European Symposium on Computer Aided Process Engineering – 15
L. Puigjaner and A. Espuña (Editors)

Use of Perfect Indirect Control to Minimize the State Deviations

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Abstract

An important issue in control structure selection is plant ”stabilization”. By the term ”stabilize” we here include both, modes that are mathematically unstable (modes with RHP poles) as well as ”drifting” modes that need to be kept within limits to avoid operational problems. By this definition, we can include states $\mathbf{x}$ as variables that should be ”stabilized”, i.e., we want to avoid them to drift too far away from their desired (nominal) values. An advantage of this approach is that we are able to avoid problems resulting from nonlinear effects. Therefore, as the objective function can, usually, be considered as a combination of states, the control system obtained by this approach is not tied too closely to a particular primary control objective (which may change with time) because it allows the designer to change the control objective. This paper presents a way to reduce the effects of disturbances and measurement errors in the states and the results show the effectiveness of this approach.

Keywords: perfect indirect control; minimization; state deviations

1. Introduction

In the regulatory control layer, the main objective is to ”stabilize” the plant. Here we put the word stabilize in quotes because we use it with the same meaning as used by Skogestad (2004): ”stabilization” includes both modes which are mathematically unstable (modes with RHP poles) as well as ”drifting” modes which need to be kept within limits to avoid operational problems. Doing this we are able to avoid problems resulted from, for example, nonlinear effects. By this definition, we include any states $\mathbf{x}$ as variables that should be “stabilized”, i.e., we want to avoid them to drift too far away from their desired (nominal) values. An advantage of keeping all states close to their nominal values is that we are able to avoid problems resulting from nonlinear effects. Therefore, an important point in the control structure selection is the choice of the operational objectives (Skogestad, 2004). The problem is that the operational objectives may change with time, according to necessities, e.g. market, safety constraints, etc. Due to these changes, we don’t want to tie the control system too

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closely to a particular primary control objective. As, usually, the objective function can be considered a combination of states, a good approach would be to define the objective function in this way ($\mathbf{y}_1 = \mathbf{Wx}$). This approach has the advantage of allowing the controller designer to easily change the control objective only changing the combination of the states. Another advantage is that the minimization of $\mathbf{Wx}$ includes both stabilization of RHP-poles and disturbance rejection. In summary, the good of this paper is to discuss in more detail the approach introduced in Skogestad (2004) of selecting secondary controlled variables ($\mathbf{c} = \mathbf{y}_2$) such that we minimize the effect of disturbances ($\mathbf{d}$) on the weighted states ($\mathbf{y}_1 = \mathbf{Wx}$).

2. Perfect Indirect Control

Consider that we have the following linear model:

$$\Delta\mathbf{y}_1 = \mathbf{G}_1\Delta\mathbf{u} + \mathbf{G}_{d1}\Delta\mathbf{d} \tag{1}$$

$$\Delta\mathbf{y} = \mathbf{G}^{y}\Delta\mathbf{u} + \mathbf{G}_d^{y}\Delta\mathbf{d} + \mathbf{n}^{y} \tag{2}$$

where $\mathbf{G}$s are steady-state models.

By definition, indirect control is when we cannot control the primary outputs ($\mathbf{y}_1$) and, instead, we aim at indirectly controlling $\mathbf{y}_1$ by controlling the ”secondary” variables $\mathbf{c}$ (Skogestad and Postlethwaite, 1996). If the number of measurements ($\#\mathbf{y}$) is equal or larger than the sum of the number of inputs ($\#\mathbf{u}$) and the number of disturbances ($\#\mathbf{d}$), it is possible to obtain a combination of these measurements ($\mathbf{c}$) that ensures a perfect indirect control of the ”primary” controlled variables (in this case $\mathbf{c}$ is used as ”secondary” controlled variable). Then we have:

