# Optimal Operation of closed cycles for heating and cooling

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## Abstract

Cycles for heating and cooling have been studied in detail when it comes to thermodynamics and design. However, there are few publications on optimal operation with given equipment which is the theme of this paper. One important issue is which variable to control during operation, for example super-heating, pressure, liquid level or valve set-point. Unlike open systems the initial charge to the cycle may have a steady state effect, and it is discussed how different designs are affected by this factor. Numerical results are provided for a ammonia and $CO_2$ cycle.

**Keywords:** Operation, vapor compression cycle, cyclic processes, charge.

## 1. Introduction

Cyclic processes for heating and cooling are widely used in many applications and their power range from less than 1 kW to above 100 MW. All of these applications use the vapor compression cycle to "pump" energy from a low to a high temperature level.

The first application in 1834 was cooling to produce ice for storage of food , which led to the refrigerator found in every home (Nagengast, 1976). Another well known system is the air-conditioner (A/C). In colder regions a cycle operating in the opposite direction, the "heat pump", has become popular recently. These two applications have also merged together to give a system able to operate in both heating and cooling mode. Common for these examples are that they consist of a simple cycle, and that the required duty is relatively small.

A schematic drawing of simple cycle is shown in figure 1 together with a typical pressure-enthalpy diagram for a sub-critical cycle. The cycle works in the following manner:

*The low pressure vapor (4) is compressed by supplying work $W_s$ to give a high pressure vapor with high temperature (1). This stream is cooled to the saturation temperature in the first part of the condenser, condensed in the middle part and possibly sub-cooled in the last part to give the liquid (2). In the expansion choke, the pressure is lowered to its original value, resulting in a two-phase mixture (3). This mixture is vaporized and heated through the evaporator giving a super-heated vapor (4) closing the cycle*

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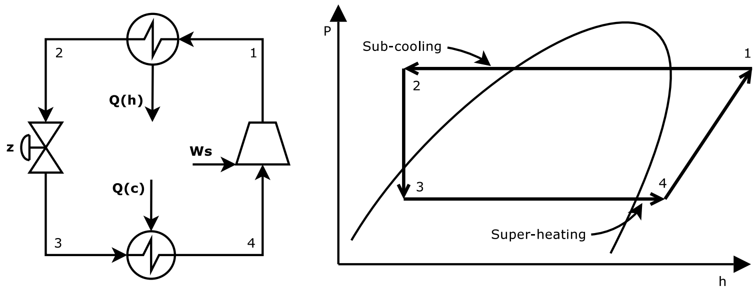


*Figure 1. Schematic illustration of a simple vapor compression cycle with corresponding pressure-enthalpy diagram*

The coefficient of performance for a heating cycle (heat pump) and a cooling cycle (refrigerator, A/C) is defined as

$$COP_h = \frac{Q_h}{W_s} = \frac{h_1 - h_2}{h_1 - h_4} \qquad \text{and} \qquad COP_c = \frac{Q_c}{W_s} = \frac{h_4 - h_3}{h_1 - h_4} \tag{1}$$

respectively. Heat pumps typically have a COP of around 3 which indicates that 33% of the gained heat is addet as electricity.

In industrial processes, especially in cryogenic processes as air separation and liquefaction of natural gas (LNG process), more complex cycles are used in order to improve the thermodynamic efficiencies. These modifications lower the temperature differences in the heat exchangers and include cycles with mixed refrigerants, several pressure levels and cascaded cycles. The Mixed Fluid Cascade process developed by the Statoil Linde Technology Alliance is being built at the LNG plant in northern Norway and incorporate all of the above modifications. The resulting plant has three cycles, all with mixed refrigerant and the first with two pressure levels. Our long term objective is to study the operation of such processes. However, as a start we need to understand the simple cycle in figure 1.

Two cases are used to get numerical results for simple cycles. The first is a steady state model of an ammonia cycle with simple thermodynamics. The latter is a dynamic model of a $CO_2$ heat pump with rigorous thermodynamics.

## 2. Operation of simple vapor compression cycles

### 2.1. Design versus operation

Table 1 shows typical specifications for simple cycles in design (find equipment) and in operation (given equipment). Note that the five design specifications results in only four equipment parameters; compressor work $W_s$, valve opening z and UA for the two heat exchangers. In other words, with the four equipment parameters specified there is not a unique solution in terms of the operation. The “un-controlled” mode is related to the pressure level, which is indirectly set by the charge of the system. This is unique for closed systems since there is no boundary condition for pressure. In practice, the “pressure level” is adjusted directly or indirectly, depending on the design, especially of the evaporator. This is considered more in detail below.

