Perfect indirect control

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Abstract

This paper considers optimal indirect control. It generalizes the work of Haggblom and Waller (1990), but is itself a special case of the work of Halvorsen et al. (2003) and Alstad and Skogestad (2002, 2003) on self-optimizing control.

1 Introduction

Indirect control (Skogestad and Postlethwaite 1996) is when we can not control the "primary" outputs y_1 (e.g., because they are not measured online), and instead we aim at indirectly controlling y_1 by controlling the "secondary" variables c (Skogestad and Postlethwaite 1996)[1] More precisely,

> **Indirect control** is when we aim at (indirectly) keeping the primary variables y_1 close to their setpoints y_{1s}, by controlling the secondary variables c at constant setpoints c_s.

Indirect control is discussed in some detail in Skogestad and Postlethwaite (1996)[page 406-407, page 422-423]. An simple example of indirect control is control of temperature (c) in a distillation column, in order to indirectly achieve composition control (y_1).

A less obvious example of indirect control, is the selection of control configurations in distillation columns. Here one aims, by keeping selected flows or flow ratios constant, at reducing the effect of disturbances on the primary outputs (product compositions). The clearest example of this is the "disturbance rejecting and decoupling" (DRD) structure of Häggblom and Waller (1990), which actually motivated the results presented below.

In general, we have a set of measurements y which are "candidate" variables for indirect control. In this paper we aim at selecting as "secondary" controlled variables c the "best" linear combination of the measurements y,

$$\Delta c = H\Delta y \tag{1}$$

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[1] The use of the terms primary and secondary controlled variables is relative and depends on at which layer you are in the control hiearchy. As seen from the top of the control system, the objective may be to control the (unmeasured) "primary" outputs y_1 and the selected controlled variables c (which are the focus of this paper) are then "secondary" outputs. However, to control c we may generally make use of the setpoints to a lower-level control system, and viewed from here, the c's are the primary outputs and the lower+level controlled variables are the secondary outputs.

In other words, we want to find matrix H. In the simplest case the variables c are directly measured and the matrix H consistis of zeros and ones. However, more generally we allow be combinations (functions) of the available measurements y, and H is a "full" matrix. In the paper we show that if we has as many independent measurements as there are independent variables (inputs plus disturbances), then we can always "perfect indirect control" with perfect disturbance rejection and a decoupled response from the setpoints c_s (the "new" inputs) to the primary variables y_1.

Indirect control may be viewed as a special case of "self-optimizing control" (Halvorsen *et al.* 2003). This is clear from the definition:

> **Self-optimizing control** (Skogestad 2000) is when we can achieve acceptable (economic) loss with constant setpoint values for the controlled variables c (without the need to reoptimize when disturbances occur).

In most cases the "loss" is an economic loss, but for indirect control it is the setpoint deviation, i.e. $L = \|y_1 - y_{1s}\|$. The implications og viewing indirect control as a special case of self-optimizing control are discussed later in the paper.

Another related idea is inferential control (Weber and Brosilow 1972). However, in inferential control the basic idea is to use the measurements y to estimate the primary variables y_1, whereas the objective of indirect control is to directly control a combination of the measurements y.

In the paper we only consider the steady-state behavior. The notation in this paper large follows that used by Halvorsen *et al.* (2003).

2 Perfect indirect control

Consider a setpoint problem where the objective is to keep the "primary" controlled variables y_1 at their setpoints y_{1s}.

Inputs (independent variables available for control of y_1): u

Disturbances: d

Available measurements: y

Problem definition: Find a set of (secondary) controlled variables $c = h(y)$ such that a constant setpoint policy ($c = c_s$) indirectly results in acceptable control of the primary outputs (y_1).

We assume that the number of controlled variables is equal to the number of inputs ($\#c = \#u$) such that it always is possible to adjust u to get $c = c_s$.

Further assumptions: Local behavior (Linearized models). Steady-state only. Will neglect the control error (including measurement noise), that is, we assume that we achieve $c = c_s$. We assume that the nominal operating point (u^*, d^*) is optimal, i.e. at the nominal point (where $d = d^*$ and $c = c_s$) we have $y_1^* = y_{1s}$.