$$\Delta\mathbf{c} = \mathbf{H}\Delta\mathbf{y} = \underbrace{\mathbf{H}\mathbf{G}^{y}}_{\mathbf{G}}\Delta\mathbf{u} + \underbrace{\mathbf{H}\mathbf{G}_d^{y}}_{\mathbf{G}_d}\Delta\mathbf{d} + \underbrace{\mathbf{H}\mathbf{n}^{y}}_{\mathbf{n}^{c}} \tag{3}$$

where $\mathbf{H}$ is the combination of measurements. Solving Equation 3 with respect to $\Delta\mathbf{u}$:

$$\Delta\mathbf{u} = \mathbf{G}^{-1}\Delta\mathbf{c} - \mathbf{G}^{-1}\mathbf{G}_d\Delta\mathbf{d} - \mathbf{G}^{-1}\mathbf{n}^{c} \tag{4}$$

In Eq. 4 we will consider $\Delta\mathbf{c} = \mathbf{0}$ because we want to keep these variables constant. In this way, Eq. 4 becomes:

$$\Delta\mathbf{u} = -\mathbf{G}^{-1}\mathbf{n}^{c} - \mathbf{G}^{-1}\mathbf{G}_d\Delta\mathbf{d} \tag{5}$$

Substituting Eq. 5 into Eq. 1 gives:

$$\Delta\mathbf{y}_1 = \underbrace{\left(\mathbf{G}_{d1} - \mathbf{G}_1\mathbf{G}^{-1}\mathbf{G}_d\right)}_{\mathbf{P}_d}\Delta\mathbf{d} - \underbrace{\mathbf{G}_1\mathbf{G}^{-1}}_{\mathbf{P}_c}\mathbf{n}^{c} \tag{6}$$

where the ”partial disturbance gain” $\mathbf{P}_d$ gives the effect of disturbances on $\mathbf{y}_1$ with closed-loop (partial) control of the variables $\mathbf{c}$, and $\mathbf{P}_c$ gives the effect on $\mathbf{y}_1$ of changes in $\mathbf{c}$ (e.g., due to setpoint changes in $\mathbf{c}_s$ or control error).

As we want to reject perfectly the effect of the disturbance in the primary variables, we will select a set of controlled variables such that the matrix $\mathbf{P}_d$ is equal to zero. As was said before this objective can be reached if we have enough measurements $\mathbf{y}$. The matrix $\mathbf{P}_c$ is a degree of freedom which can be arbitrarily specified ($\mathbf{P}_c = \mathbf{P}_{c0}$) by the designer, for example, when $\mathbf{P}_c = \mathbf{I}$ we have a decoupled response from $\mathbf{c}_s$.

To find the linear combination of variables we will make some additional assumptions:

1. $\#\mathbf{c} = \#\mathbf{y}_1 = \#\mathbf{u}$;
2. $\#\mathbf{y} = \#\mathbf{u} + \#\mathbf{d}$;
3. The matrix $\mathbf{P}_{c0}$ is invertible.

Then, we want to find a matrix $\mathbf{H}$ that gives us $\mathbf{P}_d = \mathbf{0}$ and $\mathbf{P}_c = \mathbf{P}_{c0}$. Joining Eq. 3 and 6 results in:

$$\mathbf{H}\left[\mathbf{G}^{y} \quad \mathbf{G}_d^{y}\right] = \mathbf{P}_{c0}^{-1}\left[\mathbf{G}_1 \quad \mathbf{G}_{d1}\right] \tag{7}$$

By assumption number 1 we have that the matrix $\left[\mathbf{G}^{y} \quad \mathbf{G}_d^{y}\right]$ is square and, as the measurements are independent, the matrix is invertible, then, finally:

$$\mathbf{H} = \mathbf{P}_{c0}^{-1}\underbrace{\left[\mathbf{G}_1 \quad \mathbf{G}_{d1}\right]}_{\tilde{\mathbf{G}}_1}\underbrace{\left[\mathbf{G}^{y} \quad \mathbf{G}_d^{y}\right]^{-1}}_{\tilde{\mathbf{G}}^{y^{-1}}} \tag{8}$$

When $\mathbf{P}_{c0} = \mathbf{I}$ and using $\mathbf{c}$ as secondary controlled variables, from Eq. 6 is easy to see that $\mathbf{G} = \mathbf{G}_1$ and $\mathbf{G}_d = \mathbf{G}_{d1}$.

