Limitations imposed by lower layer control configurations.

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Abstract

A hierarchical control system arises when design is done in a sequential manner. Typically, a chemical plant has at least three layers; a regulatory (base) control layer, a supervisory control layer, and an optimizing layer. In this paper we will look at possibility for imposing fundamental limitations by improper design of the lower layer control. One tool that is useful for analyzing this problem is the concept of partial control. This tool is well suited because it shows how the plant will look as seen from the higher layer controller. Analysis of these transfer-functions shows that we cannot introduce "new" limitations, provided that we have access to the measurements of the already closed loops and can adjust the setpoints to the loops.

We find that if both y_2 and r_2 are available at the next level, then no new fundamental limitation are introduced as we can cancel the lower layer controller by a posetive feedback. However if either y_2 or r_2 are unavailable then it is possible to introduce new limitations for the higher layer:

- If r_2 and y_2 are unavailable, then it is possible to introduce new RHP-zeros in the transfer function from u_1 to y_1. However, the zeros are generally located far into the RHP.
- If y_2 is unavailable then it is possible to introduce new RHP-zeros at the location of RHP-poles.
- If y_2 is unavailable then it is possible to introduce a sensitivity to uncertainty and disturbances into the remaining plant.
- One may introduce ill-conditioning into the problem. We have shown an example where we do the opposite (the DV-configuration).

1 Introduction

A hierarchical control system arises when design is done in a sequential manner. Typically, a chemical plant has at least three layers; a regulatory (base) control layer, a supervisory control layer with local optimization (e.g. model predictive control), and an optimizing layer (usually based on steady-state optimization). The functions in the two upper layers are often performed by humans. The regulatory (base) control layer usually consists of PID controllers. The task of this layer is to "stabilize" the plant, so that it can be operated manually, without the high level controllers in place.

Hovd and Skogestad (1993) proposed some rules for designing the lower layer. One of the points they mention is that one should not introduce fundamental limitations, like RHP-zeros and ill-conditioning, into the remaining control problem. The issue we want to address is whether this is really a problem. That is, can an improper selection of the base control layer lead to unnecessary limitations which cannot be overcome by the higher layers? By "unnecessary" we mean limitations which were not present as fundamental limitations in the original plant. Such fundamental limitations includes

- Non-minimum phase behavior (RHP-zeros).
- Instability.
- Ill-conditions (as seen from large RGA-elements).
- Sensitivity to disturbances, and input saturation.

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Outline

We first introduce the idea of partial control, and then discuss how to cancel the effect of the lower layer. We then discuss the possibility for RHP-zeros in subsystems under partial control. The next section is devoted to a high-purity distillation column, where we seem to remove a fundamental limitation by level control. In the last section, we address one case where feedback in the lower loop introduces a disturbance into a measurement that is to be used at the higher layer.

We end the introduction with an example. The example is how to switch from the outflow to the inflow as a manipulated variable for level control. This is an important example, because if a flow that is used for controlling a level saturates, then we will loose level control.

Example 1 *As an introductory example we study the simple case where we have a tank with two feeds and one exit stream. One of the feeds are a disturbance. The exit flow is used for controlling the volume. Thus if there is a step in the disturbance it will be propagate downstream. The question we want to answer is: Can a higher layer in the control layer redirect the disturbance upstream?*

The model of the system is

$$sV = F_i - F_o + F_d \tag{1}$$

$$y = \begin{bmatrix} F_o \\ V \end{bmatrix}, \quad u = \begin{bmatrix} F_i \\ F_o \end{bmatrix}, \quad d = F_d \tag{2}$$

$$y = \begin{bmatrix} 0 & 1 \\ 1/s & -1/s \end{bmatrix} u + \begin{bmatrix} 0 \\ 1/s \end{bmatrix} d \tag{3}$$

Control of level with the outflow ($F_o = k_2(y_2 - y_{2s})$, where y_{2s} is the setpoint), gives the closed loop response

$$y_1 = \frac{k_2}{s+k_2}u_1 + \frac{-k_2 s}{s+k_2}y_{2s} + \frac{k_2}{s+k_2}d \tag{4}$$

$$y_2 = \frac{1}{s+k_2}u_1 + \frac{k_2}{s+k_2}y_{2s} + \frac{1}{s+k_2}d \tag{5}$$

With the level control in place a disturbance d results in a change in $y_1 = F_o$. Assume that we want to redirect the disturbance to the input $u_1 = F_i$, but without breaking the level loop involving u_2 and y_2. We then have two inputs, F_i and V_s, that are available. The most straigth forward way would be to use u_1 to control y_1. However a change in the disturbance will have to affect the outflow befor the inflow can be adjusted. Is it possible to design a layer that avoids this, and how should it look like? We will leave it to the reader to come up with this after having read the article.

