SELECTION OF FEEDBACK VARIABLES FOR IMPLEMENTING OPTIMIZING CONTROL SCHEMES

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Abstract. This paper considers selection of controlled variables when implementing optimizing control schemes. As a special case we treat indirect control. The selection criterion derived is to maximize the smallest singular value of the selected subsystem to be controlled using feedback control. A procedure for selecting outputs according to this criterion is outlined. The selection criterion is dependent on scaling, so we discuss appropriate scaling.

1. INTRODUCTION

Control systems for continuous plants in the chemical process industry are often built in a hierarchical manner, with regulatory control at the lowest layer, a supervisory control layer above, and an optimizing control layer on top (e.g. Morari *et al.*, 1980). Additional layers are possible, as illustrated in Figure 1 which shows a typical control hierarchy for a complete chemical plant. In Figure 1 the control layer is subdivided into two layers: *supervisory control* ("advanced control") and *regulatory control* ("base control"). We have also included a scheduling layer above the optimization. In general, the information flow in such a control hierarchy is based on the higher layer sending commands to the layer below, and the lower layer reporting back any problems in achieving this. These commands includes reference values (setpoints) and values to unused inputs on the control layer, see Figure 2. The optimization tends to be performed *open-loop* with limited use of feedback. On the other hand, the control layer is mainly based on *feedback* information. The optimization is often based on nonlinear steady-state models, whereas we often use linear dynamic models in the control layer. There is usually a time scale separation with faster lower layers as indicated in Figure 1. This means that the setpoints, as viewed from a given layer in the hierarchy, are updated only periodically. Between these updates, when the setpoints are constant, it is important that the system remains reasonably close to its optimum. This observation is the basis for this paper which deals with selecting outputs on the control layer for a optimizing control hierarchy shown in Figure 2.

From a theoretical point of view, the optimal coordination of the inputs and thus the optimal performance is

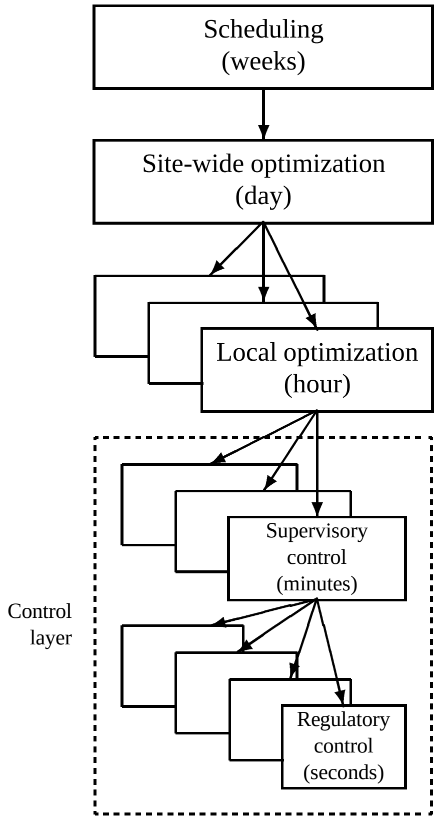

Figure 1. Typical control system hierarchy in a chemical plant

obtained with a *centralized optimizing controller*, which combines the two layers of optimization and control. All control actions in such an ideal control system would be perfectly coordinated and the control system would use on-line dynamic optimization based on a nonlinear dynamic model of the complete plant instead of infrequent steady-state optimization as considered in this paper. However, this solution is normally not used for a number of reasons; including the cost of modeling, the difficulty of controller design, maintenance and modification, robustness problems, operator acceptance, and the lack of computing power.

Notation. At the control layer we use linear time invariant transfer function models on the form

$$y(s) = G(s)u(s) + G_d(s)d(s) \qquad (1)$$

where u is the vector of manipulated inputs, d is the vector of disturbances and y is the vector of outputs. $G(s)$ and $G_d(s)$ are rational transfer function matrices of dimensions $l \times m$ and $l \times n_d$. The overall objective is to minimize some sort of performance index J_1 stated in terms of the outputs y and the inputs u, $J_1(y, u)$. As a

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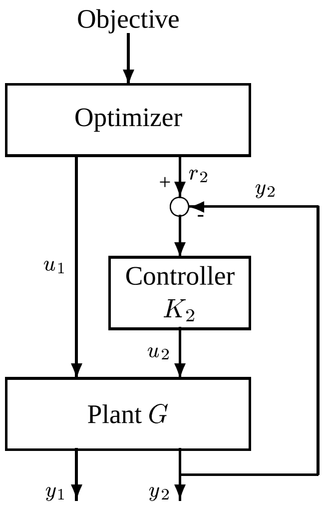

Figure 2. Optimizing control with feedback control layer

subobjective at the control layer we want keep the control error $e = y - r$ small.

