Performance limitations for unstable SISO plants

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Abstract. This paper examines the fundamental limitations imposed by instability in the plant (Right Half Plane (RHP) poles) on closed-loop performance. The main limitation is that instability requires active use of plant inputs, and we quantify this is terms of tight lower bounds on the input magnitudes required for disturbance and measurement noise rejection. These new bounds involve the $\mathcal{H}_\infty$-norm, which has direct engineering significance. The output performance in terms of disturbance rejection or reference tracking is *only* limited if the plant has RHP-zeros. It is important to stress that the derived bounds are controller independent and that they are tight, meaning that there exists controllers which achieve the lower bounds.

1 Introduction

An unstable plant, for example an unstable chemical reactor, can only be stabilized by use of feedback control which implies active use of the plant inputs. If measurement noise and/or disturbances are present (which is always the case in practical process control), then the input usage may become unacceptable.

In this paper, the above statements are quantified by deriving tight lower bounds on the $\mathcal{H}_\infty$-norm of the closed-loop transfer functions SV and TV, where S and T are the sensitivity and complementary sensitivity functions. The transfer function V can be viewed as a *generalized* "weight", which for our purpose should be independent of the feedback controller K.

One important application is that we can *quantify* the minimum input usage for stabilization in the presence of worst case measurement noise and disturbances. It appears that *even* for SISO systems this has been a difficult task, which has not been solved analytically until now.

To give the reader some appreciation of the basis of the bounds and their usefulness, we consider as a motivating example an unstable plant with a RHP-pole p. We want to obtain a lower bound on the $\mathcal{H}_\infty$-norm of the closed-loop transfer function KS from measurement noise n to plant input u. We first rewrite $KS = G^{-1}T$, which is on the form TV with $V = G^{-1}$. The basis of our bound is the use of the maximum modulus principle and the "interpolation constraint" $T(p) = 1$, which must apply to achieve internal stability. We obtain (see Theorem 2 for details)

$$\|KS(s)\|_\infty = \|G^{-1}T(s)\|_\infty \geq |G_{ms}^{-1}(p)|$$

where G_{ms} is the "stable and minimum phase" version of G (if $G(s)$ also has a RHP-zero z we get the additional penalty $\frac{|z+p|}{|z-p|}$). As an example, consider the plant $G(s) = \frac{1}{s-10}$, which has an unstable pole $p = 10$. We obtain $G_{ms}(s) = \frac{1}{s+10}$. For *any* linear feedback controller K, we find that the lower bound

$$\|KS(s)\|_\infty \geq |G_{ms}^{-1}(p)| = 2p = 20$$

must be satisfied. Thus, if we require that the plant inputs are bounded with $\|u\|_\infty \leq 1$, then we cannot allow the magnitude of measurement noise to exceed $\|n\|_\infty = 1/20 = 0.05$.

The basis for our results is the *important* work by Zames (1981), who made use of the interpolation constraint $S(z) = 1$ and the maximum modulus theorem to derive bounds on the $\mathcal{H}_\infty$-norm of S for plants with one RHP-zero. Subsequently, these results were extended to plants with one RHP-pole and then to plants with combined RHP zeros and poles, e.g. (Doyle *et al.*, 1992, pp. 93–95) and (Skogestad and Postlethwaite, 1996).

However, these generalizations to unstable plants did *not* consider the input usage which involves the closed-loop transfer function KS. An important contribution of this paper is therefore to use the "trick" $KS = G^{-1}T$, which enable us to derive lower bounds on input usage, by using the general lower bound on $\|TV(s)\|_\infty$ with $V = G^{-1}$. But when G is unstable (with RHP-pole p), then $V = G^{-1}$ has RHP-zeros for $s = p$. A second important contribution compared to earlier work is the ability to include RHP zeros and poles in the "weight" V (under the assumption that SV and TV are stable).

