Self-optimizing control: Optimal measurement selection

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1 Introduction

- **Self-optimizing control is when, using a constant set-point feedback policy, acceptable economic operation can be achieved in spite of external disturbances and measurements errors.**
- The key in self-optimizing control is to select the feedback controlled variables c.
- **The null space method is a systematic method for finding good self-optimizing variables** (Alstad and Skogestad, 2002, 2004).
- Optimal operation (steady-state):

$$\min_{\mathbf{x},\mathbf{u_0}} J_0(\mathbf{x},\mathbf{u_0},\mathbf{d})$$
$$\text{s.t.}\quad \mathbf{f}(\mathbf{x},\mathbf{u_0},\mathbf{d}) = 0, \qquad \mathbf{g}(\mathbf{x},\mathbf{u_0},\mathbf{d}) \le 0$$

  - J_0 is the scalar economic objective.
  - $\mathbf{x}$ the states.
  - $\mathbf{u}_0$ the inputs (DOF) and
  - $\mathbf{d}$ the unmeasured external disturbances.

- *Active constraints*: A subset $\mathbf{g}'$ of $\mathbf{g}(\cdot)$ active for all $\mathbf{d}$. Control the active constraints:

$$\mathbf{c_i} = \mathbf{g}'(\cdot)$$

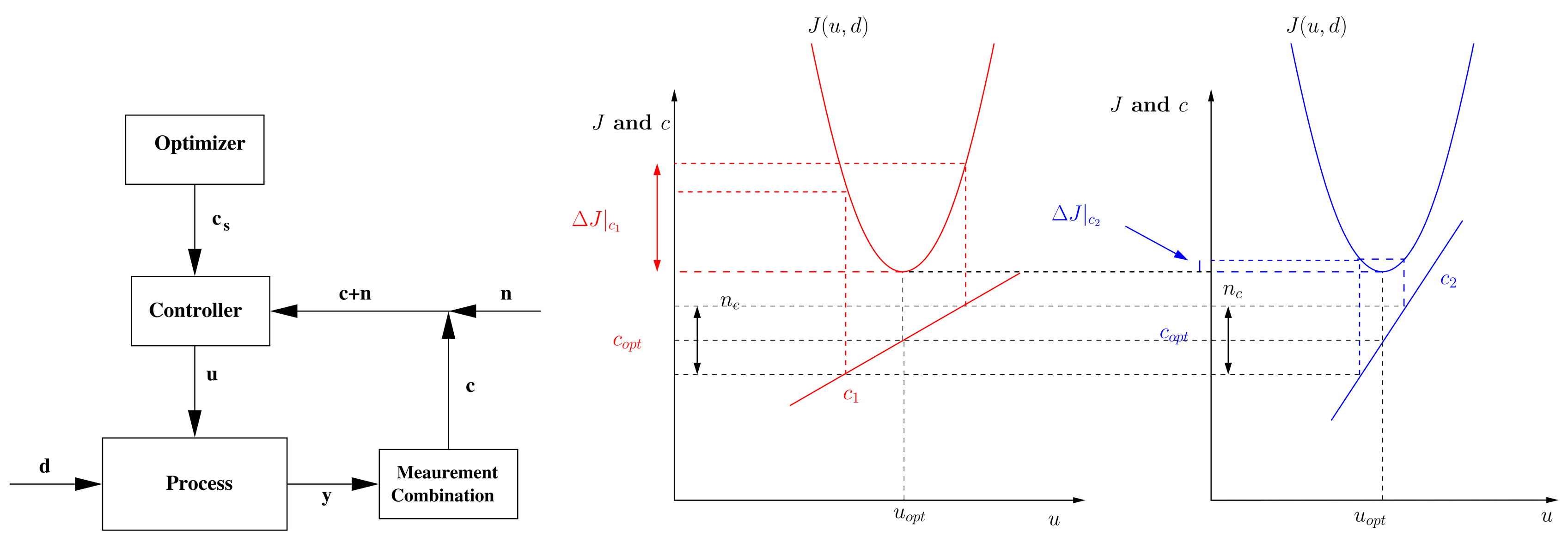
- Resulting unconstrained reduced space optimization problem:

$$\min_{\mathbf{u}} J(\mathbf{u},\mathbf{d})$$

- **Goal: Find feedback controlled variables c of the available measurements $\mathbf{y}_0$ to achieve self-optimizing control**.

1.1 Motivation

- Two sources of uncertainty in operation, see Figure 1(a):
  1. **External disturbances ($\mathbf{d}$)**: Suppress with feedback.
  2. **Measurement errors ($\mathbf{n}$)**: Always present, minimize by selecting insensitive feedback variables, see Figure 1(b).



