PROCESS DESIGN AND CONTROL

Inconsistencies in Dynamic Models for Ill-Conditioned Plants: Application to Low-Order Models of Distillation Columns

Elling W. Jacobsen† and Sigurd Skogestad*

Chemical Engineering, University of Trondheim (NTH), N-7034 Trondheim, Norway

The paper addresses the problem of obtaining consistent dynamic models for certain ill-conditioned plants like distillation columns. Due to strong interactions there is often a single dominating slow mode (pole) that results in similar first-order responses for all outputs. Typically, models are identified on the basis of fitting individual responses, but unless special care is taken this will result in an inconsistent overall model with the slow pole repeated. It is shown that such models with excessive slow poles, although a reasonable approximation for open-loop dynamics, yield a poor prediction of the closed-loop behavior of the process, in particular under partial control.

1. Introduction

The objective of this paper is to discuss some fundamental aspects of linear dynamic models for ill-conditioned multivariable processes (plants). Most published work on the identification of dynamic models from experimental data has been concentrated on the single-input–single-output (SISO) case. This is also reflected in the literature on process dynamics and control, where linear dynamic models usually are obtained by fitting input–output data from a plant or nonlinear simulation to a low-order transfer function, e.g., of the kind first order plus dead time. In cases where the process is multivariable, the transfer matrix is usually obtained by fitting the transfer matrix elements *independently*.

However, this is often a poor approach. Skogestad and Morari (1988) argue that it may easily lead to poor models for ill-conditioned processes unless one explicitly takes into account the coupling between the steady-state gains of the different elements. In particular, one is not able to obtain a good model of the low-gain direction of the plant (Skogestad and Morari, 1988; Andersen et al., 1989), and the model will easily have the wrong sign of the determinant at steady state.

Another and more fundamental problem with this identification approach is that the model may be inconsistent in that a single physical state is repeated in the model. By inconsistent we here mean that there is a fundamental modeling error in that the model contains too many slow modes. This issue is the main topic of this paper. Ill-conditioned plants often have a *single* dominating "slow" mode (with a large time constant) which is a result of interactions in the process, and is thus shared by all the transfer matrix elements. However, by fitting the elements of an $n \times n$ process *independently*, such that they all contain the dominant pole, one may get an inconsistent model with at least n poles similar to the single dominating pole of the process. As shown in this paper, the inconsistency will usually result in a poor prediction of the closed-loop behavior of the process, in particular under partial feedback control.

† Present address: Centre for Process Systems Engineering, Imperial College, London SW7 2BY, UK.

* Author to whom all correspondence should be addressed. E-mail: skoge@kjemi.unit.no.

The transfer matrices of multivariable processes are often fitted using different time constants in the different elements without considering whether the time constants actually originate from a single state (pole). This is for instance common practice in the distillation control literature (e.g., Shunta and Luyben, 1972; Hammarström et al., 1982; Waller et al., 1988). Such an approach will again result in a model with an excessive number of slow modes. The minimal realization will in this case contain n^2 large time constants, all in the order of magnitude of the dominant time constant of the process, and thus an inconsistent model.

The general literature on identification theory has so far not focused very much on multivariable issues, and the particular problems mentioned above which may be encountered when identifying the poles for ill-conditioned plants do not seem to have been discussed.

In the paper we refer to the relative gain array (RGA) (Bristol, 1966, 1978) and the condition number. The RGA matrix is at each frequency defined as $\Lambda = \mathbf{G} \times (\mathbf{G}^{-1})^T$ where $\times$ denotes element-by-element multiplication. In this paper when we refer to "the RGA" we usually mean the 1,1 element, $\lambda_{11}(\mathbf{G})$.

The condition number $\gamma(\mathbf{G})$ of the matrix $\mathbf{G}$ is the ratio between its largest and smallest singular value. *By definition an ill-conditioned plant has a large value of the condition number* $\gamma(\mathbf{G})$. Physically this means that the effect on the outputs to changes in the inputs is strongly dependent on the input *direction* and we say that the plant has strong "directionality". For example, in distillation columns the effect on the compositions (outputs) to *individual* changes in reflux and boilup (these individual changes correspond to specific "input directions") is very large. However, if we simultaneously increase reflux and boilup by the same amount (this is a another input direction), then the two inputs may counteract each other such that the effect on the outputs is small.