## 2.2. Operational (control) degrees of freedom

During operation the equipment is given. Nevertheless, we have some operational or control degrees of freedom. These include the compressor power ($W_s$), the charge (amount of vapor and liquid in the closed system), and the valve openings. The following valves may be used:

- Adjustable choke valve; see figure 1 (not avaialble in some simple cycles)
- Adjustable valve between condenser and storage tank (for designs with a separate liquid storage tank before the choke; see design III.a in figure 3

In addition, we might install bypass valves on the condenser and evaporator, but this is not normally used because use of bypass gives suboptimal operation.

Some remarks:

- The compression power $W_s$ sets the “load” for the cycle, but it is otherwise not used for optimization, so in the following we do not consider it as a degree of freedom.
- The charge has a steady-state effect in some cases because the pressure level in the system depends on the charge. A. typical example is a household refrigeration systems. However, such designs are generally undesirable. First, the charge can usually not be adjusted continuously. Second, the operation is sensitive to the initial charge and later to leaks.
- The overall charge has no steady-state effect for some designs. This is when we have a storage tank where the liquid level has no steady-state effect. This includes designs with a liquid storage tank after the condenser (III.a, figure 3), as well as flooded evaporators with variable liquid level (II.a, figure 2). For such designs the charge only effects the level in the storage tank. Note that it may be possible to control (adjust) the liquid level for these designs (II.a figure 2), and this may the be viewed as a way of continuously adjusting the charge to the rest of the system (condenser and evaporator).
- There are two main evaporator designs; the dry evaporator (I) and the flooded evaporator (II) shown in figure 2. In a dry evaporator we generally get some superheating, whereas there is no (or little) super-heating in a flooded evaporator. The latter design is better thermodynamically, because super-heating is undesirable from an efficiency (COP) point of view. In a dry evaporator one would like to control the super-heating, but this is not needed in a flooded evaporator. In addition, as just mentioned, a flooded evaporator with variable liquid level is insensitive to the charge.
- It is also possible to have flooded condensers. and thereby no sub-cooling, but this is not desirable from a thermodynamic point of view.

*Table 1. Specifications in design and operation*

| | Given | # |
|---|---|---|
| Design | Load (e.g. $Q_h$), $P_l$, $P_h$, $\Delta T_{sup}$ and $\Delta T_{sub}$ | 5 |
| Operation | $W_s$, z and $UA$ in two heat exchangers | 4 |

### 2.3. Use of the control degrees of freedom

In summary, we are during operation left with the valves as degrees of freedom. These valves should generally be used to optimize the operation, In most cases "optimal operation" is defined as maximizing the efficiency factor, COP. We could then envisage an on-line optimization scheme where one continuously optimizes the operation (maximizes COP) by adjusting the valves. However, such schemes are quite complex and sensitive to uncertainty, so in practice one uses simpler schemes where the valves are used to control some other variable. Such variables could be:

- Valve position setpoint (that is, the valve is left in a constant position)
- Low pressure ($P_l$)
- High pressure ($P_h$)
- Temperature out of condenser ($T_2$) or degree of sub-cooling ($\Delta T_{sub}$).
- Temperature out of evaporator ($T_4$) or degree of super-heating ($\Delta T_{sup}$).
- Liquid level in storage tank (to adjust charge to rest of system)

The objective is to achieve "self-optimizing" control where a constant setpoint for the selected variable indirectly leads to near-optimal operation (Halvorsen et al., 2003).

Control (or rather minimization) of the degree of super-heating is useful for dry evaporators. However, it consumes a degree of freedom , for example, the adjustable choke valve. In order to retain the degree of freedom, we need to add a liquid storage tank after the condenser. In a flooded evaporator, the super-heating is minimized by design.

With the degree of super-heating fixed (by control or design), there is only one degree of freedom left that needs to be controlled in order to optimize COP. To see this, recall that there are 5 design specifications, so optimizing these give an optimal design. During operation, we assume the load is given ($W_s$), and that the maximum areas are used in the two heat exchangers (this is optimal). This sets 3 parameters, so with the super-heating controlled, we have one parameter left that effects COP.

In conclusion, we need to set one variable, in addition to $\Delta T_{sup}$, in order to completely specify (and optimize) the operation. This variable could be selected from the above list, but there are also other possibilities Some common control schemes are discussed in the

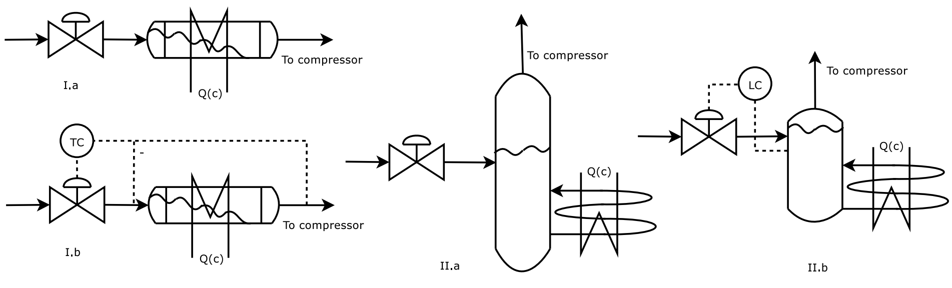


*Figure 2. Four different evaporator configurations; I.a Dry evaporator, I.b with TEV, II.a Flooded evaporator, II.b with level control*

following. We will later evaluate the self-optimizing properties for three of the schemes for the $CO_2$ case study.

