PROBABLY
THE BEST SIMPLE PID TUNING RULES
IN THE WORLD

Sigurd Skogestad
Norwegian University of Science and Technology (NTNU)
N-7491 Trondheim, Norway

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Objective:

- Present analytic tuning rules which are as simple as possible and still result in good closed-loop behavior.

Starting point:

- IMC PID tuning rules of Rivera, Morari and Skogestad (1986)

New SIMC tuning method:

- Integral term modified to improve disturbance rejection for integrating processes.
- Any process is approximated as first-order plus delay processes using "half method"
- One single tuning rule – easily memorized!

PROCESS INFORMATION

- Plant gain, k
- Dominant time constant, τ_1
- Effective time delay, θ
- Second-order time constant, τ_2 (use only for dominant second-order process with $\tau_2 > \theta$, approximately)

For slow (integrating processes):

- Slope, $k' \stackrel{\text{def}}{=} k/\tau_1$

Resulting model:

$$g(s) = \frac{k}{(\tau_1 s + 1)(\tau_2 s + 1)} e^{-\theta s} = \frac{k'}{(s + 1/\tau_1)(\tau_2 s + 1)} e^{-\theta s}$$

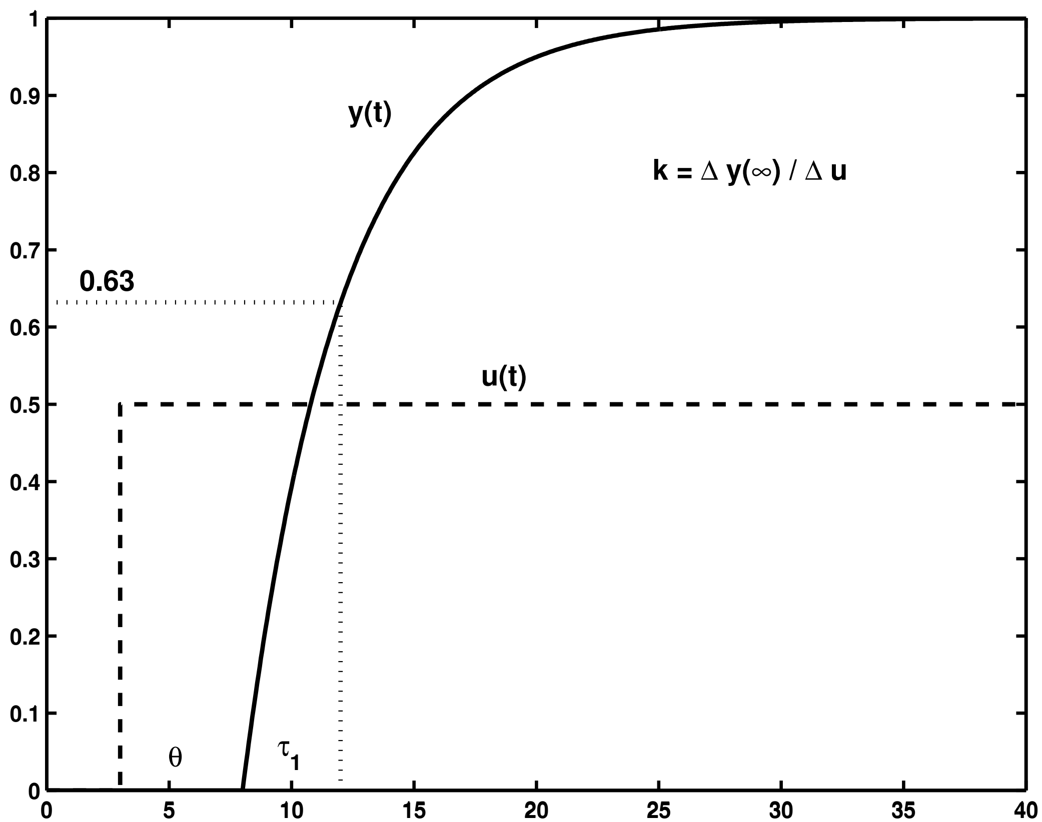


Figure 1: Step response of first-order with delay system, $g(s) = ke^{-\theta s}/(\tau_1 s + 1)$.

