



Optimal operation of heat exchanger networks

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Received 11 February 1998; received in revised form 3 August 1998

Abstract

The paper discusses optimal operation of a general heat exchanger network with given structure, heat exchanger areas and stream data including predefined disturbances. A formulation of the steady state optimization problem is developed, which is easily adapted to any heat exchanger network. Using this model periodically for optimization, the operating conditions that minimize utility cost are found. Setpoints are constant from one optimization to the next, and for implementing the optimal solution special attention is paid to the selection of controlled variables such that the operation is insensitive to uncertainties (unknown disturbances and model errors). This is the idea of self-optimizing control. In addition to heat exchanger networks, the proposed method may also be applied to other processes where the optimum lies at the intersection of constraints. © 1999 Elsevier Science Ltd. All rights reserved.

1. Introduction

Methods for heat exchanger network (HEN) synthesis have been developed during the last decades and these methods aim to design a HEN that yields a reasonable trade-off between capital and operating cost in the nominal case. Since the mid 1980s several authors have also investigated flexibility of HENs, e.g. Kotjabasakis and Linnhoff (1986) who introduced sensitivity tables to find which heat exchanger areas should be increased in order to make a nominal design sufficiently flexible. In Papalexandri and Pistikopoulos (1994), HEN synthesis and flexibility are considered simultaneously using mathematical programming.

The total design effort (on a systems level) required for a HEN typically involves the following three stages:

- Nominal design. Synthesize one or more networks with good properties for nominal stream data.
- Flexibility and controllability. Investigate the networks with regard to flexibility and controllability, and possibly introduce some modifications (e.g. increased area) such that at least one HEN shows satisfactory results.

- Operation. Design a control system to operate the HEN properly. This involves control structure selection and possibly some method for on-line optimization

For each step, some networks may be rejected or the designer may go back to the preceding step to find other alternatives. The steps are usually carried out in a sequential manner, however, the design may also be of a more simultaneous character, depending on the methods used.

Compared to synthesis of nominal and flexible HENs, much less effort has been dedicated to find methods for the operation of HENs (step c). Mathisen, Skogestad and Wolff (1992) investigated bypass selection for control of HENs, without considering the utility consumption. In Mathisen, Morari and Skogestad (1994) method for operation of HENs that minimizes utility consumption is proposed. The method is based on structural properties of the network, however, the variable control configuration may result in poor dynamic performance. A method based on repeated steady state optimization is suggested by Boyacı, Uz-türk, Konukman and Akman (1996), but their focus is not on the control structure for closed loop implementation. Recently, Aguilera and Marchetti (1998) proposed a method for on-line optimization and control of HENs. They also discussed degrees of freedom with respect to optimization of HENs during operation.

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In this paper, we consider optimal operation of HENs. This is normally done in two steps:

1. Obtaining the optimal solution at regular time intervals (normally using steady state data).
2. Implementing the optimal solution by specifying the optimal values (setpoints) for some variables with the aim of achieving ‘self-optimizing control’.

Step 1 is usually solved by numerically optimizing the degrees of freedom in the model, and in Section 5 we present a simple formulation of the optimization problem where the bypass fractions do not explicitly appear. This makes the problem less nonlinear. An alternative way of solving the optimization problem, which works in some cases, is to use structural information only. This procedure is discussed in Mathisen et al. (1994) and Glemmestad (1997), and is not discussed any further here.

The importance of selecting the right variables in step 2 is often overlooked. It will be shown that the choice of optimization variables affects the steady state performance of the (controlled) HEN when unknown disturbances are present, and a procedure for optimal selection of these variables is presented. The procedure extends ideas for selection of controlled outputs presented in Skogestad and Postlethwaite (1996).

In the following, it is assumed that the stream data (heat capacity flowrates and supply/target temperatures), network structure and heat exchanger areas are given and that the HEN is sufficiently flexible. To manipulate the network it is assumed that utility duties can be adjusted and that a variable bypass is placed across each process-to-process heat exchanger. In case of stream splits, we may also assume that split fractions can be varied. Variable split fractions introduce nonlinearities in the steady state optimization problem.

The remaining part of the paper is organized as follows: First, the complete method is outlined. In Section 3 the procedure for selection of optimization variables will be described in detail, and robust optimum is explained in Section 4. The steady state optimization model is presented in Section 5, and the complete method is applied to an example in Section 6. Finally some conclusions are drawn in Section 7.

2. Outline of method

With the term optimal operation, we mean that the following two goals are fulfilled:

- Primary goal: Satisfy targets (usually outlet temperatures).
- Secondary goal: Minimize operating cost.

A prerequisite for performing a meaningful on-line optimization is that there is at least one extra degree of freedom during operation, and most HENs have this feature. As an example, consider the network in Fig. 1

where there are four manipulations (two bypasses and heater and cooler duties) to control the three outlet temperatures to their targets (primary goal). Hence we have one manipulation ‘in excess’ which implies one degree of freedom. This extra degree of freedom can be used to minimize utility cost, i.e. to achieve the secondary goal.

Note that the number of degrees of freedom during operation is different from the synthesis stage. Within the ‘synthesis terminology’, the HEN in Fig. 1 has minimum number of units and *no* degrees of freedom (constraints on ΔT_{\min} , etc. have no relevance during operation). In some cases the degrees of freedom that affects the objective during operation may be less than the number of excess manipulations, however, this is not discussed any further in this paper (see Glemmestad, 1997).

Figure 2a shows a schematic block diagram of the method that will be described. The optimizer contains a scalar objective function (criterion) J which indicates how well the HEN is operated and a steady state model of the HEN. As objective function we will typically use total utility cost which is a linear function, see Eq. (4) in Section 5. The model is optimized regularly and reference values for the optimization variables are passed to the controller K_2 . The reference values (setpoints) are constant in the period between each optimization.

All inputs (manipulations) u and outputs (measurements) y are separated into $u = [u_1 \ u_2]^T$ and $y = [y_1 \ y_2]^T$, respectively. y_1 are those outputs which have given target (reference) values and u_1 are those manipulations dedicated to keep y_1 at their target values. Satisfying the targets for y_1 , is simply the primary goal of optimal operation. Now, we close the control loops for the primary outputs with controller K_1 (‘base control’) and assume integral action in the loops (no steady state control error). This leads to Fig. 2b where focus is on the remaining part of the system, i.e. the secondary variables (u_2 , y_2 and setpoints r_2) and the optimizer. It is simply assumed that the base control is implemented and that it works. The problem can be formulated as

$$\min_{u_2} J(x, d)$$

$$f(x, d) = 0$$

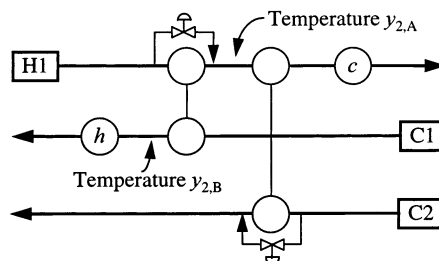


Fig. 1. Simple HEN with one degree of freedom during operation.

