

# Evaluation of self-optimising control structures for an integrated Petlyuk distillation column

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## ABSTRACT

In a Petlyuk distillation column, two extra degrees of freedom can be used for optimisation purposes. It has been reported that a typical energy saving of 30% is achievable with a Petlyuk distillation column, compared to conventional distillation arrangements. However, the optimal steady-state operation point can be difficult to maintain in practice. In this work we have studied the performance of some self-optimising control configurations for the Petlyuk distillation column in presence of disturbances and uncertainties. The results show that self-optimising control can be used to improve the robustness of optimal operation by adjusting a degree of freedom in a feedback control loop by keeping a suitable measurement variable at a setpoint.

## 1 INTRODUCTION

In most processes there are some extra degrees of freedom that can be used for optimisation purposes. The optimal operation point can be difficult to maintain if disturbances and model uncertainty are present. Self-optimising control is an approach to solve this problem by turning the optimisation problem into a set point problem. The key idea is to find a measurable variable with constant value at optimal operation. If this variable can be found, a feedback control loop is closed to keep the variable at the set point, and to keep indirectly the process at optimal operation. Since self-optimising control results in a feedback control loop, it will be robust against disturbances

and model uncertainties compared to any open loop model based optimisation methods. The application of self-optimising control to the Petlyuk distillation column was already addressed in (Halvorsen and Skogestad, 1998). Some candidate measurable feedback variables for the Petlyuk distillation column were proposed and analysed in a qualitative way. This work has to be seen as a continuation of that one in which a more careful evaluation is performed. New candidate feedback variables have been proposed and a quantitative study has been done to see the performance of the controlled system in face of various process disturbances and model uncertainties.

## 2 ENERGY OPTIMISATION IN THE PETLYUK COLUMN

The thermally coupled distillation column known as Petlyuk column (Petlyuk 1965), shown in figure 1, is a complex distillation arrangement to separate a ternary mixture of A (the more volatile), B (intermediate volatility) and C (the less volatile). The Petlyuk column has been given special attention due to very high reported energy savings. (Triantafyllou and Smith, 1992) reported savings of 30% comparing the Petlyuk column with the conventional trains of columns. Considerable investment capital savings can be obtained if the arrangement is implemented in a single shell (Divided Wall Column). The complex design of the Petlyuk column offers some extra degrees of freedom which permit an optimisation that is not possible in the conventional ternary distillation designs.

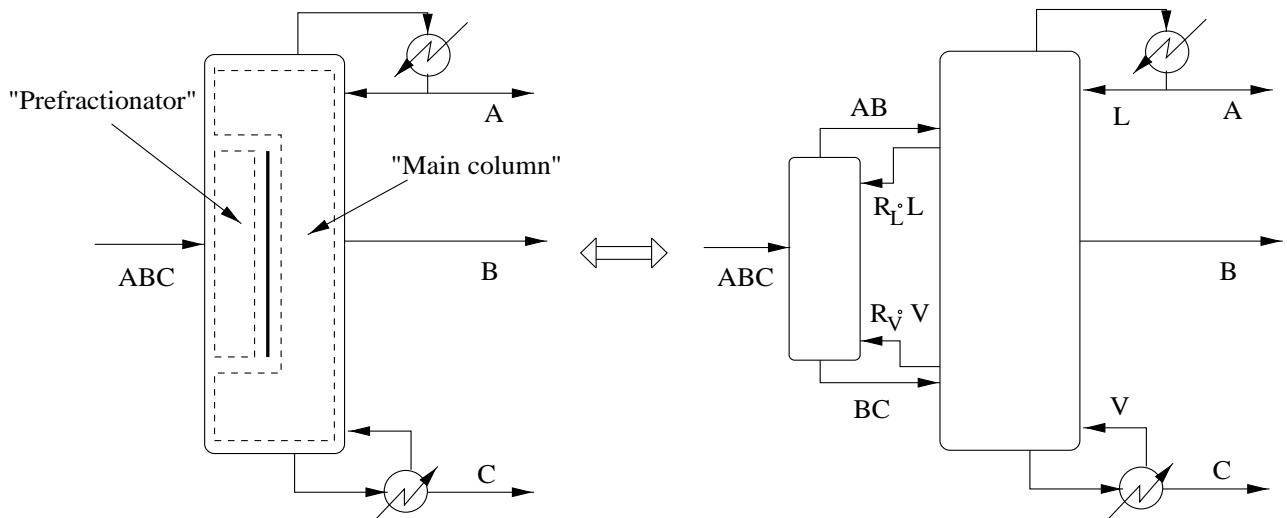


Figure 1. The Dividing Wall Column (left) and the fully thermally coupled column (right) are thermodynamically equivalent.