A third important contribution is that we show that the lower bounds are *tight*. That is, we give analytical expressions for stable controllers which *achieves* an $\mathcal{H}_\infty$-norm of the closed-loop transfer function which is equal to the lower bound.

The bounds on $\|S(s)\|_\infty$ for plants with RHP-zero derived by Zames (1981) are also valid for multivariable systems. It is important to note that all the results given in this paper have been generalized to multivariable systems (Havre and Skogestad, 1997; Havre, 1998).

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2 Basics from linear control theory

We consider linear time invariant transfer function models on the form

$$y(s) = G(s)u(s) + G_d(s)d(s) \tag{1}$$

where u is the manipulated input, d is the disturbance, y is the output, G is the SISO plant model and G_d is the SISO disturbance plant model. The measured output is $y_m = y + n$ where n is the measurement noise.

The $\mathcal{H}_\infty$-norm of a stable rational transfer function $M(s)$ is defined as the peak value in the magnitude $|M(j\omega)|$ over all frequencies.

$$\|M(s)\|_\infty \triangleq \sup_\omega |M(j\omega)| \tag{2}$$

2.1 Factorizations of RHP zeros and poles

A rational transfer function $M(s)$ with zeros and poles in the open RHP, $\{z_j, p_i\} \in \mathbb{C}_+$, can be factorized in *Blaschke products* as follows[1]

$$M(s) = \mathcal{B}_z(M)\, M_m(s) \tag{3}$$
$$M(s) = \mathcal{B}_p^{-1}(M)\, M_s(s) \tag{4}$$
$$M(s) = \mathcal{B}_z(M)\, \mathcal{B}_p^{-1}(M)\, M_{ms}(s) \tag{5}$$

where

- M_m – Minimum phase (subscript m) version of M with the RHP-zeros mirrored across the imaginary axis.
- M_s – Stable (subscript s) version of M with the RHP-poles mirrored across the imaginary axis.
- M_{ms} – Minimum phase, stable (subscript ms) version of M with the RHP zeros and poles mirrored across the imaginary axis.
- $\mathcal{B}_z(M)$ – Stable all-pass rational transfer function ($|\mathcal{B}_z(j\omega)| = 1, \forall\omega$) containing the RHP-zeros (subscript z) of M.
- $\mathcal{B}_p(M)$ – Stable all-pass rational transfer function ($|\mathcal{B}_p(j\omega)| = 1, \forall\omega$) containing the RHP-poles (subscript p) of M as RHP-zeros.

The all-pass filters are

$$\mathcal{B}_z(M(s)) = \prod_{j=1}^{N_z} \frac{s - z_j}{s + \bar{z}_j} \tag{6}$$

$$\mathcal{B}_p(M(s)) = \prod_{i=1}^{N_p} \frac{s - p_i}{s + \bar{p}_i} \tag{7}$$

where N_z is the number of RHP-zeros $z_j \in \mathbb{C}_+$ and N_p is the number of RHP-poles $p_i \in \mathbb{C}_+$ in M.

[1] Note that the notation on the all-pass factorizations of RHP zeros and poles used in this paper is reversed compared to the notation used in (Skogestad and Postlethwaite, 1996; Havre and Skogestad, 1997). The reason to this change of notation is to get consistent with what the literature generally defines as an all-pass filter.

In most cases $M = G$ and to simplify the notation we often omit to show that the all-pass filters are dependent on G, i.e. we write $\mathcal{B}_p(s)$ and $\mathcal{B}_z(s)$ in the meaning of $\mathcal{B}_p(G(s))$ and $\mathcal{B}_z(G(s))$.

The order of the two operations $(\cdot)_m$ and $(\cdot)_s$ in the combined operator $(\cdot)_{ms}$ is arbitrary. It also follows that

$$(G^{-1})_{ms} = (G_{ms})^{-1} = G_{ms}^{-1} \tag{8}$$

And we note that

$$\|M(s)\|_\infty = \|M_m(s)\|_\infty = \|M_{ms}(s)\|_\infty \tag{9}$$

The first identity follows since $|\mathcal{B}_z(M(j\omega))| = 1, \forall\omega$, and the latter identity follows since M is stable, i.e. $M_{ms} = M_m$ and $\mathcal{B}_p(M_m) = \mathcal{B}_p(M) = 1$.