OBTAINING THE EFFECTIVE DELAY θ

Basis (Taylor approximation):

$$e^{-\theta s} \approx 1 - \theta s \quad \text{and} \quad e^{-\theta s} = \frac{1}{e^{\theta s}} \approx \frac{1}{1 + \theta s}$$

Effective delay =

"true" delay

+ inverse reponse time constant(s)

+ half of the largest neglected time constant (the "half rule")
(this is to avoid being too conservative)

+ all smaller high-order time constants

The "other half" of the largest neglected time constant is added to τ_1 (or to τ_2 if use second-order model).

Example

$$g_0(s) = k\frac{(-0.3s+1)(0.08s+1)}{(2s+1)(1s+1)(0.4s+1)(0.2s+1)(0.05s+1)^3}$$

is approximated as a first-order delay process with

$\tau_1 = 2 + 1/2 = 2.5$

$\theta = 1/2 + 0.4 + 0.2 + 3 \cdot 0.05 + 0.3 - 0.08 = 1.47$

or as a second-order delay process with

$\tau_1 = 2$

$\tau_2 = 1 + 0.4/2 = 1.2$

$\theta = 0.4/2 + 0.2 + 3 \cdot 0.05 + 0.3 - 0.08 = 0.77$

IMC TUNING = DIRECT SYNTHESIS

- Controller: $c(s) = \frac{1}{g(s)} \cdot \frac{1}{\frac{1}{(y/y_s)_{\text{desired}}} - 1}$
- Consider second-order with delay plant: $g(s) = k\frac{e^{-\theta s}}{(\tau_1 s+1)(\tau_2 s+1)}$
- Desired first-order setpoint response: $\left(\frac{y}{y_s}\right)_{\text{desired}} = \frac{1}{\tau_c s+1}e^{-\theta s}$
- Gives a "Smith Predictor" controller: $c(s) = \frac{(\tau_1 s+1)(\tau_2 s+1)}{k}\frac{1}{(\tau_c s+1-e^{-\theta s})}$
- To get a PID-controller use $e^{-\theta s} \approx 1 - \theta s$ and derive

$$c(s) = \frac{(\tau_1 s + 1)(\tau_2 s + 1)}{k}\frac{1}{(\tau_c + \theta)s}$$

  which is a cascade form PID-controller with

$$K_c = \frac{1}{k}\frac{\tau_1}{\tau_c + \theta}; \quad \tau_I = \tau_1; \quad \tau_D = \tau_2$$

