

Multiple Steady States and Instability in Distillation. Implications for Operation and Control†

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The fact that distillation columns, even in the ideal binary case, may display multiple steady states and unstable operating points has only recently been recognized. This article addresses some implications of these phenomena for the operation and control of distillation columns. Under manual operation, the multiplicity and instability will result in inability to reach separations corresponding to unstable operating points and may furthermore cause abrupt changes and hysteresis in operating conditions. It is shown that an unstable operating point may be stabilized by feedback control of a single product composition or tray temperature (one-point control). The steady-state multiplicity does, in this case, not represent any severe limitation in operation, but if the control is not sufficiently tight, the column may settle in sustained oscillations (stable limit cycle). Finally, the impact of open-loop instability on the achievable closed-loop performance with both product compositions under feedback control is discussed.

1. Introduction

Distillation is undoubtedly the most studied unit operation in the process control literature. Apart from the industrial importance of distillation, this is probably due to the fact that distillation columns possess several inherent properties, e.g., strong interactions (ill-conditioning), sluggish responses, and strong nonlinearities, that make tight control of the product compositions a challenging task. However, with level and pressure loops closed, all published work so far have assumed the columns to be open-loop stable. This is mainly a result of the fact that most authors employ dynamic models with the common assumptions of inputs, e.g., reflux and boilup, given on a *molar* rate basis (i.e., in kmol/min) as well as neglected energy balance (constant molar flows). Rosenbrock (1960, 1962) analyses a model with these assumptions and shows that, in the binary case, the steady state is always unique and asymptotically stable. Also, many other authors have claimed that the operating points of distillation columns are always unique and asymptotically stable in the binary case (see e.g., Doherty and Perkins, 1982, and references therein).

The reason for the widespread assumption of inputs given on a molar rate basis is probably that it is the molar flow rates that naturally enter in the mathematical model of a distillation column and, furthermore, directly influence the separation (e.g., recall McCabe—Thiele). However, in a recent article, we argued that almost never will distillation columns have all their inputs given on a molar rate basis (Jacobsen and Skogestad, 1991). For instance, fixing the valve position for a liquid flow corresponds closely to fixing the geometric average of volumetric and mass flow. The transformation from mass or volumetric flow rates to molar flow rates is nonlinear and may in certain cases

become singular, resulting in multiple steady-state solutions. Furthermore, if the energy balance is included in the model, even molar inputs may yield multiple steady-state solutions (Jacobsen and Skogestad, 1991). The latter result shows that previous claims in the literature that the energy balance is of little importance for distillation dynamics (e.g., Rademaker et al., 1975) are incorrect in many cases.

Both types of multiplicities referred to above correspond to *output multiplicity* in which a given set of inputs, e.g., reflux and boilup, yields several solutions in terms of the outputs, e.g., product compositions. A different type of multiplicity, *input multiplicity*, that also may occur in distillation is discussed in Jacobsen (1993) and is not treated here. When we talk about multiplicity and multiple steady states in the following, we always refer to output multiplicity unless otherwise stated.

In Jacobsen and Skogestad (1994), the stability of distillation columns is studied and it is shown that columns which display multiple steady states will have at least one solution that corresponds to an unstable operating point. It is also shown that, for a given column, instability is most likely with large internal flows, i.e., large reflux and boilup.

Jacobsen and Skogestad (1991, 1994) show that the existence of multiplicity and instability in general will depend on the specific choice of independent inputs. A two-product distillation column has many inputs (flows) that may be manipulated. Assuming given feed conditions, these are typically the product flow rates D_w and B_w , the reflux L_w (the index w denotes mass basis), the boilup V (indirectly adjusted with reboiler heat input Q_B), and the condensation rate V_T (indirectly adjusted with condenser heat removal Q_D) (see Figure 1). However, all these flows may not be specified independently. A typical two-product distillation column has 2 degrees of freedom at steady state (still assuming given feed conditions), and thus, only two of the five flows may be specified independently at steady state. From a control point of view, this may be understood from the fact that the column pressure and the liquid levels of the reboiler

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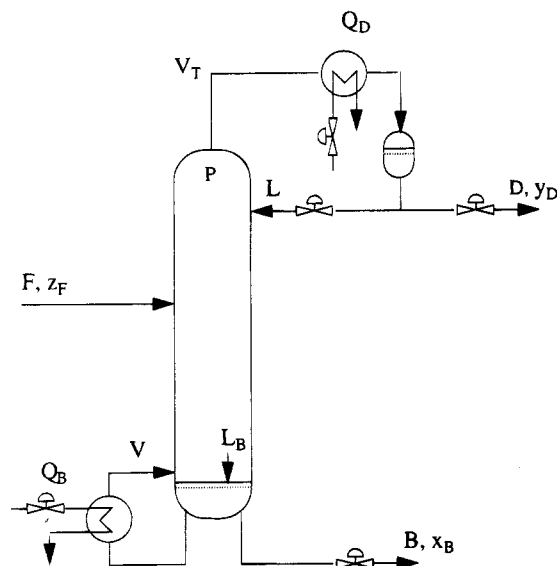


Figure 1. Two-product distillation column.

and condenser need to be controlled, and three inputs must be used for this purpose. Typically, the condensation rate V_T is used to control the pressure, and the product flows D_w and B_w are used to control the condenser and reboiler levels, respectively. This leaves reflux L_w and boilup V as the independent inputs. In control terms, this is denoted by the L_wV configuration and is probably the most widespread configuration in industry. Jacobsen and Skogestad (1991, 1994) also find that this is the configuration for which steady-state multiplicity and instability are most likely to occur. However, many other configurations are possible, and the choice of a proper configuration for composition control has been studied extensively in the literature (see e.g., Skogestad et al., 1990). The configurations considered in this paper are the L_wV configuration, where L_w [kg/min] and V [kmol/min] are used for quality control; the LV configuration, where L [kmol/min] and V [kmol/min] are used for quality control; and the D_wV configuration, where D_w [kg/min] and V [kmol/min] are used for quality control. As noted above, the LV configuration is primarily of theoretical interest because liquid flows only in rare cases may be manipulated on a molar rate basis. The L_wV and LV configurations differ if the chemical components in the distillate have different molecular weights.

Our two previous articles (Jacobsen and Skogestad, 1991, 1994) present fundamental results regarding the existence of multiple steady states and unstable operating points. In this article, we discuss some of the practical implications for operation and control. While multiplicity and instability most certainly have been experienced during industrial operation, it has not been properly understood. The aim of the present article is to explain which effects may be caused by steady-state multiplicity and instability in distillation.

We start the article by presenting an example column that will be used for illustration throughout the article. The column operates with the L_wV configuration and is shown to display multiple steady-state solutions and unstable operating points. We first discuss how the multiplicity and instability may affect the column behavior when the independent inputs are manipulated by an operator, i.e., manual operation. We then show that an unstable operating point usually may be stabilized by feedback control of a single product composition or tray temperature. Proper distillation control involves feedback control of both product compositions, and we

Table 1. Data for the Methanol-Propanol Column

z_F	F	α	N	N_F	M_1	M_2
0.50	1	3.55	8	4	32.0	60.1

feed is saturated liquid
 total condenser with saturated reflux
 liquid holdups are $M_L/F = 0.5$ min, including
 reboiler and condenser
 in Figure 11, $M_B/F = M_D/F = 2.0$ min
 liquid flow dynamics: $L_i = L_{i0} + (M_{L_i} - M_{L_{i0}})/\tau_{L_i}$, $\tau_{L_i} = M_{L_{i0}}/3L_{i0}$
 constant pressure $P = 1$ atom
 boiling point temp: $T_i^{BP} = 97.177 - 54.54x_i + 23.2x_i^2$

discuss how open-loop instability affects the achievable control performance for this case. Instability may exist also with other configurations, e.g., the D_wV configuration, and the possible implications of this type of instability are briefly discussed. Finally, we provide suggestions as to how potential open-loop instability may be detected in a column under operation.

We discuss only ideal binary separations in this article because this represents the simplest and most fundamental case. However, extending the results to the case of ideal multicomponent distillation is trivial (Jacobsen and Skogestad, 1994).