The linear models relating the variables are

$$\Delta y = G^y \Delta u + G_d^y \Delta d \tag{2}$$

$$\Delta y_1 = G_1 \Delta u + G_{d1} \Delta d \tag{3}$$

$$\Delta c = G \Delta u + G_d \Delta d \tag{4}$$

where $\Delta u = u - u^*$, etc. Solving (4) with respect to Δu yields

$$\Delta u = G^{-1} \Delta c - G^{-1} G_d \Delta d$$

which upon substitution into (3) yields

$$\Delta y_1 = \underbrace{G_1 G^{-1}}_{P_c} \Delta c + \underbrace{(G_{d1} - G_1 G^{-1} G_d)}_{P_d} \Delta d \tag{5}$$

The "partial disturbance gain" P_d gives the effect of disturbances on y_1 with closed-loop ("partial") control of the variables c, and P_c gives the effect on y_1 of changes in c (e.g.., due to setpoint changes in c_s or control error).

Ideally, we would like to find a set of controlled variables such that $P_d = 0$. Somewhat surprisingly, it turns out that this is always possible provided we have enough measurements y, and that we in fact have additional degrees of freedom left which we may use, for example, to specify P_c. For example, it may be desirable to have $P_y = I$, because this (at least at steady state) gives a decoupled response from c_s (which are our "new inputs") to the primary controlled variables y_1.

> *Refined problem definition ("perfect indirect control"):* Find an linear measurement combination, $\Delta c = H\Delta y$, such that at steady state $P_d = 0$ and $P_c = P_{c0}$, where P_{c0} is a constant specified matrix.

We make the following additional assumptions:

1. The number of (independent) controlled variables, primary variables and independent variables (inputs) is equal ($\#c = \#y_1 = \#u$), and P_{c0} is invertible.
2. The number of (independent) measurements is equal to the number of inputs plus disturbances ($\#y = \#u + \#d$).

Solution to refined problem definition: We have that

$$G = HG^y, \; G_d = HG_d^y$$

and we look for an H such that $P_d = 0$. This generally has an infinite number of solutions in H. We therefore have additional degrees of freedom which may use to specify P_y. This gives the additional contraint $G_1 G^{-1} = P_{c0}$, or equivalently

$$G = HG^y = P_{c0}^{-1} G_1 \tag{6}$$

The requirement $P_d = 0$ then becomes $G_{d1} - P_{c0} G_d = 0$ or equivalently

$$G_d = HG_d^y = P_{c0}^{-1} G_{d1} \tag{7}$$

Combining (6) and (7) gives

$$H\tilde{G^y} = P_{c0}^{-1} \tilde{G}_1 \tag{8}$$

where

$$\tilde{G}_1 = \begin{pmatrix} G_1 & G_{d1} \end{pmatrix}, \; \tilde{G^y} = \begin{pmatrix} G^y & G_d^y \end{pmatrix} \tag{9}$$

represent the combined effect of u and d on the primary outputs y_1, and the measurements y, respectively. By assumption, we have as many independent measurements as there inputs and disturbances, so $\tilde{G^y}$ is invertible. Solving (8) then gives the following unique optimal choice for H that gives $P_d = 0$:

$$H = P_{c0}^{-1} \tilde{G}_1 \tilde{G^y}^{-1} \tag{10}$$

which is the solution to the refined problem definition.

More generally, we may specify $P_d = P_{d0}$ (where P_{d0} is given and may be nonzero) and the resulting choice for H is

$$H = P_{c0}^{-1} \hat{G}_1 \tilde{G^y}^{-1} \tag{11}$$

where

$$\hat{G}_1 = \begin{pmatrix} G_1 & G_{d1} - P_{d0} \end{pmatrix} = \tilde{G}_1 - \begin{pmatrix} 0 & P_{d0} \end{pmatrix} \tag{12}$$

3 Application to distillation

The results of Haggblom and Waller (1990) on "control structures for disturbance rejection and decoupling of distillation" provide an interesting special case of the above results, and actually motivated their derivation. Haggblom and Waller (1990) showed that one could derived a DRD control configuration that achieved

1. Perfect disturbance rejection (with the new loops closed)
2. Decoupled response from the new manipulators to the primary outputs