- τ_c is the sole tuning parameter

INTEGRAL TIME

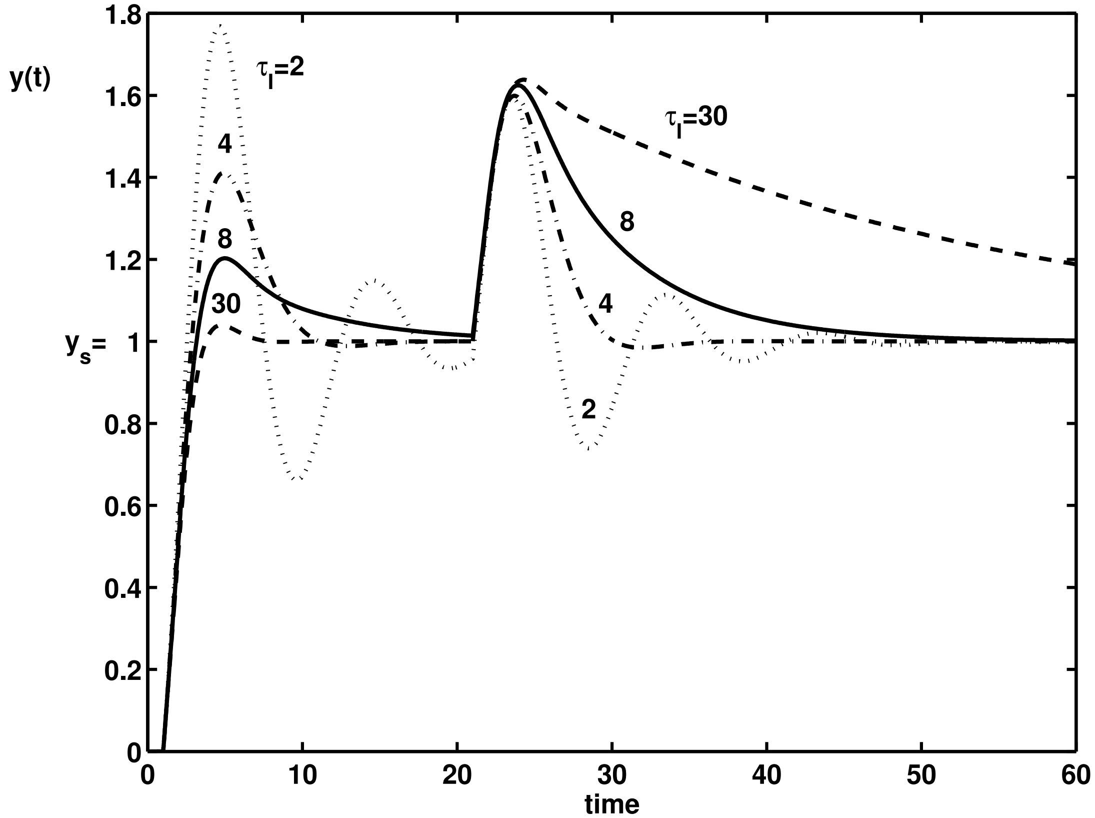


Figure 2: Effect of changing the integral time τ_I for PI-control of "slow" process $g(s) = e^{-s}/(30s+1)$ with $K_c = 15$.
Load disturbance of magnitude 10 occurs at $t = 20$.

Too large integral time: Poor disturbance rejection
Too small integral time: Slow oscillations

SIMC-PID TUNING RULES

For cascade form PID controller:

$$K_c = \frac{1}{k}\frac{\tau_1}{\tau_c + \theta} = \frac{1}{k'} \cdot \frac{1}{\tau_c + \theta} \tag{1}$$

$$\tau_I = \min\{\tau_1, \frac{4}{k'\ K_c}\} = \min\{\tau_1, 4(\tau_c + \theta)\} \tag{2}$$

$$\tau_D = \tau_2 \tag{3}$$

Derivation:

1. First-order setpoint response with response time τ_c (IMC-tuning = "Direct synthesis")
2. Reduce integral time to get better disturbance rejection for slow or integrating process (but avoid slow cycling $\Rightarrow \tau_I \geq \frac{4}{k'\ K_c}$)

TUNING FOR FAST RESPONSE WITH GOOD ROBUSTNESS

$$\text{SIMC}: \quad \tau_c = \theta \tag{4}$$

Gives:

$$K_c = \frac{0.5}{k}\frac{\tau_1}{\theta} = \frac{0.5}{k'} \cdot \frac{1}{\theta} \tag{5}$$

