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MINIMIZATION OF STATE DEVIATIONS USING PERFECT INDIRECT CONTROL

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ABSTRACT – An important issue in control structure selection is the plant ”stabilization”. By the term ”stabilize” we here include both modes which are mathematically unstable (modes with RHP poles) as well as ”drifting” modes which need to be kept within limits to avoid operational problems. By this definition, we can include the states $\mathbf{x}$ as variables that should be ”stabilized”, i.e., we want to avoid them to drift too far away from their desired (nominal) values. An advantage of this approach is that we are able to avoid problems resulted from nonlinear effects. Therefore, as the objective function can, usually, be considered as a combination of the states, the control system obtained by this approach is not tied too closely to a particular primary control objective (which may change with time) because it allows the designer to change the control objective. This paper presents a way to reduce the effects of disturbances and measurement errors in the states and the results show the effectiveness of this approach.

KEYWORDS: perfect indirect control; minimization of state deviations; distillation column.

1.INTRODUCTION

In the regulatory control layer the main objective is to ”stabilize” the plant. Here we put the word stabilize in quotes because we use it with the same meaning as used by Skogestad (2003): ”stabilization” includes both modes which are mathematically unstable (modes with RHP poles) as well as ”drifting” modes which need to be kept within limits to avoid operational problems. Doing this we are able to avoid problems resulted from, for example, nonlinear effects.

By this definition, we include any states $\mathbf{x}$ as variables that should be “stabilized”, i.e., we want to avoid them to drift too far away from their desired (nominal) values. An advantage of keeping all states close to their nominal values is that we are able to avoid problems resulting from nonlinear effects.

Therefore, an important point in the control structure selection is the choice of the operational objectives. According to Skogestad (2003), this is the first step to be done. The problem is that the operational objectives may change with time, according to the necessities, e.g. market, safety constraints, etc. Due to these changes, we don't want to tie the control system too closely to a particular primary control objective. As, usually, the objective function can be considered a combination of the states a good approach would be to define it (the objective function) in this way ($\mathbf{y}_1 = \mathbf{W}\mathbf{x}$). This approach has the advantage of allowing the controller designer to easily change the control objective only changing the combination of the states. Another advantage is that the minimization of $\mathbf{W}\mathbf{x}$ includes both stabilization of RHP-poles and disturbance rejection.

In summary, the good of this paper is to discuss in more detail the approach introduced in Skogestad (2002) of selecting secondary controlled variables ($\mathbf{c} = \mathbf{y}_2$) such that we minimize the effect of disturbances ($\mathbf{d}$) on the weighted states ($\mathbf{y}_1 = \mathbf{W}\mathbf{x}$).

2.PERFECT INDIRECT CONTROL

Consider that we have the following linear model:

$$\Delta\mathbf{y}_1 = \mathbf{G}_1\Delta\mathbf{u} + \mathbf{G}_{d1}\Delta\mathbf{d} \quad (1)$$

$$\Delta\mathbf{y} = \mathbf{G}^y\Delta\mathbf{u} + \mathbf{G}_d^y\Delta\mathbf{d} + \mathbf{n}^y \quad (2)$$

where:

$\mathbf{G}_1$, $\mathbf{G}_{d1}$, $\mathbf{G}^y$, and $\mathbf{G}_d^y$ are steady-state models

$\Delta\mathbf{u}$ - manipulated variables

$\Delta\mathbf{d}$ - disturbances

$\Delta\mathbf{y}_1$ - primary controlled variables

$\Delta\mathbf{y}$ - available measurements.

$\mathbf{n}^y$ - measurement noise

By definition, indirect control is when we cannot control the primary outputs ($\mathbf{y}_1$) (e.g., because they are not measured online) and, instead, we aim at indirectly controlling $\mathbf{y}_1$ by controlling the "secondary" variables $\mathbf{c}$ (Skogestad and Postlethwaite, 1996). It is proven (Halvorsen et al., 2003) that if the number of measurements ($\#\mathbf{y}$) is equal or larger than the sum of the number of inputs ($\#\mathbf{u}$) and the number of disturbances ($\#\mathbf{d}$), then we can obtain a combination of these measurements ($\mathbf{c}$) that ensures a perfect indirect control of the "primary" controlled variables (in this case $\mathbf{c}$ is used as "secondary" controlled variable). Then we have:

