Offset-free tracking with MPC with model mismatch: Experimental results

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Abstract

In this paper, a laboratorial experiment has been used to investigate some aspects related to integral action in model predictive control (MPC). Simulations using the same model as used for control design may indicate that integral action is present and that disturbances are handled well with no steady-state offset, but in practice unmodelled phenomena may give a poor response, including steady-state offset. The reason is that the controller may not contain feedback with integral action, although the zero offset sems to indicate it. The experiments on a two-tank process verify that output feedback with input disturbance estimation is efficient, provided that the disturbances to estimate are correctly chosen.

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1 Introduction

In this paper, we use an experiment to illustrate some important aspects regarding model predictive control (MPC) under uncertainty. MPC uses a process model to predict the future behaviour of the process, and uses this to determine an optimal sequence of adjustments of the manipulated variables. At a given time, the first value of this optimal sequence is applied to the process. Since the model is not perfect, measurements are used. When a new set of measurements is available to the controller, a re-optimization is performed, and the first value of this new optimal sequence of manipulated variables is implemented.

In many cases, one would like certain process variables (outputs) to follow given references, i.e., to obtain offset-free tracking. In most MPC applications this is achieved by simply adding the difference between the measurements and the model prediction. However, for many processes, especially those with long time constants, it has been shown that this approach is not efficient, and that estimation of input disturbances in such cases improved the performance (Muske and Rawlings, 1993; Lee *et al.*, 1994; Lundström *et al.*, 1995; Muske and Badgwell, 2002). Furthermore, simulations may indicate offset-free control even if this is not the case when the controller is applied to the actual plant. Recently, several papers have described how to rectify these problems (Muske and Badgwell, 2002; Pannocchia and Rawlings, 2003; Åkesson and Hagander, 2003; Faanes and Skogestad, 2003). In this paper, we use an experiment to illustrate that when input disturbance estimation is not correctly done, one may get steady-state offset.

An MPC controller is applied since this is the most commonly used multivariable controller in the process industry, even though the constraints never are exceeded and LQG could equally well have been used.

The experimental set-up is shown in Figure 1. The aim of the process is to keep the temperature in the circulation loop (as measured by TI2) constant by adjusting the cold-water flow-rate (marked with u in the figure) despite disturbances (marked d_1 and d_2). The level in the mixing tank is kept constant with an overflow drain, whereas in the main tank the level is kept within a band with an on/off-valve. A detailed description of the equipment is given in Appendix A.

The experimental work was carried through during October 2001 and the main contents of the paper was written at that time, and the work was therefore not motivated by the work of Muske and Badgwell (2002),Pannocchia and Rawlings (2003) and Åkesson and Hagander (2003). This also explains why the theory derived in these references has not been analyzed in the present paper.

2 Process model

We assume perfect mixing in both tanks, constant volumes, constant density, constant heat capacity and no heat loss. We model the main tank with its circulation loop as one (mixing) tank. Combination of mass and energy balance for the mixing tank (numbered 1) and the main tank (numbered 2) yields