### 3 OPTIMISING CONTROL REQUIREMENT FOR THE PETLYUK COLUMN

We assume that the Petlyuk column reboiler and accumulator levels are stabilised by the distillate flow ( $D$ ) and the bottoms flow ( $B$ ). Then it has five degrees of freedom: boilup ( $V$ ), reflux ( $L$ ), side stream flow ( $S$ ), liquid split ( $R_l$ ) and vapour split ( $R_v$ ). Of these five degrees of freedom, three are used to control the compositions of the three products (composition of component A in the distillate, composition of B in the side stream and composition of C in the bottoms stream). (Wolff and Skogestad, 1996) showed that the LSV control structure give acceptable performance. It consists in the control of A composition by the reflux ( $L$ ), the control of the B composition by the side stream flow ( $S$ ) and the control of C composition by the boilup ( $V$ ). *LSV* is the control structure assumed in this work. Therefore, liquid split ( $R_l$ ) and vapour split ( $R_v$ ) are the two extra variables to be used for optimisation purposes. The energy consumption, here represented by the boilup vapour rate ( $V$ ) will be used as the criterion. When the composition loops are closed and the products purity ( $x_{DA}, x_{SB}, x_{BC}$ ) are controlled to their specifications, the product specifications setpoints ( $x_{DAS}, x_{SBS}, x_{BCS}$ ) will replace the composition control loop inputs ( $L, S$  and  $V$ ), as degrees of freedom. These setpoints will affect the optimal operation point in addition to the disturbances in the feed flow rate ( $F$ ), feed composition ( $z$ ) and feed liquid fraction ( $q$ ).

It was shown (Halvorsen and Skogestad, 1997, 1998) that the optimal operation point of the Petlyuk column is not robust when no optimising control is applied in addition to the product composition control. The optimal values of the two degrees of freedom ( $R_l, R_v$ ) used for optimisation are sensitive to feed disturbances and product set points changes. The objective function surface  $V(R_l, R_v)$  is very steep in some directions and if no adjustment of these remaining degrees of freedom (DOF) is applied, the operation may get far from optimal. Therefore, some control is required to maintain the optimal operation when disturbances and uncertainties are present. However, in accordance with the work of (Halvorsen and Skogestad, 1998), we will fix  $R_v$  and use  $R_l$  as the only manipulated variable to indirectly achieve the energy control. Two reasons justify this decision. First the energy surface  $V(R_l, R_v)$  is quite flat close to the minimum in a narrow long region in a certain direction in the  $(R_l, R_v)$ -plane, permitting that for any given constant  $R_{vo}$ , we can find a  $R_{l,opt,I}$  that makes the value of  $V_{opt,I}=V(R_{l,opt,I}, R_{vo})$  be close to the absolute minimum when both values of the remaining DOFs are optimised:  $V_{opt}=V(R_{l,opt}, R_{v,opt})$ .  $R_{vo}$  must be set in a reasonable neighbourhood to  $R_{v,opt}$ . The flat region was shown by (Fidkowski 1986) for infinite

stages and sharp product splits. The extent of the flat region is determined by the feed properties (composition and liquid fraction), and the relative volatility of the components. Second, if we consider a dividing wall column (DWC) (Wright 1949),  $R_v$  would be a difficult variable to manipulate in normal operation since its value will be naturally given by the pressure equalisation on each side of the dividing wall.

#### 4 SELF-OPTIMISING CONTROL FOR THE PETLYUK COLUMN

The concept of self-optimizing control is presented in (Skogestad et. al. 1998 and 1999). A brief introduction for our Petlyuk column case study will be given here. The idea behind self-optimising control is to find a variable which characterise operation at the optimum, and the value of this variable at the optimum should be less sensitive to variations in disturbances than the optimal value of the remaining degrees of freedom. Thus if we close a feedback loop with this candidate variable controlled to a setpoint, we should expect that the operation will be kept closer to optimum when a disturbance occur.

