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On-line optimization and choice of optimization variables for control of heat exchanger networks

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Abstract - The paper discusses optimal operation of a general heat exchanger network with given structure, heat exchanger areas and stream data including predefined disturbances. A method that combines the use of steady state optimization and decentralized feedback control is proposed. A general steady state model is developed, which is easily adapted to any heat exchanger network. Using this model periodically for optimization, the operating conditions that minimize utility cost are found. Setpoints are constant from one optimization to the next, and special attention is paid to the selection of measurements such that the utility cost is minimized in the presence of disturbances and model errors. In addition to heat exchanger networks, the proposed method may also be applied to other processes where the optimum lies at the intersection of constraints.

INTRODUCTION

Methods for heat exchanger network (HEN) synthesis have been developed during the last decades and these methods aim to design a HEN that yields a reasonable trade-off between capital and operating cost in the nominal case. Since the mid 80's several authors have also investigated flexibility of HENs, e.g. Kotjabasakis and Linnhoff (1986) which introduced sensitivity tables to find which heat exchanger areas should be increased in order to make a nominal design sufficiently flexible. In Papalexandri and Pistikopoulos (1994), HEN synthesis and flexibility are considered simultaneously using mathematical programming.

The total design effort (on a systems level) required for a HEN typically involves the following three stages:

a) Nominal design. Synthesize one or more networks with good properties for nominal stream data.
b) Flexibility and controllability. Investigate the networks with regard to flexibility and controllability, and possibly introduce some modifications (e.g. increased area) such that at least one HEN shows satisfactory results.
c) Operation. Design a control system to operate the HEN properly. This involves control structure selection and possibly some method for on-line optimization.

For each step, some networks may be rejected or the designer may go back to the preceding step to find other alternatives. The steps are usually carried out in a sequential manner, however, the design may also be of a more simultaneous character, depending on the methods used.

Compared to synthesis of nominal and flexible HENs, much less effort has been dedicated to find methods for the operation of HENs (step c). Mathisen *et al.* (1992) investigated bypass selection for control of HENs, without considering the utility consumption. In Mathisen *et al.* (1994) a method for operation of HENs that minimizes utility consumption is proposed. The method is based on structural properties of the network, however, the variable control configuration may result in poor dynamic performance. A method based on repeated steady state optimization is suggested by Boyaci *et al.* (1996), but their focus is not on the control structure for closed loop implementation.

In this paper, a method for optimal operation of HENs is proposed. The method uses steady-state optimization which is carried out on-line with regular time intervals. The results of this optimization are then implemented by specifying the optimal value (setpoint) of some variable ("optimization variable"). It will be shown that the choice of optimization variables affects the performance of the (controlled) HEN when disturbances are present, and a procedure for optimal selection of these variables is presented.

With the term optimal operation, we mean that the following two goals are fulfilled:

- Primary goal: Satisfy targets (usually outlet temperatures).
- Secondary goal: Minimize operating cost.

In the following, it is assumed that the stream data (heat capacity flowrates and supply/target temperatures), network structure and heat exchanger areas are given and that the HEN is sufficiently flexible. To manipulate the network it is assumed that utility duties can be adjusted and that a variable bypass is placed across each process-to-process heat exchanger. In case of stream splits, we may also assume that split fractions can be varied.

The remaining part of the paper is organized as follows: First, the complete method is outlined. Then, the procedure for selection of optimization variables will be described in detail and applied to an illustrating example. Next, the steady state optimization model is presented, then the complete method is applied to an example and finally some conclusions are drawn.

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OUTLINE OF METHOD

In order to perform a meaningful on-line optimization, it is required that there is at least one extra degree of freedom during operation, and most HENs have this feature. As an example consider the network in figure 5 where there are four manipulations (bypasses u_A and u_B and heater and cooler duties) to control the three outlet temperatures to their targets (primary goal). Hence we have one manipulation "in excess" which implies one degree of freedom. This extra degree of freedom can be used to minimize utility cost, i.e. to achieve the secondary goal. Note that the number of degrees of freedom during operation is different from the synthesis stage. Within the "synthesis terminology", the HEN in figure 5 has minimum number of units and *no* degrees of freedom. (Constraints on ΔT_{min} etc. have no relevance during operation). In some cases the degrees of freedom during operation may be less than the number of excess manipulations, however, this is not discussed any further in this paper.