Outline. First, we derive some general results, applicable to both optimizing and indirect control. We discuss appropriate scaling of inputs and outputs, and we outline a procedure for selecting outputs and inputs. Next, we consider measurement selection for indirect control. Finally we give an example and a summary.

Previous work. The paper by Morari *et al.* (1980) is the first in a series of papers studying the synthesis of control structures for chemical processes. They classify the control objectives into regulatory and optimizing control, partition the process for practical implementation of the control structures and show how to analyze optimizing control structures. Maarleveld and Rijnsdorp (1970) argue that optimum operation of a process is often not at "the top of the hill", but at the intersection of constraints. During operation the active constraints may change, so a control system making use of the constraint principle should be capable of switching between constraint intersections. The idea is worked out for a distillation column where the column pressure and feed preheating are the degrees of freedom. Tyreus (1987) discusses the possibility of simplifying the traditional structure with optimizer in conjunction with multivariable regulatory control by considering alternate control structures and by intergrating the steady-state optimization into the regulatory control. According to Tyreus the resulting systems are easy to implement and perform nearly optimally. Kim *et al.* (1991) presents an on-line dynamic optimizing control procedure for operation of a binary distillation column; the performance was examined experimentally.

The present paper extends and provides an example for the results given in Skogestad and Postlethwaite (1996). Related work can also be found in Morud (1995, Chapter 8).

2. SELECTION OF CONTROLLED OUTPUTS

We rearrange and partition the outputs $y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$, into uncontrolled outputs y_1 and controlled outputs y_2, and the inputs $u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$, into unused inputs u_1 and inputs u_2 used for control of y_2. The model (1) becomes

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + \begin{bmatrix} G_{d1} \\ G_{d2} \end{bmatrix} d \quad (2)$$

y_1 includes outputs which can not directly be controlled but has an impact on the performance objective J_1.

Two distinct questions arise:

(1) What variables y_2 should be selected as the controlled variables?
(2) What is the optimal reference value ($y_{2,\mathrm{opt}}$) for these variables?

The second problem is one of dynamic optimization and is extensively studied. Here we want to gain some insight into the first problem. We make the assumptions:

(a) The overall goal can be quantified in terms of the scalar cost function J_1 which we want to minimize.
(b) For a given disturbance d, there exists an optimal value $u_{\mathrm{opt}}(d)$ and corresponding value $y_{\mathrm{opt}}(d)$ which minimizes the cost J_1.
(c) The reference values r_2 for the controlled outputs y_2 should be constant, i.e. r_2 should be independent of the disturbances d. Typically, some average value is selected, e.g. $r = y_{2,\mathrm{opt}}(\bar{d})$.

By inserting the model (1) in the cost function J_1 it can be expressed in terms of u and d, $J_1(u,d)$. However, seen from the optimizer the degrees of freedom are r_2 and u_1, see Figure 2. When the feedback controller K_2 relating u_2 to r_2 and y_2, i.e. $u_2 = K_2(r_2, y_2)$, is invertible one may look at u_2 as equivalent to r_2, and can therefore replace r_2 as a degree of freedom for the optimizer. We want to look at the variation of the cost J_1 as function of variations in the uncontrolled outputs y_1 and variations in the inputs u_2 used for control of y_2 for a given disturbance d. *We therefore write the cost function J_1 as* $J(y_1, u_2, d)$. For a given d the optimal value of the cost function is

$$J_{\mathrm{opt}}(d) \triangleq J(y_{1,\mathrm{opt}}, u_{2,\mathrm{opt}}, d) = min_u J(u,d) \quad (3)$$

Ideally, we want $u = u_{\mathrm{opt}}(d)$. However, this will not be achieved in practice, and we select controlled outputs y_2 such that:

- *The input u_2 (generated by feedback to achieve $y_2 \approx r_2$) should be close to the optimal input $u_{2,\mathrm{opt}}(d)$.*

Note that we have assumed that r_2 is independent of d. The above statement is obvious, but it is nevertheless very useful. The following development aims at quantifying the statement.