Note that plants with large elements in the RGA matrix always are ill-conditioned (i.e., $\gamma(\mathbf{G})$ is also large), but the opposite is not always true. Actually, the problems discussed in this paper mainly appear when the RGA elements are large (see section 4) and thus may not be serious for all ill-conditioned plants.

We start the paper with an example of an inconsistent low-order model of a heat exchanger. The model, although seemingly a good open-loop description of the plant, is

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Table 1. Steady-State Data for Heat-Exchanger Example (See Also Figure 1)[a]

$V_H = V_C$ (m³)	$q_C = q_H$ (m³/min)	T_{Ci} (°C)	T_{Hi} (°C)	T_C (°C)	T_H (°C)	UA (kJ/(°C min))	ρ (kg/m³)	c_P (kJ/(°C kg))
1	0.01	25	100	61.59	63.41	300	500	3.0

[a] c_P and ρ are equal for the hot and cold sides.

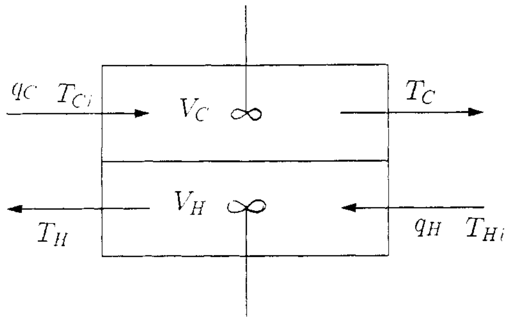

Figure 1. Simplified representation of heat exchanger with one mixing tank on each side.

shown to yield unexpected behavior when one control loop is closed ("one-point control"). The results in this example are subsequently explained using analytical results. We then briefly discuss what types of processes that are likely to be modeled with an excessive number of slow poles. The last part of the paper is devoted to the specific problem of obtaining low-order models of distillation columns.

All the results presented in this paper are for 2 × 2 processes, i.e., two inputs and two outputs. However, the results are clearly of relevance also for higher dimensional processes.

2. Introductory Heat-Exchanger Example

The objective of this section is to present a simple physically motivated example of an ill-conditioned multivariable process where a single slow pole is dominating all the responses.

Consider the simplified heat exchanger in Figure 1, which is modeled using a single mixing tank for each of the hot side and cold side. Neglecting the heat accumulated in the walls yields a model with two states. The model is derived in Appendix, and data for the example are given in Table 1. There are no significant nonlinearities in this model, and in the following we only use the linearized form, $y(s) = \mathbf{G}(s)\, u(s)$. Here $y = [y_1\, y_2]^T = [\Delta T_C\, \Delta T_H]^T$ is the cold and hot outlet temperatures and $u = [u_1\, u_2]^T = [\Delta q_C\, \Delta q_H]^T$ is the cold and hot inlet flow rates. The linear model at this operating point is (denoted the "full" model in the following)

$$\mathbf{G}(s) = \frac{1}{(1+\tau_1 s)(1+\tau_2 s)}\begin{pmatrix} k_{11}(1+z_1 s) & k_{12} \\ k_{21} & k_{22}(1+z_2 s)\end{pmatrix} \quad (1)$$

$$\tau_1 = 100;\quad \tau_2 = 2.44;\quad z_1 = z_2 = 4.76;$$
$$k_{11} = -k_{22} = -1874;\quad k_{12} = -k_{21} = 1785$$

The model is ill-conditioned as it has a steady-state condition number of 41 and diagonal steady-state RGA values of 10.8. The physical explanation for the ill conditioning is simply that the heat transfer is very effective such that the two outlet temperatures (outputs) are almost the same (61.59 and 63.41 °C in our case), and it is very difficult to change them independently. In particular, it is difficult to make one outlet stream hotter and the other colder (this is the "weak" or "difficult" or "low-gain" direction of the plant), whereas we may easily make them *both* hotter or colder (this is the "strong" or "easy" or "high-gain" direction of the plant).