Haggblom and Waller (1990) derived this for distillation column models, and made no attempt of generalizing their results. However, they can be shown to be a special case of the above results with the following choice of variables

$$y_1 = \begin{pmatrix} y_D \\ x_B \end{pmatrix}, \quad y = \begin{pmatrix} L \\ V \\ D \\ B \end{pmatrix}, \quad u = \begin{pmatrix} L \\ V \end{pmatrix}, \quad d = \begin{pmatrix} F \\ z_F \end{pmatrix} \tag{13}$$

Comments:

1. The primary outputs y_1 are the product compositions (bottoms and distillate product)
2. The measured variables are $y = u_0$ where $u_0 = (L \quad V \quad D \quad B)^T$ (flows) are the dynamic inputs for the distillation column.
3. The inputs u (a subset of u_0) are the remaining two inputs after satisfying the steady-state constraints of constant M_B and M_D (reboiler and condenser level have no steady-state effect). In this case we use $u = (L \quad V)^T$, but it actually does not matter which variables we choose to include in u, as long as the resulting matrices are well-defined.
4. The disturbances are feed flowrate and feed composition.

Note that we only allow flows as measurements, $y = u_0$. This implies that we want to achieve indirect control by keeping flow combinations at constant values. It also requires that the feed composition z_F has an effect on at least one of the flowrates. This will generally be satisfied in practice where we u_0 represents mass or volumetric flows.[2]

With this choice of variables, the use of (10) gives two controlled variables $\Delta c = H \Delta y$. This may be written in more detail as

$$\Delta c_1 = h_{11}\Delta L + h_{12}\Delta V + h_{13}\Delta D + h_{14}\Delta B$$

$$\Delta c_2 = h_{21}\Delta L + h_{22}\Delta V + h_{23}\Delta D + h_{24}\Delta B$$

which is identical to that of the DRD-configuration in Häggblom and Waller (1990). As a specific example, consider the model of a 15-plate pilot-plant ethanol-water distillation column studied by Häggblom and Waller (1990). The steady-state model for the LV-structure (with $u = (L \quad V)^T$) is

$$\begin{pmatrix} \Delta y_D \\ \Delta x_B \end{pmatrix} = G_1 \begin{pmatrix} \Delta L \\ \Delta V \end{pmatrix} + G_{d1} \begin{pmatrix} \Delta F \\ \Delta z_F \end{pmatrix}$$

[2] The feed composition will not effect the flows in the common academic case with the "constant molar flows" and the use of molar flows as inputs (u_0). Here the "constant molar flows" assumption (simplified energy balance) is reasonable in many cases, but the assumption of "molar input variables" is unrealistic because we cannot in practice mesure molar flows.

$$y = \begin{pmatrix} \Delta L \\ \Delta V \\ \Delta D \\ \Delta B \end{pmatrix} = G^y \begin{pmatrix} \Delta L \\ \Delta V \end{pmatrix} + G_d^y \begin{pmatrix} \Delta F \\ \Delta z_F \end{pmatrix}$$

where (Häggblom and Waller 1990)

$$G_1 = \begin{pmatrix} -0.045 & 0.048 \\ -0.23 & 0.55 \end{pmatrix} \quad G_{d1} = \begin{pmatrix} -0.001 & 0.004 \\ -0.16 & -0.65 \end{pmatrix} \tag{14}$$

$$G^y = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ -0.61 & 1.35 \\ 0.61 & -1.35 \end{pmatrix} \quad G_d^y = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0.056 & 1.08 \\ 0.944 & -1.08 \end{pmatrix} \tag{15}$$

From (10) we derive that the following variable combination gives perfect disturbances rejection and decoupling (DRD):

$$H = \begin{pmatrix} -0.0427 & 0.0430 & 0.0025 & -0.0012 \\ -0.5971 & 1.3625 & -0.7281 & -0.1263 \end{pmatrix} \tag{16}$$

which is identical with the DRD-structure found in Häggblom and Waller (1990).

We note that the derivation is much simpler with the approach proposed in our paper. In addition, our results generalize the results in Häggblom and Waller (1990) in two ways:

1. The results are generalized to other measurements than the choice $y = u_0$. For example, it is possible to derive a DRD-configuration based on two keeping two combinations of four temperature measurements constant.

