Perfect steady-state indirect control

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Abstract

Indirect control is commonly used in industrial applications where the primary controlled variable is not measured. This paper considers the case of "perfect indirect control" where one attempts to control a combination of the available measurements such that there is no effect of disturbances at steady-state. This is always possible provided the number of measurements is equal to the number of independent variables (inputs plus disturbances). It is further shown how extra measurements may be used to minimize the effect of measurement error. The results in this paper also provide a nice link to previous results on inferential control, perfect disturbance rejection and decoupling (DRD), and self-optimizing control.

1 Introduction

Indirect control (Skogestad and Postlethwaite 1996) is used when we for some reason cannot control the "primary" outputs y_1. Instead, we aim at indirectly controlling y_1 by controlling the "secondary" variables c (often denoted y_2) (Skogestad and Postlethwaite 1996). More precisely,

> **Indirect control** is when we aim at (indirectly) keeping the primary variables y_1 close to their setpoints y_{1s}, by controlling the secondary variables c at constant setpoints c_s.

An example is control of temperature (c) in a distillation column, in order to indirectly achieve composition control (y_1).

A less obvious example of indirect control, is the selection of "control configurations" in distillation columns. The term "control configuration" here refers to which two flows or flow combinations are left as degrees of freedom after we have closed the stabilizing loops for the condenser and reboiler levels. Ideally, keeping the selected two flow combinations (c) constant will indirectly lead to good control of the product compositions (primary outputs, y_1). For example, in the LV-configuration the condenser and reboiler levels are controlled such that the flows L (reflux) and V (boilup) are left as free variables for the layer above. However, keeping these flows constant (selecting L and V as c's) gives large changes in the product compositions (y_1) when there are disturbances in the feed flowrate. Instead, one may use the L/D V/B-configuration. In this case, keeping L/D and V/B constant (c's)

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gives almost constant product compositions (good control of y_1) when there disturbances in the feed flowrate. However, the changes in the product composition are large (poor control of y_1) for feed composition disturbances (e.g. (Skogestad *et al.* 1990)). Häggblom and Waller (1990) looked for a flow combination that handles all disturbances, and proposed the "disturbance rejecting and decoupling" configuration. This partially motivated our work, and is discussed in more detail below.

In the following, we let the set y denote the "candidate" measured variables for indirect control. We will refer to the entire set y as "measurements", but note that we in this set also include the original manipulated variables (inputs) (e.g. L, V, D and B for the distillation example). In this paper, we select as "secondary" controlled variables c a linear combination of the variables y,

$$\Delta c = H \Delta y \tag{1}$$

In other words, we want to find a good choice for the matrix H. In the simplest case individual measurements y are selected as c's, and the matrix H consists of zeros and ones. However, more generally we allow for combinations (functions) of the available measurements y, and H is a "full" matrix with all entries nonzero. In the paper, we show that if we have as many measurements as there are independent variables (inputs plus disturbances), then we can always achieve at steady state "perfect indirect control" with perfect disturbance rejection and in addition with a decoupled response from the setpoints c_s (the "new" inputs) to the primary variables y_1.

Indirect control may be viewed as a special case of "self-optimizing control" (Halvorsen *et al.* 2003). This is clear from the definition:

> **Self-optimizing control** (Skogestad 2000) is when we can achieve acceptable (economic) loss with constant setpoint values for the controlled variables c (without the need to reoptimize when disturbances occur).

In most cases the "loss" is an economic loss, but for indirect control it is the setpoint deviation, i.e. $L = \|y_1 - y_{1s}\|$. The implications of viewing indirect control as a special case of self-optimizing control are discussed later in the paper.

Another related idea is inferential control (Weber and Brosilow 1972). However, in inferential control the basic idea is to use the measurements y to estimate the primary variables y_1, whereas the objective of indirect control is to directly control a combination c of the measurements y.

In the paper we only consider the steady-state behavior. The notation in this paper largely follows that used by Halvorsen *et al.* (2003).