2. Example Column

Data for a methanol-propanol column (Jacobsen and Skogestad, 1991, 1994) are given in Table 1. We will use this column for illustration throughout the article. In most of the article, we assume that the column is operated with reflux L_w and boilup V as independent inputs, i.e., L_wV configuration. At the end of the article, we discuss briefly the use of other configurations, e.g., the D_wV configuration.

As stated in the Introduction, the flows, in particular the liquid flows, are more likely to be fixed on a mass or volume basis rather than on a molar rate basis. In this article, we assume that the reflux flow and the two product flows, which usually are liquid flows, are specified on a mass rate basis (L_w , D_w , and B_w). The boilup V is usually determined indirectly by the heat input to the reboiler Q_B , and we have $V \approx Q_B/\Delta H^{vap}$. Here the molar heat of vaporization ΔH^{vap} often depends only weakly on composition, and hence, specifying Q_B corresponds closely to specifying molar boilup V . In this article, we will therefore assume that the boilup may be specified on a molar rate basis. Jacobsen and Skogestad (1991, 1994) provide a more in-depth discussion on the effect of specifying the heat input Q_B instead of the molar boilup V .

Throughout the article, we employ a nonlinear dynamic model with the following assumptions: (1) constant relative volatility (ideal VLE); (2) constant molar flows (neglected energy balance) (note that the energy balance may act as an additional source of multiplicity and instability if included in the model (Jacobsen and Skogestad, 1991, 1994); this multiplicity and instability will have similar implications for operation and control to that caused by using mass flows); (3) varying liquid holdup (the tray hydraulics are described by a linearized Francis weir formula assuming half the liquid over weir); (4) negligible vapor holdup; (5) constant pressure. With these assumptions, we obtain a dynamic model with two states per tray; total liquid holdup and fraction of light component. In parts of the article, we employ linear dynamic models which are obtained by linearizing the nonlinear model around the operating point under discussion.

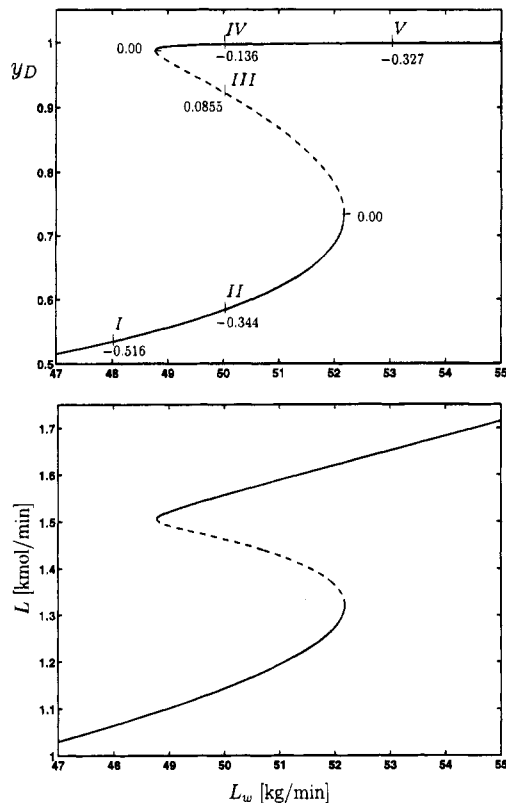


Figure 2. Steady-state solutions as a function of mass reflux L_w for the methanol–propanol column with L_wV configuration. On the upper plot, the corresponding maximum eigenvalue is shown at some of the steady-state solutions. Boilup $V = 2.0$ kmol/min.

Table 2. Steady-State Solutions for the Methanol–Propanol Column with $V = 2.0$ kmol/min, L_w in the Range 48–53 kg/min, and Constant Molar Flows (No Energy Balance)

	L , kmol/min	D , kmol/min	L_w , kg/min	y_D	x_B
I	1.064	0.936	48.00	0.5339	3.087×10^{-3}
II	1.143	0.857	50.00	0.5828	3.533×10^{-3}
III	1.462	0.538	50.00	0.9234	7.793×10^{-3}
IV	1.556	0.444	50.00	0.9969	0.1038
V	1.650	0.350	53.00	0.9984	0.2339

3. Steady-State Multiplicity and Instability

Figure 2 and Table 2 show steady-state solutions for the methanol–propanol column for different values of mass reflux L_w with boilup V fixed at 2.0 kmol/min. From Figure 2, we see that, with boilup fixed at this value, there are three steady-state solutions in terms of the product compositions for mass reflux in the range 48.8–52.2 kg/min. For example, for $L_w = 50.0$ kg/min and $V = 2.0$ kmol/min, we obtain the three solutions II, III, and IV in Table 2. Note from Table 2 that the solutions II, III, and IV have different values of the molar reflux L such that specifying the molar flows L and V would yield unique solutions for the product compositions.

Indeed, as shown in Jacobsen and Skogestad (1991), the multiplicity is in this case caused by a multiplicity between the mass reflux L_w , which is the manipulated input, and the molar reflux L , which determines separation. For a binary mixture, we have

$$L = L_w/M \quad M = y_D M_1 + (1 - y_D) M_2 \quad (1)$$

where M_1 and M_2 denote the molecular weight of the light and heavy component, respectively. Differentiating (1) with respect to L yields

$$\left(\frac{\partial L_w}{\partial L}\right)_V = M + L(M_1 - M_2)\left(\frac{\partial y_D}{\partial L}\right)_V \quad (2)$$

The transformation from L_w to L is singular when $(\partial L_w / \partial L)_V = 0$, or equivalently $(\partial L / \partial L_w)_V = \infty$, and corresponds to a limit point around which there locally exist two steady-state solutions. For further details and a discussion on operating conditions that favor multiple steady states, we refer the reader to Jacobsen and Skogestad (1991).

The maximum eigenvalue (pole), obtained from local linearizations of the full nonlinear dynamic model, are also shown at selected operating points in Figure 2. From the figure, we see that solutions with a negative slope between reflux L_w and top composition y_D corresponds to unstable operating points with poles in the right half plane (RHP). The eigenvalues at the singular points are zero, as expected. Note that the stability with the L_wV configuration is independent of the tuning of the pressure and level control loops (Jacobsen and Skogestad, 1994). For proof of the instability, we refer to Jacobsen and Skogestad (1994).

The rest of the article is devoted to a discussion on how the observed steady-state multiplicity and instability may affect the operation and control of distillation columns.

4. Manual Operation

Many industrial columns are operated manually in the sense that only the reboiler and condenser levels and column pressure are under feedback control while an operator adjusts the remaining two independent inputs to keep the product compositions close to some specified values. The multiplicity and instability presented above will have several implications for this case. First, it is very difficult to obtain product compositions corresponding to unstable operating points by manual manipulation of reflux and boilup. The reason is that stabilization requires consistent and reasonably fast feedback which the operator is unlikely to provide. Second, one should expect the operating conditions to change drastically as a manipulated variable is taken past a singular (turning) point. Finally, the fact that there are two stable operating points for certain values of the inputs implies that the product compositions will depend not only the current values of the manipulated inputs but also on the past history of operating conditions. This is, hysteresis may be experienced in operation.