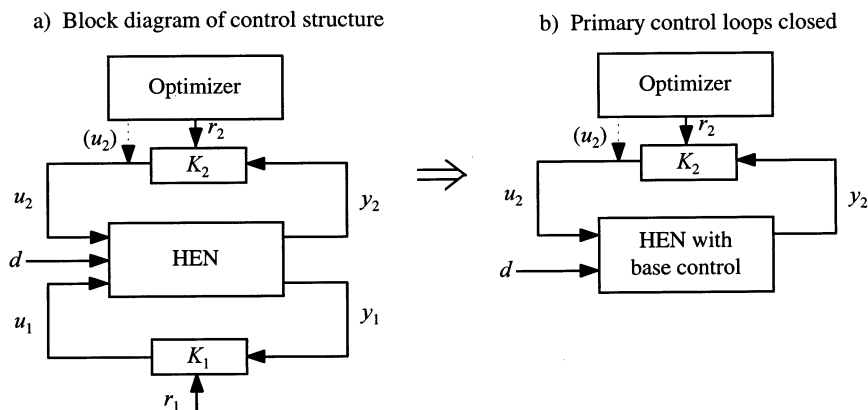


Fig. 2. General optimizing control structure.

$$g(y_2) \leq 0$$

where the latter equations represent the constraints. As explained above we assume that the constraints on y_1 are satisfied using the base control system and thus will enter the model equations $f(x, d) = 0$ (as equality constraints).

We want to focus on the secondary goal of optimal operation; utility cost minimization (variables associated with this goal have index 2). u_2 is the ‘excess’ manipulation(s) which represent the degree(s) of freedom that we will use to minimize utility cost. Of course, one could compute optimal values for u_2 and apply these directly (open-loop implementation) as indicated by the dashed line in Fig. 2a and b. Alternatively, the optimizer could pass reference values for some ‘extra’ measurements y_2 (closed-loop implementation). If the disturbance d was perfectly known (and constant), it would not matter (at steady state) which variables were chosen. However, from the explanation below it will be clear that the selection of *which* variables that are passed from the optimizer down to the control level affects how close to optimum the HEN can be operated.

The variables (setpoints) r_2 that are passed from the optimizer to the control level will be denoted optimization variables.

Let the disturbance d be partitioned into the following two contributions:

$$d = d_0 + d_u \quad (1)$$

where d_0 is the information that the optimizer has about the disturbances when it performs an optimization, and d_u (unknown disturbances) are all deviations from d_0 and the real disturbance until a new optimization is carried out. That is, d_u consists of, for example, unknown disturbances and model errors in addition to changes of the disturbances in the period between two optimizations (optimization interval). Measurement/estimation errors will not be handled explicitly in this

paper, but these errors may be included in d_u and treated as any other uncertainty.

Since the optimizer has no specific information about d_u , the optimization is based on $d = d_0$. In practice, however, d_u may vary within some known (or selected) bounds. The effect of $d_u \neq 0$ should be taken care of in the optimizer in order to avoid that the HEN becomes infeasible (primary goal cannot be satisfied) for some disturbances. Figure 3 shows a typical situation for a general plant with one degree of freedom (one extra manipulation) and an objective function J that should be minimized. The plant has one disturbance input and two candidate measurements A and B ($y_2 = [y_{2,A} \ y_{2,B}]^T$) that can be controlled to a desired value using the extra manipulation u_2 (since subscripts 1 and 2 are used to distinguish between the primary and secondary sets of inputs and outputs, we use letters A, B, etc. to denote individual elements of u and y). Also, remember that base control to keep primary outputs at fixed setpoints is already implemented. See Fig. 1 where two candidates for secondary controlled temperatures ($y_{2,A}$ and $y_{2,B}$) are indicated. The primary controlled measurements are the outlet temperatures of the three streams. Notice, however, that the HEN in Fig. 1 actually is a constrained process and the curves in Fig. 4 are more realistic (see below).

Figure 3a shows J as a function of $y_{2,A}$ with the disturbance as a parameter. The solid line is for $d_u = 0$, and the two dashed lines represent the extremes for d_u . Figure 3b shows similar curves as a function of $y_{2,B}$. Since we have to base our optimal values on $d_u = 0$, we can choose to keep either $y_{2,A} \approx 0.5$ or $y_{2,B} \approx 0.4$ using feedback control. From the figure, however, we see that when keeping $y_{2,B}$ constant, J is *less sensitive* to both variations in y_2 (control error) and to unknown disturbances, than when keeping $y_{2,A}$ constant. Therefore, we prefer to keep $y_{2,B}$ constant between the optimizations. This simple example illustrates how the choice of optimization variables affects the objective function for an unconstrained process. Figure 4 shows similar curves as

in Fig. 3 for a process where the optimum is *constrained*, which is typical for most HENs (minimum utility consumption corresponds to maximum utilization of process-to-process exchangers which again means that some bypasses are closed).

In Fig. 4a the process is infeasible when $y_{2,A}$ is specified too small (marked with ‘+’) and in Fig. 4b when $y_{2,B}$ is too large. In a HEN this typically happens when a bypass saturates such that a target temperature no longer can be met. As an example see Fig. 1. If $y_{2,A}$ is specified too low the outlet temperature of stream C1 will not be reached even if bypass around the process heat exchanger to the right is fully closed. Thus the HEN operation becomes infeasible. This represents a serious problem when implementing the optimal solution. For example, if we compute the nominally optimal value of $y_{2,A}$ (with $d_u = 0$), then we see from Fig. 4a that this will lead to infeasibility if the disturbance increases (e.g. to $d_{u,max}$). Two simple approaches can be used to avoid the infeasibility:

1. ‘Back-off’ from the nominal optimum by implementing the value where b is the back-off. This approach has been suggested by Arkun and Stephanopoulos (1980) and Narraway, Perkins and Barton (1991). The value of b should be such that the solution is feasible for all expected disturbances (as indicated by the vertical lines in Fig. 4).
2. Introduce safety margin on the constraints during the nominal optimization, i.e. replace $g(u, y) \leq 0$ by $g(u, y) \leq \varepsilon$ where again the safety margin ε should be such that the solution is feasible for all expected disturbances.

These two approaches only require that the *nominal* problem (with $d_u = 0$) is solved on-line as b or ε are assumed to be precomputed. A third more computationally demanding approach is also possible:

3. Find the *robust* optimal solution (the truly optimal value of y_2 taking into account all possible disturbances). This approach is discussed in Section 4.

The following steps summarize the main parts of the complete procedure for on-line optimization of HENs (the notation is as in Fig. 2):

1. Determine which manipulations (u_1) that should be used to control the primary outputs y_1 and design a control configuration and controllers for the primary goal (base control).
2. For each excess manipulation u_2 choose a measurement y_2 (among all candidates) such that the operation is insensitive to disturbances (see more details in next section). The additional constraints (safety margins) on u_1 are also found. Design decentralized controllers for control of y_2 .
3. Implement the steady state model including the constraints found in step 2 in the optimizer.

These three steps are carried out prior to operation. With the steady state optimizer from step 3 implemented, the optimizer computes setpoints for the optimization variables and apply these to the controller K_2 (see Fig. 2) at regular intervals during operation. Step one (base control) is considered rather trivial in most cases and is not treated in detail in this paper, see e.g. step 1 in the example in Section 6. The important step of selecting optimization variables (secondary measurements, step 2) is treated in the section below.

3. Selection of controlled outputs

This section describes a procedure for selection of the controlled variables (step 2 in the complete method given above). The selection of outputs for optimizing control is discussed by Skogestad and Postlethwaite (1996), chapter 10) where a method based on choosing outputs that maximize $\sigma(G_{22})$ (smallest singular value) for a properly scaled system is proposed. In this paper a more direct method is applied (which is also mentioned in Skogestad & Postlethwaite, 1996). In this section, we assume that the HEN is capable of reaching all targets (primary goal of optimal operation) for all disturbances that are encountered, and we also require that targets are satisfied for prespecified bounds on the unknown disturbances d_u . Before the procedure is presented, the following notation is introduced:

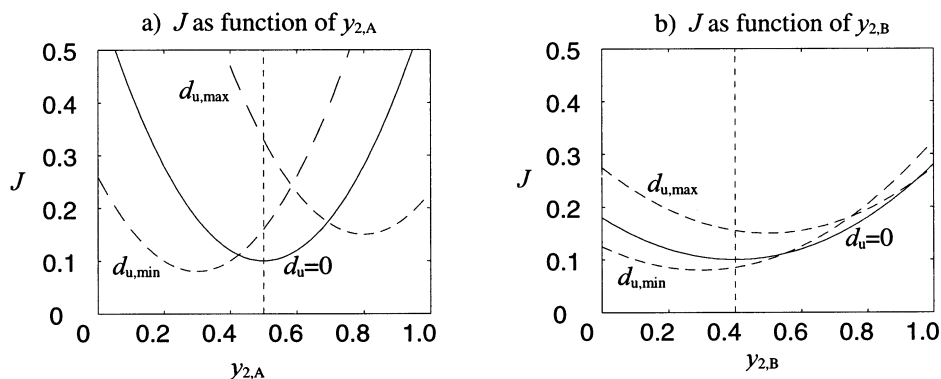


Fig. 3. Unconstrained process.