Open-loop responses obtained with model (1) to 10% step changes in the two inputs are shown by the solid lines in Figure 2. From the figure we observe that all the responses are close to first-order with a time constant around 100 min. We also note that the smallest time constant, $\tau_2 = 2.44$ min, which we later show is associated with the low-gain direction of the plant, is very difficult to observe from the open-loop responses. Indeed, as seen from the dashed lines in Figure 2, an excellent fit is obtained with the following simplified model

$$\mathbf{G}(s) = -\frac{1}{1+\tau_1 s}\begin{pmatrix} k_{11} & k_{12} \\ k_{21} & k_{22}\end{pmatrix} \quad (2)$$

Although it may appear that (2) only has a single pole, the state-space realization of course contains two poles at $-1/\tau_1$.

We now want to study the behavior of the process under partial ("one-point") feedback control, i.e., controlling one of the outlet temperatures. The cold outlet temperature T_C (y_1) is controlled with the cold inlet flow q_C (u_1) using a P controller with gain $K = 0.015$ which yields a closed-loop time constant for this loop of about 3.5 min. Figure 3 shows the responses to a 10% step change in hot inlet flow q_H (u_2) with this loop closed. The solid lines are obtained with the "full" linear model (1), whereas the dashed lines are obtained with the fitted model (2). As seen from the figure, the responses for the uncontrolled output, T_H (y_2) differ significantly. The full model yields a "fast" response in T_H (similar to that of the controlled output T_C), whereas the fitted model yields a slow settling toward the new steady state. The reason for the large difference in behavior is, as we shall see, the different number of slow poles in the two models (1) and (2).

3. Minimum Number of States and Inconsistency

Consider a linear system described by the model

$$\dot{x} = \mathbf{A}x + \mathbf{B}u;\qquad y = \mathbf{C}x + \mathbf{D}u \quad (3)$$

Here x denotes states, u inputs, y outputs and $\dot{x}$ the time derivative of x. Laplace transformation of (3) yields the transfer matrix

$$\mathbf{G}(s) = \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B} + \mathbf{D} \quad (4)$$

For a system with n states, m inputs, and p outputs we have $\dim(\mathbf{A}) = n \times n$, $\dim(\mathbf{B}) = n \times m$, $\dim(\mathbf{C}) = p \times n$, and $\dim(\mathbf{D}) = p \times m$. The maximum rank of $\mathbf{G}(s)$ is $r_{max} = \min(p, m)$. Assume that $\mathbf{G}(0)$ has rank $r > 1$. With $\mathbf{D} \neq 0$ we may define a model with a single state (time constant) by letting the dynamic part of the model, $\mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B}$, have rank equal to 1 and use $\mathbf{D}$ to make the rank of $\mathbf{G}(0) = r$. However, such a model yields an incorrect initial response for most processes and is therefore not considered. With $\mathbf{D} = 0$, which is more reasonable from a physical point of view, it is easily seen from (4) that we need at least r states for $\mathbf{G}(0)$ to have rank r.

Heat Exchanger Example, Continued. In the heat-exchanger example we had a nonsingular steady-state matrix $\mathbf{G}(0)$ with rank $r = 2$, and consequently we need at least two states to describe the system using a state-space description with $\mathbf{D} = 0$. Thus, when attempting to