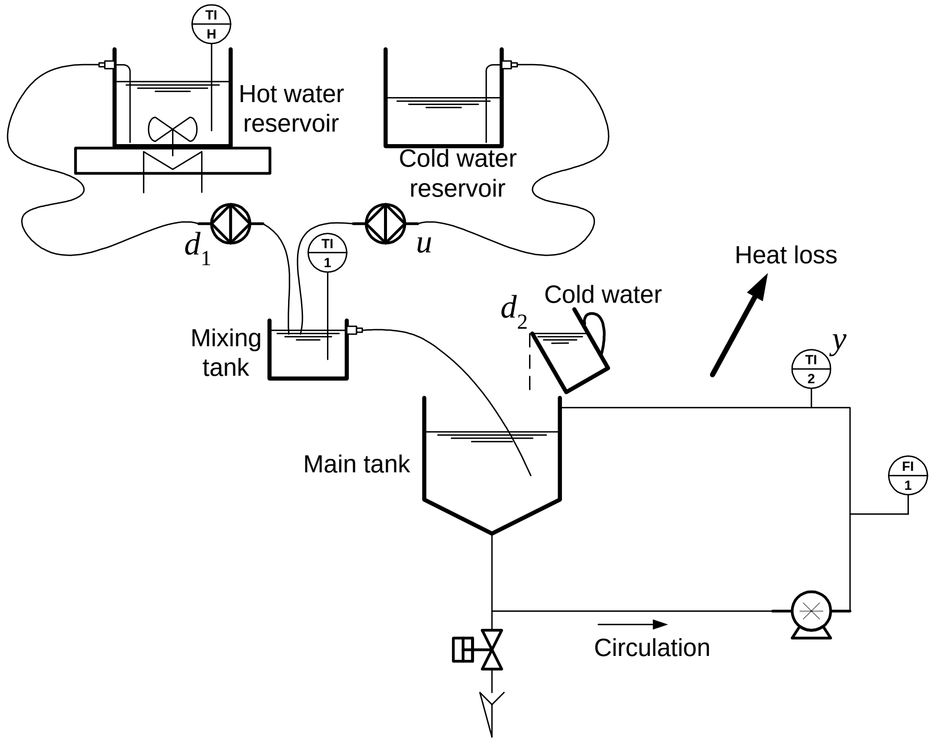


Figure 1: The experimental set-up

$$
\begin{aligned}
\frac{dT_1(t)}{dt} &= \frac{1}{V_1}\left[q_{C,1}(t)\left(T_{C,1}(t) - T_1(t)\right) + q_H(t)\left(T_H(t) - T_1(t)\right)\right] && \text{(1a)}\\
\frac{dT_2(t)}{dt} &= \frac{1}{V_2}\left[\left(q_{C,1}(t) + q_H(t)\right)\left(T_1(t-\theta_1) - T_2(t)\right) + q_{C,2}(t)\left(T_{C,2}(t) - T_2(t)\right)\right] && \text{(1b)}\\
T_1^m &= T_1(t-\theta_1) && \text{(1c)}\\
T_2^m &= T_2(t-\theta_2) && \text{(1d)}
\end{aligned}
$$

where t is time and the other variables are explained in Table 1. Here we have assumed that the outlet flow from the mixing tank is identical to the inflow (i.e. constant level in the tank). Superscript *m* denotes measurement. There is a delay θ_1 in tank 1 and a delay θ_2 in tank 2, representing transportation delays and neglected dynamics.

Linearization around a nominal point, denoted with an asterisk, yields:

$$
\begin{aligned}
\frac{d}{dt}x_1(t) &= -\frac{q^*}{V_1^*}x_1(t) + \frac{T_{C,1}^* - T_1^*}{V_1^*}u(t) + \frac{T_H^* - T_1^*}{V_1^*}d_1(t) && \text{(2a)}\\
\frac{d}{dt}x_2(t) &= \frac{q^*}{V_2^*}x_1(t-\theta_1) - \frac{q^* + q_{C,2}^*}{V_2^*}x_2(t) + \frac{T_{C,2}^* - T_2^*}{V_2^*}d_2(t) && \text{(2b)}\\
y_1^m(t) &= x_1(t-\theta_1) && \text{(2c)}\\
y_2^m(t) &= x_2(t-\theta_2) && \text{(2d)}\\
y(t) &= x_2(t-\theta_2) && \text{(2e)}
\end{aligned}
$$

where the model variables are given in Table 2 and the model parameters are given in Table 3 in Appendix B. The linear model is discretized with zero-order hold, using the Matlab Control

Table 1: Variables in nonlinear model (1)

Name	Explanation	Unit
T_1	Temperature mixing tank	°C
T_2	Temperature main tank	°C
V_1	Volume mixing tank	ml
V_2	Volume main tank	ml
$T_{C,1}$	Temperature cold-water into mixing tank	°C
T_H	Temperature hot-water into mixing tank	°C
$T_{C,2}$	Temperature cold-water into main tank	°C
$q_{C,1}$	Flow rate cold-water into mixing tank	ml / min
q_H	Flow rate hot-water into mixing tank	ml / min
$q_{C,2}$	Flow rate cold-water into main tank	ml / min

Toolbox routine c2d, with a sample time of 1 s. The delays are implemented as extra poles in the origin in the model (by delay2z in Matlab Control Toolbox). Note that this is an exact representation of the delays. The linear discrete model has 27 states, of which 25 last states are related to the delays. We define $x_k = \begin{bmatrix} x_1 & x_2 \end{bmatrix}_k^T$ as the state vector, $y_k = y_{2,k}$ as the output vector, $y_k^m = \begin{bmatrix} y_1^m & y_2^m \end{bmatrix}_k^T$ as the measurement vector, $d_k = \begin{bmatrix} d_1 & d_2 \end{bmatrix}_k^T$ as the disturbance vector, u_k as the control input u, all taken at sample number k. Then the linear discrete model may be formulated as

$$x_{k+1} = Ax_k + Bu_k + E_d d_k \tag{3a}$$

$$y_k = Cx_k \tag{3b}$$

$$y_k^m = C^m x_k \tag{3c}$$

where A, B, C, C^m and E_d are time independent matrices.