We define  $u$  to be our remaining degrees of freedom which we will use as manipulative variables for optimising control, and  $d$  to include the external disturbances, the setpoint specifications for all the closed control loops and any remaining degrees of freedom not used as manipulative variables. In our general case  $u=(R_l, R_v)$  and  $d=(z, q, x_{DA}, x_{SB}, x_{BC})$ , but when we fix  $R_v=R_{vo}$  and use  $R_l$  as the only manipulative variable we will have  $u=R_l$  and  $d=(z, q, x_{DA}, x_{SB}, x_{BC}, R_{vo})$ . The optimal solution is found by minimising  $V(u, d)$  with respect to  $u$ . Thus both the optimal value of the criterion function  $V_{opt}$  and the corresponding solution  $u_{opt}$  will be a function of  $d$ .

$$V_{opt}(d) = \min_u V(u, d) = V(u_{opt}(d), d) \quad (1)$$

The combined set of  $(u, d)$  determines an operation point uniquely, and also the values of any internal states and measurements. (In this simplified presentation we do not consider any bifurcations.) Assume now that we choose a measurement variable  $c=g(u, d)$ , and that the inverse function  $u=g^{-1}(c, d)$  exists. Then we may apply  $u=g^{-1}(c_s, d)$ , where  $c_s$  is the setpoint for  $c$ . The ideal relation would of course be to find a function  $g(.)$  where: If  $c_s=g(u_{opt,0}, d_0)$ , then

$u = g^{-1}(c_s, d) = u_{opt}(d)$ . These properties imply that want the nominal setpoint  $c_s$  to be insensitive to the disturbances, and that  $c$  characterise the optimum so that  $(c - c_s)$  is proportional to  $(u - u_{opt})$  for any disturbance in the region where  $c$  is close to  $c_s$ . An example of an ideal function  $g(\cdot)$  is the gradient of  $V(u, d)$  with respect to  $u$ :

$$g(u, d) = \nabla_u V(u, d) \quad (2)$$

In the real world, we cannot expect such an ideal function to exist, but there may be variables ( $c$ ) where  $V(g^{-1}(c_{s0}, d), d) \ll V(u_0, d)$ , when we compare the case of keeping  $c$  constant at the nominal value  $c_{s0}$ , to the case where we keep  $u$  at the nominal  $u_0$ , for a set of disturbances  $d$  around a nominal  $d_0$ .

So why not look around for the candidates?

A very important feature of the feedback implementation in self-optimising control is that we do not need to know the function  $g^{-1}(c_s, d)$  accurately since the feedback controller will adjust the input  $u$  until  $c = c_s$  in spite of uncertainties and unknown disturbances. Thus we may find the best variable  $c = g(u, d)$  with the wanted properties by using a rigorous model and advanced optimisation, but the realisation of  $g^{-1}(c_s, d)$  in the plant may simply be with a conventional PID controller, neither with the need for an on-line model nor any on-line optimisation. The task of finding a good candidate for self-optimising control is primarily a control structure problem (e.g. the task of selecting variables for inputs and outputs). When a self-optimising feedback variable is found, this variable can be treated like any other output in the task of finding the best regulatory design (e.g. finding the best input output pairing, choosing PID controllers and/or model predictive control etc.).

In the rest of this paper we will present results from a quantitative evaluation of  $V(u, d)$  and  $V(g^{-1}(c_s, d), d)$  compared to  $V(u_{opt}, d)$  for a set of candidate measurement variables and a set of disturbances around a nominal operational point.

## 5 SELF OPTIMISING CONTROL: A PETLYUK COLUMN CASE STUDY

### 5.1 *The Nominal Optimal Solution*

The non-linear model used to simulate the column behaviour in presence of disturbance and uncertainties was described in (Skogestad and Halvorsen, 1998). It is a stage by stage model where the main assumptions are: constant pressure, constant relative volatility, constant molar flows and constant tray efficiency. The relative volatilities are assumed to be (4:2:1). The number of stages is 8 in each of the 6 sections plus a reboiler and total condenser. (Note that the number of stages are not based on any rigorous column design. Our optimal boilup is about 40-50% higher than a theoretical minimum boilup with infinite number of stages, which indicates that our number should probably have been increased. However, detailed design of the column is not an issue in this paper.) The nominal operation point is selected with equimolar feed,  $z=(1/3,1/3,1/3)$ , partly vaporised, liquid fraction  $q=0.477$ , and 97% purity for all three products. The nominal optimal solution is found as  $V_{opt} = 1.497$ , for  $R_l = 0.450$  and  $R_v = 0.491$ .