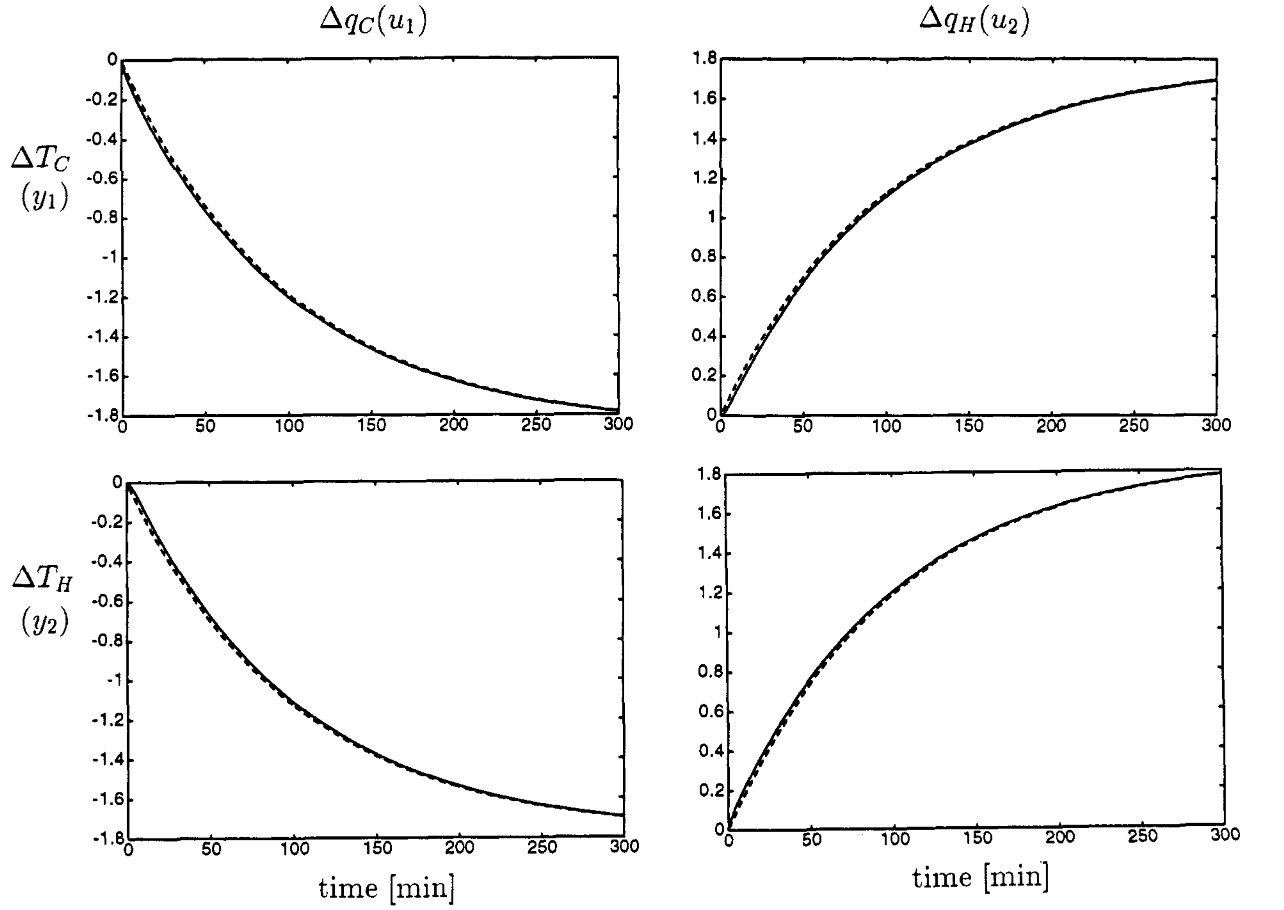

Figure 2. Open-loop step responses of heat exchanger. Left: $\Delta q_c = u_1 = 0.001$ (10% increase in cold flow). Right: $\Delta q_H = u_2 = 0.001$. Solid line: response of full model (1). Dashed line: response of fitted model (2).

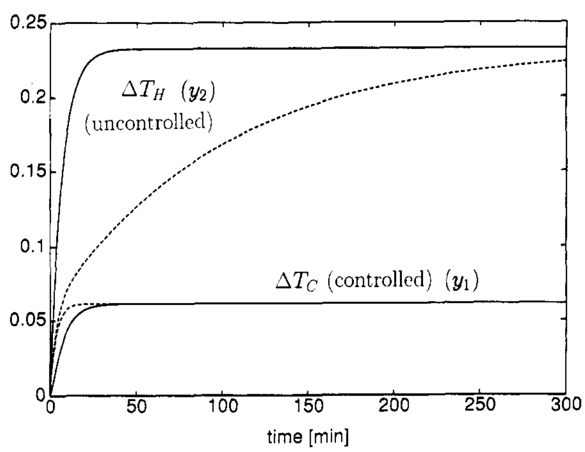

Figure 3. Dynamic response of heat exchanger with one loop closed. Responses in outlet temperatures to a 10% step increase in q_H (u_2). Cold outlet temperature T_C (y_1) is controlled by q_C (u_1) using a pure proportional controller with gain $K = 0.015$. Solid line: response of full model (1). Dashed line: response of fitted model (2).

describe the system using only one time constant, we obtained the simplified model (2) with two poles at $-1/\tau_1$.

