

SELF-OPTIMIZING CONTROL

Sigurd Skogestad

Ivar J. Halvorsen

John C. Morud

Department of Chemical Engineering

Norwegian University of Science and Technology

N-7034 Trondheim, Norway

AICHE Annual Meeting, Miami Beach, Nov. 1998

ABSTRACT

A chemical plant may have thousands of measurements and control loops. By the term plantwide control is not meant the tuning and behavior of each of these loops, but rather the formulation of the overall control problem, and how to decompose the overall problem into smaller blocks, that is, selection of the structure of the control system (control structure design).

25 year ago Alan Foss challenged the process control research community in his paper "Critique of chemical process control theory" (AIChE J., 1973). He wrote:

"The central issue to be resolved ... is the determination of control system structure. Which variables should be measured, which inputs should be manipulated and which links should be made between the two sets?"

And he added

"There is more than a suspicion that the work of a genius is needed here, for without it the control configuration problem will likely remain in a primitive, hazily stated and wholly unmanageable form."

Maybe this last statement has worked as a deterrent, because there has only been limited activity in this field over the last 25 years. Actually, the approach to plantwide control is still very much along the lines described by Page Buckley in his book from 1964.

Of course, the control field has made many advances over these years, for example, in methods for and applications of on-line optimization and predictive control. Advances has also been made in control theory and in the formulation of tools for analyzing the controllability of a plant. These latter tools can be most helpful in screening alternative control structures.

Maybe the most important reason for the slow progress in plantwide control theory is that most people do not realize that there is an issue. But ask the question: Why are we controlling hundreds of temperatures, pressures and compositions in a chemical plant, when there is no specification on most of these variables? Is it just because we can measure them or is there some deeper reason?

The concept of "self-optimizing control" seems to provide the answer to the above question, and the idea will be explained in more detail in the talk.

OUTLINE

1. Introduction and Motivation
2. The idea of self-optimizing control
3. Procedure for selecting controlled outputs
4. Special linear cases
5. Examples
6. Conclusion

Which variables should we select to control?

The Trondheim **Plantwide control group** :

- **Audun Faanes** (pH-neutralization; control structure design)
- **Marius Govatsmark** (Refinery; HDA plant)
- **Ivar Halvorsen** (Petlyuk distillation; self-optimizing control)
- **Truls Larsson** (methanol plants; control structure design)
- **Tore Lid** (Refinery)
- **Sigurd Skogestad**

plus

- Morten Hovd, *Inst. for teknisk kybertenikk*
- Stig Strand, *Statoil*

Written material:

- *Multivariable Feedback Control*, S. Skogestad and I. Postlethwaite, Wiley, 1998 (Chapter 10).
- *A review of plantwide control*, S. Skogestad and T. Larsson, (Internal report, Aug. 1998). Available at:
<http://www.chembio.ntnu.no/users/skoge/publications>
- Ph.D. theses of Morud (1995), Glemmestad (1997) and Havre (1998).

The work is planned to be published as a series of papers with the following tentative titles:

Part 1. The basic issues in self-optimizing control (selection of controlled outputs to make implementation of the optimal solution insensitive to uncertainty).

Part 2. Taylor series analysis.

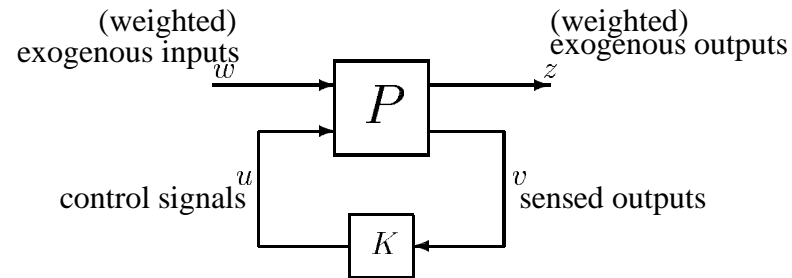
Part 3. Theoretical basis for using the minimum singular value for output selection.

Part 4. Partial and indirect control with application to selection of temperature measurements in distillation.

Part 5. Constraints and feasibility.

CONTROL THEORY

General controller design formulation



- w : Disturbances (d) and setpoints (r)
- v : Measurements (y_m, d_m) and setpoints (r)
- u : Manipulated inputs (u)
- z : Control error, $y - r$
- Find a controller K which based on the information in v , generates a control signal u which counteracts the influence of w on z , thereby minimizing the closed-loop norm from w to z .