Table 2: Variables in linear model (2)

Name	Explanation	Unit
x_1	Variation in temperature mixing tank ($T_1 - T_1^*$)	°C
x_2	Variation in temperature main tank ($T_2 - T_2^*$)	°C
$y_1^m = (T_1 - T_1^*)^m$	Measurement 1 (deviation from nominal value)	°C
$y_2^m = (T_2 - T_2^*)^m$	Measurement 2 (deviation from nominal value)	°C
$y = y_2$	Primary output that we want to control (deviation from set-point)	°C
u	Variation in cold-water flow rate into mixing tank ($q_{C,1} - q_{C,1}^*$)	ml / min
d_1	Variation in hot-water flow rate into mixing tank ($q_H - q_H^*$)	ml / min
d_2	Variation in cold-water flow rate into main tank ($q_{C,2} - q_{C,2}^*$)	ml / min

In this work we have used the linear model (3) for the controller, whereas the nonlinear model (1) is used as the process for the simulations in section 5.

Most of the process parameters can be determined directly by inspection or individual measurements. The delays θ_1 and θ_2 and the nominal volume V_2^* of the main tank are more difficult to quantify, since they represent more than one phenomena. The main tank volume includes the recirculation loop, and the delays represent both transportation of water and other neglected dynamics. Therefore, three open loop experiments have been performed to determine these three parameters, see Figure 2.

The linear model (2) was simulated with the actual u and d_1 as inputs. The nominal volumes V_2^* and the delays θ_1 and θ_2 were determined by trial and error. Simulation results with the final model are compared with the experiments in Figure 2. The resulting parameter values are given in Table 3 in Appendix B.

3 Controller

The MPC used for temperature control is based on the controller proposed by Muske and Rawlings (1993) with a discrete model on the form

$$x_{k+1} = Ax_k + Bu_k \tag{4a}$$
$$y_k = Cx_k \tag{4b}$$

This model is the same as (3), except that the disturbance term is omitted. The control input u_k is found by minimizing the infinite horizon criterion:

$$\min_{u_k^N} \sum_{j=0}^{\infty} \left(y_{k+j}^T Q y_{k+j} + u_{k+j}^T R u_{k+j} \right) \tag{5}$$

where y_{k+j} is the deviation in the main tank temperature at sample number $k+j$, and $u_k^N = \begin{bmatrix} u_k & u_{k+1} & \dots & u_{k+N-1} \end{bmatrix}^T$ is a vector of N future moves of the control input, of which only the first is actually implemented. The control input u_{k+j} is assumed zero for all $j \geq N$. A term for the control input change may also be included, but this is omitted here. Q and R are time independent weight matrices.

Muske and Rawlings (1993) show how to formulate (5) as a finite optimization problem. Upon the assumption that the constraints never are active, the optimal control input is given by the state feedback law

$$u_k = Kx_k \tag{6}$$

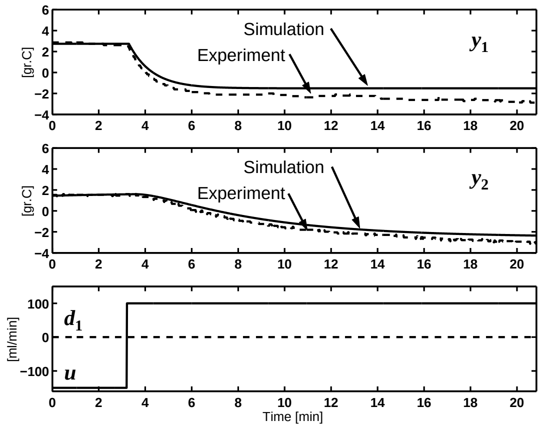
The control input u_k is assumed constant from k to $k+1$. The matrix K is time invariant, and is given by the model matrices, A, B and C and the weight matrices Q and R.

However, the control law (6) has no integral action, so we get a steady-state offset if we have a nonzero reference y_r for y or we have external disturbances. There are many ways to obtain integral action, and one is to use the modification

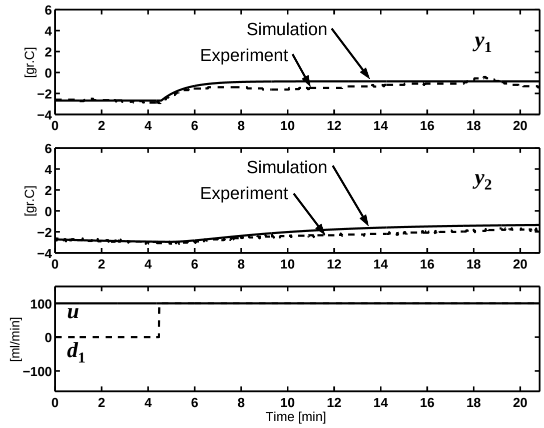
$$u_k = K(x_k - x_s) + u_s \tag{7}$$

where x_s is the state corresponding to the desired steady-state value of y_k ($y_r = Cx_s$) and u_s is the corresponding steady-state control input. The variables x_s and u_s are both functions of the reference y_r and the disturbances. In our case, y_r is known, and is held constant during the experiments. Disturbances, however, are here assumed unknown, and must therefore be estimated from the temperature measurements.