2.2 Closing the loop

A typical control problem is shown in Figure 1. In the figure possible performance weights are given in dashed lines.

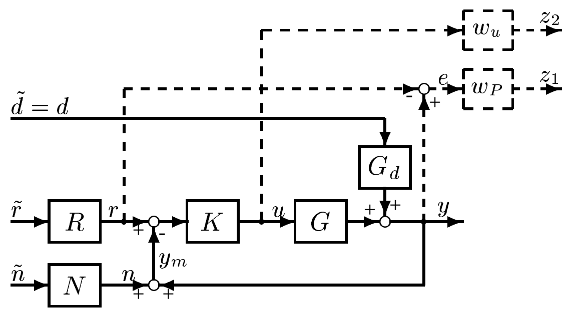

Figure 1: One degree-of-freedom control configuration

Without loss of generality the performance weights w_P and w_u may be assumed to be stable and minimum phase. We have included both the reference r and the measurement noise n, in addition to disturbances d as external inputs. The transfer functions, G_d, R and N can be viewed as weights on the inputs, and the inputs: $\tilde{d}$, $\tilde{r}$ and $\tilde{n}$ are normalized in magnitude. Normally, N is the inverse of signal to noise ratio.

We apply negative feedback control

$$u = K(r - y_m) = K(r - y - n) \tag{10}$$

The closed-loop transfer function F from

$$v = \begin{bmatrix} \tilde{r} \\ \tilde{d} \\ \tilde{n} \end{bmatrix} \quad \text{to} \quad z = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} w_P(y - r) \\ w_u u \end{bmatrix}$$

is

$$F(s) = \begin{bmatrix} -w_P SR & w_P S G_d & -w_P T N \\ w_u SKR & -w_u KSG_d & -w_u KSN \end{bmatrix} \tag{11}$$

where the sensitivity S and the complementary sensitivity T are defined as

$$S \triangleq (1 + GK)^{-1} = \frac{1}{1 + GK} \tag{12}$$

$$T \triangleq 1 - S = \frac{GK}{1 + GK} \tag{13}$$

To have good control performance (keep z_1 small) with a small input usage (keep z_2 small) we would like to have $\|F(s)\|_\infty$ small. That is we want all the SISO transfer functions in (11) small. In addition, there are robustness issues. For example, we wish to have $\|w_{\rm unc}T(s)\|_\infty$ small, where $w_{\rm unc}$ is the magnitude of the relative plant uncertainty.

2.3 Interpolation constraints

If G has a RHP-zero z or a RHP-pole p then for internal stability of the feedback system the following interpolation constraints must apply (e.g. Skogestad and Postlethwaite, 1996):

$$T(z)=0; \qquad S(z)=1 \tag{14}$$

$$S(p)=0; \qquad T(p)=1 \tag{15}$$

3 Lower bounds on the $\mathcal{H}_\infty$-norm of closed-loop transfer functions

In this section we will give the main results, which are lower bounds on the $\mathcal{H}_\infty$-norm of closed-loop transfer functions which can be written on the form SV or TV. The generalized "weight" V is assumed to be independent of the feedback controller K. V may be unstable but SV and TV must be stable. That is, it must be possible to stabilize all transfer functions by controlling the output y using the input u (this implies that all unstable modes of N, R and G_d also are modes of G).

Some examples. Consider the six transfer functions in (11). The first two can be written on the form SV by selecting $V_{11}=w_PR$ and $V_{12}=w_PG_d$. The remaining four can be written on the form TV by selecting $V_{13}=w_PN$, $V_{21}=w_uG^{-1}R$, $V_{22}=w_uG^{-1}G_d$ and $V_{23}=w_uG^{-1}N$. From this we see that the "weight" V may be unstable (if one or both of G_d and G^{-1} are unstable) and may contain RHP-zeros (if one or both of G_d and G^{-1} contain RHP-poles).