Figure 1 shows a schematic block diagram of the method that will be described. The optimizer contains a scalar objective function (criterion) J which indicates how well the HEN is operated, and a steady-state model of the HEN. As the objective function we will use total utility cost of the HEN. The model is optimized regularly and reference values for the optimization variables are passed to the controller K_2. The reference values (setpoints) are constant in the period between each optimization.

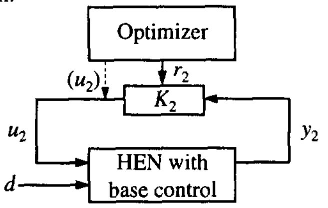

Fig. 1. General optimizing control structure.

All inputs (manipulations) u and outputs (measurements) y are separated into $u = [u_1 \ \ u_2]^T$ and $y = [y_1 \ \ y_2]^T$, respectively. y_1 are those outputs which have given target (reference) values and u_1 are those manipulations dedicated to keep y_1 at their target values. Satisfying the targets for y_1 is simply the *primary goal*, and note that in figure 1, this is not even drawn since it is assumed that this "base control" has been implemented.

We want to focus on *the secondary goal*; utility cost minimization (variables associated with this goal have index 2). u_2 is the "excess" manipulation(s) which represent the degree(s) of freedom that we will use to minimize utility cost. Of course, one could compute optimal values for u_2 and apply these directly (open-loop implementation) as indicated by the dashed line in figure 1. Alternatively, the optimizer could pass reference values for some "extra" measurements y_2 (closed-loop implementation). If the disturbance d was perfectly known (and constant), it would not matter (at steady state) which variables were chosen. However, from the explanation below it will be clear that the choice of *which* variables that are passed from the optimizer down to the control level affects how close to optimum the HEN can be operated.

The variables r_2 that are passed from the optimizer to the control level will be denoted *optimization variables*.

Let the disturbance d be partitioned into the following two contributions:

$$d = d_0 + d_u$$

where d_0 is the information that the optimizer has about the disturbances when it performs an optimization, and d_u (unknown disturbances) are all deviations from d_0 and the real disturbance until a new optimization is carried out. That is, d_u consists of for example unknown disturbances and model errors in addition to changes of the disturbances in the period between two optimizations (optimization interval). Measurement/estimation errors will not be handled explicitly in this paper, but these errors may be included in d_u and treated as any other deviation.

Since the optimizer has no specific information about d_u, the optimization is based on $d = d_0$. In practice, however, d_u may vary within some known bounds. The effect of $d_u \neq 0$ should be taken care of in the optimizer in order to avoid that the HEN becomes infeasible (primary goal can not be satisfied) for some disturbances. Figure 2 shows a typical situation for a general plant with one degree of freedom (one extra manipulation) and an objective function J that should be minimized. The plant has one disturbance input and two candidate measurements A and B ($y_2 = [y_{2,A} \ \ y_{2,B}]^T$) that can be controlled to a desired value using the extra manipulation u_2. (Since subscripts 1 and 2 are used to distinguish between the primary and secondary sets of inputs and outputs, we uses letters A, B etc. to denote individual elements of u and y). Also, remember that base control to keep primary outputs at fixed setpoints is already implemented.

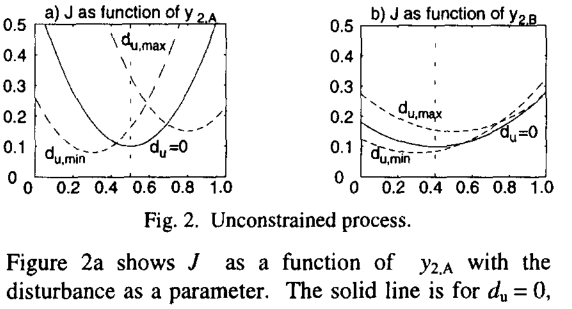

Fig. 2. Unconstrained process.