We illustrate the potential difficulties involved in the manual operation of a column with multiple steady states and instability through nonlinear simulations of the methanol–propanol column. The desired operating point is operating point III in Table 2, i.e., $y_D = 0.9234$ and $x_B = 0.0078$, which is unstable with the L_wV configuration. Assume that the required reflux and boilup have been computed by means of some steady-state simulator (note that the inputs are uniquely determined when y_D and x_B are specified). Thus, the column is started up with

$$L_w = 50.0 \text{ kg/min and } V = 2.0 \text{ kmol/min}$$

However, according to Table 2, there are three possible steady-state solutions for these values of the inputs. Only two of these are stable, namely, operating points II and IV. Assume that the column initially settles at operating point II with $y_D = 0.5828$ and $x_B = 0.0035$. Because the top product is too unpure and the bottom

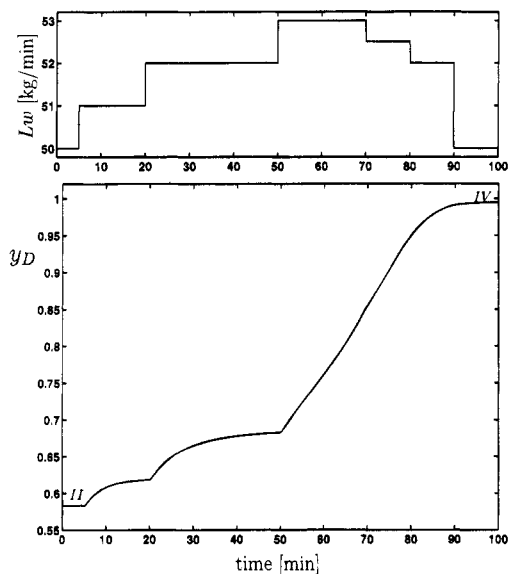


Figure 3. Nonlinear open-loop response of the methanol-propanol column to step changes in mass reflux L_w . Boilup $V = 2.0$ kmol/min.

product too pure, the operator decides to increase the reflux L_w in a stepwise fashion. This is illustrated in Figure 3 together with the response in top composition y_D . In the beginning, the top composition increases slightly with increasing reflux, as expected. However, as the operator increases the reflux from 52.0 to 53.0 kg/min (at $t = 50$ min), the top composition starts to increase drastically. The reason is that the reflux has been increased past the lower singular point in Figure 2 and the column goes through what is known as a catastrophic jump (see e.g., Poston and Stewart, 1978). The operator observes that the top product has become too pure and the bottom product too impure and decides to reduce reflux first to 52.5 kg/min (at $t = 70$ min) and then to 52.0 kg/min (at $t = 80$ min). However, as seen from Figure 3, this does not have the desired effect, and the operator finally decides to reduce the reflux all the way to the initial value of 50.0 kg/min. However, due to the steady-state multiplicity, the column now settles at operating point IV with $y_D = 0.9969$ and $x_B = 0.1038$. Thus, the operator is unable to reach the desired operating point.

Indeed, for the methanol-propanol column, it is almost impossible to obtain purities in the top product y_D in the range 0.733–0.987 by manual manipulation of the reflux L_w while keeping V constant at 2.0 kmol/min (see Figure 2). Similarly, it is almost impossible to obtain purities in the bottom product $1 - x_B$ in the range 0.9765–0.9948 by manipulation of boilup V while keeping L_w fixed at 50.0 kg/min.

In conclusion, the simulations in Figure 3 illustrate three different effects that may be observed under manual operation of columns with multiple steady states: (1) inability to reach separations corresponding to unstable operating points, (2) catastrophic jumps as the column goes through a singular point, and (3) hysteresis in operation.

Note that, in some columns, either one or both of the stable solution branches do not completely overlap the unstable branch (see, e.g., Figure 8 in Jacobsen and Skogestad (1991)). In this case, the column is likely to go globally unstable at some point in the sense that the reboiler (missing lower branch) or condenser (missing upper branch) runs dry. The reason is that there exists no stable solution as the column is taken past one of the two singular points.

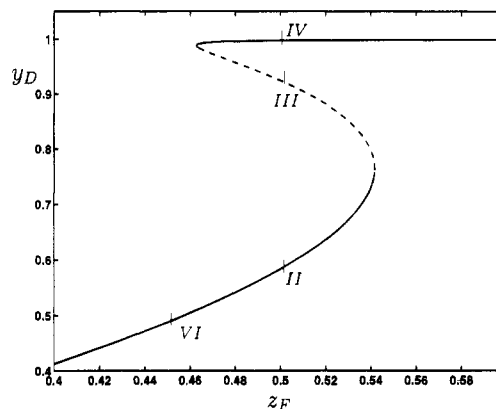


Figure 4. Steady-state solutions as a function of feed composition z_F for the methanol-propanol column. The dashed line corresponds to unstable operating points. Reflux $L_w = 50$ kg/min, boilup $V = 2.0$ kmol/min. Roman numbers II–IV refer to Table 2 with $z_F = 0.5$ and VI to $z_F = 0.45$.

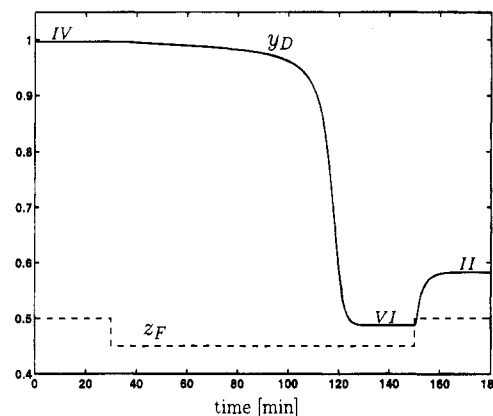


Figure 5. Nonlinear open-loop response of the methanol-propanol column to disturbances in feed composition z_F . Reflux $L_w = 50$ kg/min and boilup $V = 2.0$ kmol/min. Roman numbers II–IV refer to Table 2 with $z_F = 0.50$. Operating point VI corresponds to $z_F = 0.45$.

Effect of Disturbances. Jacobsen and Skogestad (1991, 1994) discuss multiple steady states and instability with respect to the manipulated inputs only. However, when there are multiple steady states with respect to the manipulated inputs, then there will also be multiple steady states with respect to other independent parameters like feed flow rate, feed composition, feed liquid fraction, tray efficiency, etc. (often denoted *bifurcation parameters* in the nonlinear dynamics literature). All these parameters will vary to some extent during operation and may, similarly to the inputs, cause the column operation to go from open-loop stable to open-loop unstable.

To illustrate this, consider Figure 4, which shows steady-state solutions for the methanol-propanol column with $L_w = 50.0$ kg/min, $V = 2.0$ kmol/min, and feed composition z_F in the range 0.40–0.60. From the figure, we see that there are multiple steady-state solutions in terms of the product compositions for z_F in the range 0.46–0.54. This implies that disturbances in the feed composition may take the column through a singular point and thus cause a sudden “jump” to another solution branch. This is illustrated in Figure 5, which shows the response in the top composition y_D to a change in the feed composition z_F from 0.50 to 0.45 (assuming that the column initially is at operating point IV in Table 2). The simulation illustrates how the disturbance causes the top composition to “jump” to the lower solution branch and settle at operating point VI (see Figure 4). When the feed concentration returns to z_F

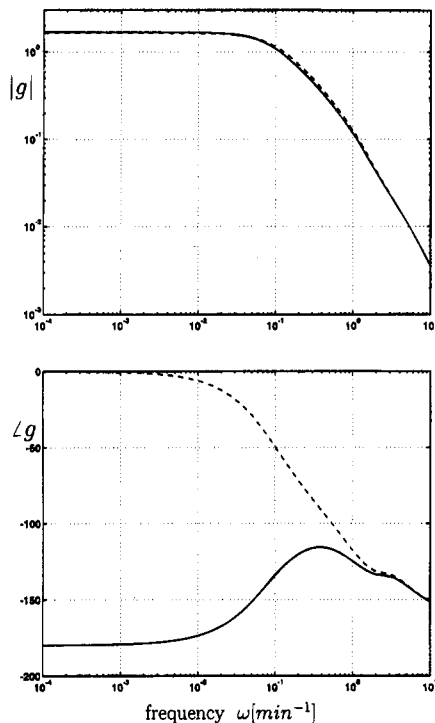


Figure 6. Frequency response for the transfer function from reflux to top composition y_D with boilup V constant for operating point III of the methanol-propanol column. Solid line: $(\partial y_D / \partial L_w)_V(s)$. Dashed line: $(\partial y_D / \partial L)_V(s)$. The magnitude of $(\partial y_D / \partial L_w)_V(s)$ is scaled by the molecular weight M^* .

= 0.50, the product composition remains on the lower solution branch in Figure 4 and settles at operating point II.

5. Operation with Feedback Control

We have seen that manual operation of distillation columns with multiple steady states is a very difficult task. In this section, we consider feedback control of unstable distillation columns and show that an open-loop unstable operating point may be stabilized using feedback control of a single product composition or tray temperature. We also discuss to what extent the existence of an open-loop unstable pole influences the achievable closed-loop performance. The latter is important, as the existing literature on distillation control assumes open-loop stable models. However, before we go on to discuss the application of feedback control, we need to understand how the use of mass reflux L_w rather than molar reflux L affects the overall dynamics, apart from the stability, of a column.