### 2.4. Some alternative designs and control schemes

Some designs are here presented and the pro's and con's are summarized in table 2. Table 3 gives performance for the ammonia cycle with a fixed condenser duty for each design. This table is discussed more in detail later.

#### *2.4.1. Dry evaporator (I)*

This is the design shown in figure 2 I.a/b where generally will have some super-heating.

**I.a** In residential refrigerators it is common to use a capillary tube, a small diameter tube designed to give a certain pressure drop. On-off control of compressor is also common, but we will consider capacity control to get comparable results.

**I.b** Larger refrigeration systems, air-conditioners and heat pumps usually have a thermostatic expansion valve (TEV) (Dossat, 2002) and (Langley, 2002). The TEV controls the super-heating to a typical value of 10 $^{\circ}C$ .

#### *2.4.2. Flooded evaporator (II)*

A flooded evaporator differs from the normal, dry evaporator, in that it only provides vaporization and no super-heating.

**II.b** In flooded evaporator systems the valve is used to control the level in either evaporator or condenser (figure 2 II.b).

**II.a** We propose a design where the volume of the flooded evaporator is so large that there is no need to control the level in one of the heat exchangers (figure 2 II.a). This design will have the valve as degree of freedom since the excess charge will be stored in the evaporator.

#### *2.4.3. Other designs (III)*

In this section extra design elements are included. The effects are discussed qualitatively.

**III.a** To reduce the sensitivity to charge in design I.b and II.b it is possible to include a liquid receiver before the valve as seen in design III.a in figure 3. To obtain an extra degree of freedom a valve is added before the receiver.

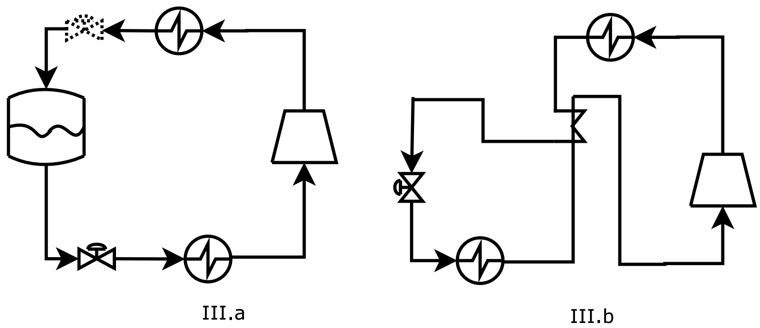


*Figure 3. Special design features; III.a Liquid receiver, III.b Internal heat exchanger*

*Table 2. Operation of alternative designs*

| | Pro's | Con's |
|---|---|---|
| I.a | Simple design | Sensitive to charge |
| | | No control of super-heating |
| I.b | Controlled super-heating | Super-heating |
| | | Sensitive to charge |
| II.a | No super-heating by design | |
| | Not sensitive to charge | |
| | Valve is free | How to use valve? |
| II.b | No super-heating by design | Sensitive to charge |
| III.a | Not sensitive to charge | Complex design |
| | | How to use valve? |

**III.b** It is possible to add a internal heat exchanger as shown in figure 3 III.b. This will super-heat the vapor entering the compressor and sub-cool the liquid before expansion. The latter is positive because of reduced expansion losses, whereas the first is bad for performance because of increased suction temperature. This can be used if a degree of superheat is desirable for other reasons, because in this configuration the super-heat is taken from the cycle and not from the surroundings which would affect performance even more.

## 3. Ammonia case study

A typical ammonia cycle in a pressure enthalpy diagram is shown in figure 4.This is based on the simplified thermodynamic model given in appendix A. In table 3 we see how the cycle efficiency depends on the degree of super-heating, which again depends on the design. In design I.a a certain degree of super-heat is needed to assure vapor out of evaporator. With super-heat control (TEV) (I.b) we can reduce this amount by somewhat, whereas the flooded evaporator (II.a/b) totally removes the super-heating.

From the table it is clear that super-heat should be avoided, which favours the use of flooded evaporators. It also shows the importance of considering operation.