Figure 2a shows J as a function of $y_{2,A}$ with the disturbance as a parameter. The solid line is for $d_u = 0$, and the two dashed lines represent the extremes for d_u. Figure 2b shows similar curves as a function of $y_{2,B}$. Since we have to base our optimal values on $d_u = 0$, we can choose to keep either $y_{2,A} \approx 0.5$ or $y_{2,B} \approx 0.45$ using feedback control. From the figure, however, we see that when keeping $y_{2,B}$ constant, J is *less sensitive to both variations in* y_2 (control error) *and to unknown disturbances*, than when keeping $y_{2,A}$ constant. Therefore we prefer to keep $y_{2,B}$ constant between the optimizations. This simple example illustrates how the choice of optimization variables affects the objective function for an unconstrained process. Figure 3 shows similar curves as in figure 2 for a process where the optimum is constrained, which is typical for most HENs. (Minimum utility consumption corresponds to maximum utilization of process-to-process exchangers which again means that some bypasses are closed).

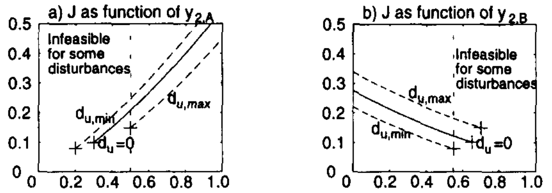

Fig. 3. Constrained process (typical for HENs).

In figure 3a the process is infeasible when $y_{2,A}$ becomes too small (marked with "+"). In a HEN this typically happens when a bypass saturates such that a target temperature no longer can be met. When $y_{2,B}$ is kept at a given value (figure 3b) the process is infeasible when the value becomes too *large*. More interesting in the constrained case, however, is that the *nominal* optimum ($d_u = 0$) is *infeasible* for some unknown disturbances. That is, we have to "back off" from the nominal optimum and find the optimal values that are feasible for all unknown disturbances. These values are indicated with the vertical dashed lines in figure 3a and 3b. Again we see that it is preferred to keep $y_{2,B}$ (rather than $y_{2,A}$) constant. However, a new obstacle has occurred; we need a remedy in the optimizer to find the values that are optimal also in the presence of the unknown disturbances. We do not want to compute optimum for all possible disturbances during operation since this may be too time consuming. This problem can be solved by computing proper constraints ("safety margins") on u_1 that are implemented in the optimizer. The optimal value of these safety margins is strongly correlated to the choice of optimization variables.
The following steps summarize the main parts of the complete procedure for on-line optimization of HENs:

1. Determine which manipulations (u_1) that should be used to control the primary outputs y_1 and design a control configuration and controllers for the primary goal (base control).
2. For each excess manipulation u_2 choose a measurement y_2 (among all candidates) such that the operation is insensitive to disturbances (see more details in next section). The additional constraints (safety margins) on u_1 are also found. Design decentralized controllers to control y_2.
3. Implement the steady state model including the constraints found in step 2 in the optimizer.

During operation, the optimizer computes setpoints for the optimization variables and apply these to the controller K_2 at regular intervals.

CHOICE OF OPTIMIZATION VARIABLES

This section describes a procedure for selection of optimization variables (step 2 in the complete method given above). The selection of outputs for optimizing control is discussed in Morud (1995, chapter 8) and in Skogestad and Postlethwaite (1996, chapter 10). In the latter a method that is based on choosing outputs that maximizes $\underline{\sigma}(G_{22})$ (smallest singular value) for a properly scaled system is proposed. In this paper a more direct method is applied (which is also mentioned in Skogestad and Postlethwaite, 1996). Before the procedure is presented, the following notation is introduced:

- $y_{2,\text{cand}}$ is a vector containing all candidates to y_2.
- y_{opt} is the optimal value of $y_{2,\text{cand}}$ for a given d_u.
- y^s_{opt} is a fixed value of $y_{2,\text{cand}}$ such that the objective function is minimized while the network is feasible for all d_u.
- J^s is $J(y^s_{\text{opt}})$ for a given value of d_u.
- Δu_1 is the constraint imposed on u_1 such that an optimization problem based on $d_u = 0$ gives feasibility for all d_u.