5.1. Overall Dynamics. The analysis presented in Jacobsen and Skogestad (1994) shows that the dominant pole (corresponding to the largest time constant), and thus the low-frequency dynamics, is strongly influenced by the transformation between mass and molar reflux. However, Jacobsen and Skogestad (1994) do not discuss the effect of the transformation on the higher frequency dynamics which are important for feedback control.

Figure 6 shows the magnitude and phase of the transfer functions for both molar and mass reflux at operating point III of the methanol-propanol column. Note that, in order to get comparable units, the magnitude for mass reflux L_w [kg/min] is multiplied by the molecular weight $M^* = y_B^* M_1 + (1 - y_B^*) M_2$, where superscript * denotes a nominal value. The most notable thing to observe from Figure 6 is that the phases of the two transfer functions differ by 180° at low frequencies. This is as expected since the transfer

function for L_w is unstable. However, the phases approach each other at intermediate frequencies and become identical at high frequencies. The magnitudes of the two transfer functions are also almost identical at intermediate and high frequencies. The fact that they are almost identical also at low frequencies is a coincidence for this operating point, because the dominant poles happen to be of similar magnitude (-0.0986 and 0.0855 min^{-1}). However, the dynamics of the two systems differ since the phases differ. In general, one will find that the magnitudes, similar to the phases, differ at low frequencies and become almost identical at high frequencies.

The main conclusion to draw from Figure 6 is that it is mainly the low-frequency dynamics corresponding to the dominant pole that is influenced by the transformation between mass and molar reflux. The initial response (high-frequency dynamics) is almost unaffected. To explain this, consider the relation between the magnitude of two transfer functions

$$\left| \left(\frac{\partial y_D}{\partial L} \right)_V(j\omega) \right| = \left| \left(\frac{\partial y_D}{\partial L_w} \right)_V(j\omega) \right| \left| \left(\frac{\partial L_w}{\partial L} \right)_V(j\omega) \right| \quad (3)$$

Differentiating the transformation $L_w = LM$ with respect to L yields

$$\left| \left(\frac{\partial L_w}{\partial L} \right)_V(j\omega) \right| = |M^*| + |L^*(M_1 - M_2)| \left| \left(\frac{\partial y_D}{\partial L} \right)_V(j\omega) \right| \quad (4)$$

Thus, at high frequencies, where the magnitude of $(\partial y_D / \partial L)_V(j\omega)$ is small, the main difference between the magnitude of the two transfer functions is simply the factor $|M^*|$. The magnitudes of the two transfer functions are significantly different only at low frequencies, where the magnitude of $(\partial y_D / \partial L)_V(j\omega)$ is large. Similar arguments apply also to the phases of the two transfer functions.

Note that also the transfer function zeros will be affected by the transformation between mass and molar flows. This is not discussed here, but Jacobsen (1993) finds that the transformation even may cause nonminimum-phase (RHP) zeros in the transfer function $(\partial x_B / \partial V)_{L_w}(s)$, i.e., from boilup V to bottoms composition x_B . However, for most columns, including the methanol-propanol column discussed in this article, the effect of the transformation on the transfer function zeros is relatively small.

5.2. Performance Limitations Imposed by RHP Poles. The analysis presented above shows that the main difference between a model with molar reflux and a model with mass reflux is the location of the dominant pole, which may even be unstable in the latter case. When a column is open-loop unstable, feedback control (in addition to level and pressure control) is required for stabilization. From control theory, it is well-known that RHP poles alone do not impose any upper limitation on the bandwidth of the closed-loop system; on the contrary, they impose a lower limit on the allowable bandwidth. Problems with respect to stabilization will therefore only arise if there simultaneously are upper bandwidth limitations (e.g., RHP zeros or delays) at frequencies comparable to the RHP pole ("the system goes unstable before we are able to do anything about it") or if there are input constraints ("we do not have sufficient power to counteract the instability").

Although unstable systems usually may be stabilized by feedback control, the existence of a RHP pole in the open-loop system does have some impact on the achiev-

able control performance. Here we consider scalar systems, but similar results, although more limited, are obtained for multivariable systems if one considers the maximum singular value of the sensitivity function $\bar{\sigma}(S)$ instead of $|S|$.

Consider the sensitivity function $S = (1 + gc)^{-1}$ of a closed-loop system. Here $S = (r - y)/r = y/d$, where y is the controlled output, r is the reference signal, and d is a disturbance acting on the output y . Note that the feedback control improves the system response for frequencies where $|S(j\omega)| < 1$ but deteriorates the system response where $|S(j\omega)| > 1$. Ideally, we want $S = 0$ at all frequencies, that is, perfect following of reference signal and complete disturbance rejection. However, for all real systems, $|S(j\omega)| = 1$ at high frequencies. In addition, for an open-loop system with a pole excess of at least two (satisfied for any real system) and a single real RHP pole p , the following constraint applies to $|S(j\omega)|$ (e.g., Freudenberg and Looze, 1985)

$$\int_0^{\infty} \log|S(j\omega)| d\omega = \pi p \quad (5)$$

(With no RHP pole, $p = 0$, eq 5 reduces to the well-known *Bode Integral*.) From (5), we see that we need a frequency range with $|S| > 1$ and that the presence of a RHP pole increases the area where $|S| > 1$. However, (5) does not impose any practical design limitation, as the area for $|S| > 1$ may be smoothed out over an arbitrarily large frequency range, and the peak of S may accordingly be made arbitrarily small. Thus, the RHP pole will not represent a control limitation if there are no other bandwidth constraints present in the system.

For an open-loop system with a real RHP pole p and a real RHP zero z , the following constraint applies (Freudenberg and Looze, 1985, 1988)

$$\int_0^{\infty} \log|S(j\omega)| W(z, \omega) d\omega = \pi \log \left| \frac{p+z}{p-z} \right| \quad (6)$$

(With no RHP pole, $p = 0$, the integral equals zero.) The weight W is given by

$$W(z, \omega) = \frac{2z}{z^2 + \omega^2} \quad (7)$$

The form of W (asymptotically it equals $2/z$ for frequencies up to $\omega = z$, where it cuts off with a -2 slope) implies in most practical cases that essentially all the area for $|S| > 1$ has to be at frequencies lower than z , and the sensitivity function must have a peak $|S| > 1$ at $\omega < z$. The peak will have to become increasingly large as the bandwidth frequency (where $|S|$ first reaches 1) approaches z . From (6), we also see that, as the RHP zero approaches the RHP pole, the peak goes to infinity.

Note that the right-hand side of (6) is symmetric in the sense that the same area is obtained regardless of whether p is to the left or to the right of z in the RHP. Thus, it may seem that the control problem is similar for the two cases. However, this is somewhat misleading. If a system has a single RHP pole p , a single RHP zero z , and $p > z$, then stabilization is impossible using a linear stable controller (Youla et al., 1974). Thus, in practice, we must require $p < z$ in order to be able to stabilize an open-loop unstable plant. With a restricted structure of the controller, e.g., a PI controller, we must require the RHP pole to be some distance inside the RHP zero in order to be able to stabilize the column.

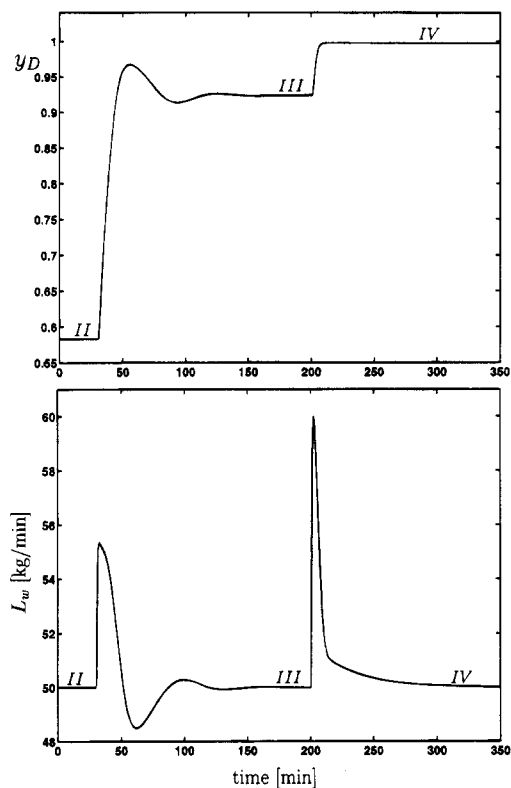


Figure 7. Nonlinear simulation of the methanol-propanol column with PI control of top composition y_D using mass reflux L_w . Setpoint changes from operating point II to III and from III to IV. Simulations includes a 1-min measurement delay. Controller parameters: $K_c = 3.0$ and $\tau_I = 20.0$ min. Gain is for logarithmic composition, i.e., $\log(1 - y_D)$. Boilup $V = 2.0$ kmol/min.