3. Minimum State Deviation

To keep the states close to their desired (nominal) values in the presence of disturbances and implementation error, we will define a matrix $\mathbf{W}$, which represents a linear combination of the states. It can also be interpreted as the objective of the controller defined by the controller's designer. Consider the following linear model:

$$\Delta\mathbf{x} = \mathbf{G}^{x}\Delta\mathbf{u} + \mathbf{G}_d^{x}\Delta\mathbf{d} \tag{9}$$

Substituting Eq. 5 into Eq. 9:

$$\Delta\mathbf{x} = \underbrace{\left(\mathbf{G}_d^{x} - \mathbf{G}^{x}\mathbf{G}^{-1}\mathbf{G}_d\right)}_{\mathbf{P}_d^{x}}\Delta\mathbf{d} - \underbrace{\mathbf{G}^{x}\mathbf{G}^{-1}}_{\mathbf{P}^{x}}\mathbf{n}^{c} \tag{10}$$

Matrices $\mathbf{P}^{x}$ and $\mathbf{P}_d^{x}$ represent the effect of the disturbances and implementation errors in the states when we control combinations of variables. To avoid problems related to non-linearities, it is important that these matrices be as small as possible. Then, to minimize the effect of the disturbances ($\mathbf{d}$) and the implementation errors ($\mathbf{n}^{c}$), we want to minimize the norms $\left\|\mathbf{P}^{x}\right\|$, $\left\|\mathbf{P}_d^{x}\right\|$, and $\left\|\mathbf{P}^{x} \quad \mathbf{P}_d^{x}\right\|$. It is important to emphasize that we will not control the states $\mathbf{x}$ directly but, instead, we will "control" them using indirect control as presented in section 2. When we have perfect indirect control and when $\mathbf{P}_{c0} = \mathbf{I}$, matrices $\mathbf{G}$ and $\mathbf{G}_d$ become equal to $\mathbf{G}_1$ and $\mathbf{G}_{d1}$, respectively.

Defining the primary variables as linear combinations of the states ($\mathbf{y}_1 = \mathbf{W}\mathbf{x}$):

$$\Delta\mathbf{y}_1 = \mathbf{W}\Delta\mathbf{x} = \underbrace{\mathbf{W}\mathbf{G}^{x}}_{\mathbf{G}_1}\Delta\mathbf{u} + \underbrace{\mathbf{W}\mathbf{G}_d^{x}}_{\mathbf{G}_{d1}}\Delta\mathbf{d} \tag{11}$$

Eq. 10 then becomes:

$$\Delta\mathbf{x} = \underbrace{\left(\mathbf{G}_d^{x} - \mathbf{G}^{x}\left(\mathbf{W}\mathbf{G}^{x}\right)^{-1}\mathbf{W}\mathbf{G}_d^{x}\right)}_{\mathbf{P}_d^{x}}\Delta\mathbf{d} - \underbrace{\mathbf{G}^{x}\left(\mathbf{W}\mathbf{G}^{x}\right)^{-1}}_{\mathbf{P}^{x}}\mathbf{n}^{c} \tag{12}$$