PRACTICE

Typical base level control structure

PRACTICE

Typical control hierarchy

Alan Foss (“Critique of chemical process control theory”, AIChE Journal, 1973):

The central issue to be resolved ... is the determination of control system structure.

Which variables should be measured, which inputs should be manipulated and which links should be made between the two sets?

There is more than a suspicion that the work of a genius is needed here, for without it the control configuration problem will likely remain in a primitive, hazily stated and wholly unmanageable form.

The gap is present indeed, but contrary to the views of many, it is the theoretician who must close it.

Carl Nett (1989):

Minimize control system complexity subject to the achievement of accuracy specifications in the face of uncertainty.

Recent developments

- Many case studies (Luyben and others; Tennessee Eastman process)
- Some theoretical tools (e.g. chapter 10 in book by Skogestad and Postlethwaite, Wiley, 1996)
- Several ad-hoc procedures for plantwide control
- BUT: No unified approach (which is the goal of our work)

PLANTWIDE CONTROL

The *control philosophy* for the overall plant with emphasis on the *structural decisions*:

- Which “boxes” (controllers; decision makers) do we have and what information (signals) are send between them

NOT:

- The inside of the boxes (design and tuning of all the controllers)

The most important sub-problem: CONTROL STRUCTURE DESIGN

(“Plantwide control” is almost the same as the “process control version of control structure design”)

CONTROL STRUCTURE DESIGN

Tasks:

1. *Selection of controlled outputs*
2. *Selection of manipulations (degrees of freedom for control)*
3. *Selection of measurements (for control purposes)*
4. **Selection of control configuration**
(a structure interconnecting measurements/commands and manipulated variables)
5. *Selection of controller type*
(control law specification, e.g., PID, decoupler, LQG, etc.).

Note distinction between control *structure* (all tasks) and *configuration* (task 3).

Tasks 1, 2 and 3 combined: **input/output selection**

Task 4 (configuration): **input/output pairing**

Shinskey (1967, 1988); Morari (1982); Stephanopoulos (1984); Balchen and Mumme (1988)

Nett (1989); van de Wal and de Jager (1995); Skogestad and Postlethwaite (1996)

Approach Control structure design:

- Top-down consideration of control objectives and available degrees of freedom to meet these (tasks 1 and 2)
- Bottom-up design of the control system, starting with stabilization (tasks 3, 4, 5).

This talk:

Focus on the least studied, least understood and frequently overlooked task 1:

1. Selection of controlled outputs

QUESTION 1:

- Why do we in a plant control a lot of internal variables, like temperatures, pressures and compositions, for which there are no control requirements?

ANSWER:

- We have degrees of freedom that need to be specified to achieve optimal operation

QUESTION 2:

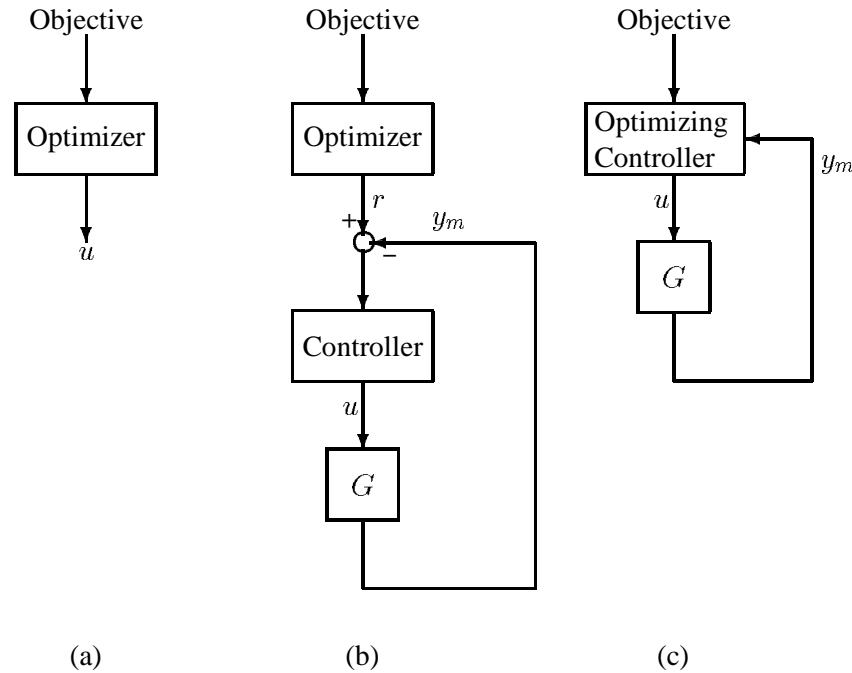
- But why do we select a particular set of controlled variables?