The steps in the procedure are listed below and some of the points will be further explained. For simplicity, we will assume there is only one degree of freedom (one optimization variable).

a) Select (i) minimum and maximum values for d_u, (ii) the objective function J, (iii) the entries of $y_{2,\text{cand}}$, (iv) the values for d_u that should be included in the computations and (v) define the type of J_{mean} that will be used for choosing optimization variable.
b) Compute y_{opt} and J_{opt} for "all" cases of d_u (i.e. the values from step iv in the previous point), see table 1. This table may also include row(s) for $u_{2,\text{opt}}$ (open-loop implementation). Note that $d_{u,j}$ is *case j* of d_u while $y_{\text{opt},i}$ denotes *element i* in y_{opt}.

	$d_{u,1}$	$d_{u,2}$	$d_{u,j}$
$y_{\text{opt},A}$	$y_{\text{opt},A}(d_{u,1})$	$y_{\text{opt},A}(d_{u,2})$	$y_{\text{opt},A}(d_{u,j})$
$y_{\text{opt},i}$	$y_{\text{opt},i}(d_{u,1})$	$y_{\text{opt},i}(d_{u,2})$	$y_{\text{opt},i}(d_{u,j})$
J_{opt}	$J_{\text{opt}}(d_{u,1})$	$J_{\text{opt}}(d_{u,2})$	$J_{\text{opt}}(d_{u,j})$

Table 1. y_{opt} and J_{opt} for all cases of d_u.

c) Keep $y_{2,\text{cand},i} = y^s_{\text{opt},i}$ for each output candidate, and evaluate $J^s_i(d_{u,j})$ and the resulting J_{mean}. In general, the setpoint $y^s_{\text{opt},i}$ should be optimized in order to minimize J_{mean}, but for constrained processes it will be some extreme value from table 1 (to ensure feasiblity for all d_u).

	$d_{u,1}$	$d_{u,2}$	$d_{u,j}$	J_{mean}
J^s_A	$J^s_A(d_{u,1})$	$J^s_A(d_{u,2})$	$J^s_A(d_{u,j})$	$J_{\text{mean},A}$
J^s_i	$J^s_i(d_{u,1})$	$J^s_i(d_{u,2})$	$J^s_i(d_{u,j})$	$J_{\text{mean},i}$

Table 2. J^s for all cases of d_u.

d) Choose the variable that gives the smallest J_{mean} from the last column in table 2 as optimization variable, i.e. this measurement should be controlled to a set-point which is updated periodically by the optimizer.

We have now found the best optimization variables. To simplify the on-line optimization we may want to use only the nominal disturbance set, $d_u = 0$. To ensure that we find the correct value of y^s_{opt} (which ensures feasiblity for all disturbances), we may impose some constraint ("safety margin") for the optimizer, e.g. $u_1 \geq \Delta u_1$. This will be explained in more detail for a simple example at the end of the example section. The "safety margin" on u_1 should of course not be implemented in the regulatory control level.

The next section describes the steady state model that can be implemented in the optimizer. Then the complete method with emphasis on the choice of optimization variables will be applied to a simple demonstration example.

Until now we have only considered one degree of freedom. If there were two degrees of freedom, two elements of $y_{2,cand}$ would have to be fixed at a time. Table 2 would need as many rows as there are possibilities to pick two variables out of the total number of candidate measurements. For example, if there is 6 candidate measurements and 2 degrees of freedom, the number of possibilities is $\frac{6!}{2!4!} = 15$.

REMARK 1. It is clear that the value of Δu_1 may depend on d_0. We assume that this change is small and that the value can be used for all d_0. In practice, one should carry out the procedure for selection of optimization variables for different d_0, to verify that Δu_1 does not change too much. The worst case value should be chosen if it is not acceptable to violate the primary goal, while a mean value can be used if a small violation to the targets is tolerable.