(a) Feedback control loop

(b) Effect of implementation error on loss: two candidate controlled variables, labeled c_1 and c_2 where $|G_2| > |G_1|$ and $\Delta c_i = G_i \Delta u$

- Two different controlled variables candidates c_1 and c_2 (Figure 1(b)).
- The effect of the implementation error larger for c_1 than c_2 due to lower gain $|G_1| < |G_2|$. Select controlled variables with large gains (Halvorsen et al., 2003).

1.2 Taylor series expansion of the loss function

- A second order accurate expression of the loss function:

$$L = J(\mathbf{c}_s + \mathbf{n}, \mathbf{d}) - J^{opt}(\mathbf{d})$$

where:
  - L is a scalar loss function.
  - $J(\mathbf{c}_s + \mathbf{n}, \mathbf{d})$ is the actual cost using the constant feedback policy
  - $J^{opt}(\mathbf{d})$ is the true optimal value of the objective function.
- Halvorsen et al. (2003) show that the loss is:

$$L = \frac{1}{2}\mathbf{e}_u^T \mathbf{J}_{uu} \mathbf{e}_u = \mathbf{z}^T\mathbf{z} \qquad (1)$$

$$\begin{aligned}\mathbf{z} &= \mathbf{J}_{uu}^{1/2}(\mathbf{J}_{uu}^{-1}\mathbf{J}_{ud} - \mathbf{G}^{-1}\mathbf{G}_d)\mathbf{W}_d\mathbf{d}' + \mathbf{J}_{uu}^{1/2}\mathbf{G}^{-1}\mathbf{W}_n\mathbf{n}_c' \\ &= \mathbf{M}_d\mathbf{d}' + \mathbf{M}_n\mathbf{n}_c' \\ &= [\mathbf{M}_d \quad \mathbf{M}_c]\left[\mathbf{d}' \quad \mathbf{n}_c'\right]^T\end{aligned} \qquad (2)$$

where
$\mathbf{M}_d = \mathbf{J}_{uu}^{1/2}(\mathbf{J}_{uu}^{-1}\mathbf{J}_{ud} - \mathbf{G}^{-1}\mathbf{G}_d)\mathbf{W}_d$
$\mathbf{M}_n = \mathbf{J}_{uu}^{1/2}\mathbf{G}^{-1}\mathbf{W}_n$
$\mathbf{J}_{uu}$ and $\mathbf{J}_{ud}$ the Hessian matrices
$\Delta\mathbf{c} = \mathbf{G}\Delta\mathbf{u} + \mathbf{G}_d\Delta\mathbf{d}$
$\mathbf{W}_d$ and $\mathbf{W}_n$ are scaling matrices

2 The null space method

- The null space method of Alstad and Skogestad (2004, 2002) propose to select controlled variables as functions of a subset of the measurements:

$$\Delta\mathbf{c} = \mathbf{H}\Delta\mathbf{y} \qquad \text{where} \quad \Delta\mathbf{y} = \mathbf{G}^y\Delta\mathbf{u} + \mathbf{G}_d^y\Delta\mathbf{d} \qquad (3)$$

- The matrix $\mathbf{H}$ is found by selecting the rows $\mathbf{h}_i$ of $\mathbf{H}$ to be the null vectors of the optimal sensitivity matrix $\mathbf{F}$ where

$$\Delta\mathbf{y}^{opt} = \mathbf{F}\Delta\mathbf{d} \qquad \text{where} \quad \mathbf{F} = -\left(\mathbf{G}^y\mathbf{J}_{uu}^{-1}\mathbf{J}_{ud} - \mathbf{G}_d^y\right) \qquad (4)$$

so

$$\mathbf{h}_i^T \in \mathcal{N}(\mathbf{F}^T) \qquad (5)$$

such that

$$\mathbf{HF} = 0$$

- Assumption:
  - $\mathbf{G}^y$ and $\mathbf{G}_d^y$ have full column rank.
  - The number of controlled variables (n_c) equals the number of input (n_u).
  - Implies that the number of measurements are (length of $\mathbf{y}$)

$$n_y = n_u + n_d \qquad (6)$$

for the null space of $\mathbf{F}$ to exist.

