# CONTROL STRUCTURE DESIGN

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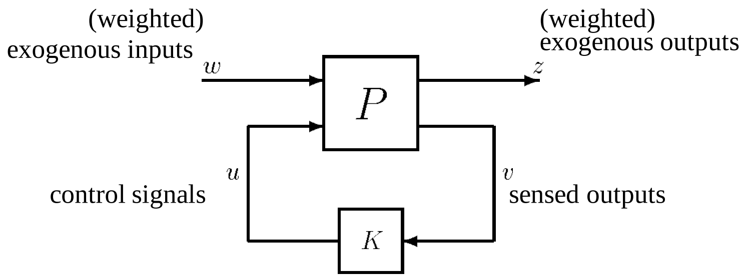
This is a copy of the transparencies used for the presentation. For more detailed information see Chapter 10 in S. Skogestad and I. Postlethwaite, *Multivariable feedback control - Analysis and design*, Wiley, 1996.

## THEORY: General formulation




- Find a controller $K$ which based on the information in $v$, generates a control signal $u$ which counteracts the influence of $w$ on $z$, thereby minimizing the closed-loop norm from $w$ to $z$.

# PRACTICE: Typical base level control structure

# PRACTICE: Typical control hierarchy

Foss (1973):

*The central issue to be resolved ... is the determination of control system structure.*

*Which variables should be measured, which inputs should be manipulated and which links should be made between the two sets?*

*There is more than a suspicion that the work of a genius is needed here, for without it the control configuration problem will likely remain in a primitive, hazily stated and wholly unmanageable form.*

*The gap is present indeed, but contrary to the views of many, it is the theoretician who must close it.*

**CONTROL STRUCTURE DESIGN**

## Tasks:

1. *Selection of controlled outputs* (a set of variables which are to be controlled to achieve a set of specific objectives)
2. *Selection of manipulations and measurements* (sets of variables which can be manipulated and measured for control purposes)
3. *Selection of* **control configuration** (a structure interconnecting measurements/commands and manipulated variables)
4. *Selection of controller type* (control law specification, e.g., PID, decoupler, LQG, etc.).

Note distinction between control *structure* and control *configuration*.

Tasks 1 and 2 combined: *input/output selection*
Task 3 (configuration): *input/output pairing*

Shinskey (1967, 1988)
Morari (1982)
Stephanopoulos (1984)
Balchen and Mumme (1988)
Nett (1989)
van de Wal and de Jager (1995)

Skogestad and Postlethwaite (1996)

**TASK 1:Selection of controlled outputs**

*Controlled output* $y$*:* Measured output with reference ($r$)

Two distinct questions:

1. What should the controlled variables $y$?
   (includes open-loop by selecting $y = u$)
2. What is their optimal values ($y_{\mathrm{opt}}$)?

Second problem: Extensively studied.

**Example 1.** *Room heating.*
$y =$ room temperature

Other cases: Less obvious.

**Example 2.** *Cake baking.*
Goal (purpose): well baked inside and nice outside
Manipulated input: $u = Q$ (assume 15 minutes).

(a) Open-loop implementation: Heat input $Q$

(b) Closed-loop implementation:

$y =$ oven temperature
"Optimizer": Cook book (look-up table)

## SELECTION OF CONTROLLED OUTPUTS

*The input $u$ (generated by feedback to achieve $y \approx r$) should be close to the optimal input $u_{\rm opt}(d)$.*

$$u - u_{\rm opt} = G^{-1}(0)(y - y_{\rm opt})$$

where

$$y - y_{\rm opt} = \underbrace{y - r}_{e} + \underbrace{r - y_{\rm opt}(d)}_{e_{\rm opt}}$$

$\Rightarrow$ Select controlled outputs $y$ such that:

1. *$G^{-1}(0)$ is small; inputs have a large effect on $y$.*
2. *$e_{\rm opt} = r - y_{\rm opt}(d)$ is small; $y_{\rm opt}(d)$ depends only weakly on disturbances.*
3. *$e = y - r$ is small; good measurement and control of $y$.*

Scale outputs such that $\|y - y_{\rm opt}(d)\| \approx 1$ (due to measurement errors and disturbances)

Note: $\bar{\sigma}(G^{-1}(0)) = 1/\underline{\sigma}(G(0))$.

**Conclusion. Simple tool:**

- Prefer a set of controlled outputs with large $\underline{\sigma}(G(0))$.

# Distillation column example from Kjetil

# Distillation column example from Kjetil

# Distillation column example from Kjetil

**TASK 2: Selection of manipulations and measurements**

- $y$ – all candidate outputs (measurements)
- $u$ – all candidate inputs (manipulations)

Combinatorial growth: Detailed analysis time consuming.