2 Perfect indirect control

Consider a setpoint problem where the objective is to keep the "primary" controlled variables y_1 at their setpoints y_{1s}. We also have

u: Inputs (independent variables available for control of y_1)
d: Disturbances (independent variables outside our control)
y: Measurements (including u and possible measured d's)

Problem definition: *Find a set of (secondary) controlled variables $c = h(y)$ such that a constant setpoint policy ($c = c_s$) indirectly results in acceptable control of the primary outputs (y_1).*

We make the following assumptions

1. The number of secondary controlled variables c is equal to the number of inputs u ($n_c = n_u$), and they are independent such that it is possible to adjust u to get $c = c_s$.

2. We consider the local behavior based on linear models.

3. We only consider the steady-state behavior.

4. We neglect the control error (including measurement noise), that is, we assume that we achieve $c = c_s$ at steady state (this assumption is relaxed later).

5. We assume that the nominal operating point (u^*, d^*) is optimal, that is, at the nominal point (where $d = d^*$ and $c = c_s$) we have $y_1^* = y_{1s}$.

The linear models relating the variables are

$$\Delta y = G^y \Delta u + G_d^y \Delta d = \tilde{G}^y \begin{pmatrix} \Delta u \\ \Delta d \end{pmatrix} \tag{2}$$

$$\Delta y_1 = G_1 \Delta u + G_{d1} \Delta d = \tilde{G}_1 \begin{pmatrix} \Delta u \\ \Delta d \end{pmatrix} \tag{3}$$

$$\Delta c = G \Delta u + G_d \Delta d = \tilde{G} \begin{pmatrix} \Delta u \\ \Delta d \end{pmatrix} \tag{4}$$

where $\Delta u = u - u^*$, etc. From (4) we can obtain the inputs Δu needed to get a given change Δc:

$$\Delta u = G^{-1} \Delta c - G^{-1} G_d \Delta d$$

where G^{-1} exists because of assumption 1. Substituting this into (3) yields the corresponding change in the primary variables

$$\Delta y_1 = \underbrace{G_1 G^{-1}}_{P_c} \Delta c + \underbrace{(G_{d1} - G_1 G^{-1} G_d)}_{P_d} \Delta d \tag{5}$$

The "partial disturbance gain" P_d gives the effect of disturbances d on the primary output y_1 with closed-loop ("partial") control of the variables c, and P_c gives the effect on y_1 of changes in c (e.g., due to a setpoint change c_s).

The controlled variables c are combinations of the measurements, $\Delta c = H \Delta y$, and it follows from (2) and (4) that

$$G = HG^y; \quad G_d = HG_d^y; \quad \tilde{G} = H\tilde{G}^y \tag{6}$$

Ideally, we would like to choose H such that $P_d = 0$. Somewhat surprisingly, at least from a physical point of view, it turns out that this is always possible provided we have enough measurements y, and that we in fact have additional degrees of freedom left which we may use, for example, to specify P_c. For example, it may be desirable to have $P_c = I$, because this (at least at steady state) gives a decoupled response from c_s (which are our "new inputs") to the primary controlled variables y_1.

"Perfect indirect control" (refined problem definition): *Find a linear measurement combination, $\Delta c = H\Delta y$, such that at steady state we have perfect disturbance rejection ($P_d = 0$) and a specified setpoint response (i.e. $P_c = P_{c0}$, where P_{c0} is given.)*

We make the following additional assumptions:

6. The number of primary outputs y_1 is equal to the number of secondary controlled variables c (i.e., $n_{y_1} = n_c$), such that P_{c0} is invertible.

7. The number of (independent) measurements y is equal to the number of inputs plus disturbances ($n_y = n_u + n_d$), such that the matrix $\tilde{G}^y$ is invertible (this assumption is relaxed later).

Solution to refined problem definition: We have $\Delta c = H\Delta y$ and want to find H such that

$$\Delta y_1 = P_{c0}\Delta c + 0 \cdot \Delta d$$

This gives $\Delta y_1 = P_{c0}H\Delta y$, and using (2) and (3) gives

$$\Delta y_1 = \tilde{G}_1 \begin{pmatrix} \Delta u \\ \Delta d \end{pmatrix} = P_{c0}H\tilde{G}^y \begin{pmatrix} \Delta u \\ \Delta d \end{pmatrix}$$

which gives $\tilde{G}_1 = P_{c0}H\tilde{G}^y$ or

$$H = P_{c0}^{-1}\tilde{G}_1\tilde{G}^{y^{-1}} \tag{7}$$

which is the solution to the refined problem definition.