$$\Delta\mathbf{c} = \mathbf{H}\Delta\mathbf{y} = \underbrace{\mathbf{H}\mathbf{G}^y}_{\mathbf{G}}\Delta\mathbf{u} + \underbrace{\mathbf{H}\mathbf{G}_d^y}_{\mathbf{G}_d}\Delta\mathbf{d} + \underbrace{\mathbf{H}\mathbf{n}^y}_{\mathbf{n}^c} \quad (3)$$

where the matrix $\mathbf{H}$ represents the combination of measurements.

As the new variable $\mathbf{c}$ is used as "secondary" controlled variable, than we can solve Equation 3 with respect to $\Delta\mathbf{u}$:

$$\Delta\mathbf{u} = \mathbf{G}^{-1}\Delta\mathbf{c} - \mathbf{G}^{-1}\mathbf{G}_d\Delta\mathbf{d} - \mathbf{G}^{-1}\mathbf{n}^c \quad (4)$$

In Equation 4 we will consider $\Delta\mathbf{c} = \mathbf{0}$ because we want to keep these variables constant. In this way, Equation 4 becomes:

$$\Delta\mathbf{u} = -\mathbf{G}^{-1}\mathbf{n}^c - \mathbf{G}^{-1}\mathbf{G}_d\Delta\mathbf{d} \quad (5)$$

Substituting Equation 5 into Equation 1 gives:

$$\Delta\mathbf{y}_1 = \underbrace{\left(\mathbf{G}_{d1} - \mathbf{G}_1\mathbf{G}^{-1}\mathbf{G}_d\right)}_{\mathbf{P}_d}\Delta\mathbf{d} - \underbrace{\mathbf{G}_1\mathbf{G}^{-1}}_{\mathbf{P}_c}\mathbf{n}^c \quad (6)$$

where the "partial disturbance gain" $\mathbf{P}_d$ gives the effect of disturbances on $\mathbf{y}_1$ with closed-loop (partial) control of the variables $\mathbf{c}$, and $\mathbf{P}_c$ gives the effect on $\mathbf{y}_1$ of changes in $\mathbf{c}$ (e.g., due to setpoint changes in $\mathbf{c}_s$ or control error).

As we want to reject perfectly the effect of the disturbance in the primary variables, we want to select a set of controlled variables such that the matrix $\mathbf{P}_d$ be equal to zero. As will be shown below, this objective can be reached if we have enough measurements $\mathbf{y}$, more specifically, when the number of measurements ($\#\mathbf{y}$) is equal or larger the sum of the number of inputs ($\#\mathbf{u}$) and the number of disturbances ($\#\mathbf{d}$). The matrix $\mathbf{P}_c$ is a degree of freedom which can be arbitrarily specified ($\mathbf{P}_c = \mathbf{P}_{c0}$) by the designer, for example, when $\mathbf{P}_c = \mathbf{I}$ we have a decoupled response from $\mathbf{c}_s$.

To find the linear combination of variables we will make some additional assumptions:

1. The number of controlled variables ($\#\mathbf{c}$), primary variables ($\#\mathbf{y}_1$), and manipulated variables ($\#\mathbf{u}$) is equal ($\#\mathbf{c} = \#\mathbf{y}_1 = \#\mathbf{u}$);
2. The number of measurements ($\#\mathbf{y}$) is equal to the number of manipulated ($\#\mathbf{u}$) plus disturbances ($\#\mathbf{d}$), i.e., $\#\mathbf{y} = \#\mathbf{u} + \#\mathbf{d}$;
3. The matrix $\mathbf{P}_{c0}$ is invertible.

