Control Design for Serial Processes

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June 30, 2003 Submitted to *Journal of Process Control*

Abstract

Conceptually, a multivariable controller uses the two basic principles of "*Feedforward*" action, based mainly on the model (for example the off-diagonal decoupling elements of the controllers), and *feedback* correction, based mainly on the measurements. The basic differences between feedback and feedforward control are well-known, and these differences also manifest themselves in the multivariable controller.

Feedforward control may improve the performace significantly, but is sensitive to uncertainty, especially at low frequencies. Feedback control is very effective at lower frequencies where high feedback gains are allowed.

In this paper we aim at obtaining insight into how a multivariable feedback controller works, with special attention to serial processes. Serial processes are important in the process industry, and the structure of this process makes it simple to classify the different elements of the multivariable controller.

An example of neutralization of an acid in a series of three tanks is used to illustrate some of the ideas.

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1 Introduction

Before designing and implementing a multivariable controller, there are some questions that are important to answer: What will the multivariable controller really attempt to do? Will a multivariable controller significantly improve the response as compared to a simpler scheme? What must the multivariable controller take into account to succeed? How accurate a model is needed?

One key issue with multivariable control is uncertainty. There is a fundamental difference between feedforward and feedback controllers with respect to their sensitivity to uncertainty. Feedforward control is sensitive to *static* uncertainty, while feedback is not. On the other hand, aggressively tuned feedback controllers are very sensitive to uncertainty in the *crossover frequency region*. Similar differences with respect to uncertainty can be found for multivariable controllers. Traditional single loop controllers are predominantly based on feedback, whereas model based multivariable controllers often combine feedback and feedforward control, and usually the component of feedforward action is significant (for example the off-diagonal "decoupling" elements of the controllers).

In this paper we discuss these issues for the important class of *serial processes*. A serial process consists of a series of one-way interacting units. The states in one unit influence the states in the downstream unit, but *not* the other way round. This is very common in the process industry, where the outlet flow of one process enters into the next. One example, which will be studied in Section 4, is neutralization performed in several tanks in series. Examples of processes that are not serial are processes with some kind of recycle of material or energy. Even for such processes, however, parts of the process may be modelled as a serial process, if the outlet variations of the last unit is dampened through other process units before it is recycled, so that no significant correlation can be found between the outlet variations and the variations in the disturbances to the first unit.

A multivariable controller often yields significant nominal improvements compared to local single-loop control. This is largely because of to the "feedforward" action, and with model error, the feedforward effect may in fact lead to worse performance. On the other hand, use of feedback from downstream measurements is much less dependent on the model, as use of high feedback gains at low frequencies removes the steady-state error. However, one must be careful about high feedback gains at higher frequencies due to potential stability problems, and it is at these higher frequencies one may have the largest benefit of the model-based "feedforward" action of the multivariable controller.

Buckley (1964) discusses control structure design for serial processes and distinguishes between material balance control (control of inventory or pressure by flow rate adjustments) and product quality control (control of quality parameters such as concentration).

Shinskey (1973) and McMillan (1982) present methods for design of pH neutralization processes. Mixing tanks are used to dampen disturbances, and they find that the total volume may be reduced by use of multiple stages with one control loop for each tank. Another advantage with multiple stages is that one may use successively smaller and smaller control valves, leading to a more precise manipulable variable in the last stage. McMillan and Shinskey both recommend different sized tanks to avoid equal resonance frequencies in the tanks, but this has later been questioned (Walsh, 1993; Faanes and Skogestad, 2000; Faanes and Skogestad, 2003).

A discussion on the open loop response of serial process is found in Marlin (1995, p. 156f). Morud and Skogestad (1996) note that the poles and zeros of the transfer function of a serial process are the poles and zeros of the transfer functions of the individual units.

Thus, the overall response may be predicted directly from the individual units, in contrast to e.g. processes with recyle. Many series connections of processing units are not really serial processes, as the response of each unit also depends on the downstream unit (for example if the outlet flow rate from a unit depends on the pressure in the subsequent unit) (Marlin, 1995), (Morud, 1995, Chapter 4), (Morud and Skogestad, 1995). Morud et al. denote the latter process structure *cascades*, whereas Marlin uses the terms *noninteracting* and *interacting* series, respectively, for the two structures.

The characteristics of serial processes can be utilized when analyzing multivariable controllers for such processes. The multivariable controller can be divided into three types of controller blocks: Local feedback, feedback from downstream units and "feedforward" from upstream units. Thus, depending on the location, the control input will be a sum of these three terms.

This division of the controller blocks has two purposes. First, it gives insight into the behavior of the control system. Second, it allows simple implementation. In some cases the multivariable controller can be implemented as combinations of conventional single loop controllers.

In Section 2 we develop the model structure for serial processes and discuss some of its properties. In Section 3 control of serial processes is discussed. One popular multivariable controller is MPC, and to be able to use theory for linear systems, we summarize in Appendix A how to express an unconstrained MPC combined with a state estimator on state space and transfer function form. This was not available for the controller we have used, so that a detailed description is given by Faanes (2003, Chapter 5). The ideas of the paper are illustrated through an example with pH neutralization in three stages (section 4). The paper is concluded by a short discussion (section 5) and the conclusions in section 6.

2 Model structure of serial processes



Figure 1: Serial process with exogenous variables u_i (manipulated) and d_i (disturbances) into unit *i*. The vector y_i represents the outflow of unit *i*, which continues into unit number i + 1.

In this section we look closer at serial processes and develop a general transfer function model. An example of a serial process is a process where mass and/or energy flows from one process unit to another, and there is no recycling of mass or energy. We define a serial process by the following (also see Figure 1):

A serial process can be divided into a series of subprocesses or units, where the states in each unit depend on the states in the unit itself (x_i) , the states in the upstream unit (x_{i-1}) , and the exogenous variables (u_i, d_i) to the unit.

The model for unit no. i can then be expressed as

$$\frac{d}{dt}x_i = f_i\left(x_i, x_{i-1}, u_i, d_i\right) \tag{1}$$

where x_i and x_{i-1} are the state vectors for unit *i* and unit *i* - 1 respectively, and the external input is divided into a vector of manipulated inputs, u_i , and disturbances, d_i . We further define the outputs from a unit as a function of the states and the external inputs for this unit

$$y_i = g_i(x_i) \tag{2}$$

It is easy to also inlude direct through t terms, i.e., define $y_i = g_i(x_i, x_{i-1}, u_i, d_i)$, but is makes the expressions below slightly more complex.

