

# PLANTWIDE CONTROL: THE SEARCH FOR THE SELF-OPTIMIZING CONTROL STRUCTURE

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**Abstract:** The following important question is frequently overlooked: Which variables should we select to control? It is shown that the idea of selecting the variables that achieve “self-optimizing control” provides a link between steady-state optimization, feedback control, time scale separation and uncertainty.

**Keywords:** Chemical process control, control structure design

## 1. INTRODUCTION

If we consider the control system in a highly automated chemical plant, then we find that it is structured hierarchially into several layers, each operating on a different time scale. Typically, layers include scheduling (weeks), site-wide optimization (day), local optimization (hour), supervisory/predictive control (minutes) and regulatory control (seconds); see Figure 1.

The layers are interconnected through the controlled variables. More precisely, the **controlled variables**  $c$  are the (internal) variables that link two layers in a control hierarchy, whereby the upper layer computes the setpoint  $c_s$  to be implemented by the lower layer.

Which should these internal variables be? That is, what should we control? Or to phrase the question in another way: Why do we in a chemical plant select to control a lot of internal variables (e.g. compositions, pressures, temperatures, etc.) for which there are no explicit control requirements?

To be more specific, consider a distillation column inside a large chemical plant. By “inside” we mean that the column is not producing any final products. Thus there are no explicit requirements on the product purities; rather their (optimal) values are determined by the overall plant economics. Still, we find in many practi-

cal cases that we select to control at least one of the product purities at a given setpoint. Why do we keep this composition constant rather than keeping a flow constant?

The method presented in this paper follows the ideas of Morari *et al.* (1980) and is very simple. The basis is to define mathematically the quality of operation in terms of a scalar cost function  $J$  to be minimized. To achieve truly optimal operation we would need a perfect model, we would need to measure all disturbances, and we would need to solve the resulting dynamic optimization problem on-line. This is unrealistic in most cases, and the question is if it is possible to find a simpler implementation which still operates satisfactorily (with an acceptable loss).

The simplest would be if we could obtain acceptable operation with constant values (setpoints) for the controlled variables, thus effectively turning the complex optimization problem into a simple feedback problem, and thus achieve “self-optimizing control”.

**Self-optimizing control** is when we can achieve an acceptable loss with constant setpoint values  $c_s$  for the controlled variables

(The reader is probably familiar with the term self-regulation, which is when acceptable dynamic control

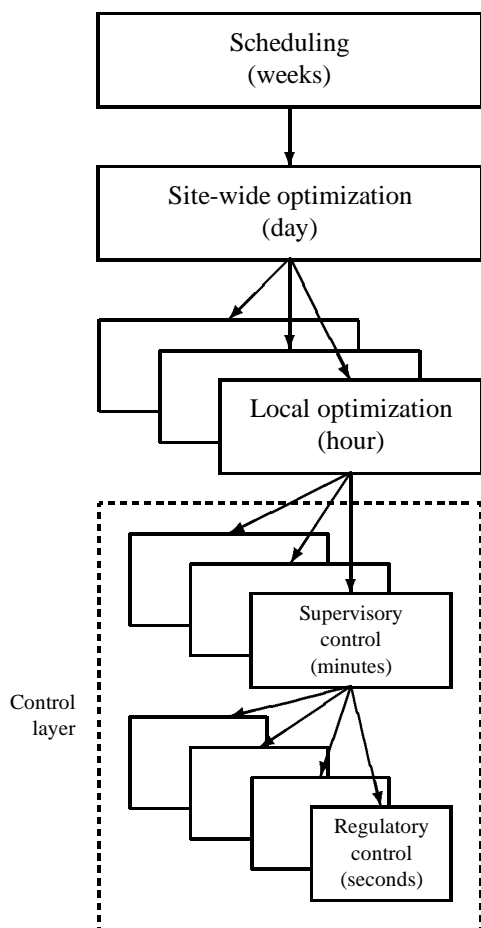


Fig. 1. Typical control hierarchy in a chemical plant.

performance can be obtained with constant manipulated inputs. Self-optimizing control is a direct generalization to the case where we can achieve acceptable (economic) performance with constant controlled variables.)