We now present the two main results, the proofs of these are given in (Havre, 1998, Chapter 4).

THEOREM 1 (LOWER BOUND ON $\|SV(s)\|_\infty$). *Consider the* SISO *plant* G *with* $N_z\geq 1$ RHP-*zeros* z_j *and* $N_p\geq 0$ RHP-*poles* $p_i\in\mathbb{C}_+$. *Let* V *be a rational transfer function, and assume that* SV *is (internally) stable. Then the following lower bound on* $\|SV(s)\|_\infty$ *applies:*

$$\|SV(s)\|_\infty \geq \max_{\text{RHP-zeros, } z_j} |\mathcal{B}_p^{-1}(z_j)|\cdot|V_{ms}(z_j)| \tag{16}$$

REMARK 1. With $|\mathcal{B}_p(z_j)|$ we mean $|\mathcal{B}_p(G(s))|_{\text{evaluated at } s=z_j}$.
REMARK 2. The assumption that SV is internally stable, means that it must be possible to stabilize the system using the feedback controller K, without having any RHP zero/pole cancellations between G and K.

THEOREM 2 (LOWER BOUND ON $\|TV(s)\|_\infty$). *Consider the* SISO *plant* G *with* $N_z\geq 0$ RHP-*zeros* $z_j\in\mathbb{C}_+$ *and* $N_p\geq 1$ RHP-*poles* p_i. *Let* V *be a rational transfer function, and assume that* TV *is (internally) stable. Then the following lower bound on* $\|TV(s)\|_\infty$ *applies:*

$$\|TV(s)\|_\infty \geq \max_{\text{RHP-poles, } p_i} |\mathcal{B}_z^{-1}(p_i)|\cdot|V_{ms}(p_i)| \tag{17}$$

Remarks on Theorems 1 and 2:

1) The lower bounds (16) and (17) are independent of the controller K, if the weight V is independent of K.
2) The bound on $\|SV(s)\|_\infty$ is caused by the RHP-zeros z_j in G, and the term $|\mathcal{B}_p^{-1}(z_j)|\geq 1$ gives an additional penalty for plants which also have RHP-poles. For the case when G has *no* RHP-poles, then $\mathcal{B}_p^{-1}(z_j)=1$.
3) The bound on $\|TV(s)\|_\infty$ is caused by the RHP-poles p_i in G, and the term $|\mathcal{B}_z^{-1}(p_i)|\geq 1$ gives an additional penalty for plants which also have RHP-zeros. For the case when G has *no* RHP-zeros, then $\mathcal{B}_z^{-1}(p_i)=1$.
4) For a plant with a single RHP-zero z and a single RHP-pole p the additional penalty is given by the term

$$|\mathcal{B}_p^{-1}(z)|=|\mathcal{B}_z^{-1}(p)|=\frac{|z+p|}{|z-p|}$$

This factor can be quite large if G contains a RHP-zero z close to the RHP-pole p.

4 Tightness of lower bounds

Theorems 1 and 2 provide lower bounds on $\|SV(s)\|_\infty$ and $\|TV(s)\|_\infty$. The question is whether these bounds are tight, meaning that there actually exist controllers which achieve the bounds? The answer is "yes" if there is only one RHP-zero or one RHP-pole. We prove tightness of the lower bounds by constructing controllers which achieve the bounds. In this short version of the paper we only give the controller minimizing $\|TV(s)\|_\infty$, the controller minimizing $\|SV(s)\|_\infty$ and the proofs are given in (Havre, 1998, Chapter 4).