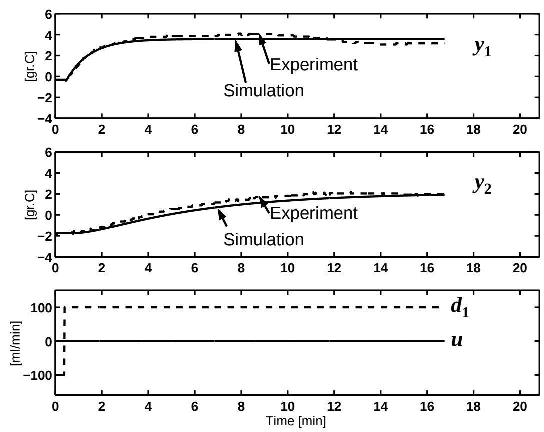
For processes with large time constants (near-integrating processes) it has been demonstrated that good performance is obtained by using estimates of disturbances $\hat{d}_k$ acting directly on the near integrating states (Lee *et al.*, 1994; Lundström *et al.*, 1995; Muske and Badgwell, 2002), and we will follow this approach here. Since we do not know the future behaviour



(a) Step in cold flow rate: u from -150 to $100\,\mathrm{ml}\,/\,\mathrm{min}$, corresponding to a change in q_{C_1} from 350 to $600\,\mathrm{ml}\,/\,\mathrm{min}$. Hot flow rate $d_1 = 0$, corresponding to $q_H = 500\,\mathrm{ml}\,/\,\mathrm{min}$.



(b) Step in hot flow rate: d_1 from 0 to $100\,\mathrm{ml}\,/\,\mathrm{min}$, corresponding to a change in q_H from 500 to $600\,\mathrm{ml}\,/\,\mathrm{min}$. Cold flow rate $u = 100$, corresponding to $q_{C_1} = 600\,\mathrm{ml}\,/\,\mathrm{min}$.



(c) Step in hot flow rate: d_1 from -100 to $100\,\mathrm{ml}\,/\,\mathrm{min}$, corresponding to a change in q_H from 400 to $600\,\mathrm{ml}\,/\,\mathrm{min}$. Cold flow rate $u = 0$, corresponding to $500\,\mathrm{ml}\,/\,\mathrm{min}$.

Figure 2: The resulting linear model: Open loop simulations compared with the open loop experiments

of the disturbance vector, we assume that it will be constant, i.e., the steady state disturbance estimate $\hat{d}_s = \hat{d}_k$. The steady-state solutions x_s and u_s can then be found by solving (Muske and Rawlings, 1993)

$$\begin{bmatrix} I-A & -B \\ C & 0 \end{bmatrix} \begin{bmatrix} x_s \\ u_s \end{bmatrix} = \begin{bmatrix} \tilde{E}_d \hat{d}_s \\ y_r \end{bmatrix} \tag{8}$$

The matrix $\tilde{E}_d$ represents the direct effect of the estimated disturbance on the state, and is specified in (12). Note that the estimated disturbance $\tilde{d}_k$ will not represent the actual disturbances in value since $\tilde{E}_d$ differs from E_d. Even the sign is opposite for one of the disturbances. Thus in the results, comparing the numerical values of d_k and $\tilde{d}_k$ has no meaning.

Provided the constraints are not active, the vectors x_s and u_s can explicitly be expressed by the disturbance estimate $\hat{d}_k$ ($= \hat{d}_s$) and the reference y_r, see Faanes and Skogestad (2003) (or Faanes (2003, Chapt. 5))):

$$\begin{bmatrix} x_s \\ u_s \end{bmatrix} = \Gamma_y y_r + \Gamma_d d_s \tag{9}$$

where Γ_y and Γ_d are given by the matrices A, B, C, and $\tilde{E}_d$.

When the states x_k are not measured, they must also be estimated since x_k is needed in the control equation (7). To obtain estimates of both x_k and d_k, we define an extended state vector:

$$\tilde{x}_k = \begin{bmatrix} x_k \\ d_k \end{bmatrix} \tag{10}$$

We assume that the disturbances are integrated white noise, and introduce the extended model

$$\tilde{x}_{k+1} = \underbrace{\begin{bmatrix} A & \tilde{E}_d \\ 0 & I_{n_d} \end{bmatrix}}_{\tilde{A}} \tilde{x}_k + \underbrace{\begin{bmatrix} B \\ 0 \end{bmatrix}}_{\tilde{B}} u_k + w_k \tag{11a}$$

$$y_k^m = \underbrace{\begin{bmatrix} C^m & 0 \end{bmatrix}}_{\tilde{C}} \tilde{x}_k + v_k \tag{11b}$$

where each disturbance acts directly on states

$$\tilde{E}_d = \begin{bmatrix} I_{n_d} \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{12}$$