We linearize (1) and (2) around a working point, introduce $A_{i,j} = \partial f_i / \partial x_j$; j = i, i - 1, $B_i = \partial f_i / \partial u_i$, $C_i = \partial g_i / \partial x_i$, and $E_i = \partial f_i / \partial d_i$ and let the variables be the deviation from their working point. Applying Laplace transformation, and recursively inserting for variables from previous tank, we obtain:

$$y(s) = G(s)u(s) + G_d(s)d(s)$$
(3)

We have defined the total output vector, y(s), as all the outputs, u(s) as all the manipulable inputs, d(s) as all the disturbances. Defining

$$M_i = (sI - A_{i,i})^{-1} (4)$$

we get

$$G(s) = \begin{bmatrix} C_1 M_1 B_1 & 0 & 0 & \cdots & 0 \\ C_2 M_2 A_{2,1} M_1 B_1 & C_2 M_2 B_2 & 0 & \cdots & \vdots \\ \vdots & \vdots & \ddots & 0 \\ C_n M_n \prod_{r=1}^{n-1} [A_{n-r+1,n-r} M_{n-r}] B_1 & C_n M_n \prod_{r=1}^{n-2} [A_{n-r+1,n-r} M_{n-r}] B_2 & \cdots & \cdots & C_n M_n B_n \end{bmatrix}$$
$$= \begin{bmatrix} G_{1,1} & 0 & 0 & \cdots & 0 \\ G_{2,1} & G_{2,2} & 0 & \cdots & \vdots \\ \vdots & \vdots & \ddots & 0 \\ G_{n,1} & G_{n,2} & \cdots & \cdots & G_{n,n} \end{bmatrix}$$
(5)

and

$$G_{d}(s) = \begin{bmatrix} C_{1}M_{1}E_{1} & 0 & 0 & \cdots & 0\\ C_{2}M_{2}A_{2,1}M_{1}E_{1} & C_{2}M_{2}E_{2} & 0 & \cdots & \vdots\\ \vdots & \vdots & \ddots & 0\\ C_{n}M_{n}\prod_{r=1}^{n-1}[A_{n-r+1,n-r}M_{n-r}]E_{1} & C_{n}M_{n}\prod_{r=1}^{n-2}[A_{n-r+1,n-r}M_{n-r}]E_{2} & \cdots & C_{n}M_{n}E_{n} \end{bmatrix}$$
$$= \begin{bmatrix} G_{d,1,1} & 0 & 0 & \cdots & 0\\ G_{d,2,1} & G_{d,2,2} & 0 & \cdots & \vdots\\ \vdots & \vdots & \ddots & 0\\ G_{d,n,1} & G_{d,n,2} & \cdots & \cdots & G_{d,n,n} \end{bmatrix}$$
(6)

where n is the number of units. G and G_d are identical except in $G_d B_i$ is replaced by E_i (the disturbances to each unit are assumed independent).

We see that G(s) and $G_d(s)$ are both lower block triangular. From (5) and (6), we can deduce the following properties:

- The state vector of a process unit is not influenced by control inputs and disturbances to downstream units.
- The influence from a control input or a disturbance which enters an upstream unit, q, is dampened by the transfer function $C_i (sI A_{i,i})^{-1} \prod_{r=1}^{i-q} [A_{i-r+1,i-r} (sI A_{i-r,i-r})^{-1}]$ before it reaches the output of unit i.
- The open loop stability of the total process is given by the stability of each unit since the elements in G and G_d consists of products of M_i 's.
- G(s) and $G_d(s)$ are block diagonal at infinite frequency $(s \to \infty)$.

Note that the nominal model of unit i can be expressed as

$$y_i = G_{i,i}u_i + G_{d,i}y_{i-1} + G_{d,i}d_i$$
(7)

where $\tilde{G}_{d,i}$ is the transfer function from "disturbances" due to variations in the upstream unit, i-1 to output y_i :

$$\tilde{G}_{d,i}\left(s\right) \stackrel{\text{def}}{=} G_{i,i-1}G_{i-i,i-1}^{-1} \tag{8}$$

3 Control structures for serial processes

In the previous section we introduced the concept of serial processes and Equations (3)-(6) summarize the linearized model. If a full, multivariable controller is used to control this process, the characteristics of each blocks of this controller can be identified. If we for simplicity assume that the set-points are zero, and we want to control all the outputs, the control inputs are given by:

$$u(s) = K(s)y(s) \tag{9}$$

where K(s) is the controller.

We divide the controller K(s) into $n \times n$ blocks of the same size as the blocks in G(s):

$$K(s) = \begin{bmatrix} K_{11} & K_{12} & \dots & K_{1n} \\ K_{21} & K_{22} & \dots & K_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ K_{n1} & K_{n2} & \dots & K_{nn} \end{bmatrix}$$
(10)

These controller blocks can be divided into three groups:

- **Blocks on the diagonal** $(K_{i,i})$ These blocks use local control, where inputs to the unit are used to control outputs of the same unit.
- **Blocks above the diagonal** $(K_{i,j}, i < j)$ These blocks represents feedback from the outputs of downstream units. Intuitively, when the effective delay through the units is large, these blocks seem ineffective since the local feedback always will be quicker. There are, however, several cases when it may prove useful:
 - 1. We have no relevant control inputs downstream so local control is impossible.

- 2. The downstream actuators are slow, so that it actually is more efficient to manipulate the upstream control inputs.
- 3. There are not enough degrees of freedom in the downstream units.
- 4. The control inputs downstream are constrained, and insufficient to compensate for the disturbances.
- 5. The downstream actuators are expensive to use.

In the latter two cases the upstream manipulated variable can be used to (slowly) drive the downstream ones to zero or to some other ideal resting value. This is called input resetting and is normally used for systems where we have more control varables than outputs (e.g., (Skogestad and Postlethwaite, 1996, page 418)).

Blocks below the diagonal $(K_{i,j}, i > j)$ Through these blocks an output from an upstream unit directly affects the input in a downstream unit. Since upstream units act as disturbances to downstream units (see (7)), these controller blocks may be viewed as "*feed-forward*" elements.

In analyzing the controller it is useful to plot the gain of the controller elements as a function of frequency, see Figures 5, 7(a), 9(a), and 11(b) presented below. A key point is to find out whether there is integral action in the feedback part of the controller or not. Integral action requires high gain at low frequencies, but it is not always straight-forward to interpret the gain plot of the controller elements. For example, in Figure 7(a) all the elements have large gains at low frequencies. In such cases the steady-state effect is better illustrated by plotting the individual gains of the sensitivity function, $S(j\omega) = (I + L(j\omega))^{-1}$ where $L(j\omega) = G(j\omega) K(j\omega)$ is the loop transfer function. The usefulness of S is seen from the following expression

$$e = -Sy_r + SG_d d \tag{11}$$

where e is the control error $(y - y_r)$, y_r is the reference, d is the disturbance and G_d is the (open loop) transfer function matrix from the disturbance to the output. To have no steady-state offset in an output we need that all elements in the corresponding row of S to be small at low frequencies. Also note that system stability is determined by the poles of S(s).

