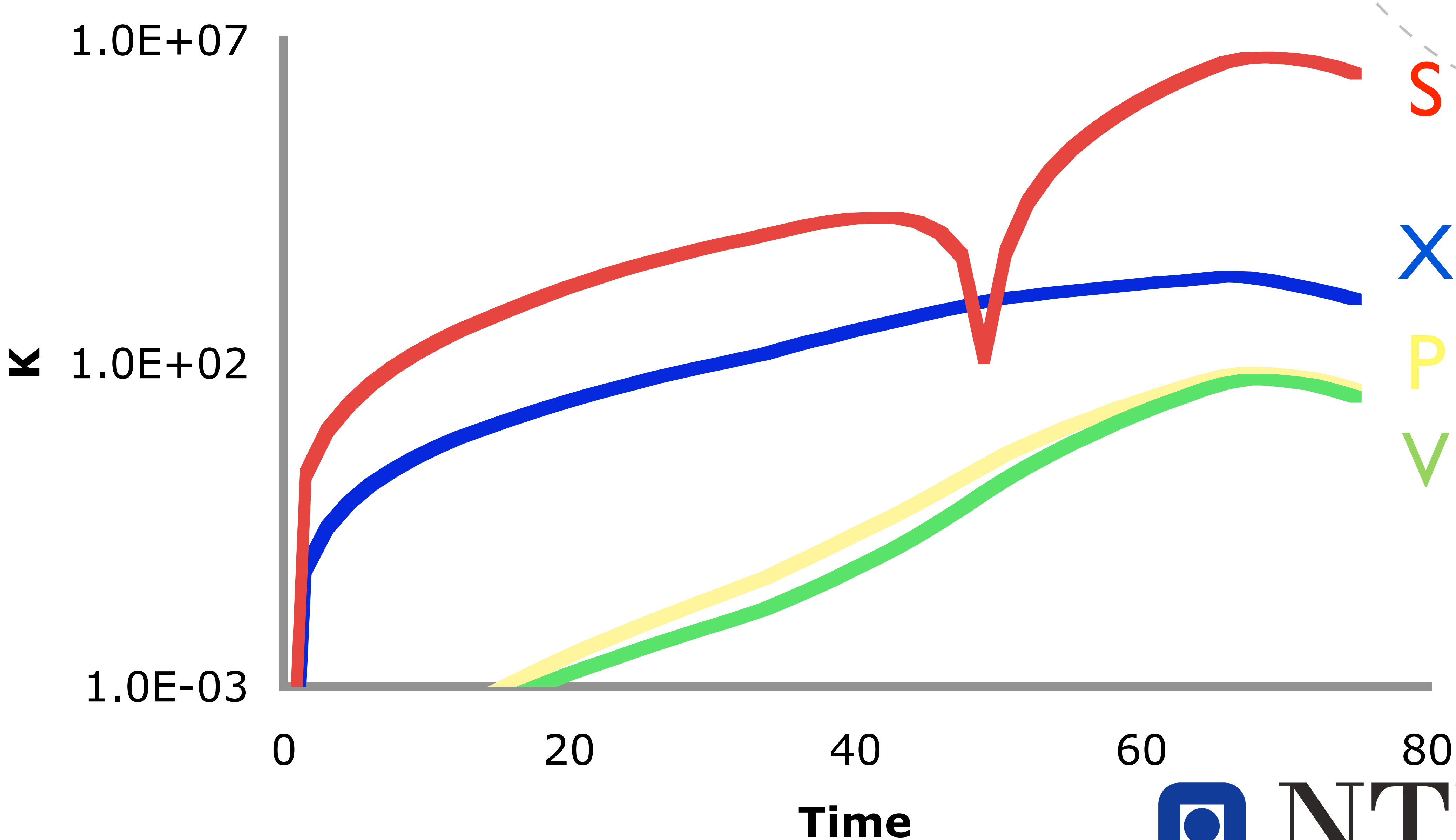
Comparison of gains

$$X \underset{S}{\overset{O_2}{\rightleftharpoons}} \alpha X + \beta P$$

Bioreactor



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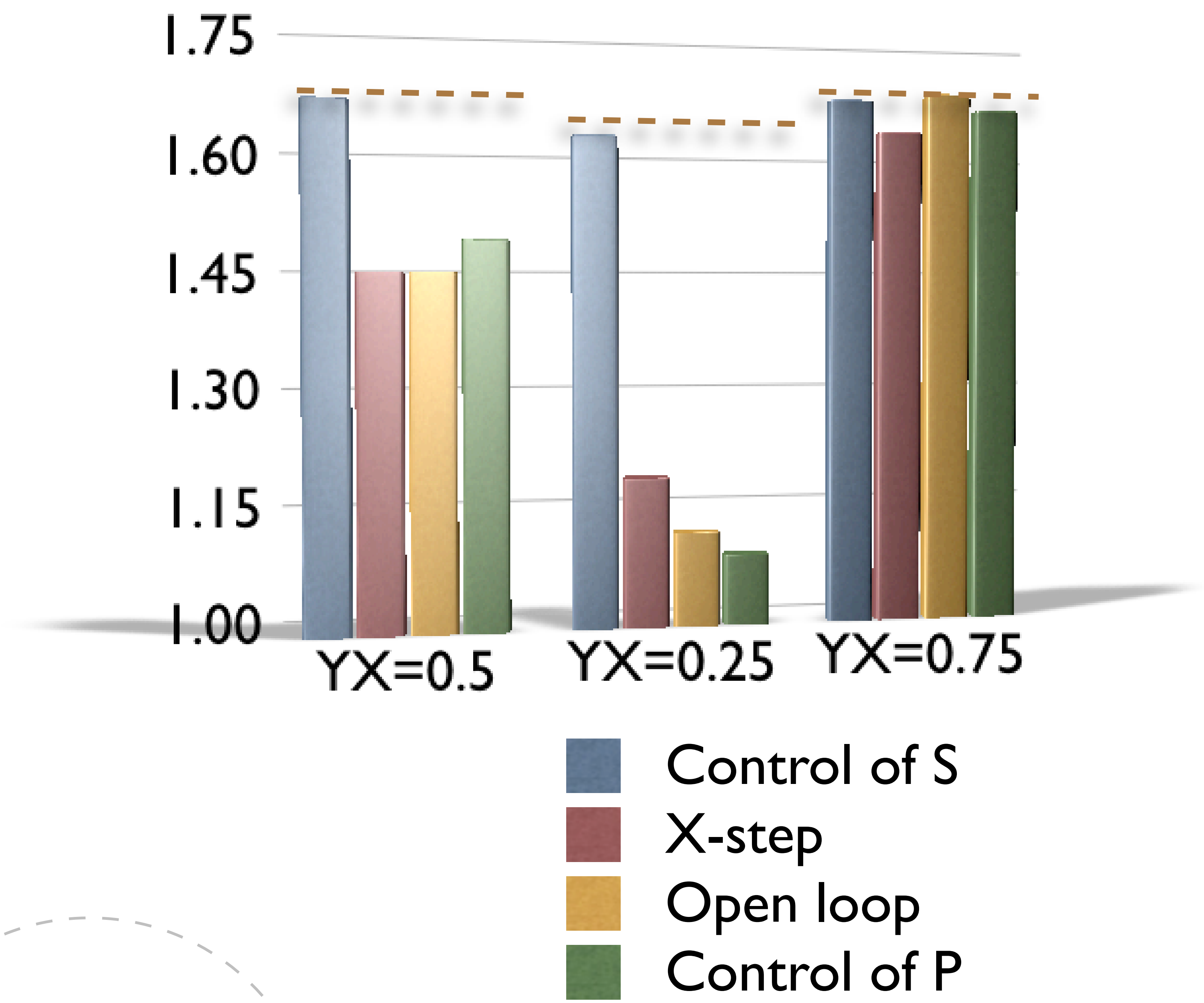
Simulation results



Summary

- The maximum gain rule can be extended to unsteady-state problems
- Using the solution of the time-varying LQR problem, it is possible to get a good estimate of the time-varying gain needed to use the maximum gain rule

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