Example: A distillation column “inside” the plant. Why control the composition rather than specifying directly the reflux flow?

ANSWER:

- To reduce the sensitivity to uncertainty and achieve self-optimizing control

Optimization and control



- (a) Open-loop optimization.
(b) HERE: Closed-loop implementation with separate control layer.
(c) Integrated optimization and control.

Hierarchical structuring:

- *optimization layer* — computes references r (e.g. once every hour)
- *control layer* — implements this in practice, $y \approx r$ (e.g., through feedback with continuous update).

Hierarchical structuring

Two questions:

1. What should be the controlled variables y ?
(includes open-loop by selecting $y = u$)
2. What is their optimal values (y_{opt}) ?

Second question: A lot of theory.

Problems with hundreds of thousands of equations, and hundreds of degrees of freedom are routinely solved (even on-line)

BUT First question: (How should the optimal solution be implemented?)

Almost no theory.

Decisions mostly made on experience and intuition.

Example 1: Cake baking

Given: Bake cake for 15 minutes

Goal (purpose): Well-baked inside and nice outside

Manipulated input (degree of freedom): Heat input $u = Q$

Implementation:

1. Open-loop implementation: Heat input Q

- Problem: Sensitive to uncertainty and optimal value depends strongly on size of oven

2. Closed-loop implementation:

y = oven temperature

“Optimizer”: Cook book (look-up table)

Used in practice. Insensitive to changes.

Example 2: Distillation column

Goal (purpose): Operate the column such that overall operating costs of the entire plant are minimized (primarily determined by steady-state considerations)

Degrees of freedom at steady state (with given feed and pressure): 2

May for example be selected as

1. Split D/F and reflux L

Optimization of the entire plant yields optimal value of these two variables (along with the corresponding values for the other column operating variables, such as compositions, temperatures, etc.)

BUT how should the optimal solution be implemented?

There are many other possible pairs (y 's) which can be kept constant

2. Two product compositions (x_D, x_B)
3. Two temperatures (T_{top}, T_{btm})
4. Reflux and one temperature (L, T_{top})
5. Reflux ratio and one temperature ($L/D, T_{top}$)
6. Two ratios (e.g. $L/D, V/B$)
7. Reflux and boilup (L, V)
8. ...

Question:

- Which of these pairs (y 's) should be kept at the given optimal setpoint?

Issues:

- Changes in operating point (disturbances)
- Accuracy of control (measurement noise)

SELF-OPTIMIZING CONTROL

- Find the set of controlled variables y , which when kept constant, result in close-to-optimal operation (in spite of disturbances and other changes)
- Steady-state analysis often sufficient

Outline of the idea of self-optimizing control

- *Basis:* Have N_{ss} degrees of freedom at steady-state which should be used to optimize the operation
- *Assume:* Have an optimizer which for a given set of nominal disturbances computes all optimal values (of flows, temperatures etc.).
- *Question:* How should this optimal solution be included in practice? (Which N_{ss} variables should be specified?)
- *Initial Answer:* Does not matter (as long as they are independent)
- *But the real world is uncertain:*
 1. The data to the optimizer may be incorrect
 2. There are unknown disturbances entering during the time between each optimization
 3. The relative accuracy in the implementation (“signal to noise ratio”) for each variable is different.

Thus the initial answer is only true in a perfect world (with no uncertainty).

- *Better Answer:* Choose as specified variables (*controlled outputs*) the set which are least sensitive to uncertainty, i.e. which achieve self-optimizing control.
- *Intuitively:* The selected set of variables (controlled outputs) should
 1. Have a small variation in optimal setpoints
 2. Be easy to control accurately
 3. Be independent (not closely correlated)

Today: Steady-state optimization is performed routinely

Thus: Have the main tool needed for a proper selection of controlled outputs!

Procedure for selecting controlled outputs

1. Define the optimal operation problem (find the cost function J to be minimized)
2. Solve the optimization problem at a given nominal operating point.

That is, find $u_{opt}(d_0)$ by solving the nominal optimization problem

$$\min_u J(u, d_0)$$

where

u – “base set” for the N_{ss} steady-state degrees of freedom

d_0 – nominal value of the parameters (disturbances)

3. This yields a table with the nominal optimal values of all variables, $y_{opt}(d_0)$.
4. Define the uncertainty:
 - (a) For the optimization: Define the magnitude or set of disturbances ($d \in \mathcal{D}$) expected to enter between each optimization (e.g. 1 hour).
Here: Treat also errors in the data and model for the optimizer as disturbances.
 - (b) For each candidate output variable (y): Define the magnitude or set of control error ($e \in \mathcal{E}$ (e.g. due to measurement error)

5. Repeat for each *candidate set* of N_{ss} output variables (y 's)

(a) Evaluate the cost function $J(y, d)$ with fixed setpoints

$$y = r + e$$

where $r = r_0 \equiv y_{opt}(d_0)$ is taken from the above table.

