System Stabilization using Minimum Energy Control

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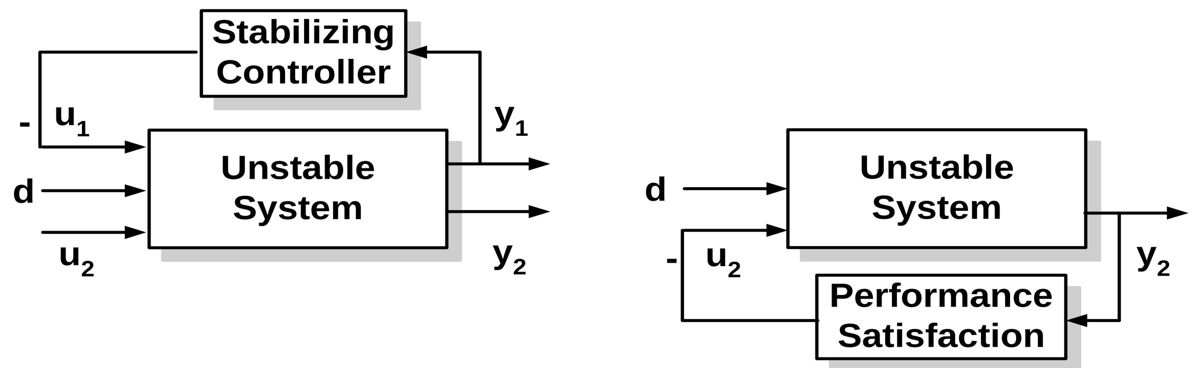
Problem Motivation

Controller design for complex unstable systems

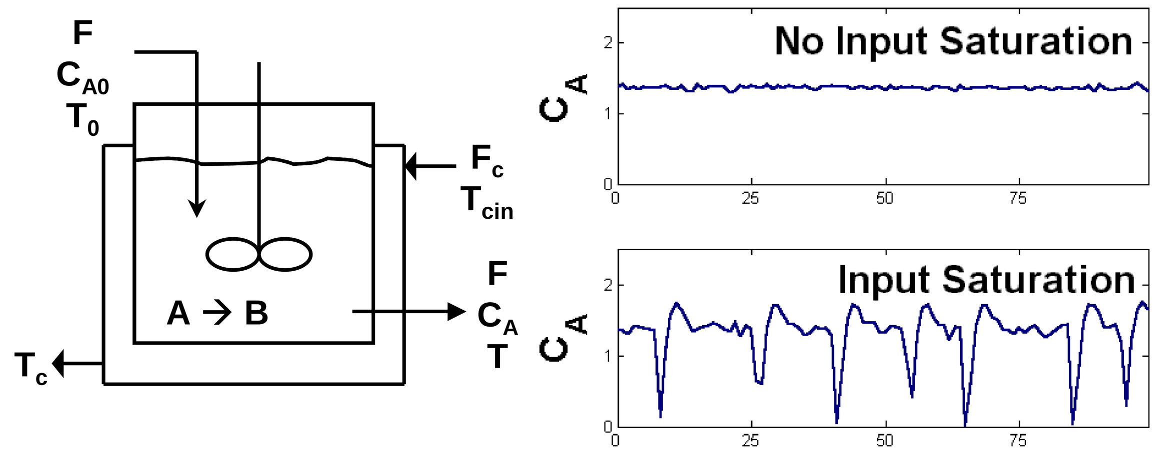


Simplified approach using division of objectives

$\mathcal{Q}$: Which outputs and inputs be used for stabilization?
$\mathcal{A}$: Choose variables which minimize input usage.

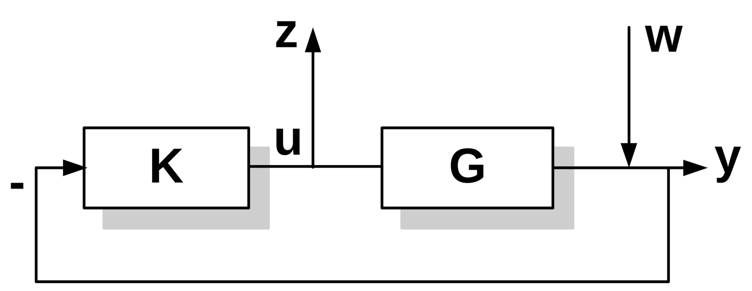
$\mathcal{Q}$: Why minimize input usage?

$\mathcal{A}$: Likelihood of input saturation is reduced
Stabilized system is least affected by stabilization layer.



Cyclic behavior of CSTR due to input saturation (Marlin, 1996)

Approach: Characterization of achievable input performance



Closed loop system

Minimize effect of disturbances on inputs

Results also useful for

- Studying interaction between design and control
- Formulation of optimal controller synthesis problem

Achievable Input Performance

Assumptions

- FDLTI system, Controllability and Observability
- Distinct unstable poles, Strictly proper system

Sensitivity function

$$\|\mathbf{KS}\|_2^{opt} = \sum_{i=1}^{n_p} \frac{2|\mathrm{Re}(\bar{\mathbf{A}}_{ii})|}{\sigma_{Hi}^2(\mathcal{U}[\mathbf{G}]^*)}$$

$$\|\mathbf{KS}\|_\infty^{opt} = \underline{\sigma}_H^{-1}(\mathcal{U}[\mathbf{G}]^*)$$