Possibilities with 1 to $M$ inputs and 1 to $L$ outputs (Nett, 1989):

$$\sum_{m=1}^{M} \sum_{l=1}^{L} \binom{L}{l} \binom{M}{m}$$

$M = L = 2$: 4+2+2+1=9 candidates

$M = L = 4$: 225 candidates, etc.

TOOLS THAT AVOID COMBINATORIAL GROWTH DESIRED.

RGA is one such tool.

**RGA for non-square plant**

$$\Lambda(G) = G \times G^{\dagger}$$

**HDA plant** (Cao, 1995).

- 5 outputs and 13 candidate inputs.
- $\binom{13}{5} = 1287$ combinations with 5 inputs / 5 outputs.
- $\binom{13}{6} = 1716$ combinations with 6 inputs / 5 outputs.

$$G^T(0) = \begin{bmatrix} 0.7878 & 1.1489 & 2.6640 & -3.0928 & -0.0703 \\ 0.6055 & 0.8814 & -0.1079 & -2.3769 & -0.0540 \\ 1.4722 & -5.0025 & -1.3279 & 8.8609 & 0.1824 \\ -1.5477 & -0.1083 & -0.0872 & 0.7539 & -0.0551 \\ 2.5653 & 6.9433 & 2.2032 & -1.5170 & 8.7714 \\ 1.4459 & 7.6959 & -0.9927 & -8.1797 & -0.2565 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.1097 & -0.7272 & -0.1991 & 1.2574 & 0.0217 \\ 0.3485 & -2.9909 & -0.8223 & 5.2178 & 0.0853 \\ -1.5899 & -0.9647 & -0.3648 & 1.1514 & -8.5365 \\ 0.0000 & 0.0002 & -0.5397 & -0.0001 & 0.0000 \\ -0.0323 & -0.1351 & 0.0164 & 0.1451 & 0.0041 \\ -0.0443 & -0.1859 & 0.0212 & 0.1951 & 0.0054 \end{bmatrix}$$

RGA may be useful in providing an initial screening:

$$\Lambda^T = \begin{bmatrix} 0.1275 & -0.0755 & 0.5907 & 0.1215 & 0.0034 \\ 0.0656 & -0.0523 & 0.0030 & 0.1294 & 0.0002 \\ 0.2780 & 0.0044 & 0.0463 & 0.4055 & -0.0060 \\ 0.3684 & -0.0081 & 0.0009 & 0.0383 & -0.0018 \\ -0.0599 & 0.9017 & 0.2079 & -0.1459 & 0.0443 \\ 0.1683 & 0.4042 & 0.1359 & 0.1376 & 0.0089 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0014 & -0.0017 & 0.0013 & 0.0099 & 0.0000 \\ 0.0129 & -0.0451 & 0.0230 & 0.1873 & -0.0005 \\ 0.0374 & -0.1277 & -0.0359 & 0.1163 & 0.9516 \\ 0.0000 & 0.0000 & 0.0268 & 0.0000 & 0.0000 \\ 0.0001 & 0.0001 & 0.0000 & 0.0001 & 0.0000 \\ 0.0002 & 0.0002 & 0.0001 & 0.0001 & 0.0000 \end{bmatrix}, \quad \Lambda_\Sigma^T = \begin{bmatrix} 0.77 \\ 0.15 \\ 0.73 \\ 0.40 \\ 0.95 \\ 0.85 \\ 0.00 \\ 0.01 \\ 0.18 \\ 0.94 \\ 0.03 \\ 0.00 \\ 0.00 \end{bmatrix}$$

- Five inputs with the largest inputs projection: $5$, $10$, $6$, $1$, and $3$ (in that order).
- For this selection $\underline{\sigma}(G_s) = 1.73$ whereas $\underline{\sigma}(G) = 2.45$ for the overall system.

**TASK 3: Selection of control configuration**

Controller $K$ connects available measurements/commands ($v$) and manipulations ($u$):

$$u = Kv$$

**Control configuration:** *The restrictions imposed on the structure of the overall controller $K$ by decomposing it into a set of local controllers (subcontrollers, units, elements, blocks) with predetermined links and with a possibly predetermined design sequence.*

Typical restriction: one degree-of-freedom controller where input is $r - y$.

Some elements used to build up configuration:

- Decentralized controllers ($K$ diagonal)
- Cascade controllers (predetermined order for tuning)
- Feedforward elements
- Decoupling elements
- Selectors

**Decentralized control.** *The control system consists of independent feedback controllers which interconnect a subset of the output measurements/commands and a subset of the manipulated inputs. These subsets should not be used by any other controller.*

Usually: Rearrange order of inputs and outputs such that $K$ is block-diagonal.