Inspired by the work of Findeisen (e.g. see Findeisen *et al.* (1980)), Morari *et al.* (1980) gave a clear description of what we here denote self-optimizing control, including a procedure for selecting controlled variables based on evaluating the loss. However, it seems that nobody, including the authors themselves, has picked up on the idea. One reason was probably that no good example was given in the paper.

More generally, the issue of selecting controlled variables is one of the subtasks in the **control structure design** problem (Foss, 1973); (Morari, 1982); (Skogestad and Postlethwaite, 1996)

- (1) *Selection of controlled variables*  $c$  (variables with setpoints  $C_s$ )
- (2) *Selection of manipulated variables*
- (3) *Selection of measurements* (for control purposes including stabilization)
- (4) *Selection of a control configuration* (structure of the controller that interconnects measurements & setpoints and manipulated variables)
- (5) *Selection of controller type* (control law specification, e.g., PID, decoupler, LQG, etc.).

Note that these *structural decisions* need to be made before we can start the actual design the controller. In most cases the control structure is designed by a mixture between a top-down consideration of control objectives and which degrees of freedom are available to meet these (tasks 1 and 2), combined with a bottom-up design of the process system, starting with the stabilization of the process (tasks 3,4 and 5). In most practical cases the problem is solved without the use of any theoretical tools.

The main objective of this paper is to demonstrate, with a few examples, that the issue of selecting controlled variables (task 1) is very important and that the concept of self-optimizing control provides a useful tool.

## 2. OPTIMIZATION AND CONTROL

### 2.1 The optimization problem

The optimizing control problem can be formulated as

$$\min_u J(u, d) \quad (1)$$

subject to the inequality constraints

$$g(u, d) \leq 0 \quad (2)$$

where  $u$  are the  $N_u$  independent variables we can affect (degrees of freedom) and  $d$  are independent variables we can not affect (disturbances). Here the constraints for instance may be

- product specifications (e.g.  $x_D \geq 0.95$ )
- manipulated variable saturations (e.g.  $0 \leq V \leq V_{max}$ )
- other operational limitation (e.g. avoid flooding)

The analysis in this paper is based on steady-state models and use of constant setpoints  $c_s$  at each steady-state (operating point). To analyze the effect of disturbances we may time-average various steady-states. The main justification for using a steady-state analysis is that the economic performance is primarily determined by steady-state considerations.

If we formulate the optimizing control problem in the usual mathematical fashion as given in (1), then we find that a centralized solution is the optimal choice. Here there is one “big” controller, which based on all available measurements and other given information (including a model of the system and expected uncertainty), computes the optimal values of all manipulated variables. Nevertheless, in practice we almost always decompose the control system into many separate parts and layers. In the simplest case we may have two layers:

- A steady-state optimization layer which computes the optimal setpoints  $c_s$  for the controlled variables, and

- A feedback control layer which implements the setpoints, to get  $c \approx c_s$ .

## 2.2 Introductory example: Distillation

With a given feed stream and a specified pressure, a conventional two-product distillation column has two degrees of freedom at steady state ( $N_u = 2$ ). (From a control point of view the column has 5 degrees of freedom, but two degrees of freedom are needed to control the reboiler and condenser holdups which have no steady-state effect, and one degree of freedom is used to control the pressure at its given value). The two remaining degrees of freedom, e.g. selected to be the reflux flow  $L$  and the distillate flow  $D$ , may be used to optimize the operation of the plant. However the question is: Which two variables  $c$  should be specified during operation?

Let us assume that the distillate product must contain at least 95% light component,  $x_D \geq x_{D,min} = 0.95$ , and that to avoid flooding the capacity of the column is limited by a maximum allowed vapor load,  $V \leq V_{max}$ .