Extension 1. More generally, we may specify $P_d = P_{d0}$ (where P_{d0} is given and may be nonzero) and the resulting choice for H is

$$H = P_{c0}^{-1}\hat{G}_1\tilde{G}^{y^{-1}} \tag{8}$$

where

$$\hat{G}_1 = \begin{pmatrix} G_1 & G_{d1} - P_{d0} \end{pmatrix} = \tilde{G}_1 - \begin{pmatrix} 0 & P_{d0} \end{pmatrix} \tag{9}$$

Extension 2. If the measurements y are not independent or closely correlated, then the matrix $\tilde{G}^y$ in (7) and (8) will be singular or close to singular, resulting in infinite or large elements in $\tilde{G}^{y^{-1}}$. In this case, one needs to consider another set of measurements y or use more measurements. This is discussed separately below.

3 Application to control configurations for distillation

The results of Häggblom and Waller (1990) on control configurations for "disturbance rejection and decoupling (DRD) of distillation" provide an interesting special case of the above results, and actually motivated their derivation. Häggblom and Waller (1990) showed that one could derive a DRD control configuration that achieved

1. Perfect disturbance rejection with the new loops closed (i.e. $P_d = 0$ in our notation).
2. Decoupled response from the new manipulators to the primary outputs (i.e. $P_c = I$ in our notation).

Häggblom and Waller (1990) derived this for distillation column models, and made no attempt of generalizing their results. However, they can be shown to be a special case of the above results when we introduce

$$y_1 = \begin{pmatrix} y_D \\ x_B \end{pmatrix}, \quad y = \begin{pmatrix} L \\ V \\ D \\ B \end{pmatrix}, \quad u = \begin{pmatrix} L \\ V \end{pmatrix}, \quad d = \begin{pmatrix} F \\ z_F \end{pmatrix} \tag{10}$$

Comments:

1. The primary outputs y_1 are the product compositions (bottoms and distillate product)
2. The measured variables are $y = u_0$ where $u_0 = \begin{pmatrix} L & V & D & B \end{pmatrix}^T$ (flows) are the original manipulated inputs for the distillation column.

3. The inputs u (a subset of u_0) are the remaining two inputs after satisfying the steady-state constraints of constant M_B and M_D (reboiler and condenser level have no steady-state effect). In (10) we have selected $u = (\,L \quad V\,)^T$, but it actually does not matter which two variables we choose to include in u, as long as the variables in u are independent.

4. The disturbances d are feed flowrate and feed composition.

Note that we in (10) only allow for flows as measurements, $y = u_0$. This implies that we want to achieve indirect control by keeping flow combinations at constant values. This implicitly requires that the feed composition z_F has an effect on at least one of the flowrates. This will generally be satisfied in practice where u_0 represents mass or volumetric flows, but it will not be satisfied in the "academic" case where we use the "constant molar flows" assumption (simplified energy balance) and assume that we manipulate molar flows.

We want to use a combination $\Delta c = H\Delta y$ of the measurements y as controlled variables,

$$\Delta c_1 = h_{11}\Delta L + h_{12}\Delta V + h_{13}\Delta D + h_{14}\Delta B$$

$$\Delta c_2 = h_{21}\Delta L + h_{22}\Delta V + h_{23}\Delta D + h_{24}\Delta B$$

From (7) we derive the choice for H that gives "perfect indirect control" at steady state, and we find that it is identical to that of the DRD-configuration in Häggblom and Waller (1990).