Important point to be discussed: what is the optimal choice of $\mathbf{W}$ that minimizes the value of $\mathbf{P}_d^{x}$ in Eq. 12? Assuming that $\mathbf{W} = \mathbf{G}^{x^T}$, results in:

$$\Delta\mathbf{x} = \left(\mathbf{G}_d^{x} - \mathbf{G}^{x}\left(\mathbf{G}^{x^T}\mathbf{G}^{x}\right)^{-1}\mathbf{G}^{x^T}\mathbf{G}_d^{x}\right)\Delta\mathbf{d} - \mathbf{G}^{x}\left(\mathbf{G}^{x^T}\mathbf{G}^{x}\right)^{-1}\mathbf{n}^{c} \tag{13}$$

The matrix $\mathbf{G}^{x}\left(\mathbf{G}^{x^T}\mathbf{G}^{x}\right)^{-1}\mathbf{G}^{x^T}$ in Eq. 13 is called projection matrix (Strang, 1980). It means that the product $\mathbf{G}^{x}\left(\mathbf{G}^{x^T}\mathbf{G}^{x}\right)^{-1}\mathbf{G}^{x^T}\mathbf{G}_d^{x}$ is the closest point to $\mathbf{G}_d^{x}$, i.e., there isn't any other matrix $\mathbf{W}$ that can result in a smaller value of $\mathbf{P}_d^{x}$ than $\mathbf{W} = \mathbf{G}^{x^T}$. Then we conclude that the choice of $\mathbf{W} = \mathbf{G}^{x^T}$ gives us the minimum value of $\mathbf{P}_d^{x}$. This will be demonstrated in the example below. It is important to notice that this is not the only optimum choice because any matrix $\mathbf{W} = \mathbf{R}\mathbf{G}^{x^T}$ is an optimum solution, where $\mathbf{R}$ is any non-singular square matrix with apropriated dimensions. The choice of $\mathbf{W} = \mathbf{G}^{x^T}$ is optimum for any choice of $\mathbf{P}_{c0}$ non-singular, i.e., this result is not restricted to $\mathbf{P}_{c0} = \mathbf{I}$. It is also important to notice that the matrix $\mathbf{W}$ can be arbitrarily chosen by the designer according to the objective of the process. For example, he can choose to make a combination of only some states or use all of them. In summary, the main result in this paper can be summarized as follows:

Theorem: *Let* $\mathbf{P}_d^{x}$ *denote the steady-state transfer function from* $\mathbf{d}$ *to* $\mathbf{x}$ *with* $\mathbf{c} = \mathbf{H}\mathbf{y}$ *kept constant. Then* $\left\|\mathbf{P}_d^{x}\right\|_2$ *is minimized by selecting* $\mathbf{H} = \mathbf{G}^{x^T}\tilde{\mathbf{G}}_1\tilde{\mathbf{G}}^{y^\dagger}$ *, where † indicates the pseudo-inverse.*

4. Application to Distillation

The proposed theory is applied to a distillation column with 82 states (41 compositions and 41 levels). As the levels don't have steady state effect, we considered that the objective function is a combination of the compositions only. This example has, after stabilization, 2 remaining manipulated variables (reflux flow rate (L) and vapor boilup (V)), and 2 disturbances (feed flow rate (F) and fraction of liquid in the feed (q_F)). Having 2 manipulated variables, we are able to control perfectly 2 combinations of the states. The measurements ($\Delta\mathbf{y}$) are the flow rates (L, V, D, and B). In this example we compared the effect of the disturbances in the states using, as primary variables, 3 different combinations of states (3 different matrices $\mathbf{W}$). The combinations used were:

- Combination 1: $\mathbf{W}$ was selected in order to select the bottom and top compositions as primary variables. This is the most common choice in distillation studies.
- Combination 2: $\mathbf{W}$ was selected as being the transpose of $\mathbf{G}^{\mathrm{x}}$ ($\mathbf{W} = \mathbf{G}^{\mathrm{x}^{\mathrm{T}}}$).
- Combination 3: $\mathbf{W}$ was calculated solving $\min_{\mathbf{W}} \left\| \begin{bmatrix} \mathbf{P}^{\mathrm{x}} & \mathbf{P}_{\mathrm{d}}^{\mathrm{x}} \end{bmatrix} \right\|_2$.