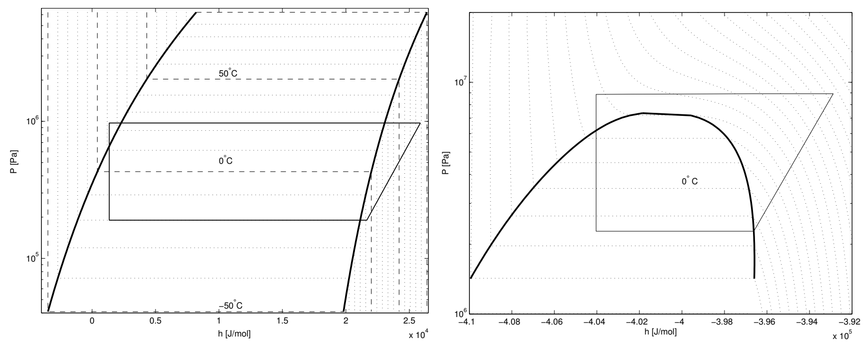


*Figure 4. Two case study examples plotted in pressure-enthalpy diagram; Ammonia cycle to the left and $CO_2$ cycle to the right*

*Table 3. Typical performance and key values of the different designs for the ammonia cycle given UA for the heat exchangers and temperature for the surroundings*

| | $COP_c$ | $W_s$ [W] | $Q_c$ [kW] | $\Delta T_{sup}$ $^\circ C$ | $\Delta T_{sub}$ $^\circ C$ | $P_1$ [MPa] | $P_2$ [MPa] |
|---|---|---|---|---|---|---|---|
| I.a | 4.81 | 4218 | 20.3 | 12 | 12.0 | 1.90 | 97 |
| I.b | 4.91 | 4138 | 20.3 | 10 | 13.6 | 1.94 | 98 |
| II.a,b | 5.08 | 3999 | 20.3 | 0 | 13.7 | 2.0 | 99 |

## 4. $CO_2$ case study

A typical trans-critical $CO_2$ cycle with a flooded evaporator (II.a) is shown in figure 4, see appendix B for details. In figure 5 we consider three different control policies; constant valve (II.a), constant high pressure (II.a) and constant evaporator level (II.b). The figure considers the main disturbance, outside air temperature, and shows the losses compared with a re-optimized case.

It is clear that fixing the pressure is not a good strategy, whereas keeping a constant valve and or a constant level both gives small losses.

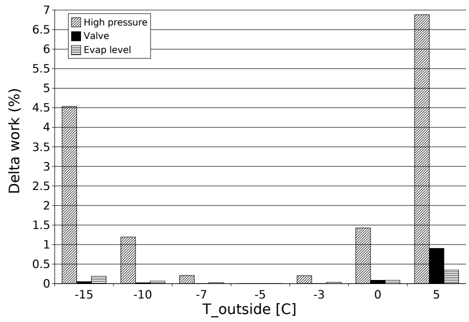


*Figure 5. Loss compared to re-optimized case using the $CO_2$ heat pump model. Outside air temperature is shown on the x-axis and three different control strategies are shown; constant valve, high pressure and evaporator level*

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## A. Ammonia cycle

A cycle operating between a hot temperature of 5 $^{\circ}C$ and a cold temperature of -5 $^{\circ}C$ is designed to remove 20.3 kW of heat from the cold source using ammonia.

This is a steady state model with simple thermodynamics. We assume constant heat capacities. Enthalpy is then given by:

$$h_l(T) = c_{P,l}(T - T_{ref}) \qquad h_v(T) = c_{P,v}(T - T_{ref}) + \Delta_{vap} h_{ref} \tag{2}$$

Pressure is coupled to the saturation temperatures, for which we use (Haar and Gallagher, 1978).

$$ln\left(\frac{P_{sat}}{P_c}\right) = \frac{1}{\omega}\sum_{i=1}^{4} A_i (1-\omega)^i \qquad \text{where} \qquad \omega = \frac{T}{T_c} \tag{3}$$

## B. $CO_2$ heat pump

A trans-critical $CO_2$ heat pump is designed to deliver 10 kW of heat to a room with 22 $^{\circ}C$ when the outside air has a temperature of 5 $^{\circ}C$ and the temperature difference in the heat exchangers are greater than 10 $^{\circ}C$ . The evaporator is of the flooded type, so there is no super-heating. A dynamic model of this heat pump has been implemented with some assumptions; Isentropic compression with constant efficiency, constant air temperature through the heat exchangers (can be realized by high airflow), constant heat transfer coefficients and one-phase valve equation.

The thermodynamics are described by the use of Span-Wagner equation of state (Span and Wagner, 2003) with data for $CO_2$ collected from (Span and Wagner, 1996).