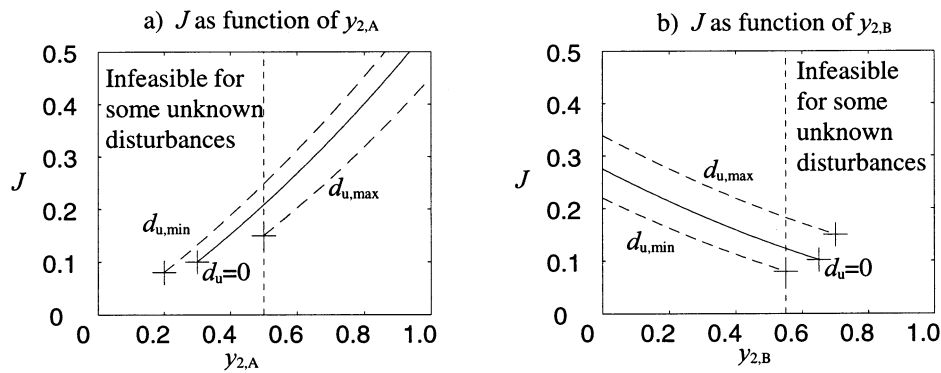


Fig. 4. Constrained process (typical for HENs).

- $y_{2,cand}$ is a vector containing all candidates to y_2
- y_{opt} is the optimal value of $y_{2,cand}$ for a given d_u
- y_{opt}^s is a fixed value of $y_{2,cand}$ such that the objective function is minimized while the network is feasible for all d_u
- J^s is $J(y_{opt}^s)$ for a given value of d_u
- Δu_1 is the constraint imposed on u_1 such that an optimization problem based on $d_u = 0$ gives feasibility for all d_u within prespecified bounds.

The steps in the procedure are listed below and some of the points will be further explained. For simplicity, we will assume there is only one degree of freedom (one optimization variable).

- a. Select (i) minimum and maximum values for d_u ; (ii) the objective function J ; (iii) the entries of $y_{2,cand}$; (iv) the values for d_u that should be included in the computations; and (v) define the type of J_{mean} that will be used for choosing optimization variable. This last step consists in selecting whether J_{mean} should be based on arithmetic mean or some weighted average of J for the different unknown disturbances d_u .
- b. Compute y_{opt} and J_{opt} for ‘all’ cases of d_u (i.e. the values from step (iv) in the previous point), see Table 1. This table may also include row(s) for $u_{2,opt}$ (open-loop implementation). Note that $d_{u,j}$ is case j of d_u while $y_{opt,i}$ denotes element i in y_{opt} .
- c. Keep $y_{2,cand,j} = y_{opt,i}^s$ for each output candidate, and evaluate $J_i^s(d_{u,j})$ and the resulting J_{mean} . In general, the setpoint $y_{opt,i}^s$ should be optimized in order to minimize J_{mean} , but for constrained processes it will be some extreme value from Table 1 (to ensure feasibility for all d_u , see remark 2 at the end of Section 6 for an explanation for the example). Notice that for the Monte Carlo method described in the next section, $y_{opt,i}^s$ will not be any extreme value from Table 1.

- d. Choose the variable that gives the smallest J_{mean} from the last column in Table 2 as optimization variable, i.e. this measurement should be controlled to a setpoint which is updated periodically by the optimizer.

We have now found the best optimization variables. To simplify the on-line optimization we may want to use only the nominal disturbance set, $d_u = 0$. To ensure that we find the correct value of y_{opt}^s (which ensures feasibility for all disturbances), we may impose some constraint (‘safety margin’) for the optimizer, e.g. $u_1 \geq \Delta u_1$. This will be explained in more detail for a simple example in Section 6 (see also remark 1 at the end of this section). The ‘safety margin’ on u_1 should of course not be implemented in the regulatory control level.

Until now we have only considered one degree of freedom. If there were two degrees of freedom, two elements of $y_{2,cand}$ would have to be fixed at a time. Tables 1 and 2 would need as many rows as there are possibilities to pick two variables out of the total number of candidate measurements, and each row would contain two parameter values. For example, if there are six candidate measurements and two degrees of freedom, the number of combinations is $6!/2!4! = 15$.

For cases with more than one degree of freedom, it may be difficult to pick the optimal parameter values based on reasoning. In such cases, the optimal parameter values may be found as follows: for each disturbance $d_{u,j}$ in Table 1, fix the parameters to the optimal value $y_{opt}(d_{u,j})$ and let a computer evaluate the steady

Table 1
 y_{opt} and J_{opt} for all cases of d_u

	$d_{u,1}$	$d_{u,2}$	$d_{u,j}$
$y_{opt,A}$	$y_{opt,A}(d_{u,1})$	$y_{opt,A}(d_{u,2})$	$y_{opt,A}(d_{u,j})$
$y_{opt,B}$	$y_{opt,B}(d_{u,1})$	$y_{opt,B}(d_{u,2})$	$y_{opt,B}(d_{u,j})$
$y_{opt,i}$	$y_{opt,i}(d_{u,1})$	$y_{opt,i}(d_{u,2})$	$y_{opt,i}(d_{u,j})$
J_{opt}	$J_{opt}(d_{u,1})$	$J_{opt}(d_{u,2})$	$J_{opt}(d_{u,j})$

Table 2
 J^s for all cases of d_u

	$d_{u,1}$	$d_{u,2}$	$d_{u,j}$	J_{mean}
J_a^s	$J_A^s(d_{u,1})$	$J_A^s(d_{u,2})$	$J_A^s(d_{u,j})$	$J_{\text{mean,A}}$
J_B^s	$J_B^s(d_{u,1})$	$J_B^s(d_{u,2})$	$J_B^s(d_{u,j})$	$J_{\text{mean,B}}$
J_i^s	$J_i^s(d_{u,1})$	$J_i^s(d_{u,2})$	$J_i^s(d_{u,j})$	$J_{\text{mean,i}}$

state equations describing the HEN for all unknown disturbances $d_{u,j}$. The parameter values that result in feasibility for all $d_{u,j}$ should be used to generate Table 2 in step c. This procedure is repeated for all (sets of) candidate measurements. It is emphasized that industrial HENs usually do not have more than one (or in some cases, two) degrees of freedom available for optimization after regulatory control is implemented. Thus, problems involving many degrees of freedom will not appear frequently in real applications.

Remark 1. It is clear that the value of Δu_1 may depend on d_0 , i.e. the value of the safety margin implemented in the optimizer to ensure feasibility may depend on the operating point. This may be due to nonlinearities in the model or due to the fact that the active constraints may not be the same for all operating points. We assume that this change in Δu_1 is small and that the value can be used for all d_0 . In practice, one should carry out the procedure for selection of optimization variables at different operating points to verify that Δu_1 does not change too much. The worst case value should be chosen if it is not acceptable to violate the primary goal, while a mean value can be used if the resulting violation of the targets is tolerable.