For the distillation column, $p = \lambda_{\max}$ and "RHP zeros" are most likely caused by delays in measurements and actuators. Using a first-order Padé approximation for a delay θ_d results in a RHP zero at $z = 2/\theta_d$. We must then approximately require $p > 2/\theta_d$, or equivalently $\theta_d < 2/p$, in order to be able to stabilize the system.

5.3. One-Point Composition Control. Tight control of distillation columns requires feedback control of both product compositions (two-point control). However, in order to simply stabilize an open-loop unstable column, one-point control will suffice. This is also the way most industrial columns with composition control are operated. An unstable column operating with the L_wV configuration may be stabilized by controlling either top or bottom composition or any other variable related to composition, e.g., a temperature on any tray inside the column. The analysis presented above for SISO systems then applies.

For operating point III of the methanol-propanol column, the RHP pole is at $p = 0.0855 \text{ min}^{-1}$ and it will be impossible to stabilize the column if the delay exceeds $2/p = 23$ min. With a simple controller, stabilization is even more difficult. Indeed, by optimizing the parameters of a PI controller such that the pole with the largest real value is minimized, we find that we are unable to stabilize the column if the deadtime θ_d exceeds 11 min. Composition measurements in industrial columns (GC analysis) may typically have deadtimes up to 30 min, and one should then use faster temperature measurements in order to stabilize the column.

Nonlinear Simulations. Figure 7 shows nonlinear simulations of the methanol-propanol column using a single-loop PI controller (the tuning $K_c = 3.0$ and $\tau_I = 20.0$, $C(s) = K_c(1 + 1/\tau_I s)$, was selected to yield reasonably fast response; note that Ziegler-Nichols tuning

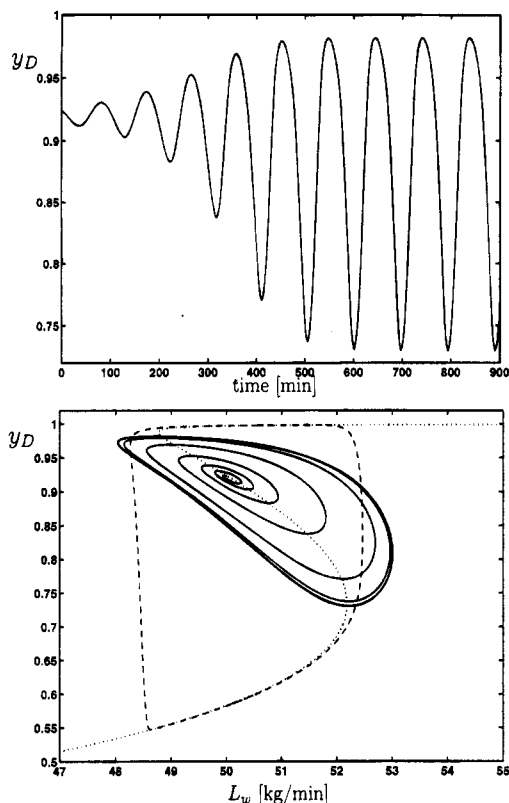


Figure 8. Nonlinear simulation of the methanol-propanol column with PI control of top composition y_D (setpoint $y_D^s = 0.920$) using mass reflux L_w . Controller gain reduced by a factor of 2 compared to Figure 7. Upper plot: Time as independent variable. Lower plot: Phase-plane plot. Dashed line: Limit cycle with controller gain reduced to 0.1. Dotted line: Steady-state solutions. Boilup $V = 2.0$ kmol/min.

rules resulted in a closed-loop unstable system) between top composition y_D and mass reflux L_w and a measurement delay $\theta_d = 1.0$ min. The figure shows the responses to setpoint changes in y_D from operating point II (open-loop stable) to operating point III (open-loop unstable) and then further on to operating point IV (open-loop stable) with boilup V fixed at 2.0 kmol/min (see Figure 2 and Table 2). A logarithmic measurement $Y_D = \log(1 - y_D)$ was used as input to the controller. From the figure, we see that the controller is able to stabilize the open-loop unstable operating point III with a RHP pole at 0.0855 min^{-1} . The simulations also demonstrate that the same controller yields reasonable performance at all of the three widely differing operating points. The reason is that the initial response (high-frequency dynamics) in terms of logarithmic composition Y_D is similar at all operating points (Skogestad and Morari, 1988). From the plot of mass reflux L_w (lower plot in Figure 7), we see that the input L_w only changes dynamically. At steady state, there is no change in the input, showing that the three operating points are multiple solutions.

One should be careful about detuning a controller in an open-loop unstable process, as the bandwidth may become lower than the minimum allowable and the operating point closed-loop unstable. This is illustrated in Figure 8, where the controller gain has been reduced by a factor of 2 (to $K_c = 1.5$) compared to the simulations in Figure 7 (note that reducing the integral action has no effect on stability since the gain $K_c = 3.0$ is sufficient for stabilization). Operating point III now becomes closed-loop unstable, and a small setpoint change causes the system to drift away. However, this does not imply that the column goes globally unstable in the sense that

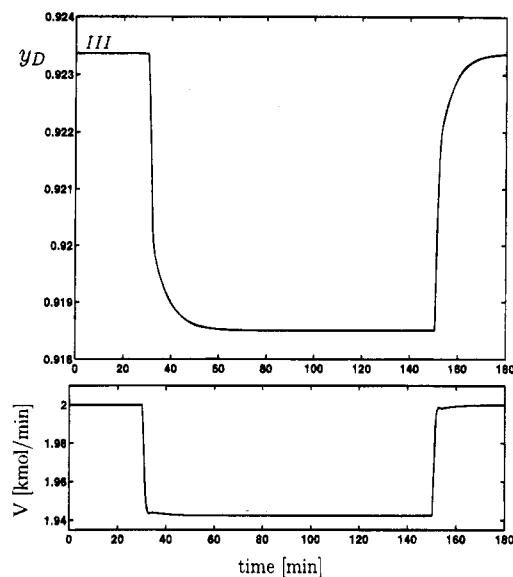


Figure 9. Nonlinear simulation of the methanol-propanol column with PI control of temperature on tray 3 using boilup V . Response to disturbances in feed composition z_F as in Figure 5. Controller gain $K_c = 0.2$. Reflux $L_w = 50.0$ kg/min.

physical constraints are violated. Since there exists stable steady-state solutions above and below the unstable solution, the column goes into a stable limit cycle. If the controller gain is reduced further, the limit cycle will continue, but now with a larger amplitude and a longer period of each cycle. As the controller gain is reduced toward zero, the limit cycle will, in phase space, closely follow the stable steady-state branches with abrupt jumps at the singular points. This is illustrated by the phase plot for the case $K_c = 0.1$ in Figure 8. Note that there are cases where no solution exists outside a singular point. In this case, the column is likely to go globally unstable, as either the condenser or reboiler runs dry.

5.4. One-Point Temperature Control. As mentioned above, composition measurements are often significantly delayed, and it may therefore prove difficult to stabilize an unstable operating point by feedback control of a composition. However, the RHP pole of an unstable operating point is shared by all outputs of the system, including the compositions and temperatures on all trays inside the column. An unstable operating point may therefore be stabilized by applying feedback control to any of these outputs. Most columns have temperature measurements on selected trays inside the column, and these may be utilized to stabilize the column when composition measurements are significantly delayed. Temperature measurements are relatively inexpensive and usually have negligible delays. Below we demonstrate through simulations of the methanol-propanol column how temperature control may be used to stabilize an unstable operating point.