Consider first a case where the distillate is the valuable product and energy costs are low. In this case it is optimal to operate the column at maximum load (Gordon, 1986) (to reduce loss of light component in the bottom) and with the distillate composition at its specification (to maximize distillate flow), i.e.

$$V_{opt} = V_{max}; x_{D,opt} = x_{D,min} = 0.95$$

Thus, the optimum lies at constraints and implementation is obvious: We should select the vapor rate  $V$  and the distillate composition  $x_D$  as the controlled variables,

$$c = \begin{bmatrix} V \\ x_D \end{bmatrix}$$

In practice, we implement this using a lower-level feedback control system where we

- adjust the boilup  $V$  to keep the pressure drop over the column, an indicator of flooding, below a certain limit
- adjust reflux  $L$  (or some other flow depending on how the level and pressure control system is configured) so that  $x_D$  is kept constant

Next, consider a case where energy costs are relatively high, and where the bottoms products is the more valuable. In this case the optimum may be unconstrained, and assume for the discussion that

$$x_{D,opt} = 0.973 > x_{D,min}; V_{opt} = 0.76V_{max}$$

Implementation in this case is not obvious. Some candidate sets of controlled variables are

$$c_1 = \begin{bmatrix} x_D \\ x_B \end{bmatrix}; c_2 = \begin{bmatrix} T_{top} \\ T_{btm} \end{bmatrix}; c_3 = \begin{bmatrix} x_D \\ V \end{bmatrix}$$

$$c_4 = \begin{bmatrix} L \\ V \end{bmatrix}; c_5 = \begin{bmatrix} L/D \\ V/B \end{bmatrix}$$

and there are many others. Controlled variables  $c_1$  and  $c_2$  will yield a “two-point” control system where we close two loops for quality control;  $c_3$  yields a “one-point” control system where only one quality loop is closed; whereas  $c_4$  and  $c_5$  are “open-loop” policies which require no additional feedback loops (except for the level and pressure loops already mentioned). All of these choices of controlled variables will have different self-optimizing control properties, as we will see from the case study below.

## 3. SELECTION OF CONTROLLED VARIABLES

In this section we present our procedure. For a given disturbance  $d$  we can solve the optimization problem

$$\min_u J(u, d) = J(u_{opt}(d), d) = J_{opt}(d)$$

and obtain the optimal value  $u_{opt}(d)$ . From this we can obtain a table with the corresponding optimal value of any other dependent variable, including  $c_{opt}(d)$ .

In actual operation we adjust  $u$  to keep  $c$  approximately at its nominally optimal value, i.e.

$$c_s = u_{opt}(d_0) \quad (3)$$

where  $d_0$  is the nominal disturbance. The difference between the actual  $u$  and the optimal  $u_{opt}(d)$  results in a loss  $L$  between the actual operating costs and the optimal operating cost,

$$L(u, d) = J(u, d) - J(u_{opt}(d), d) \quad (4)$$

Compared to the cost  $J$ , the loss  $L$  has the advantage of providing a better “absolute scale” on which to judge whether a given set of controlled variables  $c$  is “good enough”, and thus is self-optimizing.

We next present two approaches for selecting controlled variables  $c$  for use in a closed-loop policy (but note that we can actually obtain the “open-loop” policy as a special case by selecting  $c = u$ ). Approach 1 yields most insight, but is not actually used any further in the paper. Instead we use Approach 2 which is based on directly evaluating the loss.

### 3.1 Approach 1: Evaluating the error

Consider an closed-loop implementation where we attempt to keep  $c$  constant at the value  $c_s$ . With this implementation the operation may be non-optimal (with a positive loss) due to the presence of a setpoint error and an implementation error.

- (1) The setpoint error  $e_{cs} = c_s - c_{opt}(d)$  is the difference between the setpoint value and truly optimal value

- (2) The implementation error  $d_c = c - c_s$  is the difference between the actual value and the setpoint.

The overall error  $e_c = c - c_{opt}(d)$ , the difference between the actual value and the optimal value (which causes a positive loss), is the sum of the two,

$$e_c = e_{cs} + d_c \quad (5)$$

To compare various choices of controlled variables, we need to consider what effect a nonzero error  $e_c$  has on the error in the “original” (base set) degrees of freedom  $u$ , i.e. what effect  $e_c$  has on  $e_u = u - u_{opt}$ . Clearly, we would like that a large value of  $e_c$  results in only a small value of  $e_u$ , that is, we want  $u$  to be insensitive to changes in  $c$  (or equivalently, we want  $c$  to be sensitive to changes in  $u$ ).

In summary, a good candidate for a controlled variable  $c$  has the following properties:

**Property 1.** Its optimal value is insensitive to disturbances (so that the setpoint error  $e_{cs} = c_s - c_{opt}(d)$  is small)

**Property 2.** It is easy to control accurately (so that the implementation error  $d_c$  is small)

**Property 3.** Its value is sensitive to changes in the manipulated variables  $u$  (so that even a large error in the controlled variable  $c$  results in only a small error in  $u$ ).