THEOREM 3 (K WHICH MINIMIZE $\|TV(s)\|_\infty$). *Consider the* SISO *plant* G *with one* RHP-*pole* p *and* $N_z\geq 0$ RHP-*zeros* $z_j\in\mathbb{C}_+$. *Then the feedback controller* K *which minimize* $\|TV(s)\|_\infty$ *is given by*

$$K(s)=G_{ms}^{-1}K_o(s),\quad K_o(s)=PQ^{-1}(s) \tag{18}$$

where

$$P(s) = \mathcal{B}_z^{-1}(p)\,V_{ms}(p)\,V_{ms}^{-1}(s) \tag{19}$$

$$\begin{aligned} Q(s) &= (1-\mathcal{B}_z(s)\,P(s))_m \\ &= \mathcal{B}_p^{-1}(s)\,(1-\mathcal{B}_z(s)\,P(s)) \end{aligned} \tag{20}$$

With this controller we have

$$\|TV(s)\|_\infty = |\mathcal{B}_z^{-1}(p)|\cdot|V_{ms}(p)| \tag{21}$$

The controller in Theorem 3 gives constant $|TV(j\omega)|$ for all ω. We note that no properness restriction has been imposed on the controller, so the controller given in Theorem 3 may be improper. Also note that the controller $K(s)$ in Theorem 3 is always stable and minimum phase. This may seem surprising since it is known that some plants with RHP zeros and poles require an unstable controller (Youla *et al.*, 1974) to achieve closed-loop stability. However, this assumes that the controller is strictly proper, and does therefore not apply in our case, for further details see (Havre, 1998, Chapter 4).

5 Applications of lower bounds

5.1 Bounds on important closed-loop transfer functions

Consider again the six transfer functions in (11), and the weighted complementary sensitivity function $w_{\rm unc}T$. For simplicity we assume that w_P, w_u, $w_{\rm unc}$, R and N are all stable minimum phase (or have been replaced by the stable minimum phase counterparts with same magnitude). From Theorems 1 and 2 we obtain:

Output performance, reference tracking:

$$\|w_P SR(s)\|_\infty \geq \max_{\text{RHP-zeros, } z_j} |w_P(z_j)| \cdot |\mathcal{B}_p^{-1}(z_j)| \cdot |R(z_j)| \quad (22)$$

Output performance, disturbance rejection:

$$\|w_P SG_d(s)\|_\infty \geq \max_{\text{RHP-zeros, } z_j} |w_P(z_j)| \cdot |\mathcal{B}_p^{-1}(z_j)| \cdot |(G_d)_{ms}(z_j)| \quad (23)$$

Output performance, measurement noise rejection:

$$\|w_P TN(s)\|_\infty \geq \max_{\text{RHP-poles, } p_i} |w_P(p_i)| \cdot |\mathcal{B}_z^{-1}(p_i)| \cdot |N(p_i)| \quad (24)$$

Input usage, reference tracking:

$$\|w_u KSR(s)\|_\infty = \|w_u TG^{-1}R(s)\|_\infty \geq \max_{\text{RHP-poles, } p_i} |w_u(p_i)| \cdot |\mathcal{B}_z^{-1}(p_i)| \cdot |G_{ms}^{-1}R(p_i)| \quad (25)$$

Input usage, disturbance rejection:

$$\|w_u KSG_d(s)\|_\infty = \|w_u TG^{-1}G_d(s)\|_\infty \geq \max_{\text{RHP-poles, } p_i} |w_u(p_i)| \cdot |\mathcal{B}_z^{-1}(p_i)| \cdot |G_{ms}^{-1}(G_d)_{ms}(p_i)| \quad (26)$$

Input usage, measurement noise rejection:

$$\|w_u KSN(s)\|_\infty = \|w_u TG^{-1}N(s)\|_\infty \geq \max_{\text{RHP-poles, } p_i} |w_u(p_i)| \cdot |\mathcal{B}_z^{-1}(p_i)| \cdot |G_{ms}^{-1}N(p_i)| \quad (27)$$

Closed-loop sensitivity to plant uncertainty:

$$\|w_{\rm unc}T(s)\|_\infty \geq \max_{\text{RHP-poles, } p_i} |w_{\rm unc}(p_i)| \cdot |\mathcal{B}_z^{-1}(p_i)| \quad (28)$$

Note that we mainly have inherent limitations on (output) performance when the plant has RHP-zeros. The exception is for measurement noise, where the requirement of stabilizing an unstable pole may give poor performance.