This optimum, and all other optimal operating points for different sets of the disturbances ( $d$ ) are found by applying a constrained optimisation solver with the full non-linear model.

### 5.2 *Proposed Output Feedback Variables*

The set of candidate feedback variables are based on discussions in (Halvorsen and Skogestad 1998) and in (Christiansen 1997). The selection is based on qualitative evaluation and process insight. Alternative approaches based on Taylor series expansion of the criterion function is outlined in Skogestad et. al. 1996, 1997, 1998), but these methods are not considered in this study. A brief description of each of the considered feedback variables are given below.

- $D_l/F$ : The net flow from the top of the prefractionator to the main column divided by the feed flow.

$D_l = V_l - L_l$ . Thus it is not a flow but a difference between two flows.

- $\beta$ : Fractional recovery of the intermediate B-component leaving in the prefractionator top.

A similar behaviour as  $D_l$  is expected as  $D_l = z_A + \beta z_B$  with a sharp A/C split.

- $\Delta N$ : the number of trays between the tray from where the side stream is withdrawn and the

tray that has the highest B-composition. This is based on the observation that for optimised operation, the B-composition had its maximum at the sidestream withdrawal stage.

- $\Delta N'$  is the continuous variable that corresponds to a cubic interpolation of the discrete variable

$\Delta N$ .  $\Delta N'$  will be able to follow the optimum more closely. Nominal  $\Delta N$  is 0.

- $DTS$ : a measure of the temperature profile symmetry. It is defined as

$DTS = \sum(T_{1,i} - T_{4,i}) + \sum(T_{2,i} - T_{5,i})$ , where  $T_{N,i}$  is the temperature of tray i of section N. The temperature of each tray is calculated assuming the contribution of each component with its equilibrium temperature proportional to its fraction. The set point of  $DTS$  is 6.38.  $DTS$  was observed to be constant along the direction of the minimum surface  $V(R_b, R_v)$  where it was most flat.

- $y^{DI}_C$ : the C-composition of the net flow from the prefractionator distillate to the main column.

$y^{DI}_C$  is calculated as the net C in the vapour from the prefractionator to the main column minus the net C in the liquid from the main column to the prefractionator divided by the net flow from the prefractionator to the main column.

- $y^{BI}_A$  is the equivalent to  $y^{DI}_C$  in the prefractionator bottom.

- $\gamma_0$ : Ratio of net flow downwards towards the sidestream and the sidestream. This variable is implemented as a feedforward from flow measurements:  $R_l = 1 - \gamma_0 * S/L - (1 - R_v) * V/L$ .

## 6 ROBUSTNESS STUDY SIMULATION

Because of the reasons given above,  $R_l$  have been used as manipulated variable for the optimisation loop while  $R_v$  has been kept constant. Thus, the control system has become a four-loops control system. PID controllers are used to close all control loops: PID are implemented in the three composition control loops, and also PID are implemented in the optimisation loop. Since we are

interested in steady-state considerations, the tuning of the controllers to obtain good control performances has been let aside. To study the robustness of each of the proposed optimisation control structures, a set of simulations have been done. Closing the optimising loop with each of the proposed feedback variables at a time, simulations have been done for steps in each uncertainty and disturbance variable. The process was simulated from the nominal initial conditions until a new steady-state was obtained. The different control structures brought the process to different steady-state operation conditions when the disturbances were applied. The boilup values of these control-led operations are the object of our comparisons.

Feed flowrate ( $F$ ), feed composition ( $z_A, z_B$ ) and feed liquid fraction ( $q$ ) have been the considered disturbance uncertainties. Uncertainties in the measure of the product purities and in the measure of the feedback variables have also been considered. Uncertainties have been simulated through step changes in inputs and in the set points. (To simulate error in the measure of the optimisation controlled variable and in the measure of the product compositions, setpoint changes have been applied).