3.1 Local control (diagonal control)

Local control is by far the most common control element,

Local control:
$$u_i = K_{i,i}(s)y_i$$
 (12)

With only local control and three units (n = 3), the loop transfer function becomes

$$L = \begin{bmatrix} G_{11} & 0 & 0 \\ G_{21} & G_{22} & 0 \\ G_{31} & G_{32} & G_{33} \end{bmatrix} \begin{bmatrix} K_{11} & 0 & 0 \\ 0 & K_{22} & 0 \\ 0 & 0 & K_{33} \end{bmatrix}$$
$$= \begin{bmatrix} G_{11}K_{11} & 0 & 0 \\ 0 & G_{22}K_{22} & 0 \\ 0 & 0 & G_{33}K_{33} \end{bmatrix}$$
(13)

From this it follows that the stability of the closed-loop system is determined only by the blocks on the diagonal. That is, we have closed-loop stability if and only if each of the individual loops $(I + G_{i,i}K_{i,i})^{-1}$ are stable.

3.2 Pure feedforward from upstream units

The use of measurements in upstream units in the control of a unit is denoted feedforward control:

Feedforward
$$(i > j)$$
: $u_i = K_{i,j}^{\text{FF}}(s)y_j$ (14)

With "pure" feedforward control (only feedforward elements), the controller does not influence stability.

From (7) and (8) we find that perfect nominal control is obtained by selecting

$$K_{i,i-1}^{\rm FF} = -G_{i,i}^{-1}\tilde{G}_{d,i}$$
(15)

$$K_{i,i-2}^{\rm FF} = \cdots = K_{i,1}^{\rm FF} = 0$$
 (16)

The reason for the zero in (16) is that the disturbance is already eliminated by (15). If (15) cannot be realised, for example if it is not casual, (15) must be modified:

$$K_{i,i-1}^{\rm FF} = -G_{i,i,-}^{-1} \tilde{G}_{d,i} \tag{17}$$

where subcript minus indicates that negative delays and other non-causual elements of the (total) controller has been removed (this is a simplification of the \mathcal{H}_2 optimal feedforward controller given by Lewin and Scali (1988) and Scali *et al.* (1989)).

Remark: It is not necessary to make $G_{i,i}^{-1}$ causal itself. For example if $G_{i,i}$ has a delay of 10 s and $\tilde{G}_{d,i}$ a delay of 6 s the delay of the "ideal" feedforward controller would have been -4 s, which is not implementable. (17) states that the controller delay shall be trucated to 0, which means that the effect of the controller of the controller occurs 4 s too late. But, requiring $G_{i,i}^{-1}$ to be causal would have given a 6 s delay in the controller (zero in delay in $G_{i,i}$ plus 6 s in $\tilde{G}_{d,i}$), and the effect of the feedforward controller would have occured 4 + 6 = 10 s too late.

When (15) cannot be realised, feedforward from units i - 2, i - 3, ... can be useful. For example, if it is causal, the following feedforward controller from unit i - 2 eliminates the control error that "rests" after $K_{i,i-1}^{\text{FF}}$:

$$K_{i,i-2}^{\rm FF} = -G_{i,i,-}^{-1} \left(I - G_{i,i}G_{i,i,-}^{-1} \right) \tilde{G}_{d,i} \left(I - G_{i-1,i-1}G_{i-1,i-1,-}^{-1} \right) \tilde{G}_{d,i-1}$$
(18)

See Appendix B for a derivation of (18).

Feedforward control is generally sensitive to uncertainty, and we will now consider its effect. The nominal model is given by (7), and the actual model (with uncertainty) is

$$y'_{i} = G'_{i,i}u_{i} + \tilde{G}'_{d,i}y'_{i-1} + G'_{d,i}d_{i}$$
(19)

A pure feedforward controller from upsteam units then yields the following actual control error:

$$e'_{i} \stackrel{\text{def}}{=} y'_{i} - y_{r_{i}} = \tilde{G}'_{d,i}y'_{i-1} + \sum_{j=1}^{i-1} G'_{i,i}K^{\text{FF}}_{i,i-j}y'_{i-j} + G'_{d,i}d_{i} - y_{r_{i}}$$
(20)

With "ideal" feedforward control based on the nominal model, as given by (15) and (16), the actual control error becomes

$$e'_{i} = \left(\tilde{G}'_{d,i} - G'_{i,i}G^{-1}_{i,i}\tilde{G}_{d,i}\right)y'_{i-1} + G'_{d,i}d_{i} - y_{r_{i}}$$

$$= \underbrace{\left(I - G'_{i,i}G^{-1}_{i,i}\tilde{G}_{d,i}\tilde{G}'^{\dagger}_{d,i}\right)}_{E_{d,i}}\tilde{G}'_{d,i}y'_{i-1} + G'_{d,i}d_{i} - y_{r_{i}}$$
(21)

where \dagger denotes generalized inverse (Zhou *et al.*, 1996), and $E_{d,i}$ is a relative model error in $G_{i,i}\tilde{G}_{d,i}^{\dagger}$. In particular, for scalar blocks

$$E_{d,i} = 1 - \frac{G'_{i,i}/\tilde{G}'_{d,i}}{G_{i,i}/\tilde{G}_{d,i}}$$
(22)

Thus model errors at any frequency, directly influences the actual control error. Upon comparing the response with control in (21) with the response without control ($u_i = 0$ in (19)) we see that "feedforward" (decoupling) control has a positive (dampening) effect on disturbances from upstream units at frequencies ω where

$$\left\|E_{d,i}\left(\omega\right)\right\| < 1\tag{23}$$

or in words, as long as the relative error in $G_{i,i}\tilde{G}_{d,i}^{\dagger}$ is less than 1 in magnitude. Here, an appropriate norm dependent on the definition of performance is used.

External disturbances entering directly into the process at unit *i*, d_i , are (of course) not dampened by feedforward control from upstream units, but if d_i is measured, then separate feedforward controllers may be designed for d_i . Feedforward control from the reference, y_{r_i} , is also necessary to avoid control error if $y_{r_i} \neq 0$ and no feedback is applied.

3.3 Lower block triangular controller

A lower (block) triangular controller will result if we combine local feedback and feedforward from upstream units,

Local control
$$(i = j)$$
: $u_i = K_{i,i}(s)y_i$
Feedforward $(i > j)$: $u_i = K_{i,j}^{\text{FF}}(s)y_j$

The loop transfer function now becomes (n = 3):

$$L = \begin{bmatrix} G_{11} & 0 & 0 \\ G_{21} & G_{22} & 0 \\ G_{31} & G_{32} & G_{33} \end{bmatrix} \begin{bmatrix} K_{11} & 0 & 0 \\ K_{21}^{\text{FF}} & K_{22} & 0 \\ K_{31}^{\text{FF}} & K_{32}^{\text{FF}} & K_{33} \end{bmatrix}$$
$$= \begin{bmatrix} G_{11}K_{11} & 0 & 0 \\ G_{21}K_{11} + G_{22}K_{21}^{\text{FF}} & G_{22}K_{22} & 0 \\ G_{31}K_{11} + G_{32}K_{21}^{\text{FF}} + G_{33}K_{31}^{\text{FF}} & G_{32}K_{22} + G_{33}K_{32}^{\text{FF}} & G_{33}K_{33} \end{bmatrix}$$
(24)

The diagonal elements are feedback elements, where most of the control benefits are achieved simply by using sufficiently high gains, and an accurate process model is not needed. The main problem is that too high gain may give closed-loop instability.