and w_k and v_k are zero-mean, uncorrelated, normally-distributed white noise processes with covariance matrices of Q_w and R_v, respectively. I_{n_d} is the identity matrix of dimension $n_d \times n_d$, where n_d is the length of the disturbance vector. We design a Kalman filter:

$$\overline{\tilde{x}}_{k+1} = \tilde{A}\widehat{\tilde{x}}_k + \tilde{B}u_k \tag{13a}$$

$$\widehat{\tilde{x}}_k = \overline{\tilde{x}}_k + L\left(y_k^m - \tilde{C}\overline{\tilde{x}}_k\right) \tag{13b}$$

where $\overline{\tilde{x}}_k$ and $\widehat{\tilde{x}}_k$ are *a priori* and *a posteriori* estimates of $\tilde{x}_k$, respectively, and L is the estimator gain matrix given by

$$L = P\tilde{C}^T\left(\tilde{C}P\tilde{C}^T + R_v\right)^{-1} \tag{14}$$

where P the solution of the Riccati equation

$$P = \tilde{A}\left[P - P\tilde{C}^T\left(\tilde{C}P\tilde{C}^T + R_v\right)^{-1}\tilde{C}P\right]\tilde{A}^T + Q_w \tag{15}$$

By applying the *a posteriori* estimates, the following control law is obtained and used in this work:

$$u_k = \tilde{K}\widehat{\tilde{x}}_k + K_r y_r \tag{16}$$

where

$$\tilde{K} = \left[\begin{array}{cc} K & -\left(\begin{array}{cc} K & -I \end{array}\right)\Gamma_d \end{array}\right] \tag{17a}$$

$$K_r = -\left(\begin{array}{cc} K & -I \end{array}\right)\Gamma_y \tag{17b}$$

The following weight and covariance matrices were used:

$$Q = 1; \quad R = (1/6) \times 10^{-5} \tag{18a}$$

$$Q_w = \begin{bmatrix} I_n & 0 \\ 0 & 0.05 I_{n_d} \end{bmatrix}; \quad R_v = 1000 I_2 \tag{18b}$$

where n is number of states and n_d is number of estimated disturbances. The control horizon N has been selected to $40\,\mathrm{s}$.

The large difference in magnitude between Q and R is a result of not having scaled the model. For a variation in y between -0.3 and 0.3 and u between -500 and 500, the two terms are in the same order of magnitude for the limiting values:

$$y^T Q y = 0.3^2 \times 1 = 0.09 \tag{19a}$$

$$u^T R u = 500^2 \times (1/6) \times 10^{-5} = 0.42 \tag{19b}$$

4 Experimental procedure

The aim of the experiments was to investigate the effect of different disturbance vectors d_k to be estimated and used by the MPC in the calculation of steady-state control input u_s and state vector x_s. In addition to the experiments, we performed a simulation with the nonlinear model of the process (1), which was implemented in Simulink (a Matlab toolbox).

Simulation and Experiment A: MPC with estimate of disturbance d_1 only (the length of $\hat{d}_k$ is $n_d = 1$).

Experiment B: MPC with estimate of both disturbances d_1 and d_2 ($n_d = 2$).

Prior to the experiments, the process was run to a steady-state working point. The following sequence of disturbances was then introduced in each experiment
Disturbance d_1:

1a) Reduce hot flow rate from 500 to $400\,\mathrm{ml}\,/\,\mathrm{min}$

1b) Increase hot flow rate back from 400 to $500\,\mathrm{ml}\,/\,\mathrm{min}$

Disturbance d_2:

2a) Start addition of cold-water to main tank

2b) Stop addition of cold-water to main tank

The change in hot flow was done by adjusting the speed of the peristaltic pump via a Matlab user interface.

The addition of cold-water to the main tank was done by pouring water from a jug. During 7 minutes a total of $430\,\mathrm{ml}$ (Experiment A) and $450\,\mathrm{ml}$ (Experiment B) cold water was added. This gives a mean flow rate of $61.4\,\mathrm{ml}\,/\,\mathrm{min}$ and $64.3\,\mathrm{ml}\,/\,\mathrm{min}$, respectively for the two experiments.

During the two experiments the hot-water temperature varied between 48 and $51°C$, whereas during the simulations the temperature was held constant.

5 Results

In Figure 3 we show the closed loop simulation of MPC with estimate of d_1 only. Note that y_2 (solid line) is the important output (temperature), which we want to return to its set-point as quickly as possible. We see that for disturbance d_1, control of y_2 is good with (seemingly) no steady-state offset. The reason why we write “seemingly”, is that there is in fact no integral action, so in reality there will be an upset. As seen from Figure 3 we get a steady-state offset for disturbance d_2. In practice the engineer will not simulate all possible disturbances, and may incorrectly conclude (if d_2 had not been tested) that the controller has integral action.