- Explicit expression for the null space matrix $\mathbf{H}$ is:

$$\mathbf{H} = \mathbf{M}_n^{-1}\mathcal{J}[\tilde{\mathbf{G}}^y]^{-1} \qquad \text{where} \quad \tilde{\mathbf{G}}^y = [\mathbf{G}^y \quad \mathbf{G}_d^y], \quad \mathcal{J} = [\mathbf{J}_{uu}^{1/2} \quad \mathbf{J}_{uu}^{1/2}\mathbf{J}_{uu}^{-1}\mathbf{J}_{ud}] \qquad (7)$$

where $\mathbf{M}_n$ is a parameter matrix and $\tilde{\mathbf{G}}^y$ assumed invertible.
- Selecting $\mathbf{H}$ as given in equation (7) implies that $\mathbf{M}_d = 0$.
- Assume that the total number of measurements $n_{y_o} > n_y$. Issues:
  1. How to select the best minimum set of measurements in order to reduce the effect of implementation error on the operational objective.
  2. How to use the null space method when using all available measurements.

3 Selection of the best set of measurements

- Select measurements such that the effect of measurement noise $\mathbf{M}_n$ is minimized, see (1).
- Noise contribution ($\mathbf{M}_d = 0$):

$$\mathbf{z} = \mathbf{M}_n\mathbf{n}_c = \mathbf{M}_n\mathbf{H}\mathbf{n}_y \qquad (8)$$

so $\mathbf{M}_n\mathbf{H}$ should be selected as small as possible.
- Now, consider the case of which we have more measurements than the minimum necessary, thus

$$n_{y_0} > n_y = n_u + n_d \qquad (9)$$

and we may use these extra measurements to minimize the effect of the measurement error on the loss. This may be achieved in two ways
  1. **Method 1**: Select the best subset of measurements from the full set of measurements.
  2. **Method 2**: Use all measurements and select the best combination.

Best minimum subset of measurements (Method 1)

- Recognizing that from equation (7) we have

$$\mathbf{M}_n\mathbf{H} = \mathcal{J}[\tilde{\mathbf{G}}^y]^{-1} \qquad (10)$$

- $\mathcal{J}$ independent of which measurement while $\tilde{\mathbf{G}}^y$ depends on the measurements.
- Selection of the best minimum set of measurements y_i where $i \in \{1,..,n_y\}$ from the full set of measurements $y_{0,j}$ for $j = \{1,..,n_{y0}\}$.
- The choice of $\mathbf{M}_n$ does not influence the effect of the measurement noise (right hand side of equation (10) is a constant matrix).
- From equation (10) we get that in order to minimize the loss

$$\max_{||\mathbf{n}_y||_2\le 1} \tfrac{1}{2}\mathbf{z}^T\mathbf{z} = \max_{||\mathbf{n}_y||_2\le 1} \tfrac{1}{2}||\mathbf{z}||_2^2 = \tfrac{1}{2}\bar{\sigma}(\mathcal{J}[\tilde{\mathbf{G}}^y]^{-1})^2 \le \tfrac{1}{2}\left(\bar{\sigma}(\mathcal{J})\bar{\sigma}([\tilde{\mathbf{G}}^y]^{-1})\right)^2 = \tfrac{1}{2}\left(\bar{\sigma}(\mathcal{J})\underline{\sigma}([\tilde{\mathbf{G}}^y]^{-1})\right)^2 \qquad (11)$$

1. **Optimal**: Select measurements such that $\bar{\sigma}(\mathcal{J}[\tilde{\mathbf{G}}^y]^{-1})$ is minimized.
2. **Sub-optimal**: Select measurements y_i such that $\underline{\sigma}(\tilde{\mathbf{G}}^y)$ is maximized.

Best combination of all measurements (Method 2)

Using all measurement we replace the inverse of equation (10) by the pseudo-inverse (Moore-Penrose generalized inverse):

$$\mathbf{H} = \mathbf{M}_n^{-1}\mathcal{J}[\tilde{\mathbf{G}}_0^y]^{\dagger} \qquad (12)$$

where $[\tilde{\mathbf{G}}_0^y]^{\dagger}$ is the pseudo-left inverse of $\tilde{\mathbf{G}}_0^y$, such that the effect of the measurement error is $\mathbf{M}_n\mathbf{H} = \mathcal{J}[\tilde{\mathbf{G}}_0^y]^{\dagger}$.

Extensions: Fewer measurements ($n_y < n_u + n_d$)

Methods for reducing the dimension of the problem ($\mathbf{M}_d \neq 0$):
- Lump "similar" disturbances based on SVD of $G_d^y = \mathbf{U}_d\boldsymbol{\Sigma}_d\mathbf{V}_d^T$.

$$\Delta\mathbf{d} \approx \tilde{\mathbf{V}}_d\Delta\tilde{\mathbf{d}} \qquad (13)$$

corresponding to the large singular values $\{\sigma_1, \sigma_2, \ldots, \sigma_j\}$ with directions $\tilde{\mathbf{V}}_d = \left[v_{d,1} \ldots v_{d,\sigma_j}\right]$.
- Pseudo-right inverse as given by equation (12) above.