4. Robust optimum

This section introduces the term robust optimum. It will be clear that, when we have made a selection of variable(s) for y_2 , we seek the value of this variable that corresponds to the robust optimum. The robust optimal value is the optimal value when unknown disturbances and model errors are encountered. The robust optimum will often be different from the nominal optimum, where no disturbances (only nominal disturbance values) or model errors are considered.

For the unconstrained and smooth objective function in Fig. 3, the value of y_2 resulting in robust optimum is approximately equal to the value resulting in nominal optimum. For the constrained objective function in Fig. 4, the situation is different. We require that the process is feasible for the unknown disturbances. When $y_{2,A}$ is selected, the *nominal* optimal value is $y_{2,A} = 0.30$, whereas the value resulting in *robust* optimum is $y_{2,A} = 0.50$. When $y_{2,B}$ is selected, the *nominal* optimal value is $y_{2,B} = 0.65$, whereas *robust* optimum is achieved for

$y_{2,A} = 0.55$. It is clear that the nominal optimal value cannot be applied since this will result in infeasibility for some unknown disturbances.

Mathematically, if we have an objective function $J(x, d)$ where the value of the variable (argument) x can be selected/manipulated and d represent the disturbances, we can write

$$\begin{aligned} \text{Nominal optimum:} & \quad J_{\text{nopt}} = \text{opt}(J(x, d = d_0)) \\ \text{Robust optimum:} & \quad J_{\text{ropt}} = \text{opt}(J(x, d \in D)) \end{aligned}$$

In operation, we are not interested in the objective value directly, however, the argument (e.g. manipulations or setpoints) that minimize the objective is more relevant:

Nominally optimal argument:

$$x_{\text{nopt}} = \arg \text{opt}_x(f(x, d = d_0))$$

Robust optimal argument:

$$x_{\text{ropt}} = \arg \text{opt}_x(f(x, d \in D))$$

The set D represents the possible values of the unknown disturbances. D may also represent a probability distribution of the unknown disturbances. The disturbances may for instance be assumed to be normally distributed with a given mean (nominal) value and a given standard deviation. If the probability distribution is not bounded, we cannot require the process to be feasible for *all* possible disturbances. Instead, we could require feasibility with a specified probability, such as requiring feasibility in 99% of the operating time. We shall, however, use a different approach where the disturbances are specified by probability distributions: When a HEN is announced infeasible during operation, this normally does not imply that the HEN cannot be operated. Typically, the reason for infeasibility is that it is impossible to reach the targets for the primary measurements (outlet temperatures). Instead of requiring that the targets are satisfied, we choose to penalize the deviation (control error in primary control loop). In this way, the constrained problem is transformed into an unconstrained problem, however, the new objective function will often be asymmetric. This implies that the robust optimum is different from the nominal optimum.

In the search for the robust optimum, several approaches may be used depending on the assumptions made and the external conditions. Here, we shall focus on two methods: (1) a Monte Carlo method where the unknown disturbances are specified as probability distributions; and (2) requiring that targets are satisfied for some corner points of the unknown disturbances, i.e. similar to the assumptions made in the previous section. These two methods are described below.

4.1. Monte Carlo method

In this case, the unknown disturbances are given as probability distributions. If a disturbance is, for example normally distributed, we cannot require that targets are satisfied for all disturbances since this would require infinite back-off. Instead of requiring targets to be satisfied, control error is penalized. Since the constraints on the outputs (targets) are removed, there will always exist a feasible solution. The constraints on the manipulations are still present, but control errors resulting from satisfying these targets will be penalized in the objective function J . To find an (estimate) for the robust optimal value, Monte Carlo simulations are applied. That is, random disturbances are generated from the given probability distribution. For each set of random disturbances the objective J is computed as a function of the selected secondary variable. This is repeated for a large set of randomly generated disturbances, and the robust optimum is the optimum of the mean value of J for the set of disturbances. All instances of the randomly generated values, including those that result in control error is used for the computation of robust optimum.

An example is shown in Fig. 5 below. There are two different disturbances and they are both normally distributed. The thin solid line shows the objective J as a function of the selected secondary measurement y_2 for the nominal disturbances. Starting from a high value of y_2 , the objective decreases when y_2 decreases until the nominal optimum where $y_2 = 150$. Decreasing y_2 further, the primary outputs can no longer be kept at target. This is penalized in the objective function resulting in a steep increase of the objective.

The four dashed lines show the objective for four different sets of the disturbances. The location where the primary targets are violated and objective increases steeply, depends on the value of the disturbances. The most interesting curve in Fig. 5, however, is the thick

solid curve. This curve is the average of 1000 randomly generated sets of disturbances. While each fixed disturbance results in a sharp break of the corresponding curve, the average curve is smooth. To reach the robust optimum, a setpoint of $y_2 \approx 153$ should be applied. The robust optimum is slightly above 150, which is higher than the nominal optimum of $J = 145$. However, applying the nominally optimal values of $y_2 = 150$, the expected value of the objective is roughly $J \approx 170$. Since we do not know the exact value of the disturbances, only the probability distribution, the optimal setpoint for y_2 is the value corresponding to the robust optimum ($y_2 \approx 153$).

The curves in Fig. 5 are actually based on the results for the HEN in the example in Section 6, when the disturbances are normally distributed. The supply temperature of stream H1 has a mean value of 190°C and a standard deviation of 3°C, while CP_{C2} has a mean value of 0.50 kW/°C and a standard deviation of 0.01 kW/°C. The variable selected for y_2 is the setpoint for temperature T_1 (in Fig. 7) and the objective function is:

$$J = \text{utility consumption (kW)} + 25|T_{C2}^o - T_{C2}^t| + 25|y_2 - r_2|$$

That is, a deviation of 1°C in the bypass controlled temperature or in T_1 , is assumed to ‘cost’ the same as 25 kW. This type of objective function is suitable if the control error (deviation from target) corresponds approximately linearly to increase in operating cost. In such cases the weights should be selected to reflect the real change in operating cost. In other cases the relationship between control error and operating cost will be strongly nonlinear and difficult to estimate. As an example, a control error in an outlet temperature from the HEN corresponding the inlet temperature of a reactor may strongly affect the conversion or even extinguish the reaction. In such cases where the detailed correlation between control error and operating cost is not known, the control error may be given a quadratic penalty and the weights will have to be based on the best engineering judgment available.

4.2. Method requiring target satisfaction

This method is similar to the assumptions made in Section 3. That is, a prespecified (discrete) set of unknown disturbances is defined (typically corner-points), and target satisfaction is demanded for this discrete set of disturbances. If this results in infeasibility for some disturbances (during step b of the procedure) some assumptions such as the magnitude of unknown disturbances need to be relaxed. Alternatively, the plant will have to be modified.

This second method has some disadvantages compared to the Monte Carlo method:

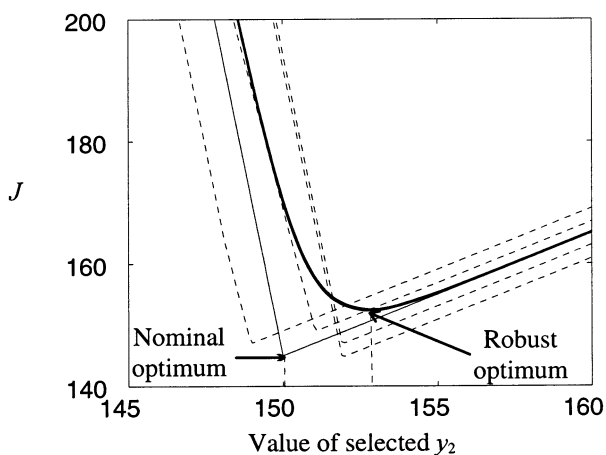


Fig. 5. Robust optimum.