STEADY STATE OPTIMIZATION MODEL

This section presents a steady state model that can be adapted to any HEN. It is developed primarily for implementation in the optimizer, however, it may also be used in the procedure for selection of optimization variables (to generate tables 1 and 2).

Before we present the general model, consider the two alternatives (equations (1) and (2), respectively) to model a single heat exchanger with bypass given below, see figure 4.

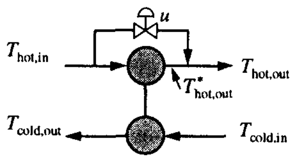

Fig. 4. Single heat exchanger with bypass.

At steady state it is of no consequence whether the bypass is placed across the hot side or cold side, and the choice in the figure is arbitrary. The temperature driving force $\Delta T_m(\cdot)$ may be logarithmic mean or some approximation, and note particularly the difference between equations (1a) and (2) regarding the arguments of $\Delta T_m(\cdot)$.

$$Q = UA\Delta T_m(T_{hot,in}, T_{cold,in}, T^*_{hot,out}, T_{cold,out}) \quad (1a)$$

$$T_{hot,out} = uT_{hot,in} + (1-u)T^*_{hot,out} \quad (1b)$$

$$Q \le UA\Delta T_m(T_{hot,in}, T_{cold,in}, T_{hot,out}, T_{cold,out}) \quad (2)$$

Equation (1) includes the hot exit temperature before it is mixed with the bypass stream and this results in bilinearities in (1b). The inequality in (2) expresses a constraint on Q when the boundary is placed *outside* the bypass splitter and mixer. The bypass fraction u does not even occur in (2), but the equality part of (2) corresponds to $u = 0$. In the optimization model, we choose the second alternative for each heat exchanger since this eliminates the bilinearities in the bypass mixer.

If u is needed, it can be found after the optimization of the network by solving one nonlinear equation for each bypass fraction. (Solving one unknown in one nonlinear equation n times is much simpler than solving n unknowns in n nonlinear equations simultaneously). As it will be shown, the value of u is often *not* required explicitly as it normally is the manipulated input in a feedback control loop.

The steady state model for a general HEN uses the following sets of heat exchangers:

PHX	: Set of all Process-to-process Heat eXchangers.
HBT	: Subset of PHX with Hot side outlet directly entering a Bypass controlled Target.
CBT	: Subset of PHX with Cold side outlet directly entering a Bypass controlled Target.
HUT	: Subset of PHX with Hot side outlet entering a Utility controlled Target (through a cooler).
CUT	: Subset of PHX with Cold side outlet entering a Utility controlled Target (through a heater).
HS	: Subset of PHX with Hot side inlet directly entering from a (hot) Supply.
CS	: Subset of PHX with Cold side inlet directly entering from a (cold) Supply.

The general HEN model shown below (eq. 3 to 11b) is an NLP problem. The variable c in equation (3) denotes the cost (pr. energy unit) for the utilities.

$$\min\left(\sum_{i\in HUT} c_i^{coolers} Q_i^{coolers} + \sum_{j\in CUT} c_j^{heaters} Q_j^{heaters}\right) \quad (3)$$

subject to

Equalities, (4) to (8)

$$Q_i = CP_i^{hot}(T_i^{hot,in} - T_i^{hot,out}) \quad i \in PHX \quad (4a)$$

$$Q_i = CP_i^{hot}(T_i^{hot,in} - T_i^{hot,out}) \quad i \in PHX \quad (4b)$$

$$Q_i^{coolers} = CP_i^{hot}(T_i^{hot,out} - T_i^t) \quad i \in HUT \quad (5a)$$

$$Q_i^{heaters} = CP_i^{cold}(T_i^t - T_i^{cold,out}) \quad i \in CUT \quad (5b)$$

$$T_i^{hot,out} = T_i^t \quad i \in HBT \quad (6a)$$

$$T_i^{cold,out} = T_i^t \quad i \in CBT \quad (6b)$$

$$T_i^{hot,in} = T_i^s \quad i \in HS \quad (7a)$$

$$T_i^{cold,in} = T_i^s \quad i \in CS \quad (7b)$$

Interconnection equations (problem specific) (8)