4 Toy example

- SISO system with one disturbance and the following objective function

$$J = (u - d)^2 \qquad (14)$$

with the nominal disturbance $d^* = 0$.
- Measurements:

$$y_1 = 0.1(u - d) \quad y_2 = 20u \quad y_3 = 10u - 5d \quad y_4 = u$$

- Optimal sensitivity matrix:

$$\Delta\mathbf{y}_0^{opt} = \mathbf{F}\Delta d = \mathbf{G}^{y\prime}\Delta u^{opt}(d) + \mathbf{G}_d^{y\prime}\Delta d = \left[0.1\ 20\ 10\ 1\right]^T\Delta d + \left[-0.1\ 0\ -5\ 0\right]^T\Delta d = \left[0\ 20\ 5\ 1\right]^T\Delta d \qquad (15)$$

- *Null space method*: Minimum number of measurements, see (6), is

$$n_y = n_u + n_d = 1 + 1 = 2$$

which results in 6 possible candidate sets of measurements to check.
- Sub-optimal rule, see Table 1: Use measurements 2 and 3 (y_2 and y_3).
- Table 2 show the worst case loss for the candidate controlled variables.
- Loss reduced with a factor of 6 using the null space method.
- Using all measurements in equation (12) the loss is marginally smaller compared to $c_{LC,4}$ due to the high implementation error of measurements y_1 and y_4 ($L_{LC}^{all} = 0.04247$).

Table 1: Minimum singular values

$c_{LC,\#}$	$y_\#$	$y_\#$	$\underline{\sigma}(\tilde{\mathbf{G}}^y)$
4	2	3	4.4490
6	3	4	0.4458
1	1	2	0.1
3	1	4	0.0995
2	1	3	0.0447
5	2	4	0

Table 2: Worst case loss

Rank	**c**	**L**
1	$c_{LC,4}$	0.0425
2	y_3	0.26
3	y_2	1.0025
4	$c_{LC,6}$	1.04
5	y_4	2
6	y_1	100
7	$c_{LC,5}$	inf

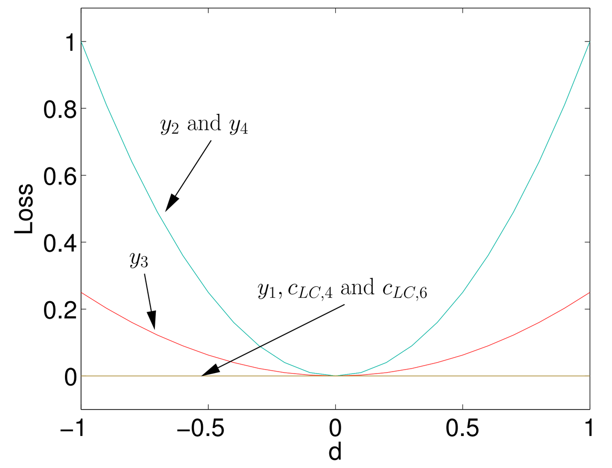


Figure 1: Loss due to disturbance d

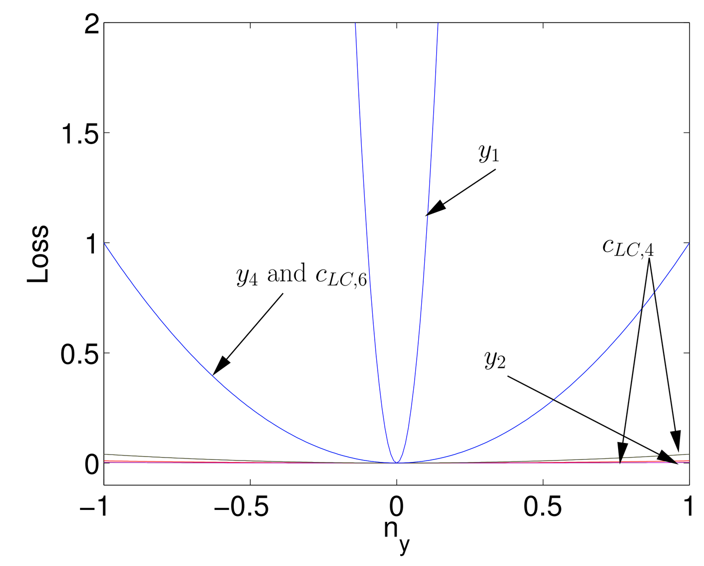


Figure 2: Loss due to measurement error n

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