Some readers might believe that also the full model (1) has two poles at $-1/\tau_1 = -1/100$ since there are two mixing tanks which isolated would have a time constant of $V/q = 100$ min each. However, an analysis of the full model (1) reveals that there is a multivariable zero that cancels one of the apparent poles at $-1/\tau_1$. The single slow pole at $-1/\tau_1$, which is shared by all the transfer function elements, is a result of the interactions between the two sides of the heat exchanger. Applying one-point feedback control to the full model (1) causes the shared pole $-1/\tau_1$ to move, and also the uncontrolled response to become fast as seen from the solid line for y_2 in Figure 3. However, this is not the case when the simplified model (2) is used (dashed line), because here only one of the two poles at $-1/\tau_1$ is moved. This is shown in the next section.

4. Analytical Treatment of Model with One Loop Closed

Consider applying the control law

$$u_1 = -K(y_1 - y_{1s}) \tag{5}$$

to the simplified model (2) (here subscript s denotes setpoint). This is denoted "one-point" control since only one output is controlled. The closed-loop transfer-matrix becomes

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \frac{1}{1 + \tau_{CL}s}\begin{pmatrix} \frac{Kk_{11}}{1+Kk_{11}} & \frac{k_{12}}{1+Kk_{11}} \\ \frac{Kk_{21}}{1+Kk_{11}} & \frac{k_{22}(1+\tau_{CL}s) - \frac{Kk_{12}k_{21}}{1+Kk_{11}}}{1+\tau_1 s} \end{pmatrix}\begin{pmatrix} y_{1s} \\ u_2 \end{pmatrix} \tag{6}$$

where

$$\tau_{CL} = \tau_1/(1 + Kk_{11}) \tag{7}$$

Thus, three of the elements are first-order with the time constant, τ_{CL}, whereas the transfer function $g_{22}(s)$ from u_2 to the uncontrolled output y_2 is second order, as it in addition contains the open-loop dominant time constant,

τ_1. To see how the two time constants contribute to the overall response in the uncontrolled output y_2, write $g_{22}(s)$ in the form

$$g_{22}(s) = \frac{X_1}{1 + \tau_1 s} + \frac{X_{CL}}{1 + \tau_{CL} s} \quad (8)$$

The ratio X_1/X_{CL} expresses the importance of the excessive slow pole $-1/\tau_1$ under partial control and is given by

$$\frac{X_1}{X_{CL}} = (1 + Kk_{11})\left(\frac{1}{Y} - 1\right); \quad Y = \frac{k_{12}k_{21}}{k_{11}k_{22}} \quad (9)$$

where Y is the ratio between the off-diagonal and diagonal steady-state gains, and is a well-known measure of interactions (e.g., Balchen, 1958; Rijnsdorp, 1965). It is also related to the 1,1 element of the RGA for 2 × 2 systems

$$\lambda_{11} = \frac{1}{1 - Y} \quad (10)$$

The model is ill-conditioned when Y is close to unity, which corresponds to a large value of λ_{11}.

From (9) we see that the ratio X_1/X_{CL} depends on the gain K used in the controller. The higher the gain is, the larger is the ratio X_1/X_{CL}. This means that the faster the response in the controlled output is, the more marked is the large time constant τ_1 in the uncontrolled output, y_2.

Consider Y in the range 0 to 1. For cases with $Y = 1$ ($\lambda_{11} = \infty$) we see from (9) that X_1/X_{CL} becomes zero, i.e., there is no gain related to τ_1, and only τ_{CL} remains in $g_{22}(s)$. This is as expected since $Y = 1$ implies that the model is singular at all frequencies and the minimal realization of (2) will only contain one state. On the other hand, if $Y = 0$ ($\lambda_{11} = 1$) we see from (9) that $X_1/X_{CL} = \infty$ and only τ_1 will be left in $g_{22}(s)$. This is also as expected since $Y = 0$ implies that the steady-state matrix is triangular or diagonal, in which case it is likely that the identified process actually contains two poles at $-1/\tau_1$ (see discussion below). For values of Y between 0 and 1 ($\lambda_{11} > 1$), both time constants will be present in $g_{22}(s)$ and their relative importance is determined by the size of X_1/X_{CL}.