As for the local feedback (diagonal) control structure the stability of the closed-loop system is determined only by the blocks on the diagonal, that is we have closed-loop stability if and only if each of the local loops $(I + G_{i,i}K_{i,i})^{-1}$ are stable.

Note that we also obtain this control structure if an inverse-based (decoupling) design method ($K(s) = k(s) G^{-1}(s)$) is used. An example of an inverse based controller is IMC decoupling (Morari and Zafiriou, 1989), $K_{IMC} = W_1 G^{-1} W_2$ where W_1 and W_2 are (block) diagonal matrices (with blocks corresponding to the blocks in G). For this controller we obtain the following diagonal and sub-diagonal blocks:

$$K_{\text{IMC},i,i} = W_{1_{i,i}} G_{i,i}^{-1} W_{2_{i,i}}$$
(25)

$$K_{\text{IMC},i,i-1}^{\text{F}F} = -W_{1,i}G_{i,i}^{-1}G_{i,i-1}G_{i-1,i-1}^{-1}W_{2_{i-1,i-1}}$$
(26)

where $W_{j_{i,i}}$ denotes block (i, i) of weight matrix W_j (this is the integrator). (25) and (26) can be verified by calculating that $GG^{-1} = I$. Since the stability is determined by the diagonal blocks, and these are the scaled inverse of the blocks of G, the weights can be selected independently for each unit, e.g. using (Rivera *et al.*, 1986) (for scalar blocks). If G is not invertible due to right half plane zeros and delays, these are essentially factored out before the inversion (for details, see (Morari and Zafiriou, 1989)).

Using (8), we note that the sub-diagonal part of the IMC controller, (26), is identical to the ideal feedforward controller (15), except for the weights. Integral action in the feedback part of the controller ($K_{IMC,i,i}$) requires an integrator in either $W_{1_{i,i}}$ or $W_{2_{i,i}}$. For example, we may choose $W_{2_{i,i}} = \frac{1}{\tau_{\text{CL}}s}I$ where τ_{CL} is the desired closed loop time constant, (Rivera *et al.*, 1986). Thus we see from (26) that also the "feedforward" gain will be amplified at low frequencies.

Let us now consider the effect of model uncertainty. The nominal model is given by (7) and the actual model by (19). A lower triangular controller yields the following actual control error:

$$e'_{i} \stackrel{\text{def}}{=} y'_{i} - y_{r_{i}} = S'_{i} \left(\tilde{G}'_{d,i} y'_{i-1} + \sum_{j=1}^{i-1} G'_{i,i} K_{i,i-j} y'_{i-j} + G'_{d,i} d_{i} - y_{r_{i}} \right)$$
(27)

where (Skogestad and Postlethwaite, 1996)

$$S'_{i} = \left(1 + G'_{i,i}K_{i,i}\right)^{-1} = S_{i}\left(1 + E_{i}T_{i}\right)^{-1}$$
(28)

where S_i and T_i are nominal sensitivity and complementary sensitivity functions, respectively, and E_i relative error in G (note that we in Section 2 let E_i denote something else).

Upon comparing the closed-loop response in (27) with the open loop response in (19) we see the following:

- 1. Effective local feedback control ($||S_i(j\omega)|| \ll 1$) dampens disturbances from the preceding tank (y_{i-1}) , external disturbances entering the process at unit *i*, and also the effect of the model error (E_i) and errors in the feedforward control.
- 2. For frequencies where the feedback control is not effective, i.e., $||S_i(j\omega)|| \ge 1$, the results from Section 3.2, (15)-(23) can be applied except that (18) must be modified due to the feedback control in unit i 1:

$$K_{i,i-2}^{\rm FF} = -G_{i,i}^{-1} \left(I - G_{i,i}G_{i,i,-}^{-1} \right) \tilde{G}_{d,i} \left(1 - G_{i-1,i-1}K_{i-1,i-1} \right)^{-1} \left(I - G_{i-1,i-1}G_{i-1,i-1,-}^{-1} \right) \tilde{G}_{d,i-1}$$
(29)

External disturbances entering the process at unit i, d_i , are not dampened by the feedforward control from upstream units, but are handled by the feedback control. If this is not sufficient, and provided d_i is measured, separate feedforward controllers may be designed for d_i .

For serial processes with a lower block triangular controller it is particularly simple to identify feedforward and feedback controller elements, but similar differences between the elements occur for most multivariable controllers. Such insights are important, e.g. when evaluating how the controller is affected by model error.

A more general analysis of feedforward control under the precence of uncertainty is given in (Faanes and Skogestad, 2002).

3.4 Full controller

With a full controller, as in (10), and three units (n = 3), the loop transfer function becomes

$$L = G(s)K(s) = \begin{bmatrix} G_{11}K_{11} & G_{11}K_{12} \\ G_{21}K_{11} + G_{22}K_{21} & G_{21}K_{12} + G_{22}K_{22} & \cdots \\ G_{31}K_{11} + G_{32}K_{21} + G_{33}K_{31} & G_{31}K_{12} + G_{32}K_{22} + G_{33}K_{32} \\ & & G_{11}K_{13} \\ & & \ddots & G_{21}K_{13} + G_{22}K_{23} \\ & & & G_{31}K_{13} + G_{32}K_{23} + G_{33}K_{33} \end{bmatrix}$$
(30)

In this case the stability of the closed-loop system is affected by all elements in the controller K (and in G).

As illustrated in the case study in Section 4, even in this case the controller block below the diagonal may be similar to feedforward control.

3.5 Final control only in last unit (input resetting)

In many serial processes, the output from the last unit is by far the most important for the overall plant economics, and the outputs in upstream units are mainly controlled to improve control performance in the final unit. The extra degrees of freedom are used for local disturbance rejection, but are otherwise typically reset to some ideal resting value by adjusting setpoints in upstream units.