2 Partial Control

One tool that is useful for analyzing this problem is the concept of partial control, (Skogestad and Postlethwaite 1996). We partition the input and output into two parts

$$y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}, \quad u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

The linear model $y = Gu + G_d d$ may then be written

$$y_1 = G_{11}u_1 + G_{12}u_2 + G_{d1}d \tag{6}$$

$$y_2 = G_{21}u_1 + G_{22}u_2 + G_{d2}d \tag{7}$$

It is assumed that the plant is partially controlled, by closing the loops involving the variables u_2 and y_2 with

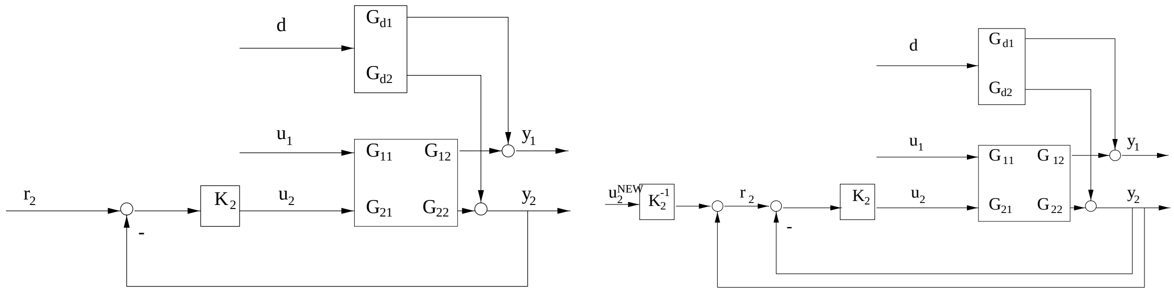
$$u_2 = K_2(r_2 - n_2 - y_2) \tag{8}$$

Where r_2 is the reference for y_2 and n_2 the measurement noise ($y_2 + n_2$ is the measurement). This partially controlled plant is shown in figure 1(a). The closed-loop response becomes

$$\begin{aligned} y_1 = {} & (G_{11} - G_{12}K_2(I + G_{22}K_2)^{-1}G_{21})u_1 + \\ & (G_{d1} - G_{12}K_2(I + G_{22}K_2)^{-1}G_{d2})d + \end{aligned} \tag{9}$$

$$
\begin{aligned}
& G_{12}K_2(I+G_{22}K_2)^{-1}(r_2-n_2) \\
y_2 \quad = \quad & (I+G_{22}K_2)^{-1}G_{21}u_1+(I+G_{22}K_2)^{-1}G_{d2}d+ \\
& (I+G_{22}K_2)^{-1}G_{22}K_2(r_2-n_2)
\end{aligned}
$$

We note that closing these loops does not reduce the degrees of freedom, at least provided that $\dim y_2 = \dim u_2$, since the setpoints r_2 introduced, may be available for the next control layer instead of u_2.



(a) The partially controlled plant.

(b) Cancellation of the partial controller.

Figure 1: The partially controlled plant and the cancellation of the controller.

Another interesting relationship is obtained by formally treating y_2 as an independent variable. Solving (7) with respect to u_2 then gives

$$u_2 = G_{22}^{-1}y_2 - G_{22}^{-1}G_{21}u_1 + G_{22}^{-1}G_{d2}d \tag{10}$$

where we assume that $\dim y_2 = \dim u_2$ and that G_{22}^{-1} exists. Substituting equation (10) into (6) then gives

$$y_1 \quad = \quad (G_{11} - G_{12}G_{22}^{-1}G_{21})u_1 + (G_{d1} - G_{12}G_{22}^{-1}G_{d1})d + G_{12}G_{22}^{-1}y_2 \tag{11}$$

Here $y_2 = r_2 + e_2$, where e_2 is the control error. If we assume perfect control then $y_2 = r_2$ and 11 becomes

$$y_1 \quad = \quad (G_{11} - G_{12}G_{22}^{-1}G_{21})u_1 + (G_{d1} - G_{12}G_{22}^{-1}G_{d1})d + G_{12}G_{22}^{-1}r_2 \tag{12}$$

Note that (12) can alternatively be derived from equation (9) by assuming $G_{22}K_2(I+G_{22}K_2)^{-1} = I$ (tight control) and $n_2 = 0$ (no measurement error).