1. The worst case may not be at the corner points for the disturbances, thus finding the worst case disturbances may be a difficult task itself.
2. Even a corner point check may be time consuming and even prohibitive when there are many independent disturbances.
3. If there are more than just a few disturbances, the probability that all have the worst case values at the same time is very small.

Taking these three points into account, it is assumed that Monte Carlo simulations will give reasonable results for most practical applications. Using the Monte Carlo approach, it is also straightforward to include dependencies between different disturbances when such information is available. Despite this preference for the Monte Carlo approach in real applications, the example in Section 6 requires that the HEN is feasible at the corner points for the two disturbances. Further, requiring targets to be satisfied also forms the basis for the steady state optimization model presented in the next section.

A disadvantage with the Monte Carlo method is that a large number of disturbance sets may have to be generated in order to get good results, and it is difficult to say in advance how many disturbance sets that are necessary. However, as it is explained in the previous section, the robust optimum is only determined once (or for a few cases) and this is done off-line. The difference in the nominal and robust optimal values are used to find a constraint on the primary manipulations u_1 . With this constraint, the (approximate) robust optimum is found periodically from the measured and inaccurate values of the disturbances.

5. A steady state formulation of the optimization problem

This section presents a simple steady state formulation of the optimization problem that can be adapted to any HEN. The formulation has the advantage of not explicitly including the bypass fractions, thus a major source to nonlinearities in the model is avoided. In addition, the formulation is based on heat balances around each heat exchanger in a way that is simple to understand and implement. It is developed primarily for implementation in the optimizer, however, it may also be used in the procedure for selection of optimization variables (to generate Tables 1 and 2).

Before we present the general formulation, consider the two alternatives to model a single heat exchanger with bypass, see Fig. 6. The first alternative is to use Eq. (2a) and Eq. (2b) while the second alternative is to use Eq. (3).

At steady state it is of no consequence whether the bypass is placed across the hot side or cold side, and

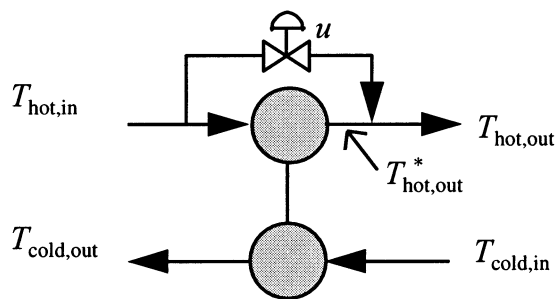


Fig. 6. Single heat exchanger with bypass.

the choice in Fig. 6 is arbitrary. The temperature driving force $\Delta T_m(\cdot)$ may be the logarithmic mean or some approximation to it. Note particularly the difference between Eq. (2a) and Eq. (3) regarding the arguments of $\Delta T_m(\cdot)$.

$$Q = UA \Delta T_m(T_{hot,in}, T_{cold,in}, T_{hot,out}^*, T_{cold,out}) \quad (2a)$$

$$T_{hot,out} = uT_{hot,in} + (1-u)T_{hot,out}^* \quad (2b)$$

$$Q \leq UA \Delta T_m(T_{hot,in}, T_{cold,in}, T_{hot,out}, T_{cold,out}) \quad (3)$$

Eqs. (2a) and (2b) includes the hot exit temperature before it is mixed with the bypass stream and this results in bilinearities in Eq. (2b). The inequality in Eq. (3) expresses a constraint on Q when the boundary is placed *outside* the bypass splitter and mixer. The bypass fraction u does not even occur in Eq. (3) but the equality part of Eq. (3) corresponds to $u = 0$. In the optimization model, we choose the second alternative for each heat exchanger since this eliminates the bilinearities in the bypass mixer. If u is needed, it can be found after the optimization of the network by solving one nonlinear equation for each bypass fraction. This equation can be found from solving Eq. (2b) for $T_{hot,out}^*$ and inserting this expression into Eq. (2a), which is solved for u through iteration (solving one unknown in one nonlinear equation n times is much simpler than solving n unknowns in n nonlinear equations simultaneously). As it will be shown, the value of u is often *not* required explicitly as it normally is the manipulated input in a feedback control loop.

The steady state formulation for a general HEN uses the following sets of heat exchangers:

- PHX: set of all process-to-process heat exchangers.
- HBT: subset of PHX with hot side outlet directly entering a bypass controlled target.
- CBT: subset of PHX with cold side outlet directly entering a bypass controlled target.
- HUT: subset of PHX with hot side outlet entering a utility controlled target (through a cooler).
- CUT: subset of PHX with cold side outlet entering a utility controlled target (through a heater).
- HS: subset of PHX with hot side inlet directly entering from a (hot) supply.

CS: Subset of PHX with cold side inlet directly entering from a (cold) supply.

The general HEN formulation shown below (Eqs. (4)–(12b)) is an NLP problem. The variable c in Eq. (4) denotes the cost (pr. energy unit) for the utilities.

$$\min \left(\sum_{i \in \text{HUT}} c_i^{\text{coolers}} Q_i^{\text{coolers}} + \sum_{j \in \text{CUT}} c_j^{\text{heaters}} Q_j^{\text{heaters}} \right) \quad (4)$$

subject to equalities (5) to (9)

$$Q_i = CP_i^{\text{cold}}(T_i^{\text{cold,out}} - T_i^{\text{cold,in}}) \quad i \in \text{PHX} \quad (5a)$$

$$Q_i = CP_i^{\text{hot}}(T_i^{\text{hot,in}} - T_i^{\text{hot,out}}) \quad i \in \text{PHX} \quad (5b)$$

$$Q_i^{\text{cooler}} = CP_i^{\text{hot}}(T_i^{\text{hot,out}} - T_i^f) \quad i \in \text{HUT} \quad (6a)$$

$$Q_i^{\text{heaters}} = CP_i^{\text{cold}}(T_i^f - T_i^{\text{cold,out}}) \quad i \in \text{CUT} \quad (6b)$$

$$T_i^{\text{hot,out}} = T_i^f \quad i \in \text{HBT} \quad (7a)$$

$$T_i^{\text{cold,out}} = T_i^f \quad i \in \text{CBT} \quad (7b)$$

$$T_i^{\text{hot,in}} = T_i^S \quad i \in \text{HS} \quad (8a)$$

$$T_i^{\text{cold,in}} = T_i^S \quad i \in \text{CS} \quad (8b)$$

$$\text{Interconnection equations (problem specific)} \quad (9)$$

Inequalities, Eqs. (10), (11), (12a) and (12b)

$$Q_i \leq \alpha_i U_i A_i \Delta T_{mi} \quad i \in \text{PHX} \quad (10)$$

$$Q_i \leq 0 \quad i \in \text{PHX} \quad (11)$$

$$Q_i^{\text{coolers}} \geq 0 \quad i \in \text{HUT} \quad (12a)$$

$$Q_i^{\text{heaters}} \geq 0 \quad i \in \text{CUT} \quad (12b)$$

Note that the index denotes heat exchangers and *not* streams (which is common in many other models), and that ΔT_m denotes the temperature driving force *outside* the bypass stream as in Eq. (3). As an example, the network in Fig. 7 will lead to the following sets: PHX = {A, B}, HUT = {B}, CUT = {A}, HBT = \emptyset , CBT = {B}, HS = {A} and CS = {A, B}, and the only interconnection Eq. (9) is $T_A^{\text{hot,out}} = T_B^{\text{hot,in}}$.