For each source of upset, some values around the nominal values have been analysed. In table 1 the specific considered upset values are shown in the second column with the values applied. For each disturbance or uncertainty, the values of the objective function (boilup  $V$ ) for each self-optimising optimisation control structure are computed. Values are compared to the pseudo optimal value where  $R_v$  is fixed at its nominal value and only  $R_l$  is adjusted for minimal boilup. We also computed the overall optimal value (where both  $R_l$  and  $R_v$  are adjusted for minimal boilup). Values are shown in the rightmost columns in Table 1. We also compare results to the values for constant  $R_l$  and  $R_v$ , that is with no control action in the self-optimising control loop.

## 7 DISCUSSION OF THE RESULTS

After doing all the simulations the following results are obtained:

- $D_l/F$  is not a good feedback variable. It handles very well disturbances in the feed flow because maintains the proportionality between internal flows. It brings the system not far to the optimum for set point changes and disturbances in  $q$ . However, it behaves very bad in front

of feed composition disturbances, worst than fixing the  $R_l$  to the nominal value. We can say

that it is not a good option because it fixes a flow, not a feature of the system.

- $\beta$  has the better behaviour in response of feed composition disturbances. With  $y^{DI}_C$  it has the best behaviour for feed vapour fraction disturbances. In front of product composition set point changes it is almost as good as  $DTS$ , the best one. As it is a recovery and not a flow, it faces feed flow disturbances quite well. Lastly, robustness against bad measurement is acceptable. It is a variable characteristic of the whole system.
- $\Delta N$  has the problem that only indicates  $R_l$  to change when the changes in profile are large because of the discreteness of the variable.  $\Delta N'$  is better than  $\Delta N$ . Other variables are better than  $\Delta N'$  for the normal disturbances and uncertainties. But surprisingly, it is the best solution for large changes in feed composition. It has to be noticed that  $\Delta N'$  takes only into account the main column.
- $DTS$  is found to be the best feedback variable for changes in the set points of the product compositions and for set point changes in its self value. It faces well disturbances in the feed flow because it is not a flow. Its behaviour in front of feed composition and vapour fraction disturbances is not bad. This feedback variable takes into account the two sides of the Petlyuk column, the prefractionator and the main column.
- $y^{DI}_C$  has shown to be a good feedback variable, too. Facing feed vapour fraction disturbances it is comparable to  $\beta$ . Its behaviour in response of changes in its setpoint value is almost as good as for the  $DTS$  variable. And its behaviour in front of set point changes in the product compositions and disturbances in the feed composition is not bad. But it does not respond well in front of feed flow disturbances because it is a composition and not a recovery. As  $DTS$ ,  $y^d_C$  is a characteristic of the whole column.
- $y^{BI}_A$  has given unstable solutions as was predicted (Christiansen 1997). This is due to that the

flat region is on the right branch of the prefractionator characteristic for our case, and this variable is expected to be best for the opposite case.

- The feed forward proposed control has also given very good results. However, it will not have the advantages and simplicity of feedback control.

When comparing the overall optimum values with the optimum values with fixed vapour split ( $R_l$  free,  $R_v$  fixed), we find the loss with a fixed  $R_v$  is quite small. This confirms it is possible to be close to the minimum by using only one of the two extra degrees of freedom as a manipulated variable.

## 8 CONCLUSIONS

Self-optimising control has been seen to be a good method for the energy optimisation of a Petlyuk column. Three output feedback variables give very good robust control of optimal operation in a self-optimising control scheme. They are  $\beta$ , DTS and  $y^{DI}_C$ . For robustness against feed flow disturbances,  $\beta$  and DTS are better than  $y^{BI}_A$  because his last variable is a composition and not a recovery. For feed composition disturbances  $\beta$  is the variable with maintains V closer to the minimum, however DTS and  $y^{DI}_C$  have also acceptable results. Facing feed vapour fraction disturbances,  $y^{DI}_C$  is the best of the three but the other two are not far from it. Facing set point changes in the product compositions, DTS is again the best feedback variable, being  $\beta$  very close and  $y^{DI}_C$  the worst of them. Lastly, DTS and  $y^{DI}_C$  behave better in response to bad measurements of themselves than  $\beta$ . In a real case, we will decide one of the three variables depending on the information we have about what are the more probable disturbances. Also technical aspects will be have to consider. It has to be remarked, for example, that DTS can be calculated with only temperature measurements, which is a great advantage and on the contrary, the measurement of  $y^{DI}_C$  and  $\beta$  involve composition measurements.