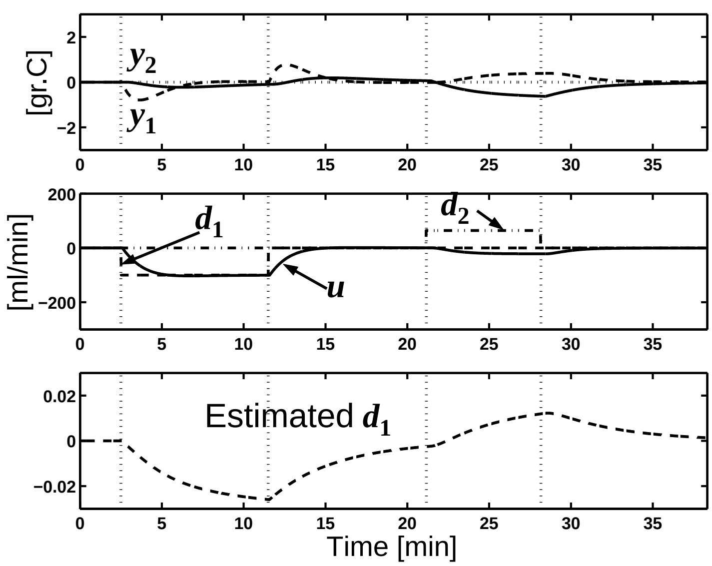


Figure 3: Simulation of MPC with estimate of d_1

In Figures 4 and 5 we show the results of the two experiments. In contrast to the simulation, the controller with estimation of only disturbance d_1 (Experiment A) fails to achieve the desired steady state, both before and after the disturbances are introduced. This is due to model error and unmodelled disturbances.

We also see that $y_1 = T_1$ is higher than $y_2 = T_2$. The reason for this is mainly heat loss, and there was also a small difference in the calibration of the temperature elements. The model does not cover these effects.

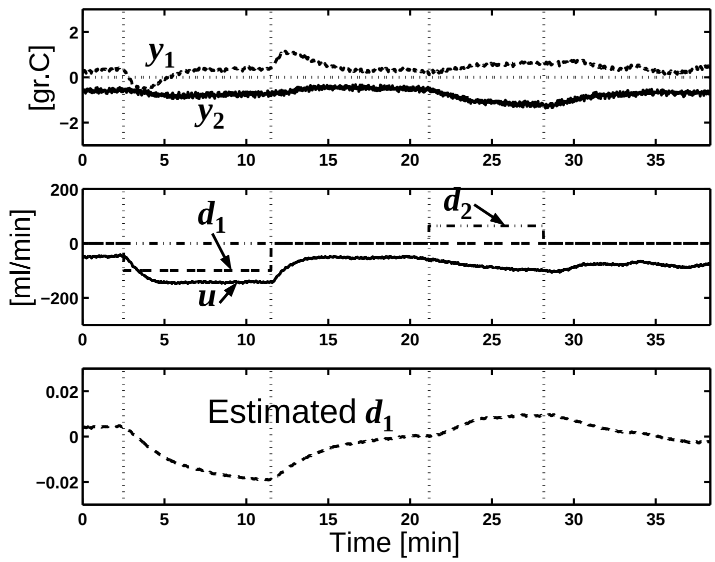


Figure 4: Experiment A: MPC with estimate of d_1. Steady-state offset in y_2.

However, in Figure 5 we can see that with estimation of both disturbances (Experiment B), we get no steady-state offset for y_2. Simulations (not shown) give the same result. To compensate for the heat loss, the controller increases the temperature in tank 1 (y_1). We see that both disturbances are handled well, in spite of the fact that the actual estimate of disturbance 2 is not very good. The large variations in u rise from the measurements noise.

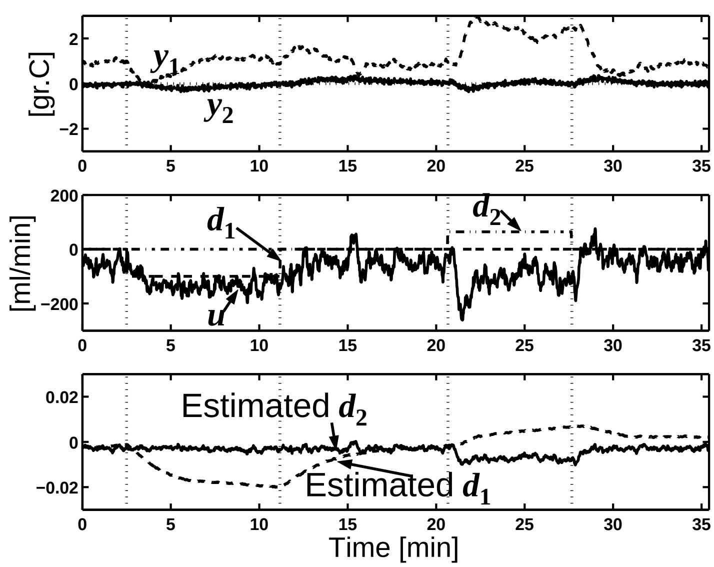


Figure 5: Experiment B: MPC with estimate of d_1 and d_2. No steady-state offset in y_2.