Unstable part

Hankel singular values

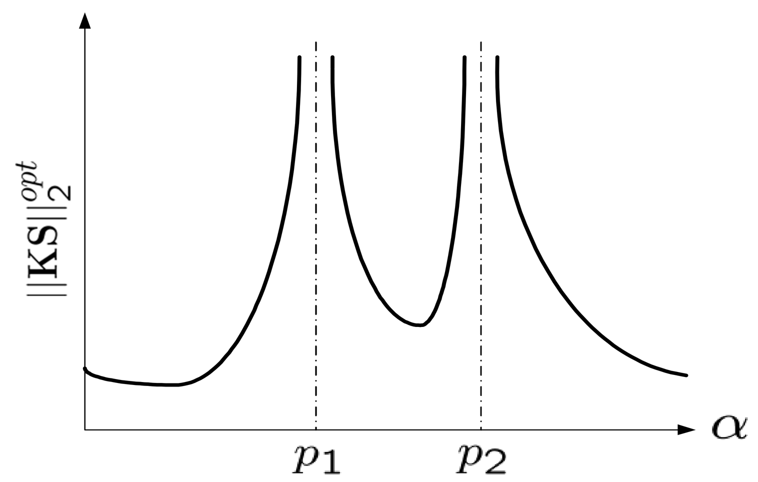
State matrix of balanced realization of $\mathcal{U}[\mathbf{G}]$

Similar results - Time delay systems, Colored noise

Limiting Factors

$$\mathbf{G} = \frac{(s-\alpha)}{(s-p_1)(s-p_2)}$$

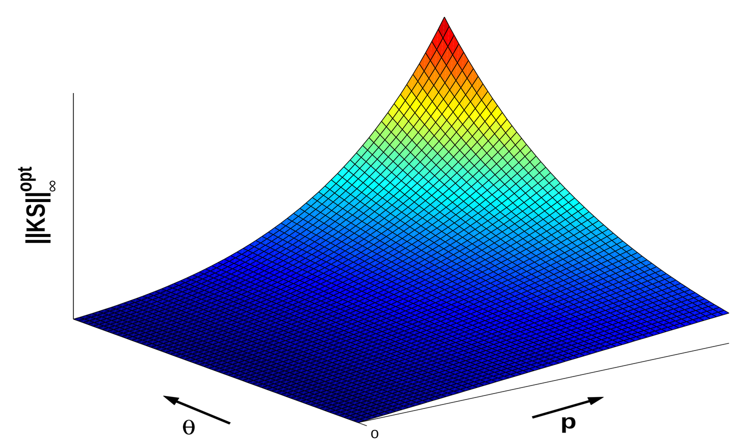
$$\|\mathbf{KS}\|_2^{opt} \propto \frac{(\alpha^2 - f(p_1,p_2))^{0.5}}{|(p_1-\alpha)(p_2-\alpha)|}$$



Effect of pole-zero location

Obstacles to detectability and stabilizability

$\Rightarrow$ Poorly separated (oriented) unstable poles and zeros



Effect of time delay

$$\mathbf{G} = \frac{e^{-\theta s}}{(s-p)}$$

$$\|\mathbf{KS}\|_\infty^{opt} = 2pe^{p\theta}$$

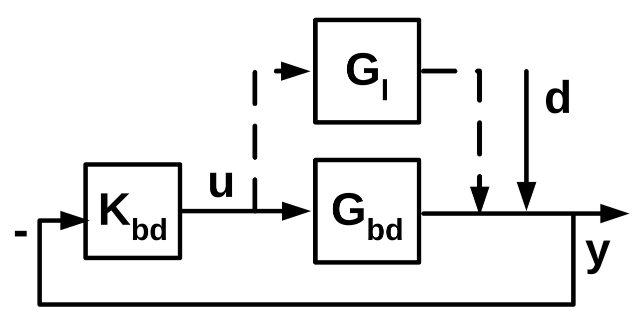
The slower the instabilities

$\Rightarrow$ The lesser is the limitation imposed by time delay

Decentralized Stabilization

$\mathcal{Q}$: Stability with independent designs of loops - feasible?
$\mathcal{A}$: If μ interaction condition is satisfied.

$\mathbf{G} = \mathbf{G}_{bd} + \mathbf{G}_I$ off-diagonal elements



Philosophy of μ-IM

$\mathbf{G}_I$ treated as uncertainty

$\mathbf{G}^{bd}$, $\mathbf{G}$ - same unstable poles

Limited to stable systems

Modified μ Interaction Measure

- Allow $\mathbf{G}_{bd}$ to be different than the diagonal elements of $\mathbf{G}$
- Treat excess poles also as uncertainty

When input performance of each loop is maximized

Hankel singular value

$$\|\mathbf{KS}\|_\infty^{opt} \leq |\underline{\sigma}_H(\mathcal{U}[\mathbf{G}_{bd}]) - \|\mathbf{G}_I\|_\infty|^{-1}$$

Unstable part

Variable selection

Optimal combination depends on choice of norm.

$\mathcal{H}_\infty$ norm addresses input saturation closely (preferred)

Tennessee Eastman Process (base case)

Havre's recommendation - Avoid using feed streams

CV	MV	$\|\mathbf{KS}\|_\infty^{opt}$
y_{22}	u_{10}	0.11
y_{21}	u_8,u_{11}	0.077
y_{12},y_{21}	u_{10}	0.0235
y_{12},y_{21}	u_{10},u_{11}	0.0222

Alternatives for stabilization using MIMO controller

Trade off between number of variables used and input usage