Then, we want to find a matrix $\mathbf{H}$ that gives us $\mathbf{P}_d = \mathbf{0}$ and $\mathbf{P}_c = \mathbf{P}_{c0}$. From Equation 6 we have that:

$$\mathbf{P}_c = \mathbf{P}_{c0} = \mathbf{G}_1\mathbf{G}^{-1} \quad (7)$$

$$\mathbf{P}_d = \mathbf{G}_{d1} - \mathbf{G}_1\mathbf{G}^{-1}\mathbf{G}_d = \mathbf{0} \quad (8)$$

Then, from Equation 3, we have that Equations 7 and 8 become:

$$\mathbf{G}_1\left(\mathbf{H}\mathbf{G}^{y}\right)^{-1} = \mathbf{P}_{c0} \quad (9)$$

$$\mathbf{G}_1\left(\mathbf{H}\mathbf{G}^{y}\right)^{-1}\left(\mathbf{H}\mathbf{G}_d^{y}\right) = \mathbf{G}_{d1} \quad (10)$$

Or, equivalently:

$$\mathbf{H}\mathbf{G}^{y} = \mathbf{P}_{c0}^{-1}\mathbf{G}_1 \quad (11)$$

$$\mathbf{H}\mathbf{G}_d^{y} = \mathbf{P}_{c0}^{-1}\mathbf{G}_{d1} \quad (12)$$

Joining Equations 11 and 12 results in:

$$\mathbf{H}\left[\mathbf{G}^{y} \quad \mathbf{G}_d^{y}\right] = \mathbf{P}_{c0}^{-1}\left[\mathbf{G}_1 \quad \mathbf{G}_{d1}\right] \quad (13)$$

By assumption number 1 we have that the matrix $\left[\mathbf{G}^{y} \quad \mathbf{G}_d^{y}\right]$ is square and, as the measurements are independent, the matrix is invertible, then, finally:

$$\mathbf{H} = \mathbf{P}_{c0}^{-1}\underbrace{\left[\mathbf{G}_1 \quad \mathbf{G}_{d1}\right]}_{\tilde{G}_1}\underbrace{\left[\mathbf{G}^{y} \quad \mathbf{G}_d^{y}\right]^{-1}}_{\tilde{G}^{y^{-1}}} \quad (14)$$

When $\mathbf{P}_{c0} = \mathbf{I}$ and we use combinations of variables as secondary controlled variables, from Equation 6 is easy to see that $\mathbf{G} = \mathbf{G}_1$ and $\mathbf{G}_d = \mathbf{G}_{d1}$.

3.MINIMUM STATE DEVIATION

To keep the states close to their desired (nominal) values in the presence of disturbances and implementation error, we will define a matrix $\mathbf{W}$ that represents a linear combination of the states. It can also be interpreted as the objective of the controller defined by the controller's designer.

We will consider that we have the following linear model:

$$\Delta\mathbf{x} = \mathbf{G}^{x}\Delta\mathbf{u} + \mathbf{G}_d^{x}\Delta\mathbf{d} \quad (15)$$

where:
$\mathbf{G}^{x}$, and $\mathbf{G}_d^{x}$ are steady-state models
$\Delta\mathbf{x}$ - states

If we substitute Equation 5 into Equation 15 we will have:

$$\Delta \mathbf{x} = \underbrace{\left(\mathbf{G}_{d}^{x} - \mathbf{G}^{x}\mathbf{G}^{-1}\mathbf{G}_{d}\right)}_{\mathbf{G}_{d}^{x^*}}\Delta \mathbf{d} - \underbrace{\mathbf{G}^{x}\mathbf{G}^{-1}}_{\mathbf{G}^{x^*}}\mathbf{n}^{c} \qquad (16)$$

The matrices $\mathbf{G}^{x^*}$ and $\mathbf{G}_{d}^{x^*}$ represent the effect of the disturbances and implementation errors in the states when we control combinations of variables. To avoid problems related to nonlinearities, it is important that these matrices be as small as possible. Then, to minimize the effect of the disturbances $(\mathbf{d})$ and the implementation errors $(\mathbf{n}^{c})$, we want to minimize the norms $\left\|\mathbf{G}^{x^*}\right\|$, $\left\|\mathbf{G}_{d}^{x^*}\right\|$, and $\left\|\mathbf{G}^{x^*} \quad \mathbf{G}_{d}^{x^*}\right\|$.

It is important to emphasize that we will not control the states $\mathbf{x}$ directly but, instead, we will "control" them using indirect control as presented in section 2.