These equations are interesting since they show us how the plant will look like as seen from a higher layer.

3 Cancellation of lower layer

Is it possible for the higher layer to cancel the effects of the control introduced at the lower layer? If this is possible, then in principle, we cannot introduce any new fundamental limitation, since the higher layer can achieve the same performance independently of the lower layer.

The answer to the question is yes and one way the lower layer can be canceled is shown in figure 1(b). Here the positive feedback cancel the negative feedback and the controller K_2 is inverted at the input (assuming that K_2 is stable and minimum phase so that $K_2K_2^{-1}$ contains no unstable hidden modes). Note that this scheme requires access to both y_2 and r_2.

Is it possible to cancel the effect of the feedback by just using feed-forward? If this was the case then we would be able to cancel the controller without using y_2. For simplicity let us assume that all outputs are controlled, then our new plant is (no measurement noise)

$$y = Tr + SG_d d \tag{13}$$

By applying the feed-forward controller $r = T^{-1}Gu'$ we would get

$$y = Tr + SG_d d = Gu' + SG_d d \tag{14}$$

As expected we were able to restore G. Note that the effect of the disturbance are now SG_d. So the answer to the second question is "no".

In conclusion cancellation of the lower layer is possible as shown in figure 1(b) if

1. y_2 is available as a measurement at the layer above.
2. r_2 is available as a degree of freedom at the layer above.
3. K_2 is stable and minimum phase so that $K_2 K_2^{-1}$ contains no unstable hidden modes.

This conclusion is rather trivial but nevertheless important.

4 RHP-zeros and partial control

In this section we will look at the possibility of introduce any new RHP-zeros in parts of a partially controlled plant. The closed loop response under partial control can be written

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \tilde{G} \begin{bmatrix} u_1 \\ r_2 \end{bmatrix} + \tilde{G}_d d + \tilde{G}_n n_2 \tag{15}$$

Where from equation (9) the new plant $\tilde{G}$ becomes

$$\tilde{G} = \begin{bmatrix} G_{11} - G_{12}K_2S_2G_{21} & G_{12}K_2S_2 \\ S_2G_{21} & G_{22}K_2S_2 \end{bmatrix}, \quad S_2 = (I + G_{22}K_2)^{-1} \tag{16}$$

It is well-known that feedback does not change the location of the RHP-zeros, see appendix A for a proof of this for a system under partial control[1]. This means that $\tilde{G}$ will have the same (multivariable) RHP-zeros as $G = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix}$. More exactly $G \begin{bmatrix} I & 0 \\ 0 & K_2 \end{bmatrix}$ and $tildeG$ have the same zeros. Thus, no new fundamental limitations are introduced (or removed) in $\tilde{G}$ by applying partial control, provided that the reference r_2 are available for manipulation. At frequencies where control is tight we have

$$\tilde{G} = \begin{bmatrix} G_{11} - G_{12}G_{22}^{-1}G_{21} & G_{12}G_{22}^{-1} \\ 0 & I \end{bmatrix} \tag{17}$$

However, in many cases we may want to use only u_1 to control y_1, or only r_2 to control y_1, and we now want to study if new RHP-zeros can occur in the resulting subsystem, $\tilde{G}_{11}$ or $\tilde{G}_{12}$.

4.1 RHP-zeros in $\tilde{G}_{11}$

The transfer function $\tilde{G}_{11} + G_11 - G_{12}K_2S_2G_{21}$ from u_1 to y_1 may contain RHP-zeros not present in the original system. For example it is well known that pairing on a negative RGA-element implies the presence of a RHP-zero in $\tilde{G}_{11}$ this was noted by sevral authors, Shinskey (1979) and Bristol 1977. For the case of negative RGA and integral action control, Grosdidier and Morari (1985) presents a proof. Which was generalized to non-integral control and MIMO systems by Jacobsen (1997).