All the bounds on input usage are caused by the presence of RHP-poles. This is reasonable since we need active use of the input in order to stabilize the plant. This is considered in more detail in the next section.

5.2 Implications for stabilization with bounded inputs

Our bounds involve the $\mathcal{H}_\infty$-norm, and their large engineering usefulness may not be immediate. In the following we will concentrate on the bounds involving input usage and we will use the lower bounds to derive and *quantify* the conclusion:

- *Bounded inputs combined with disturbances and noise may make stabilization impossible.*

Measurement noise. The transfer function from normalized measurement noise $\tilde{n}$ to the input u is KSN. Then from (27) with $w_u = 1$

$$\|u\|_\infty = \|KSN(s)\|_\infty \geq \max_{\text{RHP-poles, } p_i} |\mathcal{B}_z^{-1}(p_i)| \cdot |G_{ms}^{-1}(p_i)N(p_i)| \quad (29)$$

Thus, to have $\|u\|_\infty \leq 1$ for $\|\tilde{n}\|_\infty = 1$, we must require

$$|G_{ms}(p_i)| \geq |\mathcal{B}_z^{-1}(p_i)| \cdot |N(p_i)| \quad (30)$$

for the worst case pole p_i (we have here assumed that N is minimum phase).

EXAMPLE 1 Consider the unstable plant

$$G(s) = \frac{1}{s-p}, \qquad p > 0$$

with RHP-pole at p. From (29) we have the following lower bound on the $\mathcal{H}_\infty$-norm of the transfer function from normalized measurement noise $\tilde{n}$ to input u (we assume that N is minimum phase)

$$\|KSN(s)\|_\infty \geq |G_{ms}^{-1}(p)| \cdot |N(p)|$$

In our case $G^{-1} = s - p$, $G_{ms}^{-1}(s) = s + p$, $G_{ms}^{-1}(p) = 2p$, and the lower bound becomes

$$\|KSN(s)\|_\infty \geq 2p \cdot |N(p)| \quad (31)$$

The controller which minimizes $\|TV(s)\|_\infty$ and achieves the bound (31) is given in Theorem 3. For the special case where $N(s)$ is a constant $N(s) = N$ we get the proportional feedback controller $K(s) = 2p$.

As a numerical example, let $p = 10$, then we must have for any stabilizing feedback controller K

$$\|KSN(s)\|_\infty \geq 20\,|N(p)|$$

Thus with $\|\tilde{n}\|_\infty = 1$ we will need excessive inputs ($\|u\|_\infty > 1$) if $|N(p)| \geq |G_{ms}(p)| = 0.05$. Assume that $N(s) = N(p) = 0.05$, then $K(s) = 2p = 20$. This controller gives a “flat” frequency response, i.e. $|KSN(j\omega)| = 20, \forall\omega$. Thus, at any frequency ω_0 the closed-loop response in u due to

$$n(t) = 0.05\sin(\omega_0 t), \quad \text{is} \quad u(t) = \sin(\omega_0 t + \varphi) \quad \forall\omega$$

So, the input $u(t)$ oscillates between ± 1. The response in u and y due to $n(t) = 0.05\sin(4t)$ is shown in Figure 2.