We may then use the following control elements:

Local control
$$(i = j)$$
 $u_i = K_{i,i}(s) [y_{r_i} - y_i]$
Feedforward $(i > j)$ $u_i = K_{i,j}^{\text{FF}}(s)y_j$
Input resetting $(j = i + 1)$ $y_{r_i} = K_{i,j}^{\text{IR}}(s) [u_{r_j} - u_j]$

as illustrated in Figure 2. Note that we here have restricted input resetting to operate between neighboring units, but this is not strictly required. With local control in the three units, feed-forward from unit 1 to unit 2 and 3 and from unit 2 to unit 3, and input resetting from unit 3 to unit 2 and from unit 2 to unit 1, the resulting full multivariable controller is:

$$K(s) = \begin{bmatrix} -K_{11} \left(1 + K_{12}^{\text{IR}} K_{21}^{\text{FF}} - K_{12}^{\text{IR}} K_{22} K_{23}^{\text{IR}} K_{31}^{\text{FF}}\right) & K_{11} K_{12}^{\text{IR}} K_{22} \left(1 + K_{23}^{\text{IR}} K_{32}^{\text{FF}}\right) \\ K_{21}^{\text{FF}} - K_{22} K_{23}^{\text{IR}} K_{31}^{\text{FF}} & -K_{22} \left(1 + K_{23}^{\text{IR}} K_{32}^{\text{FF}}\right) \\ K_{31}^{\text{FF}} & K_{32}^{\text{FF}} & -K_{22} \left(1 + K_{23}^{\text{IR}} K_{32}^{\text{FF}}\right) \\ \cdots & K_{32}^{\text{FF}} \\ -K_{11} K_{12}^{\text{IR}} K_{22} K_{23}^{\text{IR}} K_{33} \\ \cdots & K_{22} K_{23}^{\text{IR}} K_{33} \\ -K_{33} \end{bmatrix} \begin{bmatrix} K_{11} K_{12}^{\text{IR}} - K_{11} K_{12}^{\text{IR}} K_{22} K_{23}^{\text{IR}} & K_{11} K_{12}^{\text{IR}} K_{22} K_{23}^{\text{IR}} K_{33} \\ 0 & K_{22} K_{23}^{\text{IR}} & -K_{22} K_{23}^{\text{IR}} K_{33} \\ 0 & 0 & K_{33} \end{bmatrix}$$
(32)

with $u(s) = K(s)y(s) + K_r(s)[u_{r_2}, u_{r_3}, y_{r_3}]^T$, where y_{r_3} is the set point for the controlled output in unit 3, whereas u_{r_2} and u_{r_3} are the ideal resting values for the inputs in tank 2 and 3.

The final controller in (31) and (32) may seem very complicated, but it can usually be tuned in a rather simple cascaded manner. The feedforward elements are normally the fastest acting and should normally be designed first. The local feedback controllers can be tuned almost independently. Finally, the slow input resetting is added, which will not affect closed-loop stability if it is sufficiently slow.

4 Case study: pH neutralization

4.1 Introduction

Neutralization of strong acids or bases is often performed in several steps (tanks). The reason for this is mainly that with a single tank the pH control is not quick enough to compensate for disturbances (Skogestad, 1996). In (McMillan, 1984), an analogy from golf is used: the difficulty of controlling the pH in one tank is compared to getting a hole in one. Using several tanks, and smaller valves for addition of reagent for each tank, is similar to reaching the hole with a series of shorter and shorter strokes. This is further discussed in (Faanes and Skogestad, 2000; Faanes and Skogestad, 2003).

In the present example we want to compare different control structures for neutralization of a strong acid in three tanks (see Figure 3). This is clearly a serial process. The aim of the control is to keep the outlet pH from the last tank constant despite changes in inlet pH and inlet flow rate. For each tank the pH can be measured, and the reagent (here base) can be added. Figure 3 shows the process with only local control in each tank (*K* diagonal).

4.2 Model

To study this process we use the models derived in Faanes and Skogestad (2003). In each tank we consider the excess H^+ concentration, defined as $c = c_{H^+} - c_{OH^-}$. This gives a bilinear model which is linearized around a steady-state working point, so that the methods from linear control theory can be used. We get two states in each process unit (tank), namely



Figure 3: Neutralization of an acid in three tanks in series with local control in each tank. Data: Outlet requirement: $pH = 7 \pm 1$, set-points tank 2 and 3: pH = 1.65 and pH = 3.8. Inlet acid flow pH = -1 (= 10 mol / l) and flow rate 0.005 m³ / s. Reactant (base): pH = 15 (= 10 mol / l), nominal flow: 0.005 m³ / s. $V_1 = V_2 = V_3 = 13.6 / \text{m}^3$.

the concentration, c, and the level. The disturbances (feed changes mainly) enter in tank 1. We here assume that there is a delay of 5 s for the effect of a change in inlet acid or base flow rate or inlet acid concentration to reach the outflow of the tank, e.g. due to incomplete mixing, and a further delay of 5 s until the change can be measured. In the discrete linear state space model these transportation delays are represented as extra states (poles in origo). We assume no further delay in the pipes between the tanks. The levels are assumed to be controlled by the outflows using a P controller such that the time constant for the level is about 1/10 of the residence time (q = 0.01 ($V - V_s$), where V_s is the volume set-point).

The volumes of the tanks are chosen to 13.6 m^3 , which are the smallest possible volumes according to the discussion in Skogestad (1996). The concentrations are scaled so that a variation of $\pm 1 pH$ corresponds to a scaled value of ± 1 . The control inputs and the disturbances are also scaled appropriately. The linear model is used for multivariable controller design, while the simulations are performed on the nonlinear model.

4.3 Model uncertainty

The model presented in the previous section was the *nominal* model, which will be used in the controller design. If the model gives an excact representation of the actual process, we say it is *perfect*. Due to simplifications in the modelling or process variations, there is often a discrepancy between the model and the actual process. Often the model is idealized, i.e., simplified, to ease the modelling work, the identification of parameters, and the controller design.

In this example we use linearized models in the MPC design. In the design of (SISO) feedforward controllers a further simplicication is that outlet flow variations are neglected. This gives a steady-state model error, but dynamically the error is small due to slow level control. What we here consider as the "actual plant", is the full nonlinear model, possibly with the following errors:

• Offset of 0.2 (in scaled value) in control input u_3 (last tank).

• pH measurement error of -1 in second tank.

4.4 Local PID-control (diagonal control)

The conventional way of controlling this process is to use local PID-control of the pH in each tank. Starting from the tunings obtained with the method of Ziegler and Nichols (1942), and employing some manual fine tuning (by trial and error), we obtained

$$K_{11} = -0.515 \frac{1+20s}{20s} \frac{1+4.8s}{1+0.48s}$$
(33)

$$K_{22} = -0.242 \frac{1+20s}{20s} \frac{1+12s}{1+1.2s}$$
(34)

$$K_{33} = -0.208 \frac{1+20s}{20s} \frac{1+14s}{1+1.4s}$$
(35)

Figure 4(a) shows the pH-response in each tank when the acid concentration in the inflow is decreased from $10 \mod / 1$ to $5 \mod / 1$. As expected (Skogestad, 1996), this control system is barely able to give acceptable control, $pH = 7 \pm 1$ in last tank. However, the nominal response can be significantly improved with feedforward or multivariable control as shown in the following.