We apply feedback control to the temperature on tray 3, T_3 , using boilup V (note that we may stabilize the system using either of the independent inputs L_w or V ; the one with the most direct effect is usually preferred) and a pure proportional controller with gain 0.2, i.e.,

$$dV(s) = \frac{0.2}{s+1} [T_3^s(s) - T_3(s)] \quad (8)$$

where we have assumed a first-order lag of 1 min as the only control limitation. Figure 9 show the nonlinear response in top composition y_D to disturbances in feed composition z_F from operating point III of the methanol-

propanol column. The disturbance sequence applied is the same as in Figure 5, and we see that the temperature control stabilizes the column around the open-loop unstable operating point III. From Figure 9, we also see that the required control action in the boilup V is relatively small.

5.5. Two-Point Composition Control. As pointed out above, one-point control is sufficient to stabilize an unstable operating point, but preferably both product compositions should be under feedback control. There exist a large amount of literature on two-point control of distillation columns, but everything is based on open-loop stable models. From the discussion above on the effect of RHP poles and RHP zeros on achievable control performance, we would expect some performance deterioration when an operating point is open-loop unstable, in particular when there are significant measurement delays. Below we design specific controllers for the methanol–propanol column to consider closer the impact of a RHP pole on the resulting control performance.

We will again consider operating point III of the methanol–propanol column. In order to compare the achievable performance for the stable model with molar reflux and the unstable process with mass reflux, we design controllers with optimized performance for both cases. We employ linear models and scale the outputs according to their nominal values so that $1 - y_D^*$ corresponds to magnitude 1 for the top composition and x_B^* corresponds to magnitude 1 for the bottom composition. As a design objective, we use the structured singular value, μ (see e.g., Skogestad et al., 1988). This implies that we may include model uncertainty in the design. We use a relative uncertainty weight in each channel given by

$$w_I(s) = 0.20 \frac{5\theta_d s + 1}{0.5\theta_d s + 1} \quad (9)$$

This means that we approximately allow for a deadtime θ_d in addition to 20% gain uncertainty in each input. The performance weight used is given by

$$w_P(s) = \frac{1}{P} \frac{\tau_{CL} s + P}{\tau_{CL} s} \quad (10)$$

This implies that the worst case peak of $\bar{\sigma}(S)(j\omega)$ should be less than P and the worst case closed-loop time constant should be less than τ_{CL} .

We design controllers for different values of θ_d , and for each design, the performance weight is adjusted until a μ value of 1 is achieved. A μ value of 1 implies that the specified performance is obtained for all plants within the uncertainty description. The performance weight is adjusted so that a reasonable trade-off between the maximum peak P and the closed-loop time constant τ_{CL} is obtained. The optimal controllers are found through so-called DK iterations, i.e., using H_∞ optimization (K step) and μ calculations (D step). For this purpose, we employ the Matlab μ -Toolbox (Balas et al., 1993). The controllers thereby obtained contain around 50 states for our designs but may be reduced to around 10 states without significant loss in performance.

Table 3 gives the results for designs with θ_d between 0.1 and 15 min. For $\theta_d = 0.1$ min, we see from Table 3 that there is almost no difference between the achieved performance for the unstable process (L_wV configuration) and the stable model (LV configuration). As

Table 3. Robust Performance Parameters (See (10)) Obtained for Stable LV Configuration and Unstable L_wV Configuration at Operating Point III of the Methanol–Propanol Column^a

θ_d , min	LV		L_wV	
	P	τ_{CL} , min	P	τ_{CL} , min
0.1	1.45	6.0	1.45	7.5
1.0	1.50	27	1.70	36
3.0	1.60	65	2.00	133
5.0	1.70	100	2.50	270
15.0	2.60	200	unstable	

^a All parameters for minimized $\mu_{RP} = 1.00$.

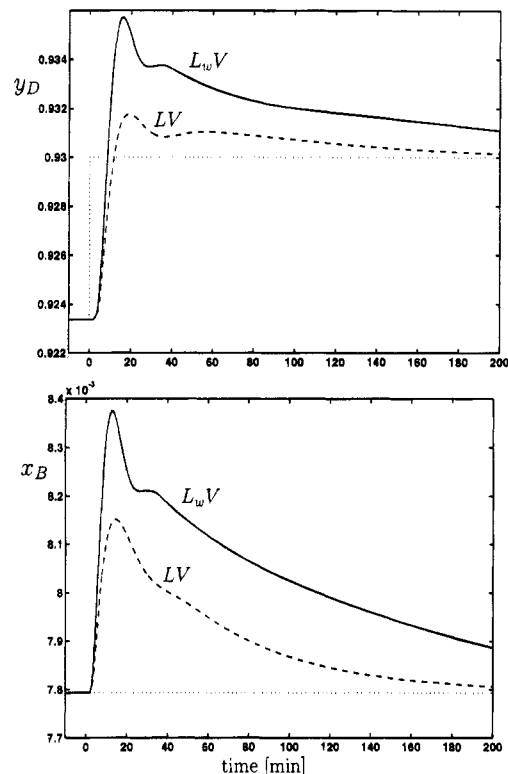


Figure 10. Nonlinear simulation of the methanol–propanol column with LV and L_wV configurations using μ -optimal controllers for $\theta_d = 5$ min. Responses to setpoint change in y_D from operating point III. Simulations include 20% gain uncertainty and measurement delay $\theta_d = 5$ min.

expected from (4), we find that the controllers obtained with this small measurement delay are very similar apart from the scaling factor M^* between L and L_w .

With larger measurement delays, we see from Table 3 that we must allow for a lower bandwidth as well as a higher peak in the sensitivity function for the open-loop unstable process compared to the open-loop stable model. This is as expected from the sensitivity integral in (6). With a delay of 3 min, we can only guarantee half the bandwidth and also get a significantly higher peak in the sensitivity function $\bar{\sigma}(S)$ for the unstable system. With a delay of 5 min (RHP zero $z \approx 0.4 \text{ min}^{-1}$), the response for the LV configuration is still reasonable ($\tau_{CL} = 100$ min and $P = 1.70$), while the response for the open-loop unstable L_wV configuration is poor ($\tau_{CL} = 270$ min and $P = 2.50$). Furthermore, the controller obtained using the model assuming molar inputs would in this case yield a closed-loop unstable system when applied to the plant with the L_wV configuration (after taking the scaling M^* into account).

Nonlinear Simulations. Figure 10 shows responses to setpoint changes in top composition y_D for the open-loop stable LV configuration and the open-loop unstable L_wV configuration using the optimized controllers for

$\theta_d = 5$ min. The simulations include 5-min measurement delay (approximated using an eighth-order lag) and 20% gain uncertainty. The simulations demonstrate the fact that the L_wV configuration has a much larger overshoot as well as a longer settling time than the LV configuration.

The results in Table 3 and Figure 10 confirm that an open-loop RHP pole indeed does influence the control performance and, in particular, when there simultaneously are control limitations such as large measurement delays. This implies that, when an operating point is open-loop unstable with the L_wV configuration and the system in addition has significant measurement delays, one should consider some modification to avoid the performance deterioration caused by the open-loop instability. One possibility is to reconfigure the level control system to obtain a different configuration, e.g., the D_wV configuration, which yields open-loop stable operating points provided the level control is sufficiently tight (see discussion below). Another possibility is to utilize a fast temperature measurement on a tray inside the column in a cascaded controller structure such that the fast inner temperature loop stabilizes the column. For instance, we could stabilize operating point III of the methanol-propanol column using a proportional controller between boilup V and the temperature on tray 3 as in (8). We are then left with reflux L_w and the setpoint T_3^s as our independent inputs, and with a composition measurement delay $\theta_d = 5$ min, we find that the achievable robust performance with the L_wV configuration is $P = 1.7$ and $\tau_{CL} = 95$. This is comparable to the values $P = 1.55$ and $\tau_{CL} = 90$ obtained with the LV configuration using the same cascaded controller structure.