For each combination we obtained the best combination of measurements (matrix $\mathbf{H}$) using Eq. 8. Then matrices $\mathbf{P}^{\mathrm{x}}$ and $\mathbf{P}_{\mathrm{d}}^{\mathrm{x}}$ were calculated. The values of the 2-norm $\left\|\mathbf{P}^{\mathrm{x}}\right\|$, $\left\|\mathbf{P}_{\mathrm{d}}^{\mathrm{x}}\right\|$, and $\left\| \begin{bmatrix} \mathbf{P}^{\mathrm{x}} & \mathbf{P}_{\mathrm{d}}^{\mathrm{x}} \end{bmatrix} \right\|$ are presented in Table 1.

Table 1. Values of $\left\|\mathbf{P}^{\mathrm{x}}\right\|$, $\left\|\mathbf{P}_{\mathrm{d}}^{\mathrm{x}}\right\|$, and $\left\| \begin{bmatrix} \mathbf{P}^{\mathrm{x}} & \mathbf{P}_{\mathrm{d}}^{\mathrm{x}} \end{bmatrix} \right\|$ for all 4 combinations.

	$\left\|\mathbf{P}^{\mathrm{x}}\right\|$	$\left\|\mathbf{P}_{\mathrm{d}}^{\mathrm{x}}\right\|$	$\left\| \begin{bmatrix} \mathbf{P}^{\mathrm{x}} & \mathbf{P}_{\mathrm{d}}^{\mathrm{x}} \end{bmatrix} \right\|$
1	48.8289	2.5182	48.8817
3	0.0252	1.0886	1.0886
4	0.2560	1.0886	1.0886

Although the choice of the top and bottom compositions as primary variables (combination 1) is able to control perfectly these two variables (the closed-loop gains relating the disturbances to the bottom and top compositions are zero), the gains of the states in the middle of the column are very large (above 0.7) (see Table 2). And also this choice doesn't give good rejection of the implementation error (see matrix $\mathbf{P}^{\mathrm{x}}$ in Table 2). As expected (session 3), the results presented in Table 1 confirm that the use of $\mathbf{W} = \mathbf{G}^{\mathrm{x}^{\mathrm{T}}}$ is an optimum choice (it has the same value of $\left\| \begin{bmatrix} \mathbf{P}^{\mathrm{x}} & \mathbf{P}_{\mathrm{d}}^{\mathrm{x}} \end{bmatrix} \right\|$ as obtained by optimization).

As we can see in Table 2, combinations 2 and 3 (obtained by minimization) are equivalent in relation to matrix $\mathbf{P}_{\mathrm{d}}^{\mathrm{x}}$ (are exactly the same in both cases). But when we analyze only $\mathbf{P}^{\mathrm{x}}$, we see that the use of $\mathbf{W} = \mathbf{G}^{\mathrm{x}^{\mathrm{T}}}$ gives us a better result. The reason is that, in the minimization, we are only interested in the norm $\left\| \begin{bmatrix} \mathbf{P}^{\mathrm{x}} & \mathbf{P}_{\mathrm{d}}^{\mathrm{x}} \end{bmatrix} \right\|$ and, in this case, the norm of the matrix $\mathbf{P}_{\mathrm{d}}^{\mathrm{x}}$ is much more important than the matrix $\mathbf{P}^{\mathrm{x}}$. This can be easily seen when we analyze Table 1 more carefully. Although the value of $\left\|\mathbf{P}^{\mathrm{x}}\right\|$, for combination 3, is quite large (0.2560), the value of $\left\| \begin{bmatrix} \mathbf{P}^{\mathrm{x}} & \mathbf{P}_{\mathrm{d}}^{\mathrm{x}} \end{bmatrix} \right\|$ is almost the same as the value of $\left\|\mathbf{P}_{\mathrm{d}}^{\mathrm{x}}\right\|$ (it is important to emphasize that although the values of $\left\|\mathbf{P}_{\mathrm{d}}^{\mathrm{x}}\right\|$ and $\left\| \begin{bmatrix} \mathbf{P}^{\mathrm{x}} & \mathbf{P}_{\mathrm{d}}^{\mathrm{x}} \end{bmatrix} \right\|$ presented in Table 1 for combinations 2 and 3 are the same, in reality the values of $\left\| \begin{bmatrix} \mathbf{P}^{\mathrm{x}} & \mathbf{P}_{\mathrm{d}}^{\mathrm{x}} \end{bmatrix} \right\|$ are slightly larger than $\left\|\mathbf{P}_{\mathrm{d}}^{\mathrm{x}}\right\|$, the difference does not appear due to truncation). As we can see in Table 2, the choice of $\mathbf{W} = \mathbf{G}^{\mathrm{x}^{\mathrm{T}}}$ doesn't give perfect control for the top and bottom compositions, but it reduces the sensitivity of the states in the middle of the column (about 0.4) to variations in the disturbances. This point is important to avoid the effects of non-linearities in the process.