$$\tau_I = \min\{\tau_1, 8\theta\} \tag{6}$$

$$\tau_D = \tau_2 \tag{7}$$

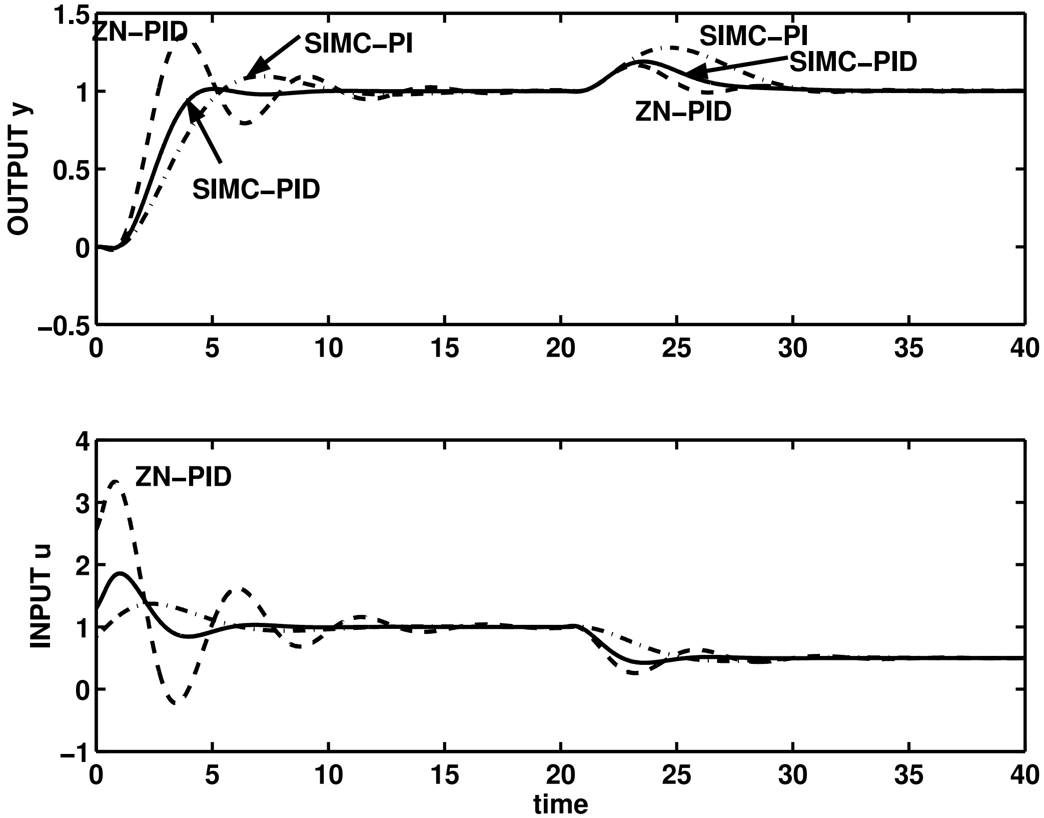
Try to memorize!

Gain margin about 3

Process $g(s)$	$\frac{k}{\tau_1 s+1}e^{-\theta s}$	$\frac{k'}{s}e^{-\theta s}$
Controller gain, K_c	$\frac{0.5}{k}\frac{\tau_1}{\theta}$	$\frac{0.5}{k'}\frac{1}{\theta}$
Integral time, τ_I	τ_1	8θ
Gain margin (GM)	3.14	2.96
Phase margin (PM)	61.4^o	46.9^o
Allowed time delay error, $\Delta\theta/\theta$	2.14	1.59
Sensitivity peak, M_s	1.59	1.70
Complementary sensitivity peak, M_t	1.00	1.30
Phase crossover frequency, $\omega_{180} \cdot \theta$	1.57	1.49
Gain crossover frequency, $\omega_c \cdot \theta$	0.50	0.51

Table 1: Robustness margins for first-order and integrating delay process using SIMC-tunings in (5) and (6) ($\tau_c = \theta$). The same margins apply to second-order processes if we choose $\tau_D = \tau_2$.

EXAMPLE



$$g_0(s) = k \frac{(-0.3s+1)(0.08s+1)}{(2s+1)(1s+1)(0.4s+1)(0.2s+1)(0.05s+1)^3}$$

EXAMPLE: Process from Astrom et al. (1998)