6. Operation with Other Configurations

So far, we have only discussed operation with reflux L_w and boilup V as independent variables. One may argue that this is the most fundamental configuration since these are the two flows that affect the separation of the column in a direct manner. This may be one reason why the L_wV configuration is the most widespread configuration in industry. However, Jacobsen and Skogestad (1991) also show that this is the configuration for which steady-state multiplicity and the related instability are most likely to occur. All other configurations are likely to yield unique operating points, but as shown by Jacobsen and Skogestad (1994), they may still yield unstable operating points. A prerequisite for instability in this case is that the operating point is unstable with the L_wV configuration and that the level control is relatively slow. While instability with the L_wV configuration is caused by a single pole crossing the imaginary axis, the instability with other configurations will be brought about by a pair of complex eigenvalues crossing the imaginary axis, i.e., a Hopf bifurcation (Jacobsen and Skogestad, 1994). Jacobsen and Skogestad (1994) found that, for most columns, the Hopf bifurcation will be supercritical, implying that a stable limit cycle (sustained oscillations) occurs as the steady state becomes unstable.

Nonlinear Simulations. We will again consider the methanol-propanol column in Table 1, but we now employ the D_wV configuration. This configuration is obtained from the L_wV configuration by changing the condenser level control from using distillate flow D_w to using reflux L_w . In the simulations, we assume holdups of 2.0 kmol in the reboiler and condenser (increased to avoid hitting constraints during oscillatory behavior).

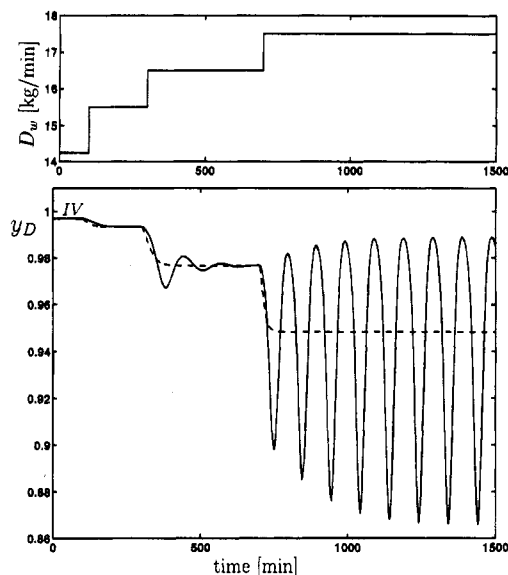


Figure 11. Nonlinear open-loop simulation of the methanol-propanol column with D_wV configuration and condenser level controller $dL_w = 0.05dM_{D_w}$. Responses to increases in distillate flow D_w with boilup $V = 2.0$ kmol/min. Dashed line: Corresponding response assuming molar reflux; i.e., $dL = 0.05dM_D$.

The condenser level is controlled using a proportional controller with gain 0.05, i.e., $dL_w = 0.05dM_{D_w}$ corresponding to a closed-loop time constant of 20 min. We assume perfect control of the reboiler level, but this assumption is not important since the tuning of the reboiler level control does not influence stability with the D_wV configuration.

Figure 11 shows responses in top composition y_D to step changes in distillate flow D_w starting at operating point IV in Table 2. The boilup V is kept fixed at 2.0 kmol/min. Initially, as D_w is increased from 14.25 to 15.5 kg/min, the response in y_D is close to first order, which is the assumed predominant behavior of distillation columns in the literature (e.g., Davidson, 1956; Moczek et al., 1965). However, as D_w is increased further to 16.5 kg/min, the response becomes more oscillatory, although still stable. Upon increasing D_w to 17.5 kg/min, however, the stability is lost and the column instead settles in a stable oscillatory behavior.

Figure 11 also shows the corresponding response in y_D using the same level controller gain but assuming the reflux flow given on a molar rate basis, i.e., $dL = 0.05dM_D$ (dashed line). We see that, in this case, the response in y_D is nonoscillatory, stable, and close to first order for all values of D_w .

The observed behavior in Figure 11 is explained by the fact that, initially, the column is at operating point IV, which is stable with the L_wV configuration (see Figure 2) and hence also stable with the D_wV configuration (Jacobsen and Skogestad, 1994). However, as y_D decreases with increasing D_w , the column enters a region of operation where it will be unstable if operated with the L_wV configuration, and a certain gain in the condenser level controller is therefore required to obtain stability with the D_wV configuration. Jacobsen and Skogestad (1994) show that a column with the D_wV configuration goes unstable approximately when the gain of the condenser level controller is smaller than the RHP pole of the column operating with the L_wV configuration. With $D_w = 17.5$ kg/min and $V = 2.0$ kmol/min, the methanol-propanol column has a RHP pole $p = 0.053$ if operated with the L_wV configuration. Thus, the column is unstable also with the D_wV config-

uration since we have a gain of 0.05 in the condenser level controller.

7. Detecting Open-Loop Instability during Operation

As we have seen, steady-state multiplicity and instability may cause several undesirable phenomena under manual operation and will also limit the achievable performance under feedback control. We provide here some suggestions as to how (potential) open-loop instability may be detected, without experiencing undesirable phenomena, in a column under operation. We discuss two different modes of operation: manual operation with configurations other than the L_wV configuration and operation with one-point composition or temperature control.

7.1. Manual Operation. Provided the level control is reasonably tight, open-loop instability is unlikely to be experienced with configurations other than the L_wV configuration. Thus, if one suspects problems with instability using the L_wV configuration, one should switch to a different configuration, e.g., to the D_wV configuration. As shown here, it will then also be possible to detect whether the column actually is unstable with the L_wV configuration.

First consider operation with the D_wV configuration. The transformation between the D_wV and L_wV configurations is for the top composition y_D given by

$$\left(\frac{\partial y_D}{\partial D_w}\right)_V(s) = \left(\frac{\partial y_D}{\partial L_w}\right)_V(s) \left(\frac{\partial L_w}{\partial D_w}\right)_V(s) \quad (11)$$

We assume no input or output multiplicity [output multiplicity is unlikely with the D_wV configuration (Jacobsen and Skogestad, 1991), while input multiplicity occurs only in special cases (Jacobsen, 1993)] with the D_wV configuration. This implies that the steady-state gain $(\partial y_D/\partial D_w)_V(0)$ always is negative (Jacobsen and Skogestad, 1991). Assuming no input multiplicity with the L_wV configuration implies that the steady-state gain $(\partial y_D/\partial L_w)_V(0)$ is positive when the L_wV configuration is stable and negative when the L_wV configuration is unstable (Jacobsen and Skogestad, 1994). By inspection of (11), we see that this implies that the steady-state gain $(\partial L_w/\partial D_w)_V(0)$ is negative when the L_wV configuration is stable and positive when the L_wV configuration is unstable. Thus, if an increase in D_w with V constant results in an increase in the reflux L_w at steady state, the column will be unstable if operated with the L_wV configuration.

Note that the increase in reflux with increasing distillate flow discussed above applies to steady state only. The reflux will always decrease initially when the distillate flow is increased because initially the top composition is unchanged and the condenser level controller thus causes L_w to decrease. Mathematically, this is understood by considering (11); if the D_wV configuration is stable, i.e., no RHP poles in $(\partial y_D/\partial D_w)_V(s)$, while the L_wV configuration is unstable, i.e., $(\partial y_D/\partial L_w)_V(s)$ has a RHP pole p , then $(\partial L_w/\partial D_w)_V(s)$ must contain a RHP zero $z = p$ that cancels the unstable pole. A RHP zero in $(\partial L_w/\partial D_w)_V(s)$ implies that there will be an inverse response in L_w to changes in D_w with V constant. This is illustrated by the simulation in Figure 12.

In conclusion, a RHP pole p with the L_wV configuration will imply an inverse response, corresponding to a RHP zero $z = p$, in reflux L_w to changes in distillate flow D_w with the D_wV configuration. This fact may be

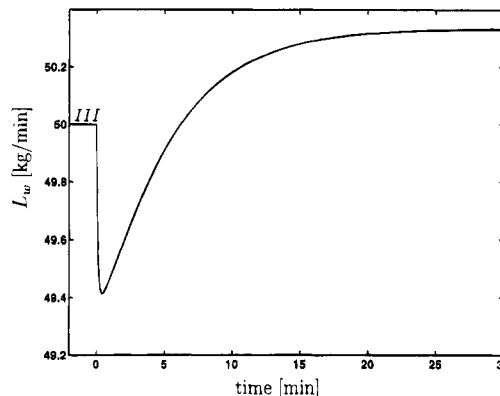


Figure 12. Nonlinear simulation of the methanol-propanol column with D_wV configuration. Response in reflux L_w to step increase in distillate flow D_w from 18.36 to 19.0 kg/min. Boilup $V = 2.0$ kmol/min.

utilized to detect a potential RHP pole with the L_wV configuration through experiments with the stable D_wV configuration.

