LIMITATIONS IMPOSED BY RHP ZEROS/POLES IN MULTIVARIABLE SYSTEMS

K. Havre and **S. Skogestad***

Chemical Engineering, Norwegian University of Science and Technology, N-7034 Trondheim, Norway
**Fax: (+47) 73 59 40 80 and e-mail: skoge@kjemi.unit.no*

This paper examines limitations imposed by Right Half Plane (RHP) zeros and poles in multivariable feedback systems. The main result is to provide lower bounds on $\|WXV(s)\|_\infty$ where X is S, S_I, T or T_I (sensitivity and complementary sensitivity). Previously derived lower bounds on the $\mathcal{H}_\infty$-norm of S and T (Zames, 1981; Skogestad and Postlethwaite, 1996; Havre and Skogestad, 1996) are thus generalized to the case with matrix-valued weights, including bounds for reference tracking and disturbance rejection. Furthermore, new bounds which quantify the minimum input usage for stabilization in the presence of measurement noise and disturbances are derived.

The basis of these new bounds is to note that the transfer function from the outputs (e.g. the measurement noise) to the inputs, KS, can be rewritten as $KS = T_I G^{-1}$. When G is unstable, G^{-1} has a RHP-zero, it is important that we can handle the case when the "weight" $V = G^{-1}$ has RHP-zeros. Otherwise, G^{-1} evaluated at the pole of G would be zero in a certain direction, and we would not derive any useful bound.

To give a specific example, consider the closed-loop transfer function from measurement noise of magnitude N to inputs u, which is $KSN = T_I G^{-1} N$. From the results in the paper we find for the special case of a SISO plant with a RHP-pole p that

$$\|KSN(s)\|_\infty \geq |N(p)| \cdot |(G^{-1})_m|_{s=p} \tag{1}$$

where $(G^{-1})_m$ is minimum-phase (it has the RHP-zeros of G^{-1} mirrored into the left half plane). If the plant also has a RHP-zero then the bound will be even higher. For example, for $G(s) = 1/(s-10)$, we have $p = 10$ and $(G^{-1})_m = s + 10$. Then $\|KSN(s)\|_\infty \geq 20 \cdot |N(p)|$ and to achieve $|u| \leq 1$ we must require $|N(p)| \leq 1/20$.

These and other bounds presented in the paper are of fundamental importance. In spite of being surprisingly simple, they appear to be new in the control literature, even for SISO systems.

References

Havre, K. and Skogestad, S. (1996). Effect of RHP zeros and poles on performance in multivariable systems, *Proc. from Control'96*, University of Exeter, UK, pp. 930–935. Also submitted to *Journal of Process Control*.

Skogestad, S. and Postlethwaite, I. (1996). *Multivariable Feedback Control, Analysis and Design*, John Wiley & Sons, Chichester.

Zames, G. (1981). Feedback and optimal sensitivity: model reference transformations, multiplicative seminorms, and approximate inverses, *IEEE Transactions on Automatic Control* **AC-26**(2): 301–320.