One approach for selecting controlled variables y_2, is to select a set of variables y_2 (with set points r_2) in order to minimize the worst case deviation from the optimal value of the loss function,

$$\text{Worst case loss :}$$

$$\Phi \triangleq \max_{d\in\mathcal{D}} |J(y_1,u_2,d) - J_{opt}(d)| \quad (4)$$

where $\mathcal{D}$ is the set of all possible disturbances. As "disturbances" we should here also include changes in operating point and model uncertainty.

To obtain some insight into the problem of minimizing the loss Φ, let us consider the term $J(y_1,u_2,d) - J_{\rm opt}(d)$ in (4) for a fixed (generally non-zero) disturbance d. We make the following additional assumptions:

(d) The cost function J is twice differentiable.
(e) The optimization problem is unconstrained. If it is optimal to keep some variable at a constraint, then we assume that this is implemented and consider the remaining unconstrained problem.
(f) We only consider low frequency dynamics where feedback control is effective.

For a fixed disturbance d we express $J(y_1,u_2,d)$ in terms of a Taylor series expansion of (y_1,u_2) around the optimal point and inserting the model $\Delta y_1 = G_{12}\Delta u_2$ (only in the first order term) gives

$$J(y_1,u_2,d) - J_{opt}(d) = \underbrace{\left(\frac{\partial J}{\partial y_1}G_{12} + \frac{\partial J}{\partial u_2}\right)}_{=0}\Delta u_2 + \frac{1}{2}\left[\,\Delta y_1^T \quad \Delta u_2^T\,\right]\begin{bmatrix} J_{y_1y_1} & J_{y_1u_2} \\ J_{u_2y_1} & J_{u_2u_2}\end{bmatrix}\begin{bmatrix}\Delta y_1 \\ \Delta u_2\end{bmatrix} + \mathcal{O}^3 \quad (5)$$

where Δy_1 and Δu_2 represents deviations from the optimal values, i.e. $\Delta y_1 = y_1 - y_{1,\rm opt}$ and $\Delta u_2 = u_2 - u_{2,\rm opt}$. We have neglected terms of third order and higher (which assumes that we are reasonably close to the optimum). The first term on the right hand side in (5) is zero at the optimal point for an unconstrained problem. It is desirable that:

- The deviation of the cost from the optimal value $J(y_1,u_2,d) - J_{\rm opt}(d)$ should be as small as possible.

In order to minimize $J(y_1,u_2,d) - J_{\rm opt}(d)$ the deviations Δy_1 and Δu_2 should be as small as possible, i.e. disturbances d should have small effect on the uncontrolled outputs y_1, and inputs used for control u_2 should have sufficient power so that they can counteract the disturbances and still stay in the neighborhood of the optimal point.

Next we take into account some variations in the disturbances, which seems reasonable since the optimizer only runs periodically. By using (2) at the optimal point with u_1 constant we get

$$\Delta y_1 = G_{12}\Delta u_2 + G_{d1}d \quad (6)$$
$$\Delta y_2 = G_{22}\Delta u_2 + G_{d2}d \quad (7)$$

Assume G_{22} is invertible (if not we can use the pseudo-inverse $G_{22}^{\dagger}$) and we solve for Δu_2 in (7) to get

$$\Delta u_2 = G_{22}^{-1}(\Delta y_2 - G_{d2}d) \quad (8)$$

Inserting (8) into (6) gives

$$\Delta y_1 = \underbrace{G_{12}G_{22}^{-1}}_{P_r}\Delta y_2 + \underbrace{G_{d1} - G_{12}G_{22}^{-1}G_{d2}}_{P_d}d \quad (9)$$

REMARK. The expressions for P_d, and P_r are similar to the expressions for partial disturbance gain and partial reference gain derived for partial control (Havre and Skogestad, 1996).

Consider Δy_2 which we want to be small. However, this is not possible in practice. To see this, write

$$\begin{aligned}\Delta y_2 &= y_2 - y_{2,\rm opt} \\ &= y_2 - r_2 + r_2 - y_{2,\rm opt} = e_2 + e_{2,\rm opt}\end{aligned} \quad (10)$$

First, we have an optimization error $e_{2,\rm opt} \triangleq r_2 - y_{2,\rm opt}$, because the algorithm pre-computes a desired r_2 which is different from $y_{2,\rm opt}$. In addition, we have a control error $e_2 = y_2 - r_2$ because the control layer is not perfect, for example due to poor control performance or an incorrect measurement or estimate of y_2. If the control itself is perfect then $e_2 = n_2$ (the measurement noise). In most cases the errors e_2 and $e_{2,\rm opt}$ can be assumed independent.