As we saw in section 2, when we have perfect indirect control and $\mathbf{P}_{c0} = \mathbf{I}$, the matrices $\mathbf{G}$ and $\mathbf{G}_{d}$ become equal to the matrices $\mathbf{G}_{1}$ and $\mathbf{G}_{d1}$, respectively. If we define the primary variables as linear combinations of the states ($\mathbf{y}_{1} = \mathbf{W}\mathbf{x}$), then we have that:

$$\Delta \mathbf{y}_{1} = \mathbf{W}\Delta \mathbf{x} = \underbrace{\mathbf{W}\mathbf{G}^{x}}_{\mathbf{G}_{1}}\Delta \mathbf{u} + \underbrace{\mathbf{W}\mathbf{G}_{d}^{x}}_{\mathbf{G}_{d1}}\Delta \mathbf{d} \qquad (17)$$

where $\mathbf{W}$ is the matrix that represents the linear combination of the states.

Equation 16 then becomes:

$$\Delta \mathbf{x} = \underbrace{\left(\mathbf{G}_{d}^{x} - \mathbf{G}^{x}\left(\mathbf{W}\mathbf{G}^{x}\right)^{-1}\mathbf{W}\mathbf{G}_{d}^{x}\right)}_{\mathbf{G}_{d}^{x*}}\Delta \mathbf{d} - \underbrace{\mathbf{G}^{x}\left(\mathbf{W}\mathbf{G}^{x}\right)^{-1}}_{\mathbf{G}^{x*}}\mathbf{n}^{c} \qquad (18)$$

One important point to be discussed in this session is:

- What is the optimal choice of $\mathbf{W}$ that minimizes the effect of the disturbances in the states, i.e., minimizes the value of $\mathbf{G}_{d}^{x^*}$ in Equation 18?

If we assume that the matrix $\mathbf{W}$ is equal to the transpose of $\mathbf{G}^{x}$ ($\mathbf{W} = \mathbf{G}^{x^T}$), then we will have:

$$\Delta \mathbf{x} = \left(\mathbf{G}_{d}^{x} - \mathbf{G}^{x}\left(\mathbf{G}^{x^T}\mathbf{G}^{x}\right)^{-1}\mathbf{G}^{x^T}\mathbf{G}_{d}^{x}\right)\Delta \mathbf{d} - \mathbf{G}^{x}\left(\mathbf{G}^{x^T}\mathbf{G}^{x}\right)^{-1}\mathbf{n}^{c} \qquad (19)$$

The matrix $\mathbf{G}^{x}\left(\mathbf{G}^{x^T}\mathbf{G}^{x}\right)^{-1}\mathbf{G}^{x^T}$ in Equation 19 is called projection matrix (Strang, 1980). It means that the product $\mathbf{G}^{x}\left(\mathbf{G}^{x^T}\mathbf{G}^{x}\right)^{-1}\mathbf{G}^{x^T}\mathbf{G}_{d}^{x}$ is the closest point to $\mathbf{G}_{d}^{x}$, i.e., there isn't any other matrix $\mathbf{W}$ that can result in a smaller value of $\mathbf{G}_{d}^{x*}$ than $\mathbf{W} = \mathbf{G}^{x^T}$. Then we conclude that the choice of $\mathbf{W} = \mathbf{G}^{x^T}$ gives us the minimum value of $\mathbf{G}_{d}^{x*}$. This will be demonstrated in the example below. But it is important to notice that this is not the only optimum choice, any matrix $\mathbf{W} = \mathbf{P}_{x}\mathbf{G}^{x^T}$ is an optimum solution, where $\mathbf{P}_{x}$ is any non-singular square matrix with apropriated dimensions.

The choice of $\mathbf{W} = \mathbf{G}^{x^T}$ is optimum for any choice of $\mathbf{P}_{c0}$ non-singular, i.e., this result is not restricted to $\mathbf{P}_{c0} = \mathbf{I}$.

It is important to notice that the matrix $\mathbf{W}$ can be arbitrarily chosen by the designer according to the objective of the process. For example, he can choose to make a combination of only some states or use all of them.