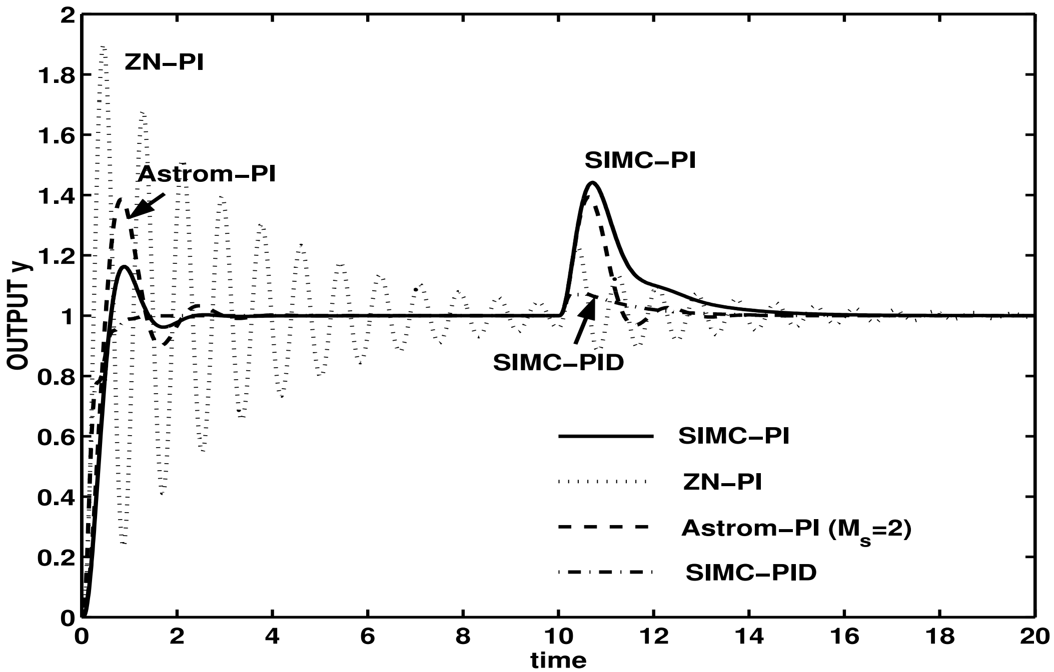


Figure 3: Load disturbance of magnitude 2 occurs at $t = 10$.

$$g_0(s) = \frac{1}{(s+1)(0.2s+1)(0.04s+1)(0.008s+1)}$$

APPLICATION: RETUNING FOR INTEGRATING PROCESS

To avoid "slow" oscillations the product of the controller gain and integral time should be increased by factor $f \approx 0.1(P_0/\tau_{I0})^2$.

Real Plant data:

$$\text{Period of oscillations } P_0 = 0.85h = 51min \Rightarrow f = 0.1 \cdot (51/1)^2 = 260$$