4.5 Feedforward control (control elements below the diagonal)

We now want to study the use of feedforward control from upstream units. As before, we let the pH in the first tank be controlled with local PID control (the same tuning as before), since we do not measure inlet disturbances to tank 1, and feedback is therefore the only possibility. We let the pH in the second and third tanks be controlled with feedforward control only, namely with feedforward from y_1 to u_2 and from y_2 to u_3 . With "ideal" feedforward control based on the nominal model we then get

$$K_{21}^{\rm FF} = -G_{22,-}^{-1}\tilde{G}_{d,2} \tag{36}$$

$$K_{32}^{\rm FF} = -G_{33,-}^{-1} \hat{G}_{d,3} \tag{37}$$

where $\tilde{G}_{d,2}$ and $\tilde{G}_{d,3}$ are given by (8) and subscript minus indicates that the net delay is increased to obtain a causal controller with zero or positive delay in the controller. The two feedforward controllers will react 5 s too late due to the measurement delays in y_1 and y_2 , and thereby introduce a transient output error. To avoid this, the last feedforward controller, K_{31}^{FF} , from y_1 to u_3 , can be used to eliminate this error by choosing $K_{3,1}$ from (29):

$$K_{31}^{\rm FF} = -G_{33,-}^{-1} \left(1 - G_{33}G_{33,-}^{-1} \right) \left(1 - G_{22}G_{22,-}^{-1} \right) \tilde{G}_{d,2}\tilde{G}_{d,3}$$
(38)

Figure 4(b) shows a simulation on the same model as used for the feedforward controller design, and we can see that perfect control is achived in tank 3 (solid line). However, when applied to a more realistic nonlinear model (incorporating flow rate changes), the feedforward controller fails (dotted lines).

4.6 Combined local PID and feedforward control (lower block triangular control)

We now combine local PID-control in all the tanks, (33)-(35) with feedforward control of tanks 2 and 3 (controllers K_{21}^{FF} , K_{31}^{FF} and K_{32}^{FF}). In K_{31}^{FF} it is now necessary to take into account the



(a) Local feedback control in all three tanks: The PID controllers must be aggressively tuned to keep the pH in the last tank within 7 ± 1 .

(b) Feedback control in tank 1 only, and feedforward control of tanks 2 and 3: With a perfect model (i.e. simulation on idealistic model) the disturbance is cancelled (solid line). With model error (i.e., simulation on a "realistic" nonlinear model), the response is very poor and drifts away (dotted line). u is only given for the nominal case.



(c) Local feedback control in all three tanks combined with feedforward control of tanks 2 and 3: Even with model error, the response in the outlet pH is good (solid line).

Figure 4: Simple control structures applied to the neutralization process in Figure 3 (tank 1 (dash-dotted), tank 2 (dashed) and tank 3 (solid)). Disturbance in inlet concentration occurs at t = 10 s.

feedback loop of tank 2 and use Equation (29):

$$K_{31}^{\rm FF} = -\frac{1}{1 - G_{22}K_{22}}G_{33,-}^{-1} \left(1 - G_{33}G_{33,-}^{-1}\right) \left(1 - G_{22}G_{22,-}^{-1}\right) \tilde{G}_{d,2}\tilde{G}_{d,3}$$
(39)

where K_{22} is the PID controller of tank 2.

Again, with perfect model (i.e. simulated on the simplified model with constant flow rates) the effect of the disturbance is eliminated (same result as in Figure 4(b)). Simulation on the more realistic model reveals an improvement compared to the pure feedback and pure feedforward structures, as expected. The feedforward controllers reduce the transient errors, whereas the PID controllers remove the steady-state errors, as illustrated in Figure 4(c).

In Figure 5 the controller gains are plotted (lower left corner). The integral actions are recognized from the high gains at low frequencies in the diagonal elements. The sub-diagonal control elements are constant, whereas $K_{31}^{\rm FF}$ only has an effect at high frequencies. This is where $K_{3,2}^{\rm FF}$ is no longer effective (error in delay, $\Delta\theta = 5 \,\mathrm{s}$ gives feedforward control error control for frequencies above $1/\Delta\theta = 0.2 \,\mathrm{rad}/\mathrm{s}$, see (Faanes and Skogestad, 2002)).

Note that with a larger model error, the positive effect of the feedforward controller may be reduced, and the feedforward action may even amplify the disturbances.



Figure 5: The controller gains of the lower block-diagonal control structure resulting from combination of feedback (PID) and feedforward control (Section 4.6)

4.7 Multivariable control

Original MPC control (full multivariable controller)

Figure 6(a) shows the response with a 3×3 MPC controller ((Muske and Rawlings, 1993); see also Appendix A). To obtain the current state at each time step for the controller, a state estimator is used. The estimated states in this "original" MPC-controller also includes the two (unmeasured) disturbances: Inlet flow rate and inlet excess concentration, modelled as integrated white noise (we will discuss this choice later). The controller design is based on a discretized model, whereas in the simulation only the controller is discrete. Even if this is a feedback controller, we see that the disturbance response is similar to that of combined local feedback and feedforward control, and the main reason for the large improvement compared to the local feedback case (Figure 4(a)) is in fact the "feedforward" effect. From the lower

plots in Figure 4 and Figure 6(a) we can see that the control input in tanks 2 and 3 acts both earlier and with a steeper slope for MPC control than for local control. Note that with MPC the control inputs for tanks 2 and 3 react before the disturbance can be measured in the two tanks. The MPC also has a higher order controller, which may explain why it reacts even faster than the combined feedback/feedforward controller (Figure 4(c)).

"Feedforward" part of MPC-controller

To study the "feedforward" effect separately, we design a MPC-controller that uses the pH measurement in the first tank only, but adjust the reactant flow rates to all three tanks as shown in Figure 6(b). The response for the nominal case is similar to the simulation with the full MPC-controller shown in Figure 6(a). If, however, a model error is introduced, e.g. by simulation on the nonlinear model instead, a steady-state error occurs for outlet pH. The reason for this is the lack of feedback control in the last two tanks.



(a) Full multivariable control: A large improvement in nominal performance is possible with a 3×3 MPC-controller compared to pure local feedback.



Figure 6: Full (3 × 3) and reduced (3 × 1) MPC (Disturbance in inlet concentration occurs at t = 10 s.)

The individual gains of the 3×3 MPC-controller are shown as a function of frequency in Figure 7(a) (solid lines). The diagonal control elements are the local controllers in each tank, whereas the elements below the diagonal represent the "feedforward" elements. From these plots we get an idea of how the multivariable controller works. For example, we see that the control input to tank 1 (row 1) is primarily determined by local feedback, while in tanks 2 and 3 (rows 2 and 3) it seems that "feedforward" from previous tank is more important for the control input. In tanks 2 and 3 the control actions are smaller, which is also confirmed in the simulation (Figure 6(a)). The local feedback control elements on the diagonal compare well with the PID controllers (dashed lines), except that the gain is reduced for tanks 2 and 3, but this depends on the tuning of the MPC. At high frequencies the "feedforward" elements are similar to the manually designed feedforward controllers.



(a) Gain of the control elements of the 3×3 MPC (solid). Also shown: Local PID controllers and manually designed feedforward elements (dashed).

(b) The gains of the sensitivity function $(|S(j\omega)|)$ with the 3 × 3 MPC: Steady-state offset can be expected since some of the elements related to control errors e_2 and e_3 have high gain at low frequencies.

Figure 7: The original multivariable MPC controller: Frequency domain analysis.