### 3.2 Approach 2: Evaluating the loss

To compare alternative choices for  $c$  it is probably simplest to directly evaluate the cost function (or equivalently the loss function) for expected values of the disturbance  $d$  and the implementation error  $d_c$ . The optimal choice for of controlled variables  $c$  is then the one that with constant values of  $c$  (more precisely,  $c = c_s + d_c$ ) minimizes some average value of the loss  $L$  for the expected set of disturbances  $d \in \mathcal{D}$ , and expected set of implementation (control) errors  $d_c \in \mathcal{D}_c$ .

### 3.3 Procedure for selecting controlled variables

Based on approach 2 we are now in a position to formulate a procedure for selecting controlled variables  $c$ .

**Step 1: Degree of freedom analysis.** Determine the number of degrees of freedom ( $N_u$ ) available for optimization, and identify a base set ( $u$ ) for the degrees of freedom.

**Step 2: Cost function.** Define the optimal operation problem by formulating a scalar cost function  $J$  to be minimized for optimal operation.

**Step 3: Optimization.** First solve the nominal optimization problem with disturbances  $d_0$ . In addition, after having specified the disturbance set in step 5, we usually solve the optimization problem for the disturbances  $d$  in question. This is needed

to check whether there exists a feasible solution  $u_{opt}(d)$  for all disturbances  $d$ , and to find the optimal cost  $J(u_{opt}, d)$  needed if we want to evaluate the loss. In addition, we could try to identify controlled variables by looking for variables which optimal value is only weakly dependent of disturbances (Fisher *et al.* (1988) p. 163; also recall property 1 presented above).

**Step 4: Candidate controlled variables.** At this stage we may identify candidate controlled variables. Typically, these are measured variables or simple combinations thereof. Insight and experience may be helpful at this stage, because the possible number of combinations may be extremely large.

**Step 5: Disturbances.** Identify the most important disturbances (uncertainty). These may be caused by

- Errors in the assumed (nominal) model (including the effect of incorrect nominal values for the disturbances used in the optimization)
- Disturbances  $d-d_0$  (including parameter changes) occurring after the optimization
- Implementation errors ( $d_c$ ) for the controlled variables (e.g. due to measurement error)

It may be important to include model uncertainty since, as pointed out by Shinnar (1981), some control structures are very sensitive to model uncertainty whereas others are not.

**Step 6: Evaluation of loss.** We compute the mean value of the loss for alternative sets of controlled variables  $c$ . This is done by evaluating the loss

$$L(u, d) = J(u, d) - J(u_{opt}(d), d) \quad (6)$$

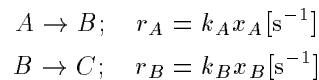
$$u = f^{-1}(c_s + d_c, d)$$

with fixed setpoints  $c_s$  for the defined set of disturbances  $d$  and implementation errors  $d_c$ . We here select the setpoints as the nominal optimal values,  $c_s = c_{opt}(d_0)$ .

**Step 7: Further analysis.** We select for further consideration the sets of controlled variables with acceptable loss (and which thus yield self-optimizing control). These could then be analyzed to see if they are adequate with respect to other criteria that may be relevant, such like the region of feasibility and the expected dynamic control performance (input-output controllability)

## 4. REACTOR CASE STUDY

We consider a continuously stirred tank reactor (CSTR) where two irreversible first-order reactions take place



Let  $z_i$  and  $x_i$  denote mole fractions of component  $i$  in the feed and reactor, respectively, and let  $F$  [mol/s] be the feed rate and  $M$  [mol] the reactor holdup. There are only three components, A, B and C, and steady-state material balances yield

$$z_A F - x_A F - k_A x_A M = 0$$

$$z_B F - x_B F + k_A x_A M - k_B x_B M = 0$$

$$x_C = 1 - x_A - x_B$$

We consider following nominal data:

$$z_A = 0.8; k_A = 1s^{-1}; k_B = 1s^{-1}; F = 1\text{mol/s}$$

and two cases

**Case 1** No C in feed ( $z_B = 1 - z_A$ ).

**Case 2** No B in feed ( $z_C = 1 - z_A$ ).

**Step 1: Degree of freedom analysis** With a given feed the reactor has one degree of freedom at steady-state, which may be selected as the reactor holdup, i.e.  $u = M$  [mol]. The value of  $M$  should be adjusted to optimize the operation.