The presence of a RHP-zero implies that there is a fundamental limitation, on the use of u_1 to control y_1. However, the significance of the limitation depends on the location of the RHP-zero, and in this case the new RHP-zero is generally located at frequencies at or beyond the bandwidth of the control system

[1] Even though RHP-zeros in the controller will be transmission zeros for the closed loop system, we will not consider them here. There reason being that although is some cases it can be optimal to include RHP-zeros in the controller, (Morari and Zafiriou 1989), is rarely used, at least in the lower layers.

involving the other outputs y_2. This means that if we require time-scale separation, i.e. that the bandwidth if the outher loops are much less than the inner loop (i.e. that $w_{B1} \ll w_{B2}$), such that we effectively can assume that y_2 is perfectly controlled, then the RHP-zero will not pose much of a limitation.

More specifically, for a plant under perfect control of y_2 we have

Theorem 1 *Given a functionally controllable plant G for which it is possible to achieve perfect control for a subset of inputs and outputs denoted by G_{22}. The remaining part of the plant, assuming perfect control, $G_{11} - G_{12}G_{22}^{-1}G_{21}$ will only contain RHP-zeros from the original plant.*

The proof is included in the appendix B. Rosenbrock (1970) presented a similar condition when all outputs but one are under perfect control.

Example 2 *To illustrate the points above, we consider the following plant*

$$G(s) = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix} \tag{18}$$

The $(1,1)$-element of the RGA for this plant is $\lambda_{11} = -1$. Pairing on this negative RGA element, by closing a loop $u_2 = 0.5/s$, gives the remaining plant

$$y_1 = \frac{2s-1}{2s+1}u_1 \tag{19}$$

Thus we get a RHP-zero in the subsystem. However for the plant with the setpoints as manipulative variables we have

$$y_1 = \frac{2s-1}{2s+1}u_1 + \frac{1}{2s+1}r_2 \tag{20}$$

$$y_2 = \frac{4s}{2s+1}u_1 + \frac{1}{2s+1}r_2 \tag{21}$$

As expected the RHP-zero in the transfer function for u_1 to y_1 is not a multivariable zero in the transfer function from (u_1, r_2) to (y_1, y_2). Thus it is only a fundamental limitation for a SISO controller. Furthermore the RHP-zero is at 0.5 which is the same as the bandwidth of the inner loop ($L_2 = G_{22}K_2 = .5/s$).

It is however easy to construct examples where the zero appears in at a point well below the bandwidth of the system. We believe that this is will occur when pairing on elements with large RGA-elements.

4.2 RHP-zeros due to RHP-poles

We here consider the case where the set u_1 is empty and we want to use u_2 to control y_1. However, the plant is unstable, and we also need to use u_2 to stabilize the plant for which we have available the extra measurement y_2. Will this instability limit the performance of the control of y_1? The answer is "yes", at least if the measurement y_2 is not available for use at the layer above (this is generally the case).

This is most easily seen by considering the transfer function $\tilde{G}_{12}$ from r_2 to y_1. For the limiting case, when y_2 is perfectly controlled $\tilde{G}_{12} = G_{12}G_{22}^{-1}$, and we see that if the instability is not detectable in y_1 (i.e. G_{12} is stable) then $\tilde{G}_{12}$ will have a RHP-zero at the location of the RHP-pole in G_{22} (because of the term G_{22}^{-1}). More generally,

$$\tilde{G}_{12} = G_{12}K_2(I + G_{22}K_2)^{-1} \tag{22}$$

and from the requierment of internal stability there has to be a RHP-zero in $K_2(I+G_{22}K_2)^{-1}$. So, unless that pole is present in G_{12}, the RHP-pole in G_{22} will appear as a RHP-zero in the subsystem $\tilde{G}_{12}$.

This is actually a fundamental problem that occurs if we attempt to use variables involved in a stabilization loop for control of a variable that does not contain this instability. To avoid this problem we would need to either stabilize the plant using some other input or to introduce more degrees of freedom u_1 (for control of y_1).

Example 3 *Consider the unstable plant*

$$y_1 = \frac{1}{s+1}u_2 \tag{23}$$

$$y_2 = \frac{1}{s-1}u_2 \tag{24}$$

Assume that we control y_2 (the pole is only observable in this output) with u_2, with a P-controller with gain $k_c = 10$, then

$$y_1 = \frac{10(s-1)}{(s+9)(s+1)} r_2 \tag{25}$$

$$y_2 = \frac{10}{s+9} r_2 \tag{26}$$

As expected the RHP-pole appeared in the transfer function from r_2 to y_1.