Since Δy_1 is related to Δu_2 through (6) we can either summarize our results in terms of keeping Δu_2 or Δy_1 small. In order to keep $\Delta u_2 = u_2 - u_{2,\rm opt}$ small we should select the controlled outputs y_2 such that:

(1) G_{22}^{-1} is small (i.e. G_{22} is large); the choice of y_2 should be such that the inputs u_2 have large effect on y_2.
(2) $e_{2,\rm opt} = r_2 - y_{2,\rm opt}(d)$ is small; the choice of y_2 should be such that its optimal value $y_{2,\rm opt}(d)$ depends weakly on the disturbances and other changes.
(3) $e_2 = y_2 - r_2$ is small; the choice of y_2 should be such that it is easy to keep the control error e_2 small.

In order to keep $\Delta y_1 = y_1 - y_{1,\rm opt}$ we should select the controlled outputs y_2 such that:

(1) $\|P_d\|$ is small; the effect of d on y_1 is small.
(2) $\|P_r\|$ is small; the choice of y_2 should be such that the effect of r_2 on y_1 is small.

Remember that $\bar\sigma(G_{22}^{-1}) = 1/\underline{\sigma}(G_{22})$, and so we want the smallest singular value of G_{22} to be large (but recall that singular values depend on scaling as is discussed below). The desire to have $\underline{\sigma}(G_{22})$ large is consistent with our intuition that we should ensure that the controlled outputs are independent of each other. Also note that the desire to have $\underline{\sigma}(G_{22})$ large (and preferably as large as possible) is here *not* related to the issue of input constraints.

We will discuss the use of P_d and P_r to select controlled outputs y_2 in section 2.1.

Scaling. To use $\underline{\sigma}(G_{22})$ to select controlled outputs, we should scale the outputs such that the expected magnitude of $y_i - y_{i,\rm opt}$ is similar in magnitude for each output, and scale the inputs such that the effect of a given deviation $u_j - u_{j,\rm opt}$ on the cost function J is similar for each input, i.e. such that $\left(\partial^2 J/\partial u^2\right)_{\rm opt}$ is close to a constant times a unitary matrix.

We must also assume that the variations in $y_i - y_{i,\rm opt}$ are uncorrelated, or more precisely:

(g) The "worst-case" combination of output deviations $y_i - y_{i,\mathrm{opt}}$, corresponding to the direction of $\underline{\sigma}(G_{22})$, may occur in practice.

Procedure for selecting controlled outputs. The use of the minimum singular value to select controlled outputs can be summarized in the procedure:

(1) From a (nonlinear) model compute the optimal parameters (inputs and outputs) for various conditions (disturbances, operating points). This yields a "look-up" table of optimal parameter values as a function of the operating conditions.
(2) From this data obtain for each candidate output y_2, the maximum variation in its optimal value

$$v_i = (y_{i_{\mathrm{opt,max}}} - y_{i_{\mathrm{opt,min}}})/2$$

(3) Scale the candidate outputs y_2, such that for each output the sum of the magnitudes of v_i and the control error (e.g. measurement noise) is similar (e.g. about 1).
(4) Scale the inputs such that a unit deviation in each input from its optimal value has the same effect on the cost function J.
(5) Select as candidates those sets of controlled outputs which correspond to a large value of $\underline{\sigma}(G_{22})$.

REMARK 1. In the above procedure for selecting controlled outputs, based on maximizing $\underline{\sigma}(G_{22})$, the variation in $y_{2,\mathrm{opt}}(d)$ with d (which should be small) enters into the scaling of the outputs.
REMARK 2. A more exact procedure, which may be used if the optimal outputs are correlated such that assumption (g) does not hold, is:
(a) Evaluate directly the cost function J for various disturbances d and control errors e_2 by solving the nonlinear equations and assuming $y_2 = r_2 + e_2$ where r_2 is kept constant at the optimal value for the nominal disturbance.
(b) The set of controlled outputs with smallest average or worst-case value of J is then preferred.