As a specific example, consider the model of a 15-plate pilot-plant ethanol-water distillation column studied by Häggblom and Waller (1990). The steady-state model in terms of $u = (\,L \quad V\,)^T$ (LV-configuration) is

$$\begin{pmatrix} \Delta y_D \\ \Delta x_B \end{pmatrix} = G_1 \begin{pmatrix} \Delta L \\ \Delta V \end{pmatrix} + G_{d1} \begin{pmatrix} \Delta F \\ \Delta z_F \end{pmatrix}$$

$$y = \begin{pmatrix} \Delta L \\ \Delta V \\ \Delta D \\ \Delta B \end{pmatrix} = G^y \begin{pmatrix} \Delta L \\ \Delta V \end{pmatrix} + G_d^y \begin{pmatrix} \Delta F \\ \Delta z_F \end{pmatrix}$$

with (Häggblom and Waller 1990)

$$G_1 = \begin{pmatrix} -0.045 & 0.048 \\ -0.23 & 0.55 \end{pmatrix} \quad G_{d1} = \begin{pmatrix} -0.001 & 0.004 \\ -0.16 & -0.65 \end{pmatrix} \tag{11}$$

$$G^y = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ -0.61 & 1.35 \\ 0.61 & -1.35 \end{pmatrix} \quad G_d^y = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0.056 & 1.08 \\ 0.944 & -1.08 \end{pmatrix} \tag{12}$$

From (7) we derive that the following variable combination gives perfect disturbances rejection and decoupling (DRD):

$$H = \begin{pmatrix} -0.0427 & 0.0430 & 0.0025 & -0.0012 \\ -0.5971 & 1.3625 & -0.7281 & -0.1263 \end{pmatrix} \tag{13}$$

which is identical with the DRD-structure found in Häggblom and Waller (1990).

We note that our derivation is much simpler. In addition, our results generalize the results in Häggblom and Waller (1990) in two ways:

1. The results are generalized to other measurements than the choice $y = u_0$ (flows). For example, it is possible to derive a DRD-configuration based on keeping two combinations of four temperature measurements constant.

2. The results are generalized to other processes than distillation.

A further extensions is discussed next.

4 Extension 2: Selection of measurements and effect of measurement error

Above we assumed that the number of independent measurements was equal to the number of independent variables, i.e. $n_y = n_u + n_d$ (Assumption 7), and neglected the effect of measurement error (noise) and control error by assuming that we can achieve perfect control of c, i.e. $c = c_s$ at steady state (Assumption 4). These assumptions are related, since the violation of Assumption 7, will lead to sensitivity the measurement error neglected in Assumption 4.

Let n^y denote the measurement error associated with the measurements y. Since $\Delta c = H\Delta y$, the effect on the controlled variables c is $n^c = c - c_s = Hn^y$. This corresponding error in the primary outputs is then

$$\Delta y_1 = P_c H n^y \tag{14}$$

From (14) we have that the effect of measurement error is large if the norm of the matrix $P_c H$ is large. With "perfect indirect control" we have from (7) that $P_c H = \tilde{G}_1 {\tilde{G^y}}^{-1}$ which is large if the measurements are closely correlated since then $\tilde{G^y}$ is close to singular and the elements in ${\tilde{G^y}}^{-1}$ are large.

If we have extra measurements, $n_y > n_u + n_d$, then we may use these extra measurements to affect $P_c H$ and thus minimize the effect of the measurement noise. This may be done in two ways as discussed below:

(a) Select the best subset of all the measurements, ("use the most independent measurements").

(b) Use all the measurements and select the best combination ("average out the measurement error").

Method (b), where we use all the measurements, it always better mathematically, but method (a), where we use only a subset, may be preferred in practice because it uses fewer measurements. In addition, there may cases where we have too few or correlated measurements, so that it is impossible to achieve "perfect" disturbance rejection. We would then like to:

(c) Select (control) a combination of the available mesurements so that the effect of disturbances on the primary variables is minimized.

(a) Best subset of measurements. This is the case discussed earlier where we select as many measurements as there are inputs and disturbances ($n_y = n_u + n_d$). The matrix $\tilde{G^y}$ is then invertible and from (7) we have for "perfect indirect control" that

$$P_c H = \tilde{G}_1 {\tilde{G^y}}^{-1} \tag{15}$$

The issue here is which subset of the measurements to select.