Special case: input resetting

A special case of the above is when $y_1 = u_2 \; (G_{12} + I)$, i.e. input resetting of a variable used for stabilization. We have the transfer function

$$u = ((I + KG)^{-1} K r \tag{27}$$

Internal stability implies that the transfer function contains a RHP-zero at location of the RHP-pole.

5 Control configurations for distillation columns

We consider here an example that has puzzled us. A high-purity distillation column, e.g. column A studied by Skogestad and Morari (1988), has large RGA-elements. But somehow by closing the level loops in a particular way to obtain the DV configuration, we are able to eliminate the large RGA-elements. Thus a fundamental limitation[2] seems to have disappeared. This is obviously not possible, and in this section we will study why this happens.

5.1 The plant

The column has four inputs, reflux (L), boil-up (V), distillate (D) and bottom product flow (B). The four outputs are, top composition of light component (y), bottom composition of light component (x), level in condenser (M_D) and level in the re-boiler (M_D). A simplified linear model is (Skogestad and Morari 1988)

$$\begin{bmatrix} y \\ x \\ M_D \\ M_B \end{bmatrix} = \begin{bmatrix} g_{yL} & g_{yV} & 0 & 0 \\ g_{xL} & g_{xV} & 0 & 0 \\ -\frac{1}{s} & \frac{1}{s} & -\frac{1}{s} & 0 \\ \frac{1}{s} & -\frac{1}{s} & 0 & -\frac{1}{s} \end{bmatrix} \begin{bmatrix} L \\ V \\ D \\ B \end{bmatrix} + \begin{bmatrix} g_{yF} & g_{yz_F} \\ g_{xF} & g_{xz_F} \\ 0 & 0 \\ \frac{k_F}{s} & 0 \end{bmatrix} \begin{bmatrix} F \\ z_F \end{bmatrix} \tag{28}$$

It is well known that different choices for controlling the level, will give different values for the RGA of the remaining plant. The RGA for the open loop plant is

$$RGA = \begin{bmatrix} \frac{g_{yL}g_{xV}}{g_{yL}g_{xV} - g_{yV}g_{xL}} & -\frac{g_{yV}g_{xL}}{g_{yL}g_{xV} - g_{yV}g_{xL}} & 0 & 0 \\ -\frac{g_{yV}g_{xL}}{g_{yL}g_{xV} - g_{yV}g_{xL}} & \frac{g_{yL}g_{xV}}{g_{yL}g_{xV} - g_{yV}g_{xL}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{29}$$

Which at steady state becomes for column A, (Skogestad and Morari 1988)

$$RGA = \begin{bmatrix} 35.1 & -34.1 & 0 & 0 \\ -34.1 & 35.1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{30}$$

Here we see that we have large RGA values that indicates problems with decoupling control.

[2] Large RGA-elements implies that decoupling control with an inverse based controller is very sensitive to input gain uncertainty, (Skogestad and Morari 1987).

5.2 The LV-configuration

By selecting D and B for level control, and assuming perfect level control we have the partially controlled plant

$$\begin{bmatrix} y \\ x \end{bmatrix} = \begin{bmatrix} g_{yL} & g_{yV} \\ g_{xL} & g_{xV} \end{bmatrix} \begin{bmatrix} L \\ V \end{bmatrix} + 0 \begin{bmatrix} M_d^r \\ M_B^r \end{bmatrix} + \begin{bmatrix} g_{yF} & g_{yz_F} \\ g_{xF} & g_{xz_F} \end{bmatrix} \begin{bmatrix} F \\ z_F \end{bmatrix} \tag{31}$$