This analysis seems to suggest that it is for weakly interactive processes; i.e., where Y is close to zero, we get the largest error when an inconsistent model with excessive slow poles is used. However, this conclusion is misleading as it is for ill-conditioned processes we most likely will identify a model with too many slow poles. To see this, consider a 2 × 2 model which is reduced to have two states. If a proper model reduction method is employed, the two poles left should be the ones with the largest effect on the input–output behavior of the full model. Each of the two poles will have an input direction related to them, that is, a set of inputs that cancels the other pole. A similarity transformation of the state-space model, so that the **A** matrix becomes diagonal, will reveal these directions in the rows of the transformed **B** matrix. Changes in one input at a time, i.e., the input vectors $[1\ 0]^T$ and $[0\ 1]^T$, will span the input space. If one of the poles dominates the responses to both these input perturbations, it means that the gain related to the "hidden" pole must be small compared to the gain related to the dominating pole. This implies that the system has two directions with widely differing gains; i.e., the system is ill-conditioned. (Note that some ill-conditioned systems may have the directions of the poles closely aligned with the input vectors of the perturbations. In this case both poles will show up in the simulations.) From this we conclude that it is only for ill-conditioned systems that the open-loop responses are likely to be well approximated using an inconsistent model

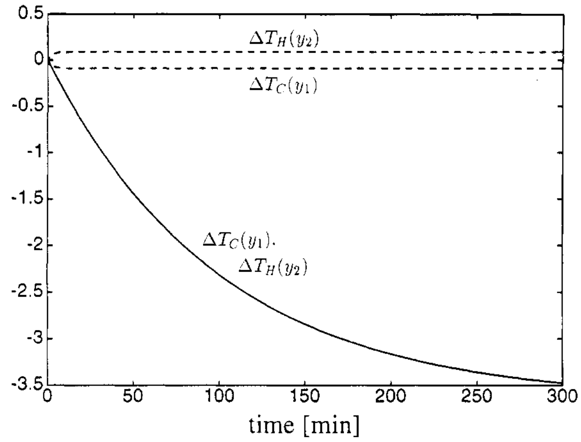

Figure 4. Open-loop step response of heat exchanger illustrating ill-conditioned behavior. Solid lines: $\Delta q_C = -\Delta q_H = 0.001$ ($u_1 = -u_2$). Dashed lines: $\Delta q_C = \Delta q_H = 0.001$ ($u_1 = u_2$). Responses obtained with full model (1).

with a single time constant. A diagonal or triangular 2 × 2 process which has $Y = 0$ ($\lambda_{11} = 1$) and is well described using only one time constant τ_1 is thus likely to actually contain two poles at $-1/\tau_1$.

Heat-Exchanger Example, Continued. For the heat-exchanger example we have $Y = 0.907$ and $Kk_{11} = 28.1$, which yields $X_1/X_{CL} = 2.98$ for the simplified model (2). That is, a major part of the response in the uncontrolled output y_2 is related to τ_1, which is confirmed by the slow settling for y_2 (dashed line) in Figure 3. For the full model (1) the single time constant τ_1 is affected by the feedback control, and y_2 has no slow settling (solid line in Figure 3).

A similarity transformation of the state-space realization of the full heat-exchanger model (1) shows that the input direction cancelling τ_2 is $[1\ -1]^T$ and the input direction cancelling τ_1 is $[1\ 1]^T$. A singular value decomposition of the model gives a (minimized) condition number $\gamma = 41$ with the high-gain input direction being $[1\ -1]^T$ and the low-gain input direction being $[1\ 1]^T$. In this case we therefore have a perfect alignment of the singular input vectors and the pole-canceling vectors, i.e., the high-gain input direction has a pole $-1/\tau_1$ and the low-gain input direction a pole $-1/\tau_2$. The gain in the direction of the slow pole $-1/\tau_1$ is consequently 41 times the gain in the direction of the fast pole $-1/\tau_2$, and the fast pole is thus only weakly visible in open-loop simulations with perturbations in single inputs. This explains why a model using only one time constant yields an excellent fit of the open-loop responses in Figure 2.

Figure 4 shows the responses in the outlet temperatures of the heat exchanger to the inputs $u_1 = -u_2$ (input direction canceling τ_2) and $u_1 = u_2$ (input direction canceling τ_1) obtained with the full model (1). From the figure