2.1 Measurement selection for indirect control

The above ideas also apply for the case where the overall goal is to keep some variable y_1 at a given value (setpoint) r_1, e.g. $J = \|y_1 - r_1\|$. However, we cannot measure y_1, and instead we attempt to achieve our goal by controlling y_2 at some fixed value r_2, e.g. $r_2 = y_{2,\mathrm{opt}}(\bar{d})$ where $\bar{d} = 0$ if we use deviation variables. In this case we have y_1 as "primary outputs", y_2 as controlled outputs, the set u_1 is empty and $u_2 = u$. The model (2) becomes

$$y_1 = G_{12}u_2 + G_{d1}d \quad (11)$$

$$y_2 = G_{22}u_2 + G_{d2}d \quad (12)$$

By using (9) with $\Delta y_1 = y_1 - r_1$ and $\Delta y_2 = e_2$, we get the effect of d and the control error e_2 on y_1

$$y_1 - r_1 = \underbrace{(G_{d1} - G_{12}G_{22}^{-1}G_{d2})}_{P_d} d + \underbrace{G_{12}G_{22}^{-1}}_{P_r} e_2 \quad (13)$$

To minimize $\|y_1 - r_1\|$ we again have the result: *the choice of y_2 should be such that $\|P_d\|$ and $\|P_r\|$ are small.* Note that P_d only depends on the scaling of disturbances d and "primary" outputs y_1. Based on (13) a procedure for selecting controlled outputs may be suggested:

Procedure for selecting controlled outputs for indirect control. Scale the disturbances d to be of magnitude 1, and scale the outputs y_2 so that the expected control error e_2 (measurement noise) is of magnitude 1 for each output (this is different from the output scaling used in step 3 in our minimum singular value procedure). Then to minimize J we should select sets of controlled outputs which:

$$\text{Minimize } \|\,[\,P_d \quad P_r\,]\,\| \quad (14)$$

REMARK 1. The choice of norm in (14) depends on the scaling, but the choice is usually of secondary importance. The maximum singular value arises if $\|d\|_2 \leq 1$ and $\|e\|_2 \leq 1$, and we want to minimize $\|y_1 - r_1\|_2$.
REMARK 2. The above procedure does not require assumption (g) on uncorrelated variations in the optimal values of $y_i - y_{i,\mathrm{opt}}$.
REMARK 3. Of course, for the choice $y_2 = y_1$ we have that $y_{2,\mathrm{opt}} = r_1$ is independent of d and the matrix P_d in (13) is zero. However, P_r is still non-zero.
REMARK 4. In some cases this measurement selection problem involves a trade-off between wanting $\|P_d\|$ small (wanting a strong correlation between measured outputs y_2 and "primary" outputs y_1) and wanting $\|P_r\|$ small (wanting the effect of control errors (measurement noise) to be small), see Example 1.
REMARK 5. One might say that (5), (8), (9) and the resulting procedure for output selection, generalizes the use of P_d and P_r from the case of indirect control to the more general case of minimizing some cost function J.

From (11), $y_1 = r_1$ is obtained with $u_2 = u_{2,\mathrm{opt}}(d)$ where $u_{2,\mathrm{opt}}(d) = G_{12}^{-1}(r_1 - G_{d1}d)$ (replace G_{12}^{-1} with the pseudo-inverse, $G_{12}^{\dagger}$, if G_{12} is not invertible). By inserting $u_{2,\mathrm{opt}}$ into (12) the optimal output for the controlled variables y_2 are

$$y_{2,\mathrm{opt}}(d) = \underbrace{(G_{d2} - G_{22}G_{12}^{-1}G_{d1})}_{P_{y_2,d}} d + \underbrace{G_{22}G_{12}^{-1}}_{P_{y_2,r_1}} r_1 \quad (15)$$

If one consider to use the procedure involving $\underline{\sigma}(G_{22})$ for selection of outputs in the case of indirect control then (15) can generate information about scaling of y_2. The disturbances should be scaled with respect to the maximum allowed change and the reference r_1 should be normalized by including a diagonal matrix R_1 such that $r_1 = R_1\tilde{r}_1$, (15) then becomes

$$y_{2,\mathrm{opt}} = \underbrace{[\,P_{y_2,d} \quad P_{y_2,\tilde{r}_1}\,]}_{P_{y_2}} \begin{bmatrix} d \\ \tilde{r}_1 \end{bmatrix}$$

where $P_{y_2,\tilde{r}_1} = P_{y_2,r_1}R_1^{-1}$. Denote the j'th row of P_{y_2} by $[\,P_{y_2}\,]^j$. A measure on the expected change in controlled output j when including measurement noise n_j, is $s_j = \|\,[\,P_{y_2}\,]^j\,\| + n_j$. A resonable scaling factor for the controlled output j is then s_j, see Example 1.