Another important point to discuss is if the choice of $\mathbf{W} = \mathbf{G}^{x^T}$ gives the minimum value of $\left\|\mathbf{G}^{x^*}\right\|$. From Equation 19 we see that the matrix $\mathbf{G}^{x}\left(\mathbf{G}^{x^T}\mathbf{G}^{x}\right)^{-1}$ is the right pseudoinverse of $\mathbf{G}^{x}$. It means that this is the solution of the problem $\mathbf{G}^{x}\Delta\mathbf{x}-\mathbf{n}^{c}=\mathbf{0}$ (Strang, 1980).

4.APPLICATION TO DISTILLATION

The proposed theory is applied to a distillation column with 82 states (41 compositions and 41 levels). As the levels don't have steady state effect, we considered that the objective function is a combination of the compositions only. This example has, after stabilization, 2 remaining manipulated variables (reflux flow rate (L) and vapor boilup (V)), and 2 disturbances (feed flow rate (F) and fraction of liquid in the feed (q_F)). Having 2 manipulated variables, we are able to control perfectly 2 combinations of the states. The measurements ($\Delta\mathbf{y}$) are the flow rates (L, V, D, and B). The variables were scaled according to Skogestad and Postlethwaite (1996).

In this example we compared the effect of the disturbances in the states using, as primary variables, 4 different combinations of states (4 different matrices $\mathbf{W}$). The combinations used were:

- Combination 1: The matrix $\mathbf{W}$ was selected in order to select the bottom and top compositions as primary variables. This is the most common choice in distillation studies.
- Combination 2: The matrix $\mathbf{W}$ was selected in order to separate the column in the following way. The compositions below the feed stage (feed stage composition included) were combined in one primary variable (with all weights equal to 1) and the compositions above the feed stage were combined in the other primary variable (also with all weights equal to 1).
- Combination 3: The matrix $\mathbf{W}$ was selected as being the transpose of $\mathbf{G}^{x}$ ($\mathbf{W}=\mathbf{G}^{x^{T}}$).
- Combination 4: The matrix $\mathbf{W}$ was calculated by the following minimization problem:

$$\min_{\mathbf{W}}\left\|\left[\mathbf{G}^{x^*}\quad \mathbf{G}_d^{x^*}\right]\right\|_2 \qquad (20)$$

where $\mathbf{G}^{x^*}$ and $\mathbf{G}_d^{x^*}$ represent the effects of the noise and the disturbances in the states, respectively (see Equation 18).

For each combination we calculated the best combination of measurements (calculated the matrix $\mathbf{H}$) using Equation 14. Then, finally we calculated the matrices $\mathbf{G}^{x^*}$ and $\mathbf{G}_d^{x^*}$. The values of the 2-norm $\left\|\mathbf{G}^{x^*}\right\|$, $\left\|\mathbf{G}_d^{x^*}\right\|$, and $\left\|\left[\mathbf{G}^{x^*}\quad \mathbf{G}_d^{x^*}\right]\right\|$ are presented in Table 1.

Table 1 - Values of $\left\|\mathbf{G}^{x^*}\right\|$, $\left\|\mathbf{G}_d^{x^*}\right\|$, and $\left\|\left[\mathbf{G}^{x^*}\quad \mathbf{G}_d^{x^*}\right]\right\|$ for all 4 combinations.

	$\left\|\mathbf{G}^{x^*}\right\|$	$\left\|\mathbf{G}_d^{x^*}\right\|$	$\left\|\left[\mathbf{G}^{x^*}\quad \mathbf{G}_d^{x^*}\right]\right\|$
1	48.8289	2.5182	48.8817
2	0.2548	1.1070	1.1080
3	0.0252	1.0886	1.0886
4	0.2560	1.0886	1.0886

Although the choice of the top and bottom compositions as primary variables (case 1) is able to control perfectly these two variables (the resulting closed-loop gains that relate the disturbances to the bottom and top compositions are zero), the closed-loop gains of the states in the middle of the column are very large (above 0.7) (see Table 2). And also this choice doesn't give good rejection of the implementation error (see matrix $\mathbf{G}^{x^*}$ in Table 2).

As expected (see session 4), the results presented in Table 1 confirm that the use of $\mathbf{W} = \mathbf{G}^{x^T}$ as the combination of variables is an optimum choice (it has the same value of $\left\| \left[\mathbf{G}^{x^*} \quad \mathbf{G}_d^{x^*} \right] \right\|$ as obtained by optimization).