2. The results are generalized to other processes than distillation.

4 Discussion: Implementation error (noise)

Above we neglected the effect of measurement error (noise) and control error, by assuming that we can achieve perfect control of c with $c = c_s$. In practice, there will be an implementation error $n^c = c - c_s$ which will result in a corresponding change $\Delta y_1 = P_c(c - c_s)$ in the primary controlled variables. If we assume that we have integral action in the controller used for controlling c, then the implementation error (nonzero n^c) must be caused by measurement error. The effect of a measurement error n^y in the measurements y on the controlled variables c is $n^c = Hn^y$, and the resulting effect on the primary variables is

$$\Delta y_1 = P_c H n^y \tag{17}$$

From this it is clear that in order to minimize the effect of measurement error we need to minimize the norm of the matrix $P_c H$. For the case with "perfect indirect control" (DRD) we have from (10) that

$$P_c H = \tilde{G}_1 \tilde{G^y}^{-1} \tag{18}$$

Interstlingly, we note that the choice of P_c has no effect on the sensitivity to measurement noise. Also, note that the choice of measurements does not influences the matrix G_1. However, the choice of measurements does affect the matrix $\tilde{G^y}$, and if we have extra measurements then we should select them such that the effect of measurement noise is minimized. To choose the best measurements we first need to scale the measured variables:

- Each measured variable y is scaled such that its associated measurement error n^y is of magnitude 1.

The induced 2-norm or maximum singular value of a matrix, $\bar{\sigma}$, provides the worst-case amplication in terms of the two norm, that is, we have from (17) and (18) that

$$\max_{\|n^y\|_2 \leq 1} \|\Delta y_1\|_2 = \bar{\sigma}(\tilde{G}_1 \tilde{G^y}^{-1}) \leq \bar{\sigma}(\tilde{G}_1)\bar{\sigma}(\tilde{G^y}^{-1}) = \bar{\sigma}(\tilde{G}_1)/\underline{\sigma}(\tilde{G^y}) \tag{19}$$

This has the folliwing implications:

1. (Optimal) In order to minimize the worst-case value of $\|\Delta y_1\|_2$ for all $\|n^y\|_2 \leq 1$, select measurements such that $\bar{\sigma}(\tilde{G}_1 \tilde{G^y}^{-1})$ is minimized.
2. (Suboptimal) From the inequality in (19) it follows that the effect of the measurement error n^y will be small when $\underline{\sigma}(\tilde{G^y})$ (the minimum singular value of $\tilde{G^y}$) is large. It is therefore reasonable to select measurements y such that $\underline{\sigma}(\tilde{G^y})$ is maximized.

We have above assumed that we use as many measurements as there are inputs and disturbances ($\#y = \#u + \#d$).

Use all measurements: If we use more (all) measurements then we have from (8) that

$$H\tilde{G}^y_{\rm all} = P_{c0}^{-1}\tilde{G}_1 \tag{20}$$

which for $\#y > \#u + \#d$ has an infinite number of solutions for H. The solution with the smallest 2-norm of H is obtained by making use of the pseudo inverse:

$$H = P_{c0}^{-1}\tilde{G}_1 \tilde{G^y_{\rm all}}^{\dagger} \tag{21}$$

With this choice and the effect of measurement noise is

$$P_c H = \tilde{G}_1 \tilde{G^y_{\rm all}}^{\dagger}$$

For the common case with $P_c = I$ the solution in (21) minimizes the effect of the measurement noise on the primary outputs, that is, it minimizes $\|P_c H\|_2 = \|H\|_2$.

Question 1: What about general P_c? Probably easy to find some weighted pseudo inverse.

Question2 : Is this always better ?? For sure we have that

$$\underline{\sigma}(\tilde{G_{\rm all}}) \geq \underline{\sigma}(\tilde{G})$$

so it is alays better in terms of the second (suboptimal) test. However, I am a bit uncertain with regards the first (optimal) test.

In conclusion, select measurements such that...

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VIDAR: Merk at resultatene over har en del interssante implikasjoner for deg.