3. EXAMPLE

EXAMPLE 1. *Selection of secondary temperature measurements in distillation control.* Indirect control of product compositions through temperature control on selected trays in distillation columns is widely used in practice.

The previous literature has focused on the benefits of using inner loops controlling the temperature at one or two selected trays with outer loops adjusting the setpoints to the temperature loops to obtain the desired product purities. In this example we will focus on the selection of the trays for temperature measurements. Related work include: Joseph and Brosilow (1978), Tolliver and McCune (1980), Yu and Luyben (1984; 1987), Moore *et al.* (1987), Mejdell (1990), Wolff (1994), Lee *et al.* (1995) and Lee and Morari (1996).

We consider a binary distillation column, LV-configuration, i.e. reflux L and boilup V is used for product composition control. The pressure in the column and the liquid holdups in the reboiler and the condenser is already controlled using condenser cooling water flow, top and bottom product flows. The model corresponds to column A studied by Skogestad and Morari (1988). The basic data are:

$\#\text{Trays}$	x_D	$1-x_B$	z_F	L/F	$M_i/F[min]$
41	0.99	0.99	0.5	2.71	0.5

The temperature difference across the column is 13.5 °C. The model includes composition and liquid flow dynamics, resulting in a 82 order model which is linearized in the operating point. For a binary mixture with constant pressure there is a direct relationship between temperature (T) and composition (x). In terms of deviation variables, $T = K_T x$, where for ideal mixtures K_T is approximately equal to the difference in pure component boiling points. Data are found in (Wolff, 1994, Chapter 4).

The objective is to keep the product compositions $y_1 = [\,x_D \quad x_B\,]^T$ at their desired values, i.e. $J = \|y_1 - r_1\|$. The secondary outputs to be considered are the temperature on all the trays, of which two shall be selected to be used for control, i.e. $y_2 = [\,T_i \quad T_j\,]^T$. This is a case of indirect control, see section 2.1. Inputs are reflux (L) and boilup (V), $u = [\,L \quad V\,]^T$. Disturbances are changes in feed flowrate (F) and feed composition (z_F), $d = [\,F \quad z_F\,]^T$. The disturbances and the product compositions have been scaled such that a magnitude of 1 corresponds to a change in F of 20%, a change in z_F of 20% and a change in x_B and y_D of 0.01 mole fraction units. The inputs u are scaled such that a magnitude of 1 corresponds to a change in u_1 and u_2 of 50%.

We consider two approaches for selecting the trays (i/j). In the first approach we maximize the smallest singular value of the subsystem G_{22} of size 2×2. In the second approach we minimize the norm $\|\,[\,P_d \quad P_r\,]\,\|_2$. We consider measurement noise of size n in both of the temperatures.

1. Maximizing $\underline{\sigma}(G_{22})$. The primary outputs ($y_1$), the disturbances ($d$) and the inputs ($u$) are scaled as described above. Since we lack data for the variations in the optimal values of the secondary outputs (y_2), we use (15) to generate the scaling factors for y_2. For each combination with two temperature measurements, we have the following effect of disturbances (d) and changes in composition setpoints (r_1) on the temperatures (y_2)

$$P_{y_2,d} = G_{d2} - G_{22}G_{12}^{-1}G_{d1}\,,\; P_{y_2,\tilde{r}_1} = G_{22}G_{12}^{-1}R_1^{-1}$$

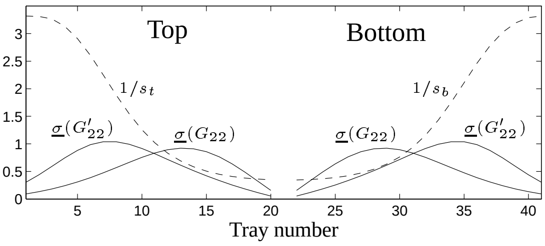

Figure 3. Effect of symmetric tray location on $\underline{\sigma}(G'_{22})$, $n = 0.3\,°\text{C}$

where $R_1^{-1} = \text{diag}\{0.01,\ 0.01\}$ such that $r_1 = R_1^{-1}\tilde{r}_1$, and $\tilde{r}_1$ is normalized in magnitude to be less than one. The combined matrix $P_{y_2} = [\,P_{y_2,d} \quad P_{y_2,\tilde{r}_1}\,]$ describes the effect of disturbances and references on the controlled outputs. Denote row j of the combined matrix with $[\,P_{y_2}\,]^j$, and compute the two scaling factors

