

# ① Selection of closed-loop time constant $\bar{\tau}_c$

## Issues

- ① Upper bound due to effective time delay (robustness)

$$\text{SNC-rule: } \bar{\tau}_c \geq \theta \quad (= \bar{\tau}_{\text{min}})$$

- ② Lower bound due to disturbance rejection (performance γ)

$$K_c \geq \frac{|u_0|}{|Y_{\max}|} \quad \leftarrow \begin{array}{l} (|u_0| = \text{input magnitude required} \\ \text{for disturbance rejection}) \\ (|Y_{\max}| = \max |y|) \end{array}$$

Gives

$$\bar{\tau}_c \leq \bar{\tau}_{c,\max}$$

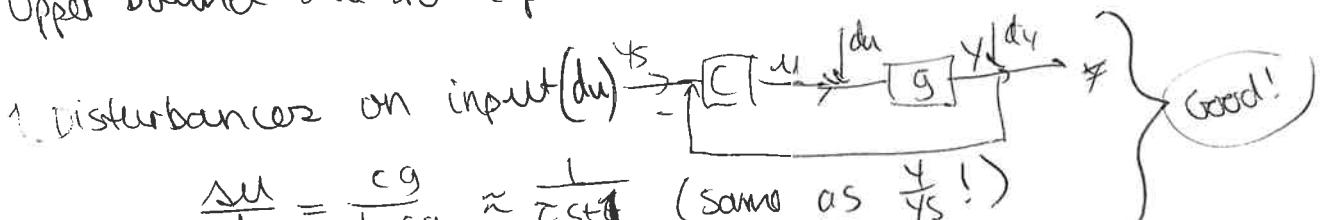
by use of

$$K_c = \frac{1}{K} \frac{\tau}{\bar{\tau}_c + \theta}$$

comment: if  $\theta \propto 0$  then  
 $\bar{\tau}_c \leq \frac{\tau}{K} \frac{1}{|Y_{\max}|} \frac{|Y_{\max}|}{|Y_0|} \tau$

where  $|Y_0| = \text{expected variation in } |y| \text{ with no control}$

- ③ Upper bound due to input saturation (avoiding too large  $u$ )



$$\frac{du}{du} = \frac{cg}{1+cg} \approx \frac{1}{\bar{\tau}_c s + 1} \quad (\text{same as } \frac{y}{ys})$$

(so ~~no~~ overshoot here!)

⑤ BUT could be that there is a requirement  $\bar{\tau}_c \geq \bar{\tau}_{c,\min}$   
 (because fast changes in  $u$  are not desired ("filtering of  $du$  required"))

2. Disturbances on output  $y$  or set point changes  $(y_s)$

$\frac{du}{dy} = \frac{du}{y_s} = \frac{c}{1+gc} = \frac{1}{g} \frac{gc}{1+gc} = \frac{1}{g} \frac{1}{\bar{\tau}_c s + 1}$

(a) Study-state ( $s \rightarrow 0$ )  $\frac{du}{dy} = \frac{1}{g(0)} = \frac{1}{K}$  (This we must be able to handle. Has nothing to do with tuning.)

Initial response + Assume  $g(s) = \frac{K}{s+1} \Rightarrow g(\infty) = \frac{K}{\bar{\tau}_c} \Rightarrow \frac{du}{dy} = \frac{1}{g(\infty)} \frac{1}{\bar{\tau}_c s} = \frac{1}{K} \frac{s}{\bar{\tau}_c}$

⇒ "Overshoot" initially is given by  $M_N$  "speed-up"  $\bar{\tau}/\bar{\tau}_c$

Get requirement  $\Delta y = \Delta y_{\max} \Rightarrow \frac{1}{K} \frac{\bar{\tau}}{\bar{\tau}_c} dy \leq M_N \Rightarrow \bar{\tau}_c \geq \frac{dy}{M_N K}$

Maximum speedup allowed is  $\frac{M_N}{M_{\max}}$

(Max. input magnitude for s.s. output disturbance rejection)

$$\bar{\tau}_c \geq \frac{M_N}{M_{\max}} \cdot \tau$$

(2)

## Summary

(1)  $\tau_c \geq \theta$  (robustness)

(2)  $\tau_c \leq \frac{|Y_{\max}|}{|Y_0|} \cdot \tau$  (~~speedup required for disturbance rejection~~)

( $|Y_0|$  = output magnitude w/o control  
(due to disturbances))

(3)  $\tau_c \geq \frac{|\mu_y|}{|\mu_{\max}|} \cdot \tau$  (maximum speedup because input may saturate when there are output disturbances)

( $|\mu_y|$  = input change required to reject output disturbance (setpoint change)  $= \frac{|\Delta y|}{K}$ )

(4)  $\tau_c \leq \tau_{c,\text{setpoint}} \leftarrow \begin{array}{l} \text{Response time required for} \\ \text{acceptable setpoint tracking} \end{array}$

↑  
Generally want as small as possible

NOTE:  $\frac{1}{Y_S} = \frac{\Delta u}{\Delta y}$   
 $\approx \frac{1}{T_S + 1}$

(5)  $\tau_c \geq \tau_{c,\text{input}} \leftarrow \begin{array}{l} \text{Response time filtering of} \\ \text{input disturbances} \end{array}$