Do this for all disturbances ($d \in \mathcal{D}$) and all control errors ($e \in \mathcal{E}$).

(b) Compute the “mean” cost, J_{mean}

6. Select as the controlled outputs the candidate set with the lowest “average” cost

Note: We always use fixed setpoints r between each optimization.

Comments on procedure

1. **Mean cost, J_{mean} .** Many possibilities:

- (a) Mean cost over a finite set (e.g., max, nominal, min for each disturbance and each control error)
- (b) Mean cost from “Monte-Carlo” evaluation of given distribution of d and e
- (c) Worst-case loss (compared to true optimal)

2. **Feasibility.** A given combination of $y = r_0 + e$ and d may yield an infeasible solution (e.g. negative flowrate), and the cost J is infinite.

This may result in discarding good candidate sets.

To avoid this one cannot use the nominal setpoints r_0 .

Two approaches:

- (a) Use back-off from nominal setpoints, $r = r_0 + b$
- (b) Add safety margins to the constraints

3. Robust optimization problem

Above: Setpoints r_0 determined by solving the nominal optimization problem.

Better: Setpoints r_{opt} determined by solving the robust optimization problem, where we minimize J_{mean} (e.g. using an outer loop in the above procedure)

- Problem: It will be very time-consuming and difficult to solve the robust optimization problem online.
- The **back-off** is defined as the difference between the robust and nominal setpoints

$$b = r_{opt} - r_0$$

- Possible implementation: Optimizer computes r_0 for a given disturbance d_0 , but during implementation we use $r = r_0 + b$ where the back-off b is fixed.
- An alternative solution is to use **safety margin for the constraints** (Glemmestad, 1997):

In the nominal optimization problem the original constraints $g(u, d) \leq 0$ are replaced by $g(u, d) \leq \epsilon$. The safety margins ϵ need to be predetermined from the robust optimization problem.

4. **Loss function.** Sometimes it is convenient to replace the cost function $J(y, d)$ by the always positive loss function

$$L(y, d) = J(y, d) - J_{opt}(d)$$

where J_{opt} is the lowest possible cost for a given disturbance d

$$J_{opt}(d) = \min_y J(y, d) = J(y_{opt}(d), d)$$

5. Computations.

- Above method well suited for comparing (ranking) a few alternative sets of controlled outputs.
- Finding the truly optimal set may be very time consuming because
 - (a) Infinite number of candidate variables (if we allow for combinations of variables, including sums, ratios, etc.)
 - (b) Combinatorial growth in the number of candidate sets
(For example, there are more than 2 millions sets of 7 variables selected from 30 candidate variables)
- But if we consider the Loss (rather than the cost) then we have a target (lower bound), and may stop the search when the output set is “good enough”

Toy Example

$$J = (u - d)^2$$

Optimal solution for any d is $J_{opt} = 0$ (corresponding to $u_{opt}(d) = d$).

Consider three alternative choices for the controlled output (measurement)

- $y_1 = 0.1(u - d)$
- $y_2 = 20u$
- $y_3 = 10u - 5d$

The losses for these three choices are

- $L_1 = (10e_1)^2$
- $L_2 = \left(\frac{e_2}{20} - d\right)^2$
- $L_3 = \left(\frac{e_3}{10} - \frac{d}{2}\right)^2$

Assume $|d| \leq 1$ and $|e_i| \leq 1$. We have

- With no control error y_1 is the best choice
- With no disturbances y_2 is the best choice.
- Output y_3 is the best overall choice for self-optimizing control.

Candidate output variables

There are an infinite number of variables we can select to keep constant, but we concentrate on choices which are easy to implement.

- flows (physical manipulated inputs; valves)
- measured outputs (through the use of feedback controllers with integral action)
- ratios, sums and differences of “similar” variables (e.g. ratio of flows, or difference between two temperatures).
- Soft control using P-controllers (resulting in a mix of input and output variables)

Soft control

Use proportional rather than integral control.

Example: P -control of top composition using reflux may be interpreted as keeping

$$y = \frac{L - L_s}{x_D - x_{D_s}} = K_c$$

where the constant K_c is the proportional gain.

This is a mix of keeping L and x_D fixed (depending on the value of K_c).