As discussed in Section 3, it is not straight-forward to interpret the steady state behaviour from the gain plots of the controller elements when all the elements have large gains at low frequencies as in Figure 7(a). In Figure 7(b) we therefore show the individual gains of the sensitivity function, $S = (I + GK)^{-1}$. To have no steady-state offset in an output we need that all elements in the corresponding row of S to be small at low frequencies. From Figure 7(b) we then see that we do have integral action for output 1, but not for outputs 2 and 3. We should therefore expect steady-state offset in tank 3. However, the simulations in Figure 6(a) show no offset. The reason is that the integral effect in the first tank removes the concentration effect, and the "feedforward" control gives the correct compensation for the flow rate disturbance. However, if some unmodelled disturbance or model error is introduced (e.g. a constant offset in u_3 or a measurement error in tank 2), then we do indeed get steady-state offset. This is shown in Figure 8. The local PID controllers give no such steady-state offset.

Modified MPC-controller with integral action

In the "original" estimator used above we only estimated the inlet disturbances. We now redesign the controller by estimating *one disturbance in each tank*: The concentration disturbance to the first tank and disturbances in the manipulable variables in tanks 2 and 3 (u_2 and u_3). The resulting controller gains are shown in Figure 9(a). With this design the gain in $|S(j\omega)|$ is low at low frequencies for all tanks (Figure 9(b)), and the simulations in this case give no steady-state offset (Figure 10). This agrees with the result from (Faanes, 2003, Chapt. 5) that the number of disturbance estimates in the controller must equal the number of measurements.

This illustrates one of the problems of the "feedforward" control block, namely its sensitivity to static uncertainty. Simulations using the perfect model may lead the designer to believe



(a) Unmodelled disturbance: Control input u_3 has got an offset of 0.2 (at time 0) compared to the model, and the steady-state pH is 7.8 in stead of 7 in last tank (disturbance in inlet concentration occurs at t = 10 s).

(b) Measurement error: At time 10 s a pH measurement error of -1 is introduced in tank 2, and the steady-state pH is 5.9 instead of 7 in the last tank.





(a) Gain of the control elements of the modified 3×3 MPC (solid). Also shown: Local PID controllers and manually designed feedforward elements (dashed).



Figure 9: Modified 3×3 MPC: Frequency domain analysis.



(a) Unmodelled disturbance: Control input u_3 has got an offset of 0.2 (at time 0) compared to the model, and with the modified MPC we get no steady-state offset. (disturbance in inlet concentration occurs at t = 10 s). Step disturbance at time 10 s.



Figure 10: Modified 3×3 MPC with integral action: Closed loop simulations with model errors.

that there is integral effect in the controller even if it is not.

4.8 MPC with input resetting

In the simulations above we gave set points for the pH in each tank. Actually we are only interested in the pH in the last tank, so that giving set points for the other two is not necessary. Since we have three control inputs, this leaves two extra degrees of freedom as described in section 3.5, which may be used for input resetting. The MPC controller is easily modified to accomodate this. Figure 11 illustrates how this works after a unit step in the disturbance: At steady-state all the required change in base addition is done in the first tank. Since we do not measure the actual base addition, offsets in the control input are not compensated for.

4.9 Conclusion case study

The case study shows a large improvement that is obtained by the introduction of a multivariable controller instead of single loop control (Figure 4(a)). The improvement is caused by "feedforward" effects (Figure 4(c)), and with model errors, the "feedforward" may in fact lead to worse performance.

Integral action or strong gain in the local controllers at low frequencies is required, even if the "feedforward" effect itself nominally give no steady-state. Feedback to upstream tanks may be used to bring the inputs to their ideal resting positions. The example indicates that it is possible to get a good performance with careful use of a multivariable controller or a combination of local control, "feedforward" from tank 1 and 2 and possibly input resetting.



(a) u_2 and u_3 are brought back to 0 (but it takes slightly more than 250 s). Step disturbance at time 10 s.

(b) Control element gains: At steady-state the dominant control element is (1,3): u_1 is used to control y_3 at low frequencies.



5 Discussion

There are several ways to avoid steady-state offsets with MPC controllers. The most common method is to estimate the bias in the outputs, i.e. the difference between the predicted and the measured outputs, and compensate for this bias. However, performance is often improved by estimating *input* biases, or disturbances (Muske and Rawlings, 1993; Lee *et al.*, 1994; Lundström *et al.*, 1995). In this paper we have followed this appoach. We ended up with estimating the concentration disturbance into first tank and input biases for tanks 2 and 3 (three input biases gives similar results). Our controller handles well both input disturbances (see Figure 6(a)) and output disturbances or measurement errors (see Figure 10(b)).

We have also tried to estimate output biases, but this gave a very slow settling in response to inlet disturbances. The reason is the long time constants in our process, which give the output bias estimates a ramp form (Lundström *et al.*, 1995). The controller then faces a problem similar to following a ramp trajectory.

In (Faanes and Skogestad, 2000) we found that the minimum volume in each tank is limited by the delays in each tank. In the present paper we found that with a multivariable controller for simultaneous control of all three tanks, these limitations are no longer valid provided a sufficiently accurate process model is used. The reason for this is that the multivariable controller does not have to wait for the measurement in last tank before it takes action (due to the "feedforward" effect). To be able to achieve a nominally perfect "feedforward" control effect, the delay from at least one control input to the output must be shorter or equal to the delay from a measurement in the disturbance to the output. The effect of model uncartainty on the feedforward control improvements must be evaluated for the process. If there is an improvement, one may design smaller tanks compared to the sizes given in (Faanes and Skogestad, 2000; Faanes and Skogestad, 2003), or reduce the instrumentation.

6 Conclusions

An example of neutralization of a strong acid with base in a series of three tanks is used to illustrate some of the ideas in the paper. This is obviously a serial process. The example illustrates that a multivariable controller yields significant nominal improvements compared to single loop PID control (compare Figure 6(a) with Figure 4(a)). This is mainly due to "feed-forward" elements (see Figure 4(c)). Due to imperfections in the process model, including unmodelled disturbances, an efficient feedback effect must also be included. To obtain this one must:

- include measurements late in the process.
- include integral action if offset free steady-state is important. For MPC control, the use of input error estimates is one efficient method, which requires that the disturbance vector is chosen with some care.

Testing of the controller on a too idealistic process model may give the impression that the feedback is better than it actually is. Simulations with the multivariable controller active must include all possible disturbances, model offsets (for example one may apply the controller on a more realistic (nonlinear) process model) and also offsets in the measurement signals.

Assuming no active constraints, a linear analysis may be used to analyze the controller. The frequency dependent gain in each channel may give insight into how the controller utilize each measurement and the magnitude of the control actions for each input. The steady-state behaviour can be seen from the low frequency gains. But often more than one channel in a row have high gain at low frequencies, for example when inversed based methods like IMC is used, and then it is difficult to interpret the result. It is then better to consider the elements of the sensitivity function matrix. An offset-free steady-state control for a specific output requires that all the elements in the corresponding row have low gain at low frequencies.