The disturbance estimates cannot be compared in value with the real disturbance since $\tilde{E}_d$ has been chosen different from E_d. If the estimate of the disturbance is of interest, one must seek to find E_d and use this in the estimator. In Figure 5 we see e.g. that the estimated disturbance d_2 has opposite sign of the real one.

6 Discussion

With estimation of two input disturbances (d_1 and d_2) an offset-free steady state was obtained, whereas with only one input estimate (d_1) insufficient integral action was obtained. This is in accordance with the theoretical results by Pannocchia and Rawlings (2003) and Faanes and Skogestad (2003) (or (Faanes, 2003, Chapter 5)). In these references, it is found that the number of estimated input disturbances must equal the number of measurements if steady-state offset shall be avoided. A similar result was also derived by Åkesson and Hagander (2003), although they proposed to use a combination of input disturbances and output bias estimation.

We have also simulated the case when y_1 is omitted, i.e. only y_2 is used by MPC. In this case it is sufficient to only estimate one disturbance in the second tank (d_2). Normally this controller will give a poorer performance, since the early information of disturbances to the first tank from $y_1 = T_1$ is not exploited, but for the controller tunings we have chosen, the performance was actually slightly improved. In Faanes and Skogestad (2005) an example with tanks in series is presented where the use of measurements from upstream tanks improves the performance.

We now return to the case when both measurements y_1 and y_2 are used, and compare our MPC controllers (with estimation of d_1 only and estimation of d_1 and d_2) in the frequency domain. This is possible since the constraints in the control input u are never active. In Faanes and Skogestad (2003) (or (Faanes, 2003, Chapt. 5)) a state-space formulation is derived for the combination of the controller and the estimator for this case. The controller may further be expressed by an approximated continuous state-space formulation (by d2c in Control Toolbox in Matlab), which is easily converted to a transfer function formulation:

$$u\left(j\omega\right) = K\left(j\omega\right) y\left(j\omega\right) \tag{20}$$

To study the magnitude of the elements in K, it is convenient to introduce scaled variables. The maximum possible variation in u in each direction is $u_{\max} = 500\,\mathrm{ml}\,/\,\mathrm{min}$, and $y_{\max} = 0.3\,°\mathrm{C}$ is the maximum desired variation in y. By defining the scaled variables $u' = u/u_{\max}$ and $y' = y/y_{\max}$, both u' and y' stay within ± 1. The corresponding controller equation for the scaled system is

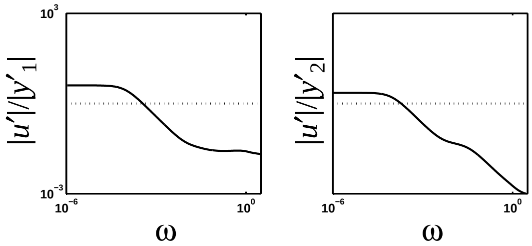
$$u'\left(j\omega\right) = K'\left(j\omega\right) y'\left(j\omega\right) \tag{21}$$

where $K'\left(j\omega\right) = K\left(j\omega\right) y_{\max}/u_{\max}$.

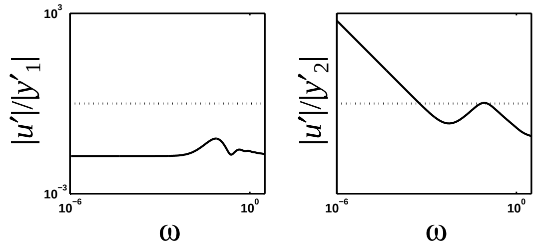
In Figure 6 we plot the magnitude of the elements in $K'\left(j\omega\right)$ for the two types of controllers. The most important is the gain from the primary output y_2' to u. We see that the controller with only one disturbance estimate has low gains at low frequencies (Figure 6(a)), whereas for the controller with two disturbances the low-frequency gain from y_2 is high beacuse of the integral action (Figure 6(b)). Figure 6(b) also reveals that the gain from y_1 is low for all frequencies, which explains why the use of y_1 in the control did not improve performance.

7 Conclusions

In a laboratory experiment, we have used MPC combined with an estimator for temperature control of a process with two tanks in series. Since this often improves performance, we used the temperature measurements of both tanks in the controller, even if we only are interested in the last temperature, and we have only one control input. To avoid steady-state offset, we have estimated input disturbances, and used these estimates in the calculation of the steady-state control input.