1. Valg av Pc har ikke noe si for sensitivitet til mlesty - bekrefter at de ekstra frihetsgradene ikke kan brukes til redusere mlesty dersom vi krever Pd=0 (dvs. Md=0).

2. Valg av mlinger: Bekrefter regelen vr om velge mlinger slik at min.singulrverdi maksimeres.

3. Ekstra mlinger: Se mer p (bde i dette tilfellet og ditt tilfelle)

5 Discussion: Link to previous work on inferential control

If we choose $P_{c0} = I$, then we find, not unexpectedly, that (10) is the same as Brosilow's static inferential estimator; see eq. (2.4) in Weber and Brosilow (1972). The advantage with the derivation in our paper is that it provides a link to control configurations, regulatory control, cascade control, indirect control and self-optimizing control, and also provides the generalization (11).

... That's good, but why do we then need our paper...?

6 Discussion: Link to previous results on self-optimizing control

The results in this paper om perfect indirect control, provide a nice generalization of the distillation results of Haggblom and Waller (1990), but are themselves a special case of the work of Alstad and Skogestad (2003) on self-optimizing control with prefect disturbance rejection.

Definition of self-optimizing control. J, L, $c = Hy$. y includes u_0 and all other measurements. $\Delta y_{\text{opt}} = F\Delta d$.

If $\#c = \#u + \#d$ then we can always get zero loss for disurbances, i.e. $M_d = 0$ (results of Alstad and Skogestad). This is done by selecting H such that $HF = 0$.

This may be written on the form considered above by defining

$$J = \frac{1}{2}(y_1 - y_{1s})^T(y_1 - y_{1s}) = \frac{1}{2}e_1^T e_1 \tag{22}$$

Differentiation gives

$$J_u = (G_1\Delta u + G_{d1}\Delta d)^T G_1, \; J_{uu} = G_1^T G_1, \; J_{ud} = G_1^T G_{d1} \tag{23}$$

and we can compute the matrix M in the exact method (**??**) and search for the optimal measurement combination. Note in particular that the term $(J_{uu}^{-1}J_{ud} - G^{-1}G_d)$ in M_d is equal to $(G_1^\dagger G_{d1} - G^{-1}G_d)$ where

$$G_1^\dagger = (G_1^T G_1)^{-1}G_1^T \tag{24}$$

is the pseudo (left) inverse of G_1. From this it is clear that $M_d = 0$ for the ideal ("uninteresting") case with $c = y_1$ (as expected). The goal of indirect control is to search for other ("interesting") choices for the controlled variables c (measurement combinations) with M_d small or even zero.

Some facts:

- $P_d = 0$ ("perfect control" with zero sensitivity to disturbances) implies $M_d = 0$ (zero loss for disturbances). To prove this postmuliply P_d by $G_1^\dagger$ and note that $G_1^\dagger G_1 = I$ since $G_1^\dagger$ is a left inverse.

- However, unless $\#y_1 \leq \#u$ we do not have $G_1 G_1^\dagger G_1 = I$, so $M_d = 0$ (zero loss) does not generally imply $P_d = 0$ (zero sensitivity). This is easily explained: We can only perfectly control as many outputs (y_1) as we have independent inputs (u).

- We have above assumed $\#y_1 = \#c = \#u$. In this case $M_d = 0$ is equaivalent to $P_d = 0$, and P_y is square and invertible.

The rest can be short, but here is something:

We want to find controlled variables (find H) such that the loss with respect to disturbances is zero ($M_d = 0$). From the above general results we know this is always possible provided the number of independent measurements is equal to the number of inputs plus disturbances, i.e. $\#y = \#u + \#d$. In fact, there are generally infinite solutions for H (the measurement combination). We therefore have additional degrees of freedom which may use to specify P_y where we assume that P_y is invertible. Alternative derivation. Require $HF = 0$.

From the above results and (23) we get

$$F = -G^y J_{uu}^{-1} J_{du} + G_d^y = -G^y G_1^{\dagger} G_{d1} + G_d^y$$

Any H satisfying $HF = 0$ will give zero loss. The requirement $HF = 0$ then gives

$$etc.. \quad (25)$$

7 Conclusion

References

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