Table 2 - Values of the matrices $\mathbf{G}^{x^*}$ and $\mathbf{G}_d^{x^*}$ for the four combinations.

Combination 1		Combination 2		Combination 3		Combination 4	
$\mathbf{G}^{x^*}$	$\mathbf{G}_d^{x^*}$	$\mathbf{G}^{x^*}$	$\mathbf{G}_d^{x^*}$	$\mathbf{G}^{x^*}$	$\mathbf{G}_d^{x^*}$	$\mathbf{G}^{x^*}$	$\mathbf{G}_d^{x^*}$
1.0000 0	-0.0000 0	0.0071 -0.0016	0.0000 -0.0270	0.0005 0.0005	0.0000 -0.0315	0.0070 -0.0018	0.0000 -0.0315
1.4851 0.0000	-0.0000 0.0000	0.0105 -0.0024	0.0000 -0.0401	0.0007 0.0007	0.0000 -0.0468	0.0104 -0.0027	0.0000 -0.0468
2.0690 0.0104	-0.0000 0.0015	0.0146 -0.0033	0.0000 -0.0547	0.0009 0.0009	0.0000 -0.0641	0.0145 -0.0037	0.0000 -0.0641
2.7643 0.0363	-0.0000 0.0053	0.0195 -0.0042	0.0000 -0.0705	0.0012 0.0012	0.0000 -0.0831	0.0193 -0.0048	0.0000 -0.0831
3.5807 0.0840	-0.0000 0.0124	0.0251 -0.0052	0.0000 -0.0872	0.0016 0.0016	0.0000 -0.1035	0.0250 -0.0060	0.0000 -0.1035
4.5229 0.1615	-0.0000 0.0237	0.0317 -0.0062	0.0000 -0.1038	0.0020 0.0020	0.0000 -0.1244	0.0314 -0.0072	0.0000 -0.1244
5.5860 0.2779	-0.0000 0.0409	0.0389 -0.0071	0.0000 -0.1193	0.0024 0.0024	0.0000 -0.1449	0.0387 -0.0084	0.0000 -0.1449
6.7521 0.4430	-0.0000 0.0651	0.0469 -0.0078	0.0000 -0.1320	0.0029 0.0029	0.0000 -0.1630	0.0465 -0.0094	0.0000 -0.1630
7.9849 0.6663	-0.0001 0.0980	0.0552 -0.0083	0.0000 -0.1399	0.0033 0.0033	0.0000 -0.1767	0.0547 -0.0102	0.0000 -0.1767
9.2268 0.9551	-0.0001 0.1405	0.0634 -0.0084	0.0000 -0.1405	0.0038 0.0038	0.0000 -0.1833	0.0629 -0.0106	0.0000 -0.1833
10.3980 1.3125	-0.0001 0.1930	0.0710 -0.0078	0.0000 -0.1315	0.0042 0.0042	0.0000 -0.1799	0.0704 -0.0104	0.0000 -0.1799
11.4015 1.7346	-0.0001 0.2551	0.0773 -0.0066	-0.0000 -0.1106	0.0045 0.0044	0.0000 -0.1639	0.0766 -0.0095	0.0000 -0.1639
12.1331 2.2095	-0.0001 0.3249	0.0816 -0.0045	-0.0000 -0.0763	0.0046 0.0046	0.0000 -0.1334	0.0808 -0.0077	-0.0000 -0.1334
12.4987 2.7164	-0.0001 0.3994	0.0832 -0.0017	-0.0000 -0.0285	0.0046 0.0045	-0.0000 -0.0877	0.0824 -0.0051	-0.0000 -0.0877
12.4326 3.2275	-0.0001 0.4746	0.0818 0.0019	-0.0000 0.0314	0.0043 0.0043	-0.0000 -0.0279	0.0809 -0.0016	-0.0000 -0.0279
11.9136 3.7119	-0.0001 0.5458	0.0772 0.0060	-0.0000 0.1005	0.0039 0.0039	-0.0000 0.0431	0.0763 0.0025	-0.0000 0.0431
10.9721 4.1409	-0.0001 0.6089	0.0697 0.0104	-0.0000 0.1748	0.0033 0.0033	-0.0000 0.1213	0.0688 0.0070	-0.0000 0.1213
9.6844 4.4928	-0.0001 0.6606	0.0600 0.0148	-0.0000 0.2496	0.0026 0.0025	-0.0000 0.2016	0.0591 0.0117	-0.0000 0.2016
8.1582 4.7557	-0.0001 0.6993	0.0487 0.0191	-0.0000 0.3207	0.0018 0.0017	-0.0000 0.2794	0.0478 0.0162	-0.0000 0.2794