Special case: Selection of secondary outputs

y_1 – primary outputs (want $y_1 = r_1$ but no online measurement)

y_2 – secondary measurements (controlled variables)

Objective: Want e_1 as small as possible ($J = \|e_1\|$).

Assume linear model

$$y_1 = G_1 u + G_{d1} d$$

$$y_2 = G_2 u + G_{d2} d$$

We then have

$$e_1 = \underbrace{(G_{d1} - G_1 G_2^{-1} G_{d2})}_{\triangleq P_d} d + \underbrace{G_1 G_2^{-1}}_{\triangleq P_y} e_2$$

Measurement selection often a trade-off between

1. Sensitivity to disturbances (want $P_d d$ small)
2. Sensitivity to control error (want $P_y e_2$ small)

Example: Measurement selection for distillation column

LINEAR ANALYSIS AND INSIGHTS

- Assume u is the “base set” of independent variables (e.g. could be the “manipulated control inputs”)
- We may solve the optimization problem in terms of these variables

$$\min_u J(u, d) = J(u_{opt}(d))$$

where J is the operating cost (\$).

Note: $u_{opt}(d)$ depends on the disturbances (operating point).

- During operation the actual inputs u (possibly generated by feedback to achieve $y = r + e$) will differ from $u_{opt}(d)$.

- **Obvious:** *The actual input u should be close to the optimal input $u_{\text{opt}}(d)$.*

$$u - u_{\text{opt}} = G^{-1}(y - y_{\text{opt}})$$

where G – steady-state gain matrix (effect of small change in u on y), and

$$y - y_{\text{opt}} = \underbrace{y - r}_{\text{Control error } e} + \underbrace{r - y_{\text{opt}}(d)}_{\text{Optimization error}}$$

⇒ Select controlled outputs y such that:

1. *Optimization error $r - y_{\text{opt}}(d)$ is small;
 $y_{\text{opt}}(d)$ depends only weakly on disturbances.*
2. *Control error $e = y - r$ is small;
good measurement and control of y .*
3. *$G^{-1}(0)$ is small; the variables y are uncorrelated.*

Use of minimum singular value for selecting controlled outputs

- Scale outputs such that $\|y - y_{\text{opt}}(d)\| \approx 1$ (due to measurement errors and disturbances)
- Then $u - u_{\text{opt}} = G^{-1} \cdot 1 \Rightarrow$ Want $\|G^{-1}\|$ as small as possible.
- Using two-norm: $\|G^{-1}\|_2 = \bar{\sigma}(G^{-1}) = 1/\underline{\sigma}(G)$
 \Rightarrow Prefer a set of controlled outputs with large $\underline{\sigma}(G)$.

EXAMPLE: Recycle around reactor (snowball effect)

Simple example (Luyben, Yu):

- Reaction $A \rightarrow B$
- Recycle of unreacted A
- Product is pure B

At steady-state

Feed of A = Generation of B in reactor = Production of B

where Generation of B in reactor is

$$G_B = k(T)x_A V_R$$

$$G_B = k(T)x_A V_R$$

Three ways to increase G_B :

1. Increase reactor temperature T
2. Increase x_A by increasing the recycle ratio RR

$$x_A = \frac{RR}{1 + RR}$$

(the “snowball effect” of Luyben is that $x_A \rightarrow 1$ as $RR \rightarrow \infty$ – occurs when the reactor is too small)

3. Increase the reactor volume V_R
 - BUT: Loose money by not operating at maximum volume (Possible trade-off between operating costs and controllability)
 - Gas phase reactor: Increasing the pressure has the same effect (larger inventory in reactor).

Conclusion: Rules for selecting controlled outputs y

Select the controlled outputs y such that:

1. Optimal value $y_{\text{opt}}(d)$ is insensitive to disturbances (changes in the operating point)
2. Result insensitive to expected control error for y .
 - (a) “Optimum is flat” and/or
 - (b) Can achieve tight control of y (need accurate measurement)
3. The outputs are weakly correlated

This is usually based on a steady-state analysis

Conclusion: Self-optimizing control

- Above ideas outlined and partly quantified in Skogestad and Postlethwaite (Wiley, 1996, Chapter 10)
- Have been somewhat further developed (Ph.D. theses of Glemmestad and Havre)
- Possibly main problem: Convince people (YOU) that output selection is actually an important issue
- (Foresee large problems in getting things published if I go for a control-oriented journal)
- Working on:
 1. Better mathematical formulation of problem
 2. Applications
 - Distillation
 - Petlyuk distillation
 - Recycle systems
 - Many other applications