Inequalities, (9) to (11b)

$$Q_i \le \alpha_i U_i A_i \Delta T_{m\,i} \quad i \in PHX \quad (9)$$

$$Q_i \ge 0 \quad i \in PHX \quad (10)$$

$$Q_i^{coolers} \ge 0 \quad i \in HUT \quad (11a)$$

$$Q_i^{heaters} \ge 0 \quad i \in CUT \quad (11b)$$

Note that the index denotes heat exchangers and *not* streams (which is common in many other models), and that ΔT_m denotes the temperature driving force *outside* the bypass stream as in (2). As an example, the network in figure 5 will lead to the following sets: PHX = {A,B}, HUT = {B}, CUT = {A}, HBT = ∅, CBT = {B}, HS = {A} and CS = {A,B}, and the only interconnection equation (8) is $T_A^{\text{hot,out}} = T_B^{\text{hot,in}}$.

During each optimization, T^t, T^s, CP and UA for each heat exchanger are treated as constants. The model is valid without modifications for networks with fixed stream split fractions since CP denotes heat flow capacity in each heat exchanger. For networks with variable stream splits, CP in the split streams can be regarded as variables, and equations that preserve the mass balance in the splitter(s) and energy balance in the mixer(s) must be included. During operation, variable stream splits can be used as manipulated inputs.

The constant α in (9) is a factor that may limit the duty of a heat exchanger somewhat below its theoretical maximum. This is simply the way that the constraint on u_1 is implemented. Instead of implementing $u_1 \geq \Delta u_1$ directly (which is impossible since the model does not include u_1), the corresponding value for α has to be computed. This is done separately for each heat exchanger that controls a primary output. For heat exchangers associated with u_2, we have $\alpha = 1$.

The model does not include any upper constraints on the duty of the utility exchangers, and this implies the assumption that these are designed to handle the required duty. If this is not the case, additional constraints have to be added to the model, e.g. an upper limit on the duty.

The only possible source of nonlinearities in the model (for networks without variable splits) is the term ΔT_m in (9). In other words, if arithmetic mean (as opposed to logarithmic mean) is used as the temperature driving force, the model can be solved as an LP problem. The following procedure for solving the model has proven to be reliable: First, use arithmetic mean in (9) for all exchangers and solve the corresponding LP problem. Second, replace arithmetic mean with logarithmic mean (or e.g. Paterson or Chen approximations) and solve the NLP problem using the LP solution as the initial value.

EXAMPLE

The HEN used in the example is shown in figure 5. The primary outputs are the outlet temperatures of each stream which should be controlled to their target values of 30, 160 and 130°C for streams H1, C1 and C2, respectively. That is, we have

$$y_1 = \begin{bmatrix} T_{\text{H1}}^{\text{o}} & T_{\text{C1}}^{\text{o}} & T_{\text{C2}}^{\text{o}} \end{bmatrix}^T$$

where superscript ° denotes outlet temperature. There is a total of four manipulations (two bypasses and two variable utility duties) which gives

$$u = \begin{bmatrix} u_{\text{A}} & u_{\text{B}} & q_c & q_h \end{bmatrix}^T$$

There are two disturbances; ±10°C in the supply temperature of stream H1 and ±0.05 kW/°C in the CP of stream C2. These values represent the maximum variations d that may be present. The smaller variations/errors (d_u) that may occur within the optimization interval is defined in step 2a) of the procedure. UA for heat exchangers A and B are 0.523 and 1.322 kW/°C, respectively. For simplicity it is assumed that the utility exchangers are able to deliver sufficient duty for all possible cases. With this assumption and the given UA-values all target temperatures can be reached for all combinations of disturbances mentioned above.

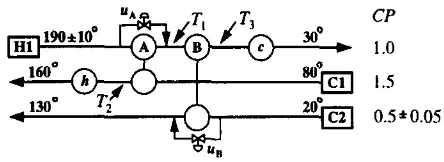

Fig. 5. Heat exchanger network used in example.

Applying the procedure step by step yields:

Step 1. Assign primary manipulations.