The RGA is as expected the same as for the open plant

$$\lambda_{11}^{LV} = \frac{g_{yL} g_{xV}}{g_{yL} g_{xV} - g_{yV} g_{xL}} \tag{32}$$

and for column A in (Skogestad and Morari 1988)

$$\lambda_{11}^{LV} = 35.1 \tag{33}$$

5.3 The DV-configuration

By using L and B for perfect level control we obtain the DV configuration. The partially controlled plant

$$\begin{bmatrix} y \\ x \end{bmatrix} = \begin{bmatrix} -g_{yL} & g_{yV} + g_{yL} \\ -g_{xL} & g_{xV} + g_{xL} \end{bmatrix} \begin{bmatrix} D \\ V \end{bmatrix} - \begin{bmatrix} g_{yL} s & 0 \\ g_{xL} s & 0 \end{bmatrix} \begin{bmatrix} M_D^r \\ M_B^r \end{bmatrix} + \begin{bmatrix} g_{yF} & g_{yz_F} \\ g_{xF} & g_{xz_F} \end{bmatrix} \begin{bmatrix} F \\ z_F \end{bmatrix} \tag{34}$$

This controller configuration has RGA values

$$\lambda_{11}^{DV} = \frac{g_{yL}(g_{xL} + g_{xV})}{g_{yL} g_{xV} - g_{yV} g_{xL}} \tag{35}$$

Thus the relationship for RGA of LV and DV configuration is

$$\frac{\lambda_{11}^{DV}}{\lambda_{11}^{LV}} = 1 + \frac{g_{xL}}{g_{xV}} = .013, \;\; \lambda_{11}^{DV} = 0.46 \tag{36}$$

Which is a considerable smaller value.

5.4 What has happend?

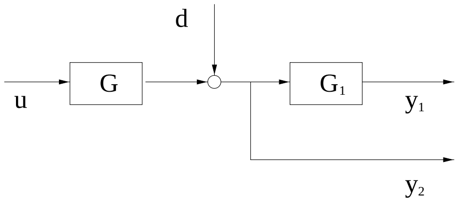
For the DV-configuration the large RGA values have disappeared. Thus, it should be possible to have decoupled control of the compositions. This seems inconsistent with the large value of the RGA for both the full plant and the LV configuration.

The reason is that the DV-configuration has interactions from the levels to the composistions see equation 34. On the other hand we see from equation (31) that for the LV configuration there is no interaction from the level to compositions loops. Apparently this means that in the DV-configuration we can have decoupling between the compositions.

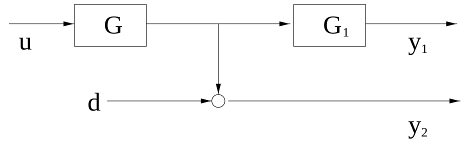
6 The effect of disturbances

This section looks at the role of disturbances. This is done using two examples. In the first we show how feedback is used to reduce the effect of the disturbance on output y_1. In the second we give an example where feedback will introduce disturbances to y_1. The question is if the next layer can cancel the effect of this disturbance introduced by the lower layer.

Both plants are shown in figure 2. In both plants there are one inout and two outputs. One common such example is the top (or bottom) of a distillation column where we have fast temperature measurements y_2 and slow composition measurements y_1. For such cases it is common to arrange the control loops in



(a) Disturbance affects both measurements.



(b) Disturbance affects only the fast measurement.

Figure 2: Different effect of the disturbance.

a cascaded manner, where the fast measurement y_2 is used in a fast inner loop. The outer loop uses the slow measurement y_1, and the setpoints of the inner loop is used as manipulative variables.

The difference between the plants are that in figure 2(a) the disturbance is added to both measurements, while in figure 2(b) the disturbance is only added to the fast measurement. In both cases we study the effect introduced by a feedback controller $u_2 = -K_2(y_2 - r_2)$.

In case (a), the closed loop transfer function from d to y_1 becomes

$$y_1 = S_2 G_d d + T_2 r_2 \approx 0d + G_1 r_2 \tag{37}$$

(where the approximation applies at low frequencies) In this case feedback will reduce the effect of the disturbance d on y_1 at frequencies where S_2 is small. However in case (b) the feedback control introduces a dependency on the disturbance that was not present in the original plant

$$y_1 = T_2 d + T_2 G_1 r_2 \approx -G_1 d + G_1 r_2 \tag{38}$$

(where the approximation applies at low frequencies) Thus in this case feedback in the lower layer will introduce variables in the output y_1 that was not present in the original plant (with $K_2 = 0$).

If the measurement y_2 is not available in the next control layer, then the dependence on d introduced by the lower layer can be a fundamental limitation for control performance. For example, G_1 is non-minimum phase, and d is such that control is needed at frequencies where the non-minimum phase of G_1 is a limitation. Note that if the measurement of y_2 is available then it is possible to achieve the same performance with a multivariable controller independent of the lower layer controller. (See 1(b).)