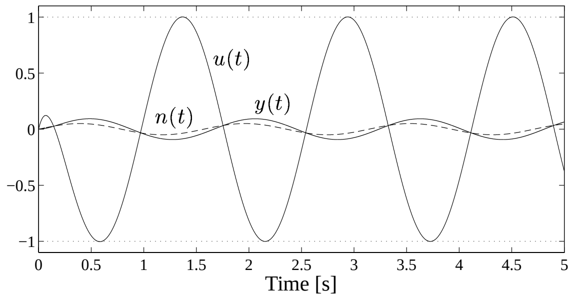

Figure 2: Closed-loop response at input u and output y of the plant G, due to $n(t) = 0.05\sin(4t)$ (dashed), with $K = 20$

Disturbances. Similar results as those for measurement noise also apply to disturbances by replacing N with G_d. From (26) with $w_u = 1$ we obtain

$$\|u\|_\infty = \|KSG_d(s)\|_\infty \geq \max_{\text{RHP-poles, } p_i} |\mathcal{B}_z^{-1}(p_i)| \cdot |G_{ms}^{-1}(G_d)_{ms}(p_i)| \tag{32}$$

To have $\|u\|_\infty \leq 1$ for $\|d\|_\infty = 1$ we must require

$$|G_{ms}(p_i)| \geq \left|\mathcal{B}_z^{-1}(p_i)\right| \cdot |(G_d)_{ms}(p_i)| \tag{33}$$

for the worst case pole p_i.

6 Stabilization with input saturation

Our results provide tight lower bounds for the required input signals for an unstable plant. Assume that we have found, from one of these bounds, that we need $\|u\|_\infty > 1$. That is, at some frequency ω_0 we need $u(t) = u_{\max}\sin(\omega_0 t)$, with $u_{\max} > 1$. Will the system become unstable in the case where input is constrained such that $|u(t)| \leq 1$ $(\forall t)$?

Unfortunately, all our results are for linear systems, and we have not derived any results for this nonlinear effect of input saturation.

Nevertheless, for simple low order systems we find as expected very good agreement between our lower bounds and the actual stability limit in systems with input saturation.

Intuitively, this agreement should be good if the input remains saturated for a time which is longer than about $1/p$, where p is the RHP-pole.

EXAMPLE 1 CONTINUED. Consider again the plant

$$G(s) = \frac{1}{s - 10}$$

with the controller $K = 20$ which minimizes $\|KSN(s)\|_\infty$ when N is constant. With this controller we get $|KS(j\omega)| = 20, \forall\omega$, from which we know that sinusoidal measurement noise

$$n(t) = n_0\sin(\omega_0 t)$$

cause the input to become

$$u(t) = 20 n_0 \sin(\omega_0 t + \varphi)$$

for any frequency ω_0. Thus, for $n_0 = f \cdot 0.05$ we have that $u(t) = f\sin(\omega_0 t + \varphi)$, and for $f > 1$ the plant input will exceed ± 1 in magnitude. The question is: what happens if the inputs are constrained to be within ± 1? Will the stability be maintained? We will investigate this numerically by considering sinusoidal measurement noise with frequency $\omega_0 = 1$ [rad/s].

Figure 3 shows the response to $n(t) = 1.01 \cdot 0.05\sin(t)$ ($\omega_0 = 1$ [rad/s], $f = 1.01$). We see that the plant becomes unstable due to the input saturation.

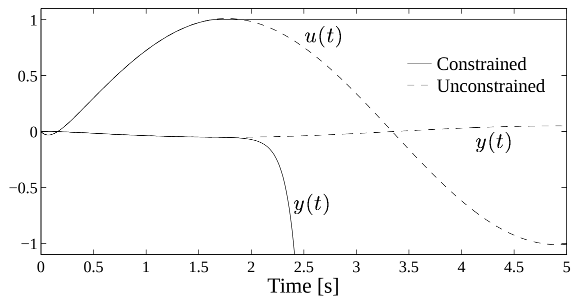

Figure 3: Closed-loop response at input u and output y of the plant G, due $n(t) = 1.01 \cdot 0.05\sin(t)$

Further reading. The full version of this paper can be found in (Havre, 1998, Chapter 4), which also contain several additional examples.

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