**Step 2: Cost function** In this example component B is the desired product and the objective is to maximize the concentration of B, i.e. we choose the cost function

$$J = -100 \cdot x_B$$

(in most cases we would recycle unreacted A, but this is not the case in this example).

**Step 3: Optimization** The optimal holdup and corresponding optimal compositions for the two nominal cases are:

$$\text{Case 1 : } M = 0.6; x_A = 0.5, x_B = 0.3125$$

$$\text{Case 2 : } M = 1.0; x_A = 0.4, x_B = 0.2$$

**Step 4: Candidate controlled variables** The following candidates for the controlled variable  $c$  have been suggested

$$c_1 = M; c_2 = \frac{M}{F}; c_3 = x_A; c_4 = x_A; c_5 = x_C; c_6 = \frac{x_B}{x_A}$$

plus the following two property variables

$$c_7 = \theta_1 = 10x_A + 20x_B + 30x_C$$

$$c_8 = \theta_2 = 10x_A + 30x_B + 20x_C$$

which may represent a boiling temperature, a viscosity, a refraction index or similar.

Which controlled variable is preferred? It seems clear that it will be better to keep  $M/F$  rather than  $M$  constant, because the optimal residence time  $M/F$  is independent of the feed rate, whereas the optimal value of the holdup  $M$  clearly depends on the feedrate. It is also rather obvious that a policy based on keeping  $x_B$  constant is most likely to fail, because  $x_B$  goes through a maximum as we increase  $M$ , and if we specify a value of  $x_B$  above this maximum, then operation is infeasible. However, otherwise it is not at all clear, even in this simple case, what the best choice of the controlled variable is.

**Step 5: Disturbances** To answer the question in a quantitative manner we need to specify the disturbances (errors). We will consider disturbance in feed rate, feed composition and in the rate constants. In addition, we have an implementation error for the controlled variable; e.g., due to measurement error, for which we use the following values

- $M$ : 10%
- $M/F$ : 20%
- $x_A, x_B$  and  $x_C$ : 5%
- $x_A/x_B$ : 10%
- $\theta_1$  and  $\theta_2$ : 1 unit (about 5%)

**Step 6: Evaluation of loss** To compare the alternatives we compute the loss

$$L = J - J_{\text{opt}} = 100(x_{B,\text{opt}} - x_B)$$

with each of the candidate variables kept constant at its nominal optimal value.

The results for case 1 with no C in feed are given in Table 1 for the 8 candidate variables and the 6 disturbances. The loss is quite small in most cases, but in some cases there is no feasible solution (marked as inf. in the table). As expected, this is the case if we specify  $x_B = c_{4s} = 0.3125$  higher than its maximum value. But note that infeasibility may occur for most choices of controlled variables if the disturbance is sufficiently large. For example, if we specify  $x_A = c_{3s} = 0.5$  then we would obviously get infeasibility with  $z_A < 0.5$ . Note that there is usually no “warning”, in terms of a large value of the loss, as we approach infeasibility.

We find that variable  $c_3 = x_A$  is the ideal variable to keep constant when there are disturbances in  $z_A$  (it can be proven analytically that the optimal value for  $x_A$  is 0.5 irrespective of the value of  $z_A$ ), and keeping  $x_A$  constant also yields a small loss when there are other disturbances. Consequently, as seen from the table, the smallest average loss (0.18) is obtained by keeping  $c_3 = x_A$  constant. Keeping  $c_6 = \theta_1$  constant also gives a very small average loss (0.20). Except for the choices  $c_4 = x_B$  and  $c_6 = x_B/x_A$ , for which we get infeasibility, the worst average loss (0.89) is obtained when we keep the holdup  $c_1 = M$  constant. This value is reduced to 0.81 if we include “feedforward” action from the feedrate  $F$  and keep the residence time  $c_2 = M/F$  constant, but the improvement is so small that we would probably not include it in this case.