Table 2. Values of the matrices $\mathbf{P}^{x}$ *and* $\mathbf{P}_{d}^{x}$ *for the four combinations.*

Combination 1				Combination 2				Combination 3			
$\mathbf{P}^{x}$		$\mathbf{P}_{d}^{x}$		$\mathbf{P}^{x}$		$\mathbf{P}_{d}^{x}$		$\mathbf{P}^{x}$		$\mathbf{P}_{d}^{x}$	
1.00	0	0.0	0	0.001	0.001	0.0	-0.032	0.007	-0.002	0.0	-0.032
1.49	0.00	0.0	0.00	0.001	0.001	0.0	-0.047	0.010	-0.003	0.0	-0.047
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
12.13	2.21	0.0	0.32	0.005	0.005	0.0	-0.134	0.081	-0.008	0.0	-0.133
12.50	2.72	0.0	0.40	0.005	0.005	0.0	-0.088	0.082	-0.005	0.0	-0.088
12.43	3.23	0.0	0.47	0.004	0.004	0.0	-0.028	0.081	-0.002	0.0	-0.028
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
6.51	4.93	0.0	0.72	0.001	0.001	0.0	0.351	0.036	0.020	0.0	0.351
4.85	5.02	0.0	0.74	0.000	0.000	0.0	0.413	0.024	0.024	0.0	0.413
4.82	6.63	0.0	0.73	-0.001	-0.001	0.0	0.354	0.021	0.035	0.0	0.354
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
0.01	1.41	0.0	0.00	-0.001	-0.001	0.0	-0.048	-0.003	0.009	0.0	-0.048
	1.00	0.0	0.00	-0.000	-0.000	0.0	-0.034	-0.002	0.007	0.0	-0.034

5. Conclusions

In this paper we showed that it is possible to control perfectly (having perfect disturbance rejection and minimizing the implementation error effects) any combination of the states if we have enough measurements available. Therefore, it is shown the importance of the use of the combination of states as primary variables. Although the choice of the top and bottom compositions of a distillation column is good to reject perfectly the disturbances, it fails in the rejection of the implementation error and also it doesn't give a good control of the states in the middle of the column. The choice of $\mathbf{W} = \mathbf{G}^{x^{T}}$ proved to be the best choice if the objective is to keep the states as close as possible to their desired (nominal) values. It rejects very well both disturbances and implementation errors, although it doesn't give perfect control of the top and bottom compositions.

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Acknowledgments

The financial support of The National Council for Scientific and Technological Development (CNPq/Brasil) and Coordenação de Aperfeiçoamento de Pessoal de Nível Superior (CAPES/Brasil) is gratefully acknowledged.