First, we note that the choice of P_c does not affect the sensitivity to measurement error $\tilde{G}_1 {\tilde{G^y}}^{-1}$, that is, the "degree of freedom" in selecting P_c is not useful in terms of measurement error. Also note that the choice of measurements y does not influences the matrix $\tilde{G}_1$. However, the choice of measurements y does affect the matrix $\tilde{G^y}$, and if we have extra measurements then we should select them such that the effect of measurement error is minimized, that is, such that $\tilde{G}_1 {\tilde{G^y}}^{-1}$ is minimized. To choose the best measurements we first need to scale the measured variables:

- Each measured variable y is scaled such that its associated measurement error n^y is of magnitude 1.

Since the induced 2-norm or maximum singular value of a matrix, $\bar{\sigma}$, provides the worst-case amplication in terms of the two-norm, we have from (14) and (15) that

$$\max_{\|n^y\|_2 \leq 1} \|\Delta y_1\|_2 = \bar{\sigma}(\tilde{G}_1 {\tilde{G^y}}^{-1}) \leq \bar{\sigma}(\tilde{G}_1)\bar{\sigma}({\tilde{G^y}}^{-1}) = \bar{\sigma}(\tilde{G}_1)/\underline{\sigma}(\tilde{G^y}) \tag{16}$$

This has the following implications:

1. (Optimal) In order to minimize the worst-case value of $\|\Delta y_1\|_2$ for all $\|n^y\|_2 \leq 1$, select measurements such that $\bar{\sigma}(\tilde{G}_1 {\tilde{G^y}}^{-1})$ is minimized.

2. (Suboptimal) Recall that the measurement selection does not affect $\tilde{G}_1$. From the inequality in (16) it then follows that the effect of the measurement error n^y will be small when $\underline{\sigma}(\tilde{G^y})$ (the minimum singular value of $\tilde{G^y}$) is large. It is therefore reasonable to select measurements y such that $\underline{\sigma}(\tilde{G^y})$ is maximized. Here $\tilde{G^y}$ represents the effect of u and d on y.

(b) Best combination of all the measurements. Let $\tilde{G}^y_{\text{all}}$ represent the effect of the independent variables on all the available measurements. A derivation similar to (7) gives that "perfect indirect control" is achieved when

$$H\tilde{G}^y_{\text{all}} = P_{c0}^{-1}\tilde{G}_1 \tag{17}$$

However, we now have $n_y > n_u + n_d$, and (17) has an infinite number of solutions for H. We want to find the solution that minimizes the effect of measurement error on the primary outputs y_1. The solution that minimizes the 2-norm of y_1 is the one with the smallest 2-norm of $P_c H$, see (14). With $P_c = P_{c0} = I$ (decoupling) this is obtained from (17) by making use of the pseudo inverse:

$$H = \tilde{G}_1 {\tilde{G^y_{\text{all}}}}^{\dagger} \tag{18}$$

In this case ${G^y_{\text{all}}}^{\dagger}$ is the left inverse of G^y_{all}. With this choice the effect of measurement error is

$$P_c H = \tilde{G}_1 {\tilde{G^y_{\text{all}}}}^{\dagger}$$

(c) Few measurements. We here consider the case with fewer measurements than indepedendent variables, i.e. $n_y < n_u + n_d$. In this case, (17) has no solution, so perfect disturbance rejection ($P_d = 0$) is not possible. One possibility, is to delete or combine disturbances such that (17) has a solution. Another possibility, is to use the pseudo inverse as shown in (18),

$$H = \tilde{G}_1 {\tilde{G^y_{\text{all}}}}^{\dagger} \tag{19}$$

but in this case the pseudo inverse is the right inverse. This corresponds to selecting H such $\|E\|_2$ is minimized, where $E = P_{c0}^{-1}\tilde{G}_1 - H\tilde{G}^y_{\text{all}}$. This seems reasonable as we can show that $P_d \Delta d = P_{c0}^{-1} E \begin{pmatrix} \Delta u \\ \Delta d \end{pmatrix}$, so a small value of E implies a small value of $P_d \Delta d$, and thus a small disturbance sensitivity.