With the numbers given above, the implementation error is not very important. However, in many cases it may be a critical factor which eliminates an otherwise good candidate controlled variable. Assume for example, that the property variable  $\theta_1$  was a temperature measurement and that the expected implementation error was 10 units rather than 1 unit used above (e.g., 1 unit could represent 0.1 K). In this case the loss for case  $d_6$  with implementation error (where we specified

$\theta_1$  at 26.875 rather than at its optimal value of 16.875) would be 20.0 and the average loss for  $\theta_1$  would be 3.49 rather than 0.20.

Consider next the evaluation of the loss for case 2 where the feed contains component C rather than component B. Otherwise all the data is the same. Variable  $c_5 = x_B/x_A$  here replaces  $c_3 = x_A$  as the ideal variable with respect to disturbances in  $z_A$  (this can be proven analytically), but keeping  $c_5$  constant is not good when there are disturbances in the rate constants. Therefore, the average loss for  $c_5$  is high as 0.74. In this case loss is smallest (0.10) when  $c_2 = M/F$  is kept constant; it is 0.52 with  $c_3 = x_A$  constant, and it is 0.82 with  $c_6 = \theta_1$  constant. Thus we find, somewhat surprisingly, that the ranking is almost reversed in case 2 compared to case 1.

**Step 7: Candidates for self-optimizing control** If we assume that the requirement for acceptable operation (self-optimizing control) is that the mean loss is less than 0.5, then none of the proposed controlled variables are acceptable if this needs to be satisfied both for case 1 and case 2. On the other hand,  $c_3 = x_A$ ,  $c_5 = x_C$  and  $c_7 = \theta_1$  are acceptable if we consider only case 1 (no C in feed), whereas  $c_1 = M$  and  $c_2 = M/F$  are acceptable if we consider only case 2 (no B in feed); at least if we evaluate the loss by considering one disturbance at a time.

It is not easy to explain why these particular variables are preferred in the two cases.

## 5. CONCLUSION

In this paper we have presented a procedure for selecting controlled variables  $c$  based on evaluating with constant setpoints  $c_s$  the loss  $L = J - J_{opt}$  for possible disturbances. If the loss is acceptable then we have “self-optimizing” control.

The procedure requires a process model and a clear definition of the cost function  $J$  to be minimized during operation. However, there usually exists very many possible control structures and going through all of

these using the procedure can be tedious. For a simpler analysis, it should be noted a good candidate for a controlled variable  $c$  should have the following properties:

**Property 1.** Its optimal value is insensitive to disturbances

**Property 2.** It is easy to control accurately

**Property 3.** Its value is sensitive to changes in the manipulated variables

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Disturbance	$x_{B,opt}$	Loss for $M = c_{1s}$ = 0.6	Loss for $\frac{M}{F} = c_{2s}$ = 0.6	Loss for $x_A = c_{3s}$ = 0.5	Loss for $x_B = c_{4s}$ = 0.3125	Loss for $x_C = c_{5s}$ = 0.1875	Loss for $\frac{x_B}{x_A} = c_{6s}$ = 0.625	Loss for $\theta_1 = c_{7s}$ = 16.875	Loss for $\theta_2 = c_{8s}$ = 18.125
Nominal	0.3125	0	0	0	0	0	0	0	0
$d_1 : F = 0.7$	0.3125	0.60	0	0	0	0	0	0	0
$d_2 : z_A = 0.6$	0.4167	2.60	2.60	0	inf.	1.37	inf.	0.42	1.37
$d_3 : z_A = 1.0$	0.2500	1.56	1.56	0	inf.	0.45	1.56	0.10	0.45
$d_4 : k_A = 1.5$	0.3624	0.05	0.05	0.52	4.99	0.02	1.75	0.19	1.32
$d_5 : k_B = 1.5$	0.2679	0.47	0.47	0.47	inf.	0.05	4.06	0.20	1.68
$d_6 : \text{impl. error}$	0.3125	0.04	0.15	0.08	inf.	0.01	0.31	0.29	1.72
Average loss		0.89	0.81	0.18	inf.	0.32	inf.	0.20	1.09
Ranking		6	4	1	8	3	8	2	5

Table 1. Loss for reactor case study; case 1