During each optimization, T^f , T^S , CP and UA for each heat exchanger are treated as constants. The model is valid without modifications for networks with fixed stream split fractions since CP denotes heat flow capacity in each heat exchanger. For networks

with variable stream splits, CP in the split streams can be regarded as variables, and equations that preserve the mass balance in the splitter(s) and energy balance in the mixer(s) must be included. During operation, variable stream splits can be used as manipulated inputs.

When the model above is used during operation, it is important to ensure a feasible solution for all possible cases that may be encountered. This may be implemented by a hierarchical strategy for constraint satisfaction. For example, if infeasibility is encountered, then the least critical target in Eqs. (7a) and (7b) is removed and a new optimization is performed. If this problem also result in infeasibility, the second least critical target is removed and so on until feasibility is achieved. Alternatively, all constraints in Eqs. (7a) and (7b) may be removed and control error may be penalized in the objective function (Eq. (4)). This is similar to the Monte Carlo approach in the previous section.

The constant α in Eq. (10) is a factor that may limit the duty of a heat exchanger somewhat below its theoretical maximum. This is simply the way that the constraint on u_1 , is implemented. Instead of implementing $u_1 \geq \Delta u_1$, directly (which is impossible since the model does not include u_1), the corresponding value for α has to be computed. This is done separately for each heat exchanger that controls a primary output. The example below explains how this can be done. Notice that the relationship between Δu_1 and α may depend on the operating point. For heat exchangers associated with u_2 , we have $\alpha = 1$.

The model does not include any upper constraints on the duty of the utility exchangers, and this implies the assumption that these are designed to handle the required duty. If this is not the case, additional constraints have to be added to the model, e.g. an upper limit on the duty.

The only possible source of nonlinearities in the model (for networks without variable splits) is the term ΔT_m in Eq. (10). In other words, if arithmetic mean (as opposed to logarithmic mean) is used as the temperature driving force, the model can be solved as an LP problem. The following procedure for solving the model has proven to be reliable: First, use arithmetic mean in Eq. (10) for all exchangers and solve the corresponding LP problem. Second, replace arithmetic mean with logarithmic mean (or e.g. Paterson or Chen approximations, see Paterson, 1984 and Chen, 1987) and solve the NLP problem using the LP solution as the initial value. In the case of one or more variable split fractions, the steady state model (Eqs. (4)–(12b)) have to be modified to incorporate this. Then the problem becomes nonlinear even if ΔT_m is approximated by arithmetic mean.

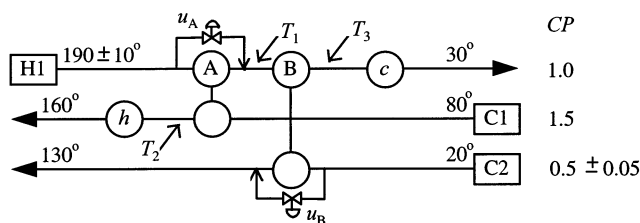


Fig. 7. Heat exchanger network used in example.

Table 3
Values for y_{opt} and J_{opt} for all cases of d_u in the example^a

	Case 1 $d_{u,1} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$	Case 2 $d_{u,2} = \begin{bmatrix} -3 \\ -0.01 \end{bmatrix}$	Case 3 $d_{u,3} = \begin{bmatrix} -3 \\ +0.01 \end{bmatrix}$	Case 4 $d_{u,4} = \begin{bmatrix} +3 \\ -0.01 \end{bmatrix}$	Case 5 $d_{u,5} = \begin{bmatrix} +3 \\ +0.01 \end{bmatrix}$
$T_{1,\text{opt}}$	150.0	149.0	151.0	151.9	151.9
$T_{2,\text{opt}}$	106.7	105.4	104.0	107.4	107.4
$T_{3,\text{opt}}$	95.0	95.1	94.9	98.0	95.8
$u_{A,\text{opt}}$	0.000	0.105	0.292	0.000	0.000
$u_{B,\text{opt}}$	0.000	0.000	0.000	0.038	0.011
J_{opt}	145.0	147.0	149.0	146.9	144.7

^a Case 1 is the nominal disturbance.

6. Example

The HEN used in the example is shown in Fig. 7. The primary outputs are the outlet temperatures of each stream which should be controlled to their target values of 30, 160 and 130°C for streams H1, C1 and C2, respectively. That is, we have

$$y_1 = [T_{H1}^o \quad T_{C1}^o \quad T_{C2}^o]^T$$

where superscript ‘o’ denotes outlet temperature. There is a total of four manipulations (two bypasses and two variable utility duties) which gives

$$u = [u_A \quad u_B \quad q_c \quad q_h]^T$$

There are two disturbances; $\pm 10^\circ\text{C}$ in the supply temperature of stream H1 and $\pm 0.05 \text{ kW}/^\circ\text{C}$ in the CP of stream C2. These values represent the maximum variations d that may be present. The smaller variations/errors (d_u) that may occur within the optimization interval is defined in step 2a of the procedure. UA for heat exchangers A and B are 0.523 and 1.322 kW/°C, respectively. For simplicity, it is assumed that the utility exchangers are able to deliver sufficient duty for all possible cases. With this assumption and the given UA-values, all target temperatures can be reached for all combinations of disturbances mentioned above. Applying the procedure step by step yields:

Step 1. Assign primary manipulations.

We use the main rule for selection of manipulations in HENs which is to choose the manipulation closest to the measurement (e.g. Mathisen, 1994, chapter 4). This implies that the primary manipulations u_1 become q_c , q_h and u_B and these control the outlet temperatures of streams H1, C1 and C2, respectively.

Step 2. Selection of optimization variable.

There is one excess manipulation, $u_2 = u_A$, and the steps (a) to (d) below illustrate the selection of optimization variable.

(2a) We assume:

- $d_u = [\pm 3^\circ\text{C}, \pm 0.01 \text{ kW}/^\circ\text{C}]^T$ (maximum variations/errors of the disturbances within the optimization interval).
- The objective function is $J = q_c + q_h$ (utility consumption).
- Possible candidates to y_2 are $y_{2,\text{cand}} = [T_1 \quad T_2 \quad T_3 \quad u_A]^T$ (see Fig. 7). Note that the open-loop implementation (u_A) is an alternative.
- The computations are done for the four ‘corner points’ of d_u , in addition to $d_u = 0$. J_{mean} , is the arithmetic mean of these five cases. (We require that target temperatures have to be reached for the five cases).

(2b) y_{opt} and J_{opt} for different d_u are shown in Table 3. The table is generated for $d_0 = [0 \quad 0]^T$, i.e. for nominal values of the disturbances (190°C and 0.5 kW/°C). Also a row for $u_{B,\text{opt}}$ is included for extra information.

(2c) Table 4 shows J for optimal fixed values of $y_{2,\text{cand}}$. Note that in this example, the values for $y_{2,\text{cand}}$ can be found without optimization, but simply from Table 3 and physical insight (see remark 2 at the end of this section). If there is a possibility that the optimum is not constrained one would have to resort to conventional optimization.

(2d) From the last column of Table 4 it is clear that keeping T_1 , constant is preferred.

Step 3. Implementation of optimizer.

The model (including the sets and connection equations) was described in the previous section. The constraint (‘safety margin’) that should be included in the optimizer is $u_B \geq 0.025$. We will explain how this value is obtained, but first we explain the details in the implementation of this constraint. To implement the constraint, we first find $q_B = 55 \text{ kW}$ for $d_u = 0$ (55 kW is the deficit heat of stream C2). Then we find $\alpha_B = 0.946$ from $q_B = \alpha_B UA_B \Delta T_{m,B}$, where the last term is the logarithmic mean for heat exchanger B for $d_u = 0$ and $T_1 = 151.9^\circ\text{C}$. Implementing $\alpha_B = 0.946$ (and $\alpha_A = 1.0$) in Eq. (10) will ensure the required safety margin on u_B when unknown disturbances d_u are present.