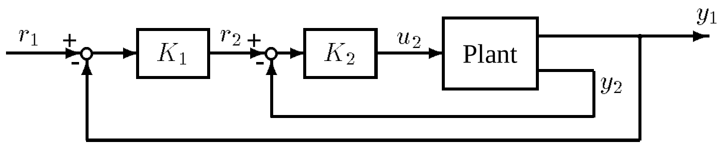
**Cascade control** *is when the output from one controller is the input to another.* This is broader then the *conventional* definition of cascade control which is that the output from one controller is the reference command (*setpoint*) to another.

**Feedforward elements** *link measured disturbances and manipulated inputs.*

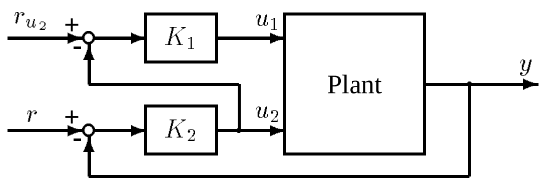
**Decoupling elements** *link one set of manipulated inputs ("measurements") with another set of manipulated inputs.* Often viewed as "feedforward" elements.

In addition to restriction on structure on $K$: Impose restrictions on *sequence* the subcontrollers are designed.

## Cascaded controllers




(a) Extra measurements $y_2$ (conventional cascade control)




(b) Extra inputs $u_2$ (input resetting)

**Why use control configurations?**

- Decomposed configurations often quite complex.
- Better performance: Optimization problem – resulting in a centralized multivariable controller.

So why use control configurations?

- Cost associated with obtaining good plant models (needed for centralized control).
- Cascade, decentralized, etc.: Controller is usually tuned one at a time with little modelling effort. Often ON-LINE tuning.

Other advantages decentralized/cascade/hierarchical configurations:

- Often easier to understand for operators
- Reduce the need for control links
- Allow for decentralized implementation
- Tuning parameters have direct and "localized" effect
- Tend to be insensitive to uncertainty
- Simpler implementation
- Reduced computation load
- Simple or even on-line tuning
- Longer sampling intervals for the higher layers
- Allow simple models when designing higher layers
- "Stabilize"[1] the plant such that it is can be controlled by operators.

[1] The terms "stabilize" and "unstable" as used by process operators may not refer to a plant that is unstable in a mathematical sense, but rather to a plant that is *sensitive* to disturbances and which is difficult to control manually.

## Partial control

| | Meas./Control of $y_1$ ? | Control objective for $y_2$ ? |
|---|---|---|
| Sequential decentralized control | Yes | Yes |
| Sequential cascade control | Yes | No |
| "True" partial control | No | Yes |
| Indirect control | No | No |

Set $y_2 = r_2$

$$y_1 = \underbrace{(G_{11} - G_{12}G_{22}^{-1}G_{21})}_{\triangleq\, P_u} u_1 + \underbrace{(G_{d1} - G_{12}G_{22}^{-1}G_{d2})}_{\triangleq\, P_d} d + \underbrace{G_{12}G_{22}^{-1}}_{\triangleq\, P_r}(r_2 - n_2)$$

$P_d$ – *partial disturbance gain*

*Some criteria for selecting $u_2$ and $y_2$ in lower-layer:*

1. Lower layer must quickly implement the setpoints from higher layers, i.e., controllability of subsystem $u_2$/ $y_2$ should be good. ($G_{22}$)
2. Provide for local disturbance rejection. ($P_d$)
3. Impose no unnecessary control limitations on problem involving $u_1$ and/or $r_2$ to control $y_1$. ($P_u$ or $P_r$)

"Unnecessary": Limitations (RHP-zeros, ill-conditioning, etc.) not in original problem involving $u$ and $y$

**Example: Control of $5 \times 5$ distillation process**

$$u = [\,L \quad V \quad D \quad B \quad V_T\,]^T$$
$$y = [\,y_D x_B \quad M_D \quad M_B \quad p\,]^T$$

Stabilize: Close three SISO loops for level and pressure:

$$y_1 = [\,y_D \quad x_B\,]^T, \quad y_2 = [\,M_D \quad M_B \quad p\,]^T$$

Many possible choices for $u_1$ and $u_2$. $LV$-configuration:

$$u_1 = [\,L \quad V\,]^T$$

$DV$-configuration:

$$u_1 = [\,D \quad V\,]^T$$

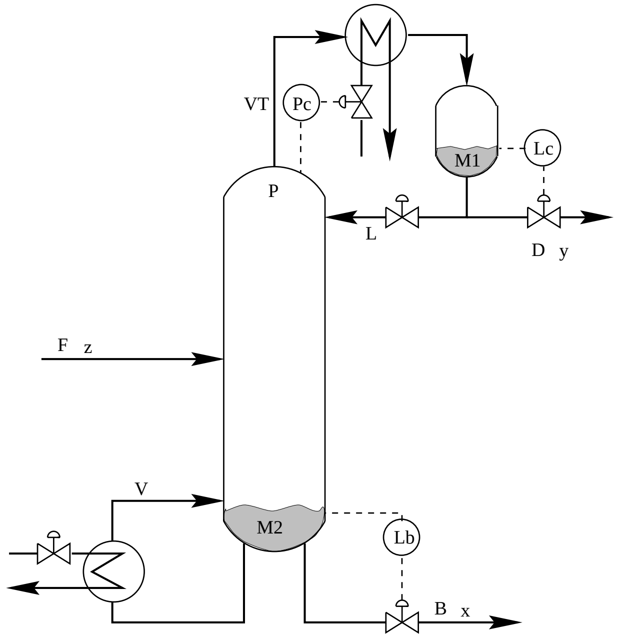



Figure 6: Typical distillation column with LV-control configuration

- $5 \times 5$ model.

Important issues:

- Disturbance sensitivity ($P_d$ should be small)
- Interactions (RGA-elements of $P_u$)

Because of interactions and cost of measurements: Often only one product composition controlled ("true" partial control).

## Decentralized feedback control

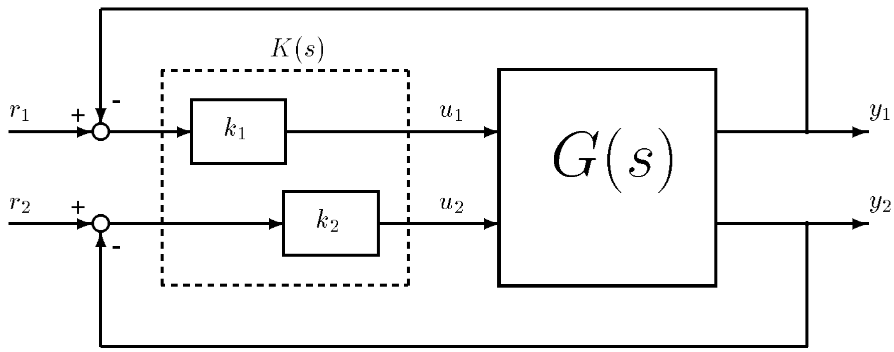



Figure 8: Decentralized diagonal control of a $2 \times 2$ plant

Design of decentralized control systems involves two steps:

1. Choice of pairings (control configuration selection)
2. Design (tuning) of each controller, $k_i(s)$.

Magnitude of off-diagonal elements in $G$ relative to its diagonal elements given by:

$$E \triangleq (G - \widetilde{G}_{diag})\widetilde{G}_{diag}^{-1}$$

Important relationship:

$$\underbrace{(I + GK)}_{\text{overall}} = \underbrace{(I + E\widetilde{T}_{diag})}_{\text{interactions}} \quad \underbrace{(I + \widetilde{G}_{diag}K)}_{\text{individual loops}}$$

ETC...........
(quite a lot of theory available, RGA etc.)

**Conclusions / Future work**

1. Control structure design: Issues must be defined
2. Tools and theory are developing; Controllability analysis
3. Use feedback effectively - feedback hierarchies
4. Balance between centralized and cascade/decentralized implementations
5. Balance between complexity and performance
6. Balance between modelling effort and performance
7. Mathematical problem formulation is difficult (non-convex):
   - Optimal design of $K$ with given no. of non-zero elements
   - Penalize links and controller complexity
   - Controllability analysis approaches; branch and bound

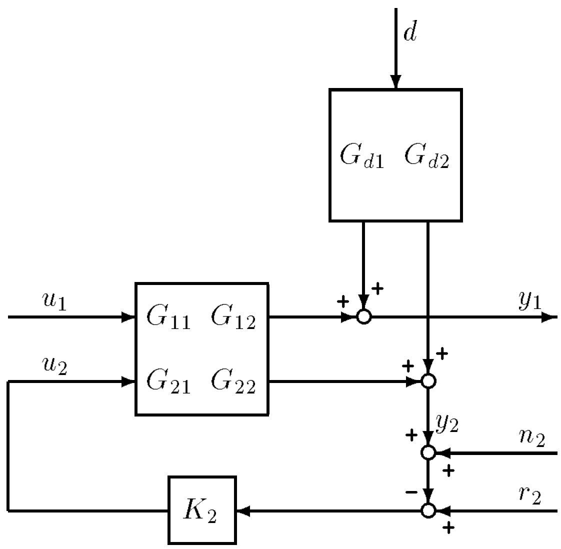



Figure 1: Block diagram of a partial control system

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