6.5112 4.9285	-0.0001 0.7247	0.0367 0.0229	-0.0000 0.3849	0.0010 0.0009	-0.0000 0.3509	0.0359 0.0203	-0.0000 0.3509
4.8528 5.0192	-0.0001 0.7381	0.0249 0.0262	-0.0000 0.4400	0.0002 0.0001	-0.0000 0.4134	0.0241 0.0239	-0.0000 0.4134
4.8185 6.6327	-0.0001 0.7328	0.0216 0.0371	-0.0000 0.3820	-0.0005 -0.0005	-0.0000 0.3541	0.0206 0.0345	-0.0000 0.3541
4.7126 8.2966	-0.0001 0.7167	0.0177 0.0486	-0.0000 0.3134	-0.0012 -0.0013	-0.0000 0.2845	0.0166 0.0456	-0.0000 0.2845
4.5197 9.8952	-0.0001 0.6874	0.0133 0.0597	-0.0000 0.2361	-0.0020 -0.0020	-0.0000 0.2066	0.0120 0.0564	-0.0000 0.2066
4.2315 11.2891	-0.0001 0.6436	0.0087 0.0696	-0.0000 0.1536	-0.0027 -0.0027	-0.0000 0.1242	0.0072 0.0660	-0.0000 0.1242
3.8513 12.3376	-0.0001 0.5857	0.0040 0.0773	-0.0000 0.0712	-0.0033 -0.0033	-0.0000 0.0426	0.0025 0.0735	-0.0000 0.0426
3.3956 12.9284	-0.0001 0.5164	-0.0003 0.0820	-0.0000 -0.0055	-0.0037 -0.0037	-0.0000 -0.0325	-0.0019 0.0782	-0.0000 -0.0325
2.8924 13.0048	-0.0001 0.4399	-0.0040 0.0833	-0.0000 -0.0709	-0.0040 -0.0040	0.0000 -0.0958	-0.0056 0.0797	-0.0000 -0.0958
2.3761 12.5800	-0.0001 0.3614	-0.0069 0.0813	0.0000 -0.1214	-0.0040 -0.0041	0.0000 -0.1435	-0.0084 0.0778	0.0000 -0.1435
1.8807 11.7311	-0.0001 0.2860	-0.0088 0.0763	0.0000 -0.1551	-0.0039 -0.0039	0.0000 -0.1742	-0.0102 0.0732	0.0000 -0.1742
1.4335 10.5778	-0.0001 0.2180	-0.0098 0.0692	0.0000 -0.1727	-0.0036 -0.0037	0.0000 -0.1887	-0.0110 0.0665	0.0000 -0.1887
1.0517 9.2533	-0.0001 0.1600	-0.0100 0.0609	0.0000 -0.1763	-0.0033 -0.0033	0.0000 -0.1895	-0.0110 0.0585	0.0000 -0.1895
0.7418 7.8796	-0.0001 0.1128	-0.0096 0.0521	0.0000 -0.1694	-0.0029 -0.0029	0.0000 -0.1799	-0.0105 0.0501	0.0000 -0.1799
0.5017 6.5517	-0.0000 0.0763	-0.0088 0.0435	0.0000 -0.1552	-0.0024 -0.0024	0.0000 -0.1634	-0.0095 0.0419	0.0000 -0.1634
0.3237 5.3327	-0.0000 0.0492	-0.0077 0.0356	0.0000 -0.1370	-0.0020 -0.0020	0.0000 -0.1432	-0.0083 0.0342	0.0000 -0.1432
0.1972 4.2566	-0.0000 0.0300	-0.0066 0.0285	0.0000 -0.1170	-0.0016 -0.0016	0.0000 -0.1217	-0.0071 0.0274	0.0000 -0.1217
0.1115 3.3346	-0.0000 0.0170	-0.0055 0.0224	0.0000 -0.0970	-0.0013 -0.0013	0.0000 -0.1005	-0.0059 0.0216	0.0000 -0.1005
0.0565 2.5628	-0.0000 0.0086	-0.0044 0.0172	0.0000 -0.0782	-0.0010 -0.0010	0.0000 -0.0808	-0.0047 0.0166	0.0000 -0.0808
0.0238 1.9280	-0.0000 0.0036	-0.0035 0.0130	0.0000 -0.0612	-0.0008 -0.0008	0.0000 -0.0630	-0.0037 0.0125	0.0000 -0.0630
0.0067 1.4132	-0.0000 0.0010	-0.0026 0.0095	0.0000 -0.0462	-0.0006 -0.0006	0.0000 -0.0475	-0.0028 0.0092	0.0000 -0.0475
0 1.0000	-0.0000 -0.0000	-0.0019 0.0068	0.0000 -0.0333	-0.0004 -0.0004	0.0000 -0.0342	-0.0020 0.0065	0.0000 -0.0342