We use the main rule for selection of manipulations in HENs which is to choose the manipulation closest to the measurement (e.g. Mathisen, 1994, chapter 4). This implies that the primary manipulations u_1 become q_c, q_h and u_B and these control the outlet temperatures of streams H1, C1 and C2, respectively.

Step 2. Choice of optimization variable.

There is one excess manipulation, $u_2 = u_A$, and the steps a) to d) below illustrate the selection of optimization variable.

2a) We assume:

(i) $d_u = [\pm 3°\text{C}, \pm 0.01\text{kW/°C}]^T$ (maximum variations/ errors of the disturbances within the optimization interval).

(ii) The objective function is $J = q_c + q_h$ (utility consumption)

(iii) Possible candidates to y_2 are $y_{2,\text{cand}} = [T_1 \;\; T_2 \;\; T_3 \;\; u_A]^T$ (see figure 5). Note that the open-loop implementation (u_A) is an alternative.

(iv) The computations are done for the four "corner points" of d_u in addition to $d_u = 0$.

(v) J_{mean} is the arithmetic mean of the five cases in step (iv). (We require that target temperatures have to be reached for the five cases).

2b) y_{opt} and J for different d_u are shown in table 3. The table is generated for $d_0 = [0 \;\; 0]^T$, i.e. for nominal values of the disturbances (190°C and 0.5kW/°C). Also a row for $u_{\text{B,opt}}$ is included for extra information.

2c) Table 4 shows J for optimal fixed values of $y_{2,\text{cand}}$. Note that in this example, the values for $y_{2,\text{cand}}$ can be found without optimization, but simply from table 3 and physical insight (see remark 2). If there is a possibility that the optimum is not constrained one would have to resort to conventional optimization.

2d) From the last column of table 4 it is clear that keeping T_1 constant is preferred.

Step 3. Implementation of optimizer.

The model (including the sets and connection equations) was described in the previous section. The constraint ("safety margin") that should be included in the optimizer is $u_B \geq 0.105$. We will explain how this value is obtained, but before that we explain the details in the implementation of this constraint. To implement the constraint, we first find $q_B = 55$kW for $d_u = 0$ (55kW is the deficit heat of stream C2). Then we find $\alpha_B = 0.946$ from $q_B = \alpha_B \, UA_B \Delta T_{m,B}$, where the last term is the logarithmic mean for heat exchanger B for $d_u = 0$ and $T_1 = 151.9$°C. Implementing $\alpha_B = 0.946$ (and $\alpha_A = 1.0$) in eq. (9) will ensure the required safety margin on u_B when unknown disturbances d_u are present.

The actual value for the safety margin ($\Delta u_B = 0.105$) is obtained as follows: The values of u_A and u_B for the five cases in table 4 corresponding to $T_1 = 151.9$°C are given in table 5.

	Case 1 $d_{u,1}=\begin{bmatrix}0\\0\end{bmatrix}$	Case 2 $d_{u,2}=\begin{bmatrix}-3\\-0.01\end{bmatrix}$	Case 3 $d_{u,3}=\begin{bmatrix}-3\\+0.01\end{bmatrix}$	Case 4 $d_{u,4}=\begin{bmatrix}+3\\-0.01\end{bmatrix}$	Case 5 $d_{u,5}=\begin{bmatrix}+3\\+0.01\end{bmatrix}$
$T_{1,opt}$	150.0	149.0	151.0	151.9	151.9
$T_{2,opt}$	106.7	105.4	104.0	107.4	107.4
$T_{3,opt}$	95.0	95.1	94.9	98.0	95.8
$u_{A,opt}$	0.000	0.105	0.292	0.000	0.000
$u_{B,opt}$	0.000	0.000	0.000	0.154	0.049
J_{opt}	145.0	147.0	149.0	146.9	144.7

Table 3. Values for y_{opt} and J_{opt} for all cases of d_u in the example. Case 1 is the nominal disturbance.