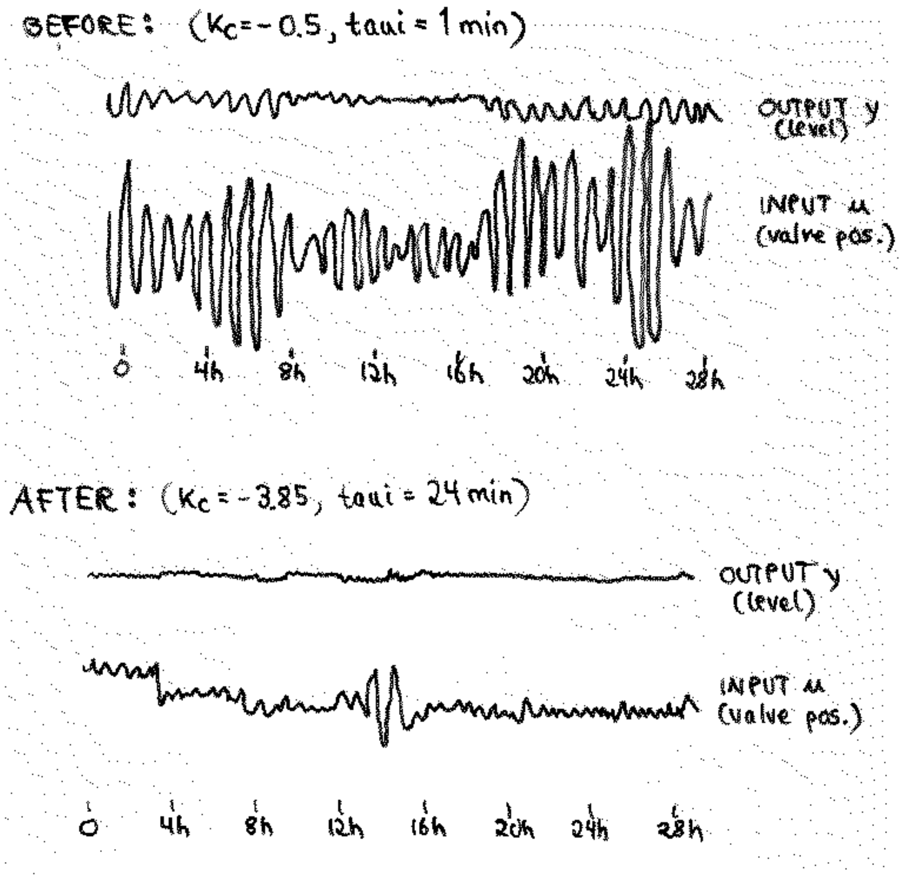


Figure 4: Industrial case study of retuning reboiler level control system

DERIVATIVE ACTION ?

First order with delay plant ($\tau_2 = 0$) with $\tau_c = \theta$:

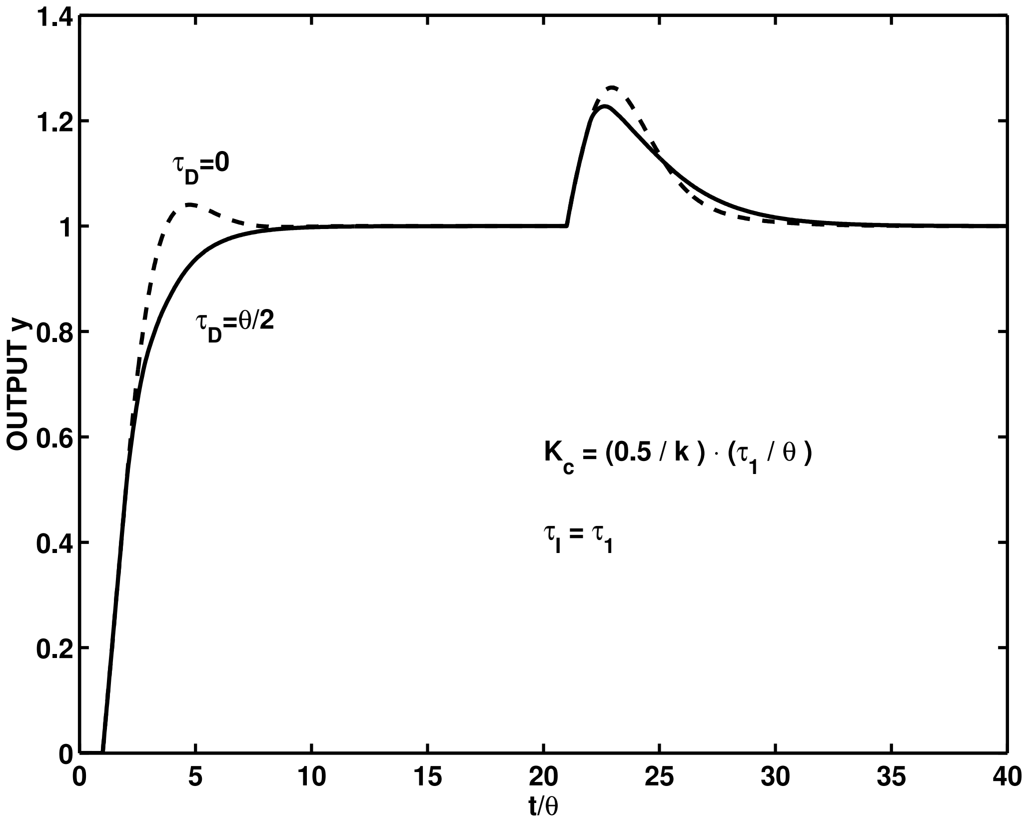


Figure 5: Setpoint change at $t = 0$. Load disturbance of magnitude 0.5 occurs at $t = 20$.

- Observe: Derivative action (solid line) has only a minor effect.
- Conclusion: Use second-order model (and derivative action) only when $\tau_2 > \theta$ (approximately)

CONCLUSION

- It is simple (one single rule for all processes)
- It is excellent for teaching (analytical)
- It works very well for all of "our" processes

Full paper with many additional examples available at:

http://www.chembio.ntnu.no/users/skoge/publications/2001/tuningpaper_reno/