We do not discuss operation with other configurations in detail here, but similar arguments to those used for the D_wV configuration apply also to other configurations. For instance, if $(\partial V/\partial B_w)_{L_w}(0)$ is positive with the L_wB_w configuration, then the column will be unstable if operated with the L_wV configuration. Note that also in this case will there be an inverse response in boilup V to changes in bottoms flow B_w .

7.2. Operation with One-Point Control. One of the surprising features of the steady-state multiplicity discussed in this article is that, at unstable operating points, the steady-state gains

$$\left(\frac{\partial x_i}{\partial L_w}\right)_V(0) < 0; \quad \left(\frac{\partial T_i}{\partial L_w}\right)_V(0) > 0 \quad (12)$$

Here x_i denotes the fraction of light component and T_i the temperature on tray i , including the reboiler and condenser. Thus, at an unstable operating point, separation in the top of the column becomes worse as the reflux is increased. This is definitely counterintuitive. Similarly, the gains

$$\left(\frac{\partial x_i}{\partial V}\right)_{L_w}(0) > 0; \quad \left(\frac{\partial T_i}{\partial V}\right)_{L_w}(0) < 0 \quad (13)$$

at an open-loop unstable operating point. This unexpected behavior may be used to detect potential open-loop instability in a column operating with one-point feedback control. For example, with V fixed, increasing a tray temperature through feedback control will at steady state result in an increase in the reflux L_w . Note that, similar to what was discussed above, there will also in this case be an inverse response for L_w .

8. Conclusions

This article discusses the implications of steady-state multiplicity and open-loop instability for the operation and control of distillation columns.

In a column operated manually with reflux and boilup as the manipulated inputs, the following phenomena may be observed due to multiplicity and instability: inability to reach certain separations because they correspond to unstable operating points; large and abrupt changes in operating conditions for relatively small disturbances or changes in the manipulated inputs; hysteresis in operation, that is, the separation

depends not only on the present value of the manipulated inputs and feed conditions but also on the past history of these.

With configurations other than the L_wV configuration, e.g., the D_wV configuration, steady-state multiplicity is unlikely. However, if the level control is relatively slow, an operating point may become unstable and sustained oscillations corresponding to a limit cycle will then occur.

An unstable column may usually be stabilized by feedback control of a product composition or tray temperature. In this case, the following phenomena may be observed at open-loop unstable operating points: (1) increasing purity in the top, with boilup constant, results in decreased reflux; similarly, increasing purity in the bottom, with reflux constant, results in decreased boilup; this is opposite to what one intuitively would expect; (2) sustained oscillations if the composition or temperature control is not sufficiently tight.

With both product compositions under control, the existence of open-loop instability will limit the achievable control performance to some extent. This effect becomes increasingly marked with large measurement delays.

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Nomenclature (See also Figure 1)

B = bottoms flow (kmol/min)
 c = controller transfer function
 D = distillate flow (kmol/min)
 d = disturbance acting on output
 F = feed rate (kmol/min)
 g = process transfer function
 L = reflux flow rate (kmol/min)
 M = molecular weight, usually of top product (kg/kmol)
 M_1 = pure component molecular weight of most volatile component (kg/kmol)
 M_2 = pure component molecular weight of least volatile component (kg/kmol)
 M_L = tray liquid holdup (kmol)
 M_D = condenser holdup (kmol)
 M_B = reboiler holdup (kmol)
 N = number of theoretical stages in column
 N_F = feed stage location (1 reboiler)
 P = maximum peak of sensitivity function
 p = right half plane pole (min^{-1})
 Q_B = heat input to reboiler
 Q_D = heat removal in condenser
 q_F = liquid fraction in feed
RHP = right half plane
 r = reference signal/setpoint
 S = sensitivity function
 T_i = temperature on tray i
 V = boilup from reboiler (kmol/min) (determined indirectly by heating Q)
 V_T = vapor flow to condenser (kmol/min)
 x_B = mole fraction of most volatile component in bottom product
 x_i = mole fraction of most volatile component at tray i
 y = process output
 y_D = mole fraction of most volatile component in distillate (top product)
 z = right half plane zero (min^{-1})

z_F = mole fraction of most volatile component in feed

Greek Symbols

$\alpha = [y_i/x_i]/[(1 - y_i)/(1 - x_i)]$ = relative volatility (binary mixture)
 $\lambda_i(A)$ = i th eigenvalue of A
 $\lambda_{\max} = \max_i |\lambda_i(A)|$ = maximum eigenvalue = dominant pole
 μ = structured singular value
 $\bar{\sigma}$ = maximum singular value
 τ_{CL} = closed-loop time constant (min)
 θ_d = deadtime (min)
 ω = frequency (min^{-1})

Superscripts

* = nominal steady-state value

Subscripts

w = flow rate (kg/min)

Literature Cited

- Balas, G. J.; Doyle, J. C.; Glover, K.; Packard, A. K.; Smith, R. *Matlab μ -Analysis and Synthesis Toolbox*, Version 2.0; The Mathworks Inc.: 1993.
- Davidson, J. F. The Transient Behavior of Plate Distillation Column. *Trans. Inst. Chem. Eng.* **1956**, *34*, 44–52.
- Doherty, M. F.; Perkins, J. D. On the Dynamics of Distillation Processes—IV. Uniqueness and Stability of the Steady-State in Homogeneous Continuous Distillation. *Chem. Eng. Sci.* **1982**, *37*(3), 381–392.
- Freudenberg, J. S.; Looze, D. P. Right Half Plane Poles and Zeros and Design Tradeoffs in Feedback Systems. *IEEE Trans. Autom. Control* **1985**, *AC-30*, 555–565.
- Freudenberg, J. S.; Looze, D. P. *Frequency Domain Properties of Scalar and Multivariable Feedback Systems*; Springer-Verlag: Berlin, 1988.
- Jacobsen, E. W. Input Multiplicity and Right Half Plane Zeros in Binary Distillation. Proceedings European Control Conference, Groningen, Holland; 1993; pp 661–666.
- Jacobsen, E. W.; Skogestad, S. Multiple Steady-States in Ideal Two-Product Distillation. *AIChE J.* **1991**, *37*, 4, 499–511.
- Jacobsen, E. W.; Skogestad, S. Instability of Distillation Columns. *AIChE J.* **1994**, *40* (9), 1466–1478.
- Moczek, J. S.; Otto, R. E.; Williams, T. J. Approximation Models for the Dynamic Response of Large Distillation Columns. *Chem. Eng. Prog. Symp. Ser.* **1965**, *61* (55), 136–146.
- Poston, T.; Stewart, I. *Catastrophe Theory and its Applications*; Pitman: London, 1978.
- Rosenbrock, H. H. A Theorem of “Dynamic Conservation” for Distillation. *Trans. Inst. Chem. Eng.* **1960**, *38* (20), 279–287.
- Rosenbrock, H. H. A Lyapunov Function with Applications to Some Nonlinear Physical Problems. *Automatica* **1962**, *1*, 31–53.
- Rademaker, O.; Rijnsdorp, J. E.; Maarleveld, A. *Dynamics and Control of Continuous Distillation Units*; Elsevier: Amsterdam, 1975.
- Skogestad, S.; Morari, M. Understanding the Dynamic Behavior of Distillation Columns. *Ind. Eng. Chem. Res.* **1988**, *27* (10), 1848–1862.
- Skogestad, S.; Morari, M.; Doyle, J. C. Robust Control of Ill-Conditioned Plants: High-Purity Distillation. *IEEE Trans. Autom. Control* **1988**, *33* (12), 1092–1105.
- Skogestad, S.; Lundström, P.; Jacobsen, E. W. Selecting the Best Distillation Control Configuration. *AIChE J.* **1990**, *36* (5), 753–764.
- Youla, D. C.; Bongiorno, J. J.; Lu, C. N. Single-Loop Feedback-Stabilization of Linear Multivariable Dynamical Plants. *Automatica* **1974**, *10* (2), 159–173.

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