Table 4
 J^s for the possible choices of measurement and for all cases of d_u in the example

	Case 1	Case 2	Case 3	Case 4	Case 5	J_{mean}
$J(T_1^s = 151.9)$	148.9	153.0	150.8	147.0	144.8	148.9
$J(T_2^s = 104.0)$	153.0	151.2	149.0	159.2	155.0	153.0
$J(T_3^s = 98.0)$	151.0	152.9	155.1	146.9	149.1	151.0
$J(u_A^s = 0.292)$	151.1	151.2	149.0	153.2	151.0	151.1

Note that all values for u_B (and the constraint Δu_B) differ from the values given in Glemmestad, Skogestad and Gundersen (1997) for the same example. The values in that paper actually refer to the bypass fractions on the hot side of heat exchanger B (which is inconsistent with Fig. 7), while the correct values for the bypass fractions on the cold side are given here.

The actual value for the safety margin ($\Delta u_B = 0.025$) is obtained as follows: The values of u_A and u_B for the five cases in Table 4 corresponding to $T_1 = 151.9^\circ\text{C}$ (first row in Table 4) are given in Table 5. For cases 4 and 5, u_A saturates at zero which implies that u_A just is capable of maintaining $T_1 = 151.9^\circ\text{C}$ for these disturbances. The computations in the optimizer is based on $d_u = 0$ (case one, see Table 3) where u_B takes the value of 0.025, see Table 5. Thus, in order to handle cases 4 and 5 without violating $T_1 = 151.9^\circ\text{C}$, a safety margin of $\Delta u_B = 0.025$ has to be used by the optimizer. Note that if we accepted that T_1 deviated from its setpoint (due to saturation in u_A) it would be possible to further reduce utility consumption somewhat. Then the setpoint for T_1 , could be reduced slightly below 151.9°C until u_B saturated for some disturbance. In this example we require that setpoints for secondary measurements have to be satisfied.

The reason for implementing the ‘safety margin’ on u_B as an inequality constraint is that other values of d_0 (i.e. other operating points) may give $u_{B,\text{opt}} > 0.025$. Requiring $u_B = 0.025$ in such cases will result in infeasibility.

The value for J_{mean} of 148.9 kW in Table 4 should be compared to the mean value of J_{opt} from Table 3 which is 146.5 kW. That is, it costs 1.6% of the utility consumption to guarantee feasibility for the worst case unknown disturbance.

Figure 8 shows the results for the example when T_1 is selected as secondary measurement (optimization variable). Figure 8a shows that T_{C2}° can be controlled to its setpoint for all unknown disturbances around the nominal operating point $d_0 = [0 \ 0]^T$. Note from Fig. 8b and c, that the time up to zero corresponds to case 1 of d_u , from 0 to 20 min corresponds to case 2, from 20 to 40 min is case 3 and so on. Bypass fractions are shown in Fig. 8d, and u_A drops to close to zero after 40 minutes (case 4 and 5). The perhaps most important curves are shown in Fig. 8e. The steady state values for

the utility consumption corresponds to the values in Table 4 (upper row). The optimal values (when d_u is perfectly known) from the lower row of Table 3 is also plotted for comparison. The utility consumption should be compared with the ‘traditional’ scheme *without* optimization also shown in Fig. 8e. The latter is implemented by fixing u_A at a value such that the network is just feasible for *all* possible disturbances, i.e. $d = [\pm 10, \pm 0.05]^T$, using u_B only (this requirement gives $u_A = 0.680$). From the results given in Fig. 8 and Table 4, it is clear that the main reduction in utility consumption compared to the traditional case is due to the periodic optimization (about 13% nominally), whereas the selection of optimization variable constitutes 2.75% (between best and worst case).

Figure 8 has shown results around the nominal operating point corresponding to the cases 1–5 in Tables 3 and 4. Only one optimization is done prior to the simulations, and the optimal setpoint for T_1 found here is maintained during the simulation (see Fig. 8f). Figure 9 shows similar results, but for a larger part of the operating region (with respect to disturbances) and with optimizations updating the setpoint for T_1 at 0, 20 and 40 min.

At $t < 0$, we have $T_{\text{HI}}^s = 200^\circ\text{C}$ and $CP_{C2} = 0.45 \text{ kW}/^\circ\text{C}$ (i.e. $d = [10 \ -0.05]^T$). For this operating state u_A is saturated at zero. For the steady state optimization carried out before the simulations started the constraint on u_B was not active. From Fig. 9d we see that u_B (≈ 0.13) is larger than the safety margin of 0.025 for $t < 0$, thus u_B will not saturate if unknown disturbances (within the prespecified bound of $d_u \leq [\pm 3 \pm 0.01]^T$) should occur. At time equal to zero, nominal operating conditions are encountered and an optimization is performed immediately after. Figure 9f shows how T_1 is controlled to its new setpoint (which now has the same value as in Fig. 8). At $t = 20$, the (known) disturbance $d = [-7 \ 0.04]^T$ is applied and a new optimization is done. At $t = 40$, an *unknown* disturbance of

Table 5
 Values of u_A and u_B when $T_1 = 151.9^\circ\text{C}$

	Case 1	Case 2	Case 3	Case 4	Case 5
u_A	0.207	0.354	0.354	0	0
u_B	0.025	0.038	0.012	0.038	0.012