(a) MPC with estimates of d_1 only



(b) MPC with estimates of d_1 and d_2

Figure 6: Controller gain elements $|K'(j\omega)|$

Simulations may indicate that disturbances are handled well with no steady-state offset. However, if apparent integral action is actually due to a model-based "feedforward correction", then unmodelled phenomena may give poor results in the actual plant, also at steady state.

To obtain integral action, the number of disturbance estimates must equal the number of measurements (Pannocchia and Rawlings, 2003; Åkesson and Hagander, 2003; Faanes and Skogestad, 2003). In our experiment, the use of estimates of input disturbances to both tanks gave satisfactory performance with no steady-state error.

8 Acknowledgements

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Appendix A Experimental set-up

A.1 Equipment

The exprimental set-up is illustrated in Figure 1. Hot and cold water from two reservoirs are mixed in a mixing tank. The water flow rates are controlled with peristaltic pumps (Watson Marlow 505Du/RL). There is an overflow drain, and the mixed water flows through a flexible tube to the main tank, which is situated at a lower altitude.

The main tank has a circulation loop with a pump (Johnson Pump F4B-8) and a flow-rate measurement (tecfluid SC-250). The main tank temperature measurement is placed in the circulation loop, which gives an adjustable delay in the measurement. In addition, the circulation serves for mixing.

In the circulation loop, below the main tank, there is a drainage. The drainage flow rate is controlled with an on-off valve (Asco SCE030A017). The drainage keeps the level in the main tank approximately constant despite the inflow from the mixing tank.

The reservoirs and the tanks are all modified beakers. The pipes of the circulation loop are made of glass.

The experiments take place at room temperature (about $20\,^{\circ}\mathrm{C}$). Since the hot-water temperature ($48 - 51\,^{\circ}\mathrm{C}$) deviates considerable from this, the hot-water reservoir is placed on a hot-plate with thermostat to keep the hot-water temperature approximately constant. Since the two reservoirs do not contain a sufficient amount for the whole experiment, refill is necessary. The cold-water is about $13 - 15\,^{\circ}\mathrm{C}$, which is considered fairly close to room temperature.

Magnetic stirrers are placed in the hot-water reservoir and in the mixing tank.

A.2 Instrumentation and logging

Pt-100 elements (class B, 3 wire, single, diameter 3mm, length 150mm) are placed in the hot-water reservoir, the mixing tank and in the circulation loop of the main tank. The main tank level is measured with a capacitance probe (Endress+Hauser Multicap DC11 TEN). The instruments are connected to National Instruments Fieldpoint modules, which are further connected to a PC via the serial port. In the PC, Bridgeview (National Instruments) is used for data display and basic control. Bridgeview also provides an OPC server interface, such that an OPC client may read off measured data, and give values to the actuators. The temperature controller is implemented in Matlab. The temperature measurements are read into Matlab, and the flow rate for the peristaltic pumps are determined in Matlab, and provided to Bridgeview via the OPC interface. Matlab is also used to plot the results.

A.3 Basic control

The following basic control is implemented in Bridgeview on the connected PC:

1. The level in the main tank is controlled by opening the drainage valve when the main tank level reaches above $2.0\,\mathrm{l}$, and closing it when it is below $1.9\,\mathrm{l}$. A manually adjustable valve is installed on the drainage tube to reduce the drainage flow (otherwise the main tank empties too quickly compared to the response time of the level control loop).

2. The rotational speed of the circulation pump is set to a constant value, which in this set-up gives a constant circulation flow-rate.

3. The speed of the peristaltic pumps is determined from the desired flow rate by a linear relation. A two-point calibration is used.

Appendix B Model parameters

The model parameters of the linear model (2) is given in Table 3 below.

Table 3: Model parameters

Name	Explanation	Value	Unit
T_1^*	Nominal temperature mixing tank	$31.75, 31.08^1$	°C
T_2^*	Nominal temperature main tank (=set-point)	$31.75, 31.08^1$	°C
V_1^*	Nominal liquid volume of mixing tank (tank no.1)	1000	ml
V_2^*	Nominal liquid volume of main tank, including circulation loop (tank no. 2)	5000	ml
$T_{C,1}^*$, $T_{C,2}^*$	Cold-water temperatures (assumed constant)	13.5	°C
T_H^*	Hot-water temperature	$48 - 51$	°C
q^*	Nominal total flow from mixing tank $(= q_H^* + q_{C,1}^*)$	1000	ml / min
q_H^*	Nominal flow rate from hot reservoir	500	ml / min
$q_{C,1}^*$	Nominal flow rate from cold reservoir into mixing tank	500	ml / min
$q_{C,2}^*$	Nominal flow rate from cold reservoir into main tank	0	ml / min
θ_1	Transportation and measurement delay in T_1	5	s
θ_2	Transportation and measurement delay in T_2	15	s

[1] For Experiment A and B, repectively.