$$s_t = \|\,[\,P_{y_2}\,]^1\,\|_2 + n, \quad s_b = \|\,[\,P_{y_2}\,]^2\,\|_2 + n$$

where n is the amount of measurement noise in the temperatures. The subscripts t and b stands for *top* and *bottom*. The scalings of the outputs y_2 is then taken to be $D_{y_2} = \text{diag}\{1/s_t, 1/s_b\}$, i.e. $G'_{22} = D_{y_2}G_{22}$ where G_{22} is the lower part of the model and G'_{22} is the corresponding rescaled model using the scalers s_t and s_b. Figure 3 show $\underline{\sigma}(G_{22})$, $\underline{\sigma}(G'_{22})$, s_t and s_b for $n = 0.3\,°\text{C}$ with temperature measurements symmetric around the feed tray, i.e. two temperature measurements with equal distance from the feed tray (one above and one below the feed tray). The curve $\underline{\sigma}(G'_{22})$ in Figure 3 indicates that the optimal tray combination is $8/34$. Note that if rescaling is left out, curve $\underline{\sigma}(G_{22})$ in Figure 3, the result is far from tray combination $8/34$. So, it is important to scale the secondary outputs y_2 properly when using this selection procedure. When considering all $\binom{41}{2} = 820$, we find that tray combination $7/34$ maximizes $\underline{\sigma}(G'_{22})$ when $n = 0.3\,°\text{C}$. The upper part of Table 1 summarizes our results for different levels of measurement noise. From the table

TABLE 1: Optimal tray combinations for different noise levels.

Measurement noise, n [°C]		0.1	0.3	0.7	1.0
$\underline{\sigma}(G'_{22})$	Sym [a]	$5/37$	$8/34$	$9/33$	$10/32$
	All [b]	$5/37$	$7/34$	$9/33$	$10/32$
$\|\,[\,P_d \quad P_r\,]\,\|_2$	Sym [a]	$5/37$	$8/34$	$10/32$	$11/31$
	All [b]	$5/37$	$7/34$	$9/32$	$11/31$

[a] Tray combinations symmetric around the feed tray are considered.
[b] All 820 tray combinations are considered.

we see, as expected, that the optimal location for temperature measurement is closer to the column ends with decreasing measurement noise.

2. Indirect control, minimizing $\|[\,P_d \quad P_r\,]\|$. In this case we consider to select outputs which minimize $\|[\,P_d \quad P_r\,]\|_2$. Both P_d and P_r depends on output scaling (y_1), P_d depends on input scaling (d) and P_r on e_2, which represents the control error in the secondary outputs, which for perfect steady-state control, is equal to the measurement noise n. The primary outputs, the disturbances and the inputs are scaled as described above. The secondary outputs are scaled relative to the noise n. The results for the tray combinations symmetric around the feed tray, are

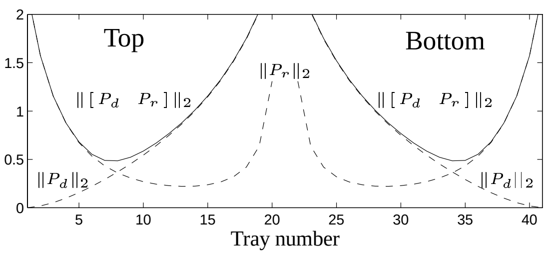

Figure 4. Effect of sym. tray location on $\|\,[P_d\ P_r]\,\|_2$, $n = 0.3\,^\circ\mathrm{C}$

shown in Figure 4. If we have zero control error and perfect temperature measurements ($n = 0$), then it is optimal to measure the temperature at the ends of the column, see lines for $\|P_d\|_2$ in Figure 4. To be practical, we need to consider some measurement noise, perfect control can easily be achieved at steady-state using integral action in the loops. The effect of noise in the temperature measurements on the primary outputs is given by the line $\|P_r\|_2$ in Figure 4. Measuring to close to the column ends yields a finite non-zero $\|P_r\|_2$ (because changes in temperature imply changes in composition) and measuring close to the feed trays yields strong interactions in G_{22}. This describes the characteristic shape of $\|P_r\|_2$. The combined effect of the disturbance and the control error due to measurement error, is given by $\|\,[\,P_d \quad P_r\,]\,\|_2$ in Figure 4. When considering all possible combinations we find that tray combination $7/34$ minimize $\|\,[\,P_d \quad P_r\,]\,\|_2$ when $n = 0.3\,^\circ\mathrm{C}$, which is equal to what we obtained for $\underline{\sigma}(G_{22})$. The lower part of Table 1 gives the results for the other noise levels.