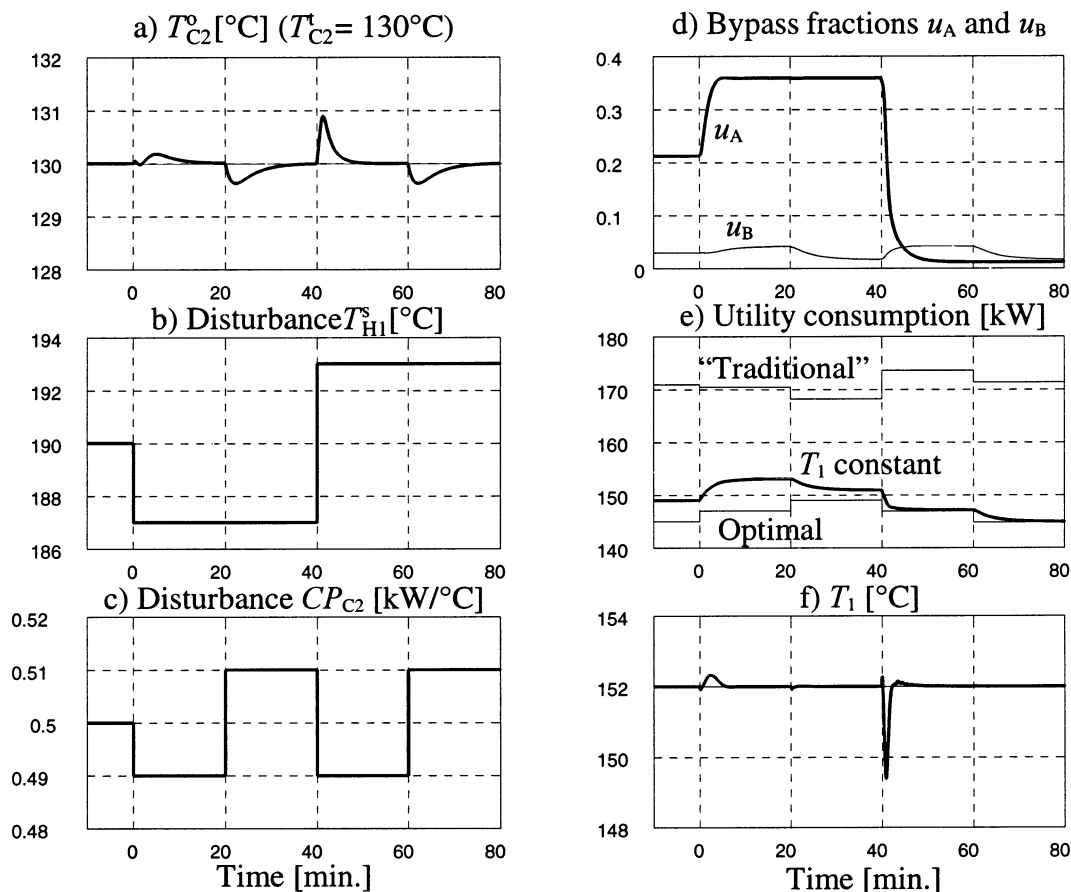


Fig. 8. Results for example. (a) Controlled output T_{C2} with setpoint 130°C . (b) Disturbance T_{H1}^s . (c) Disturbance CP_{C2} . (d) Bypass u_A (thick line) and bypass u_B (thin line). (e) Utility consumption. (f) Temperature T_1 (secondary measurement), thick line shows actual value while thin line shows setpoint (constant at 152°C).

$d_u = [-3 \ 0.01]^T$ is applied which brings the operating state to the opposite corner of where we started. Since the new disturbance is unknown, the optimizer now returns the same setpoint for T_1 as it computed at $t = 20$. The steady state value for u_B at this corner point is close to zero.

Note that the utility consumption at the last part of the simulation is similar for the ‘traditional’ approach (u_A is fixed at 0.680) and the proposed method. This is because this corner point is limiting u_A for the traditional approach, thus this approach is optimal for this corner of the operating region. After 40 min (the extreme corner point) the traditional approach actually has lower utility consumption than the proposed method. This can be explained from the requirement we have made in this example that also the setpoint for T_1 should be satisfied when unknown disturbances are present. As mentioned above, relaxing this requirement could reduce the utility consumption somewhat further for this example.

Remark 2. From Fig. 7, it is clear that decreasing T_1 , T_3 (by decreasing u_A) or u_A will reduce utility consumption (J), i.e. optimal values for these variables in Table

3 are minimum values (smaller values will violate the primary goal). Therefore, the case with the largest value has to be chosen as this is the smallest value feasible for all d_u . For T_2 , a similar (but opposite) argument leads to choosing the *smallest* value in Table 3.

7. Conclusions

A method for optimal operation of heat exchanger networks based on periodic steady state optimization is proposed. A fixed control structure for the outlet temperatures (primary measurements) is selected prior to operation. Thus, all outlet temperatures are usually controlled by the heat exchanger located immediately upstream, and a fast response is obtained. The periodic steady state optimization concerns the setpoints for measurements *internally* in the HEN, which are controlled by the remaining manipulations. An important issue is to make the implementation insensitive to uncertainty (self-optimizing control): *which* outputs (measurements) should be kept constant between each optimization. The outputs are selected such that the sensitivity to unknown disturbances and model errors is

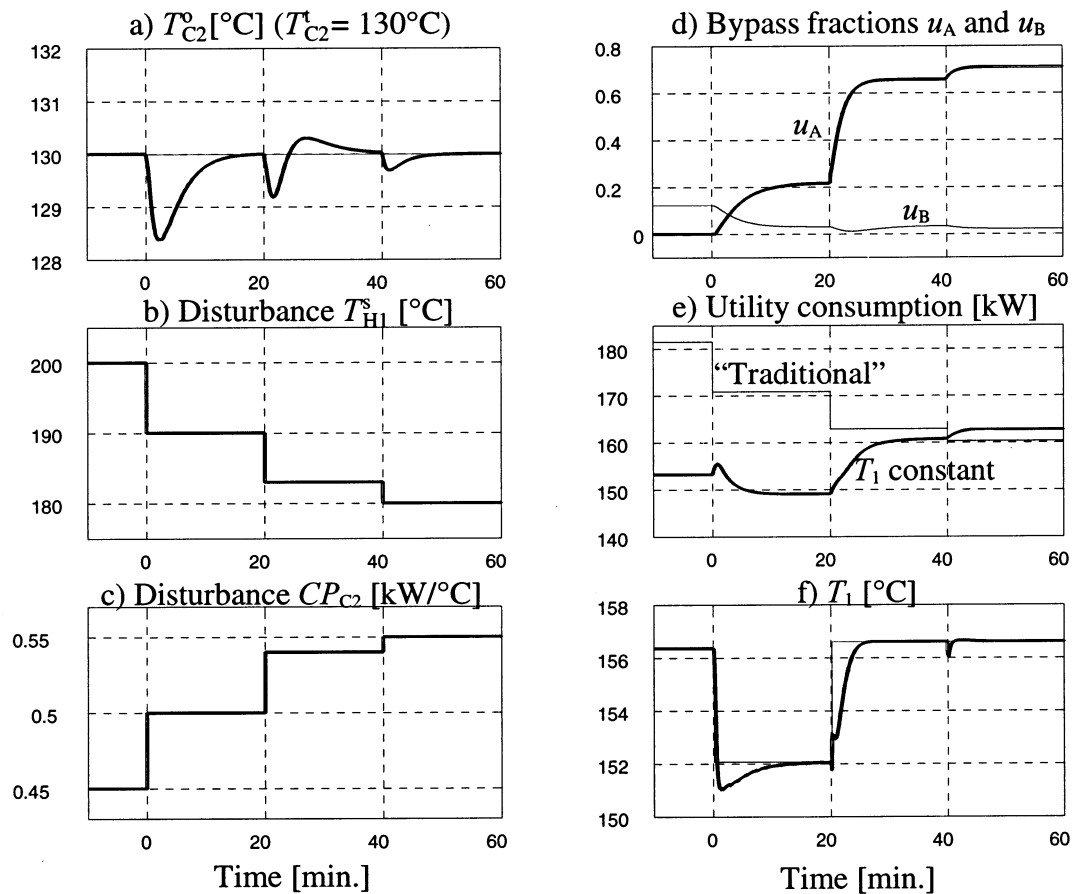


Fig. 9. Results for example. The different curves show the same as Fig. 8 but for a larger part of the operating region, and with steady state optimization done at 0, 20 and 40 min.

as small as possible. Optimal operating conditions for heat exchanger networks are normally located at the intersection of constraints. Implementing the setpoints computed by the optimizer without considering uncertainty (nominal optimum) will cause infeasible operation for certain disturbances. This infeasibility is avoided by introducing constraints in the steady state formulation in the optimizer. It is proposed to compute the constraints such that the optimizer finds the robust optimal solution, i.e. the optimal setpoints taking into account all possible uncertainties. This provides an optimal back-off from the nominal optimum.

A steady state formulation for heat exchanger networks avoiding the nonlinearities due to bypass fractions is proposed. This general model concerns heat exchanger networks in particular, however, the methodology comprising periodic steady state optimization combined with self-optimizing control is well suited also for other processes than heat exchanger networks.

8. Notation

c cost data

CP heat capacity flowrate
 d disturbance
 e control error
 g element in transfer function/matrix
 G process transfer function
 J objective function
 K transfer function for controller
 Q heat load (duty) of heat exchanger
 r reference (setpoint)
 T temperature
 t time
 u manipulated input (bypass fraction)
 y output (measurement)

Superscripts

BP bypass
 U utility
 o actual output or outlet (temperature)
 s supply (temperature)
 t target or reference (temperature)

Subscripts

0 nominal
 1 primary
 2 secondary

C_i	cold stream I
H_j	hot stream j
m	mean value
nopt	nominal optimum
ropt	robust optimum
u	unknown

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