Another advantage of this choice is that it reduces the effect of the implementation error (reduces the norm $\left\| \mathbf{G}^{x^*} \right\|$, see Table 1) in the states. As we can see in Table 2, combinations 3 ($\mathbf{W} = \mathbf{G}^{x^T}$) and 4 (obtained by optimization) are equivalent in relation to matrix $\mathbf{G}_d^{x^*}$, the matrices are exactly the same in both cases. But when we analyze $\mathbf{G}^{x^*}$ separately we see that the use of $\mathbf{W} = \mathbf{G}^{x^T}$ gives us a better result. The reason is that in the optimization we are only interested in the norm $\left\| \left[\mathbf{G}^{x^*} \quad \mathbf{G}_d^{x^*} \right] \right\|$ and, in this case, the norm of the matrix $\mathbf{G}_d^{x^*}$ is much more important than the matrix $\mathbf{G}^{x^*}$. This can be easily seen when we analyze Table 1 more carefully. Although the value of $\left\| \mathbf{G}^{x^*} \right\|$, for combination 4, is quite large (0.2560), the value of $\left\| \left[\mathbf{G}^{x^*} \quad \mathbf{G}_d^{x^*} \right] \right\|$ is almost the same as the value of $\left\| \mathbf{G}_d^{x^*} \right\|$ (it is important to emphasize that although the values of $\left\| \mathbf{G}_d^{x^*} \right\|$ and $\left\| \left[\mathbf{G}^{x^*} \quad \mathbf{G}_d^{x^*} \right] \right\|$ presented in Table 1 for combinations 3 and 4 are the same, in reality the values of $\left\| \left[\mathbf{G}^{x^*} \quad \mathbf{G}_d^{x^*} \right] \right\|$ are slightly larger than $\left\|\mathbf{G}_d^{x*}\right\|$, the difference does not appear due to the approximation that was done).

As we can see in Table 2, the choice of $\mathbf{W} = \mathbf{G}^{x^T}$ doesn't give perfect control for the top and bottom compositions, but it reduces the sensitivity of the states in the middle of the column (about 0.4) to variations in the disturbances. This point is important to avoid the effects of non-linearities in the process.

5.CONCLUSIONS

In this paper we showed that it is possible to control perfectly (having perfect disturbance rejection and minimizing the implementation error effects) any combination of the states if we have enough measurements available.

Therefore, it is shown the importance of the use of the combination of states as primary variables. Although the choice of the top and bottom compositions of a distillation column is good to reject perfectly the disturbances, it fails in the rejection of the implementation error and also it doesn't give a good control of the states in the middle of the column.

The choice of $\mathbf{W} = \mathbf{G}^{x^T}$ proved to be the best choice if the objective is to keep the states as close as possible to their desired (nominal) values. It rejects very well both disturbances and implementation errors, although it doesn't give perfect control of the top and bottom compositions.

6.REFERENCES

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7.ACKNOWLEDGMENTS

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