In summary, we see that the two approaches yield similar results. Increasing the amount of measurement noise (control error), moves the measurements towards the middle of the column. We also see that the optimal locations for temperature measurements are close to the best locations obtained when considering only the tray combinations symmetric around the feed tray. This does not apply in general but is merely a result of requiring equal product purities and feed composition $z_F = 0.5$. Tray combination $7/34$ compares well with (Lee and Morari, 1996) who found the choice $7/35$ to be the best, however they only considered 15 possible combinations of two temperatures.

4. SUMMARY

Generally, the optimal values of all variables will change with time during operation (due to disturbances and other changes). For practical reasons, we have considered a hierarchical strategy where the optimization is performed only periodically. The question is then:

- Which variables (*controlled outputs*) should be kept constant (between each optimization)?

Essentially, we found that we should select variables y_2 for which the variation in optimal value and control error is small compared to their controllable range (the range y_2 may reach by varying the input u_2). This is hardly a big surprise, but it is nevertheless useful and provides the basis for our procedure for selecting controlled outputs.

The objective of the control layer is then to keep the controlled outputs at their reference values (which are computed by the optimization layer). The controlled outputs are often measured, but we may also estimate their values based on other measured variables. We may also use other measurements to improve the control of the controlled outputs, for example, by use of cascade control. Thus, the selection of controlled and measured outputs are two separate issues, although the two decisions are obviously related. The measurement selection problem is discussed in (Havre and Skogestad, 1996).

5. REFERENCES

Havre, K. and S. Skogestad (1996). Input/output selection and partial control. In: *Proc. from 13th IFAC World Congress*. San Francisco, USA.

Joseph, B. and C. B. Brosilow (1978). Inferential control of processes - I: Steady state analysis and design. *AIChE Journal* **24**, 485–492.

Kim, Y. H., T. W. Ham and J. B. Kim (1991). On-line dynamic optimizing control of a binary distillation column. *J. Chem. Eng. of Japan* **24**(1), 51–57.

Lee, J. H. and M. Morari (1996). Control structure selection and robust control system design for a high-purity distillation column. *IEEE Trans. on Cont. Sys. Tech.*, In print.

Lee, J. H., R. D. Braatz, M. Morari and A. Packard (1995). Screening tools for robust control structure selection. *Automatica* **31**(2), 229–235.

Maarleveld, A. and J. E. Rijnsdorp (1970). Constraint control on distillation columns. *Automatica* **6**, 51–58.

Mejdell, T. (1990). Estimators for Product Composition in Distillation Columns. PhD thesis. Norwegian University of Science and Technology, Trondheim.

Moore, C., J. Hackney and D. Canter (1987). Selecting sensor location and type for multivariable processes. *Shell Process Control Workshop (book), Butterworth*.

Morari, M., Y. Arkun and G. Stephanopoulos (1980). Studies in the synthesis of control structures for chemical process, Part I: Formulation of the problem. Process decomposition and the classification of the control tasks. Analysis of the optimizing control structures. *AIChE Journal* **26**(2), 220–232.

Morud, J. (1995). Studies on the Dynamics and Operation of Integrated Plants. PhD thesis. Norwegian University of Science and Technology, Trondheim.

Skogestad, S. and I. Postlethwaite (1996). *Multivariable Feedback Control, Analysis and Design*. John Wiley & Sons. Chichester.

Skogestad, S. and M. Morari (1988). LV-control of a high-purity distillation column. *Chemical Engineering Science* **43**(1), 33–48.

Tolliver, T. L. and L. C. McCune (1980). Finding the optimum temperature control trays for distillation columns. *InTech*.

Tyreus, B. D. (1987). Optimization and multivariable control of distillation columns. *Adv. Instrum.* **42**, 25–44.

Wolff, E. (1994). Studies on Control of Integrated Plants. PhD thesis. Norwegian University of Science and Technology, Trondheim.

Yu, C. C. and W. L. Luyben (1984). Use of multiple temperatures for the control of multicomponent distillation columns. *Ind. Eng. Chem. Process Des. Dev.* pp. 590–597.

Yu, C. C. and W. L. Luyben (1987). Control of multicomponent distillation columns using rigourous compositon estimators. *I. Cheme. E. Symp. Ser.*