Comment. It is appropriate at this point to make a comment about the pseudo inverse of a matrix. Above we are looking for the best solution for H that satisfies the equation set $H\tilde{G}^y_{\text{all}} = P_{c0}^{-1}\tilde{G}_1$. In general, we can write the solution of $HA = B$ as $H = BA^{\dagger}$ where

- $A^{\dagger} = (A^T A)^{-1} A^T$ is the left inverse for the case when A has full column rank (we have extra measurements). In this case there are an infinite number of solutions and we seek the solution that minimizes H.

- $A^\dagger = A^T(AA^T)^{-1}$ is the right inverse for the case when A has row column rank (we have too few measurements). In this case there is no solution and we seek the solution that minimizes the two-norm of $E = B - HA$ ("regular least squares").

- In the general case with extra mesurements, but where some are correlated, A has neither full column or row rank, and the singular value decomposition may be used to compute the pseudo inverse.

5 Discussion: Link to previous work

Inferential control. If we choose $P_{c0} = I$, then we find, not unexpectedly, that (7) is the same as Brosilow's static inferential estimator; see eq. (2.4) in Weber and Brosilow (1972). To more clearly see the link, recall that the idea in inferential control is to first "infer" from the measurements Δy the inputs and disturbances, and from this estimate the primary output. From (2) the inferred input and disturbance is

$$\begin{pmatrix} \Delta u \\ \Delta d \end{pmatrix} = \tilde{G^y}^{-1} \Delta y$$

and from (3) the resulting estimated value of the primary output is

$$\Delta y_1 = \tilde{G}_1 \tilde{G^y}^{-1} \Delta y$$

On the other hand, in indirect control, the idea is to control a measurement combination, and from (7) with $P_c = I$ (that is, we want $\Delta y_1 = \Delta c$) the resulting measurement combination is

$$\Delta c = H \Delta y = \tilde{G}_1 \tilde{G^y}^{-1} \Delta y$$

which is identical to the estimated primary output found with inferential control. The advantage with the derivation in our paper is that it provides a link to control configurations, regulatory control, cascade control, indirect control and self-optimizing control, and also provides the generalization (8).

Self-optimizing control. The results in this paper on perfect indirect control provide a nice generalization of the distillation results of Häggblom and Waller (1990), but are themselves a special case of the work of Alstad and Skogestad (2002) on self-optimizing control with perfect disturbance rejection (Alstad and Skogestad 2002) (Alstad and Skogestad 2003). To see this link we need to write the cost function as

$$J = \frac{1}{2}(y_1 - y_{1s})^T (y_1 - y_{1s}) \tag{20}$$

Differentiation gives

$$J_u = (G_1 \Delta u + G_{d1} \Delta d)^T G_1, \; J_{uu} = G_1^T G_1, \; J_{ud} = G_1^T G_{d1} \tag{21}$$

and we can compute the matrix M in the exact method of Alstad and Skogestad (2002) and search for the optimal measurement combination. We find that:

- $P_d = 0$ ("perfect control" with zero sensitivity to disturbances) implies $M_d = 0$ (zero loss for disturbances). To prove this premuliply P_d by $G_1^\dagger$ and note that $G_1^\dagger G_1 = I$ since $G_1^\dagger$ is a left inverse.

- However, unless $n_{y_1} \leq n_u$ we do not have $G_1^\dagger G_1 = I$, so $M_d = 0$ (zero loss) does not generally imply $P_d = 0$ ("perfect control"). This is easily explained: We can only perfectly control as many outputs (y_1) as we have independent inputs (u).

6 Conclusion

Indirect control is commonly used in industrial applications where the primary controlled variable is not measured. In this paper we considered the case of "perfect steady-state indirect control" where one attempts to control a combination of the available measurements such that there is no effect of disturbances at steady-state. This is always possible provided the number of measurements is equal to the number of independent variables (inputs plus disturbances). It is further shown how extra measurements may be used to minimize the effect of measurement error. This paper generalizes the work of Häggblom and Waller (1990), but is itself a special case of the work of Halvorsen *et al.* (2003) and Alstad and Skogestad (2002) on self-optimizing control.

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