When designing the controller one must also consider which of the outputs that is really important. If the number of inputs exceed the number of (important) outputs, one may either give set-points to other (less important) outputs, or one may let the controller bring some of the inputs back to ideal resting positions.

In this study we used multivariable MPC, but very similar results have also been found for a multivariable \mathcal{H}_{∞} -controller (Faanes and Skogestad, 1999).

7 Acknowledgements

Financial support from the Research Council of Norway (NFR) and the first author's previous employer Norsk Hydro ASA is gratefully acknowledged.

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Appendix A State space MPC used in case study

Here we briefly describe the MPC controller of Muske and Rawlings (1993) under the assumption that the constraints are not active. For details we refer to (Faanes, 2003, chapt. 5).

The MPC controller uses an estimate of the current states of the process and a state space model to predict future responses to control input movements. By letting the control input change each time step over a certain horizon, and thereafter held constant, the optimal sequence of control inputs is calculated. The criterion for the optimization is

$$\min_{u_k^N} \sum_{j=0}^{\infty} \left(y_{k+j}^T Q y_{k+j} + u_{k+j}^T R u_{k+j} + \Delta u_{k+j}^T S \Delta u_{k+j} \right)$$
(40)

where u_k^N is the vector of N future control inputs, the first at sample number k, y_k is the output vector at time k, u_k is the control input at time k, Δu_k is the change in u_k since last time step and Q, R and S are weight matrices. Note that in the crierion we assume that the set-point for the output, $y_r = 0$. Non-zero set-points are handled by a steady-state solver. Only the first control input is applied, since at next time step the whole sequence is recalculated, starting from the states actually obtained at that moment.

Without constraints the MPC can be represented as state feedback control, i.e. the control input u_k at time step no. k can be expressed by

$$u_k = Kx_k + K_u u_{k-1} \tag{41}$$

where x_k is the state vector at time k and K and K_u are constant matrices, independent of time provided the model is assumed time invariant. The dependence of the control input at the previous step, u_{k-1} , comes from the weight on change in u in the optimization criterion.

Since all the states are not measured, we estimate them for example with a Kalman filter. For the MPC algorithm we use a discretized model with time step 1 second and use a zero order hold method for the discretization since the inputs are held constant between the time steps. In the discretized model time delays are represented exactly, as long as they are multiples of the time step.

In (Faanes, 2003, chapt. 5) we derive a state space formulation for the controller and the estimator:

$$x_{k+1}^{K} = Ax_{k}^{K} + By_{k}^{m} + Ey_{r}$$
(42)

$$u_k = Cx_k + Dy_k^m + Fy_r \tag{43}$$

where u_k is the control input at sample number k, x_k^K is the controller/estimator state vector, y_k^m is the measurement vector and y_r the reference, which may be seen as a disturbance to the controller. A, B, C, D, E and F are constant matrices.

For frequency analysis of the controller we may convert this discrete controller into a continuous one using d2c in Matlab (Tustin method), and Laplace transform yields:

$$u(s) = K(s) y^{m}(s) + K_{r}(s) y_{r}(s)$$
(44)

We have chosen weights in the MPC optimization criterion as Q = diag (100, 1, 1), R = Iand S = 0 in the MPC optimization criterion (40). For the estimator the co-variance matrices are $Q_w = I$ (process noise) and $R_v = I$ (measurement noise).

Appendix B Derivation of equations (18) and (29)

With pure feedforward control we get the following control error

$$e_{i} = \tilde{G}_{d,i}y_{i-1} + G_{i,i}K_{i,i-1}^{\text{FF}}y_{i-1} + G_{i,i}K_{i,i-2}^{\text{FF}}y_{i-2}$$

= $\left(I - G_{i,i}G_{i,i,-}^{-1}\right)\tilde{G}_{d,i}y_{i-1} + G_{i,i}K_{i,i-2}^{\text{FF}}y_{i-2}$ (45)

where we have inserted feedforward from unit i-1 from (17). With a combination of feedback and feedforward control we get (with (17))

$$e_{i} = \left(1 - G_{i-1,i-1}K_{i-1,i-1}\right)^{-1} \left(I - G_{i,i}G_{i,i,-}^{-1}\right) \tilde{G}_{d,i}y_{i-1} + G_{i,i}K_{i,i-2}^{\text{FF}}y_{i-2}$$
(46)

In both cases "ideal" feedforward requires $e_i = 0$ for all y_{i-1} and y_{i-2} :

$$\left(I - G_{i,i}G_{i,i,-}^{-1}\right)\tilde{G}_{d,i}y_{i-1} + G_{i,i}K_{i,i-2}^{\text{FF}}y_{i-2} = 0$$
(47)

We consider first pure feedforward, $K_{i,i} = K_{i-1,i-1} = 0$, and find the transfer function from y_{i-2} to y_{i-1} :

$$y_{i-1} = \left(\tilde{G}_{d,i-1} + G_{i-1,i-1}K_{i-1,i-2}^{\text{FF}}\right)y_{i-2}$$
(48)

 $K_{i-1,i-2}^{\rm FF} = -G_{i-1,i-1,-}^{-1} \tilde{G}_{d,i-1}$ yields

$$y_{i-1} = \left(I - G_{i-1,i-1}G_{i-1,i-1,-}^{-1}\right)\tilde{G}_{d,i-1}y_{i-2}$$
(49)

and upon inserting (49) into (47) we obtain

$$G_{i,i}K_{i,i-2}^{\rm FF} + \left(I - G_{i,i}G_{i,i,-}^{-1}\right)\tilde{G}_{d,i}\left(I - G_{i-1,i-1}G_{i-1,i-1,-}^{-1}\right)\tilde{G}_{d,i-1} = 0$$

leading to (18).

Second, we find the transfer function from y_{i-2} to y_{i-1} for a combination of local feedback and feedforward,

$$y_{i-1} = G_{i-1,i-1}K_{i-1,i-1}y_{i-1} + \left(\tilde{G}_{d,i-1} + G_{i-1,i-1}K_{i-1,i-2}^{\text{FF}}\right)y_{i-2}$$
(50)

where $K_{i-1,i-2}^{\rm FF} = -G_{i-1,i-1,-}^{-1} \tilde{G}_{d,i-1}$. Then

$$y_{i-1} = \left(1 - G_{i-1,i-1}K_{i-1,i-1}\right)^{-1} \left(I - G_{i-1,i-1}G_{i-1,i-1,-}^{-1}\right) \tilde{G}_{d,i-1}y_{i-2}$$
(51)

and by inserting this into (47) it follows

$$G_{i,i}K_{i,i-2}^{\text{FF}} + \left(I - G_{i,i}G_{i,i,-}^{-1}\right)\tilde{G}_{d,i}\left(1 - G_{i-1,i-1}K_{i-1,i-1}\right)^{-1}\left(I - G_{i-1,i-1}G_{i-1,i-1,-}^{-1}\right)\tilde{G}_{d,i-1} = 0$$
(52)
which gives (29).