	Case 1	Case 2	Case 3	Case 4	Case 5	J_{mean}
$J(T_1^s=151.9)$	148.9	153.0	150.8	147.0	144.8	148.9
$J(T_2^s=104.0)$	153.0	151.2	149.0	159.2	155.0	153.0
$J(T_3^s=98.0)$	151.0	152.9	155.1	146.9	149.1	151.0
$J(u_A^s=0.292)$	151.1	151.2	149.0	153.2	151.0	151.1

Table 4. J^s for the possible choices of measurement and for all cases of d_u in the example.

	Case 1	Case 2	Case 3	Case 4	Case 5
u_A	0.207	0.354	0.354	0	0
u_B	0.105	0.155	0.051	0.155	0.051

Table 5. Values of u_A and u_B when T_1 = 151.9°C.

For cases 4 and 5, u_A saturates at zero which implies that it is no longer possible for u_A to keep T_1 = 151.9°C. The optimizer uses d_0 (case 1) where u_B takes the value of 0.105. Thus, in order to handle cases 4 and 5, a safety margin of $\Delta u_B = 0.105$ has to be used by the optimizer. Note that if we accepted that T_1 deviated from its setpoint (due to saturation in u_A) it would be possible to further reduce utility consumption somewhat. Then the setpoint for T_1 could be reduced slightly below 151.9°C. until u_B saturated for some disturbance. In this example we require that setpoins for secondary measurements have to be satisfied.

The reason for implementing the "safety margin" on u_B as an <u>inequality</u> constraint is that other values of d_0 may give $u_{B,opt} > 0.105$. Requiring $u_B = 0.105$ in such cases will result in infeasibility.

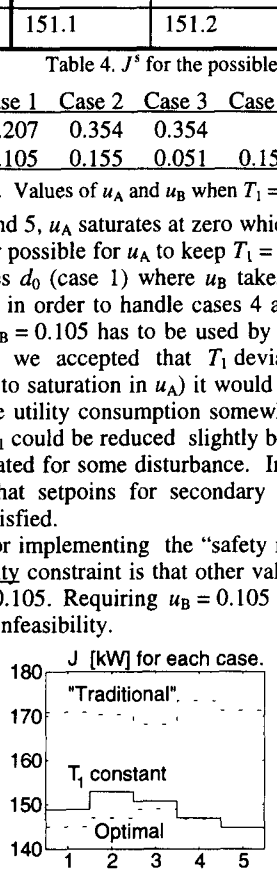

Figure 6. Results for example.

The value for J_{mean} of 148.9 kW in table 4 should be compared to the mean value of J_{opt} from table 3 which is 146.5 kW. Figure 6 visualizes the results for the example when T_1 is the optimization variable (solid curve). The different cases are the same as in tables 3 and 4. For comparison, the optimal values (when d_u is perfectly known) are shown in the lower dashed curve, and the upper dashed curve shows the result for the "traditional" scheme <u>without</u> optimization. (The latter is implemented by fixing u_A at a value such that the network is feasible for <u>all</u> possible disturbances, i.e. $d = [\pm 10, \pm 0.05]^T$, using u_B only). From the results given in figure 6 and table 4, it is clear that the main reduction in utility consumption compared to the traditional case is due to the periodic optimization, whereas the choice of optimization variable contributes less. For other examples, the choice of optimization variable can have a more significant effect on the utility reduction.

REMARK 2. From figure 5, it is clear that decreasing T_1, T_3 (by decreasing u_A) or u_A will reduce utility consumption (J). I.e. optimal values for these variables in table 3 are minimum values (smaller values will violate the primary goal). Therefore, the case with the <u>largest</u> value has to be chosen as this is the smallest value feasible for all d_u. For T_2, a similar (but opposite) argument leads to choosing the <u>smallest</u> value in table 3.

SUMMARY AND CONCLUSIONS

A method for optimal operation of heat exchanger networks based on periodic steady state optimization is proposed. An important issue is optimal choice of measurements that are kept constant between each optimization using feedback control. The objective functions used during operation and for choice of optimization variables are identical. Optimal operating conditions for heat exchanger networks are normally located at the intersection of constraints, and additional constraints ("safety margins") have to be implemented in the optimizer in order to maintain the target temperatures when unknown disturbances are present.

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