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Disturbances			DTS		$y^{DI}_C$		$b$		$R_l=0.450$	Optimum for $R_{vo}=0.491$	Optimal reference		
Name	value		V	$R_l$	V	$R_l$	V	$R_l$	V	$V_{opt,1}$	$R_l$	$V_{opt}$	
$F$	1.2		1.797	0.450	1.804	0.457	1.797	0.450	1.797	1.797	0.450	1.797	
	1 (Nominal)		1.498	0.450	1.498	0.450	1.498	0.450	1.498	1.498	0.450	1.498	
	0.8		1.198	0.450	1.198	0.441	1.198	0.450	1.198	1.198	0.450	1.198	
$z_A/z_B$	0.333	0.333	1.498	0.450	1.498	0.450	1.498	0.450	1.498	1.498	0.450	1.498	
	0.399	0.333	1.602	0.412	1.560	0.421	1.536	0.430	1.550	1.532	0.436	1.531	
	0.333	0.399	1.580	0.418	1.562	0.426	1.557	0.432	1.585	1.557	0.433	1.554	
	0.267	0.333	1.541	0.485	1.504	0.477	1.487	0.469	1.510	1.482	0.463	1.481	
	0.333	0.267	1.444	0.481	1.430	0.473	1.428	0.469	1.489	1.428	0.470	1.425	
	0.379	0.379	1.664	0.400	1.601	0.411	1.567	0.424	1.601	1.564	0.429		
	0.379	0.286	1.476	0.446	1.475	0.447	1.472	0.449	1.471	1.470	0.453		
	0.286	0.379	1.532	0.453	1.532	0.453	1.529	0.451	1.528	1.526	0.446		
	0.286	0.286	1.505	0.496	1.455	0.485	1.440	0.477	1.539	1.438	0.474		
$1-q$	0.627		1.451	0.483	1.449	0.475	1.451	0.482	1.554	1.448	0.478	1.446	
	0.575		1.473	0.467	1.472	0.463	1.473	0.466	1.498	1.472	0.464	1.472	
	0.523		1.498	0.450	1.498	0.450	1.498	0.450		1.498	0.450	1.498	
	0.471		1.526	0.433	1.525	0.437	1.526	0.433	1.546	1.525	0.436	1.524	
	0.418		1.557	0.416	1.554	0.424	1.557	0.416	1.626	1.554	0.421	1.552	
Purity	0.97	0.97	0.97	1.498	0.450	1.498	0.450	1.498	0.450		1.498	0.450	1.498
	0.97	0.97	0.98	1.727	0.455	1.775	0.440	1.727	0.455	1.734	1.726	0.457	
	0.97	0.97	0.96	1.371	0.446	1.382	0.456	1.371	0.446	1.373	1.370	0.445	
	0.97	0.98	0.97	1.551	0.446	1.552	0.448	1.556	0.452	1.554	1.551	0.443	
	0.97	0.96	0.97	1.467	0.453	1.467	0.452	1.469	0.449	1.468	1.467	0.453	
	0.98	0.97	0.97	1.564	0.456	1.584	0.445	1.569	0.451	1.57	1.564	0.457	
	0.96	0.97	0.97	1.466	0.448	1.47	0.453	1.467	0.451	1.467	1.466	0.447	
Error	10		1.499	0.453	1.500	0.455	1.512	0.462					
	-10		1.499	0.448	1.499	0.446	1.513	0.439					

Table 1: Data show the boilup ( $V$ ) and liquid split ( $R_l$ ) with self-optimising control, using one DOF ( $R_l$ ), to keep each of the three candidate feedback variables  $DTS$ ,  $y^{DI}_C$  or  $\beta$  at the nominal setpoint, for the given set of disturbances. The results are compared to the optimal values for each case and to the case of no control (constant  $R_l=0.450$ ). In all cases, except for the optimal reference solution in the rightmost column, the vapour split ratio ( $R_v$ ) is kept constant at the nominal value:  $R_v=R_{vo}=0.491$ .