In summary, we have found that if we close a lower loop involving the output y_2, and we assume y_2 is not available at the higher layer (this is a reasonable assumption in most cases), then the presence of the lower layer may introduce a sensitivity to disturbance which cannot be counteracted at a higher layer.

This is an important result, which we have not seen explicitly stated befor. The results follows more generally from the discussion in section 3, and was nicely illustrated by the above example. **Comment**: Much of this discussion here is also valid for model uncertainty and for noise.

7 Conclusion

In this paper we have followed up a point made by Hovd and Skogestad (1993), that the lower layer should be designed in such a manner that it will not impose any new fundamental limitation that was not present in the original plant.

We find that if both y_2 and r_2 are available at the next level, then no new fundamental limitation are introduced as we can cancel the lower layer controller by a posetive feedback. However if either y_2 or r_2 are unavailable then it is possible to introduce new limitations for the higher layer:

- If r_2 and y_2 are unavailable, then it is possible to introduce new RHP-zeros in the transfer function from u_1 to y_1. However, the zeros are generally located far into the RHP.
- If y_2 is unavailable then it is possible to introduce new RHP-zeros at the location of RHP-poles.

- If y_2 is unavailable then it is possible to introduce a sensitivity to uncertainty and disturbances into the remaining plant.
- One may introduce ill-conditioning into the problem. We have shown an example where we do the opposite (the DV-configuration).

The fact that we theoretically can counteract poor choices for base layer control does not mean that the design of this layer is not important. First, we want the higher layer to be as simple as possible, and we may want to avoid that the higher layer must use setpoints in the lower layer as degrees of freedom. Second, and more importantly, we want to avoid the need for fast control and a detailed dynamic model in the higher layer. Thus, a lower-layer control system which is effectively self-regulating (at least at higher frequencies) is strongly desirable.

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A Zeros in the partially controlled plant.

From linear systems theory we have that for a system under feedback control the zeros of the closed loop system is given by the zeros of the individual elements in the loop. However this statement is only valid for the transfer function from the reference to the measurements. So it is not given that it is valid for a partially controlled system.

It is possible to rewrite the equation 9 to

$$y = \left[I + G \begin{bmatrix} 0 & 0 \\ 0 & K_2 \end{bmatrix} \right]^{-1} G \begin{bmatrix} I & 0 \\ 0 & K_2 \end{bmatrix} \tag{39}$$

Let us assume that all the elements above is functionally controllable then the zeros of the system above is given by the zeros of the elements. Which means that zeros of G and K_2 will be zeros in the closed loop system. However if there are zeros in $\left[I + G \begin{bmatrix} 0 & 0 \\ 0 & K_2 \end{bmatrix} \right]^{-1}$ then that is due to poles in $\left[I + G \begin{bmatrix} 0 & 0 \\ 0 & K_2 \end{bmatrix} \right]$. Those poles are also poles of G or K_2, and hence they are not zeros of the system. This proves that the only zeros of the partially controlled system is the zeros of either G or K_2.

B Proof of Theorem 1

The partially controlled plant is

$$\hat{G}_{11} = G_{11} - G_{12} G_{22}^{-1} G_{21} \tag{40}$$

The determinant of the partially controlled plant is

$$\det \hat{G}_{11} = \det(G_{11} - G_{12}G_{22}^{-1}G_{21}) \tag{41}$$

Using Schurs formula gives

$$\det \hat{G}_{11} = \frac{\det G}{\det G_{22}} \tag{42}$$

From (MacFarlane and Karcanias 1976) we know that the zero polynomial, corresponding to a minimal realization, is the greatest common divisor of all the numerators of all minors of the system of the same size as the normal rank. For a functionally controllable system, this means that the zeros of the system is given by the numerator of determinant when it is written with the pole polynomial as denominator.

Since we assume that we have perfect control then G_{22} must be functionally controllable, and it cannot have RHP-zeros. This means that it cannot cancel any RHP-zeros in $\det G$. Furthermore any RHP-poles in $\det G_{22}$ must be canceled by the same pole in $\det G$. If there is a pole in $\det G_{22}$ that does not appear in $\det G$, then that is due to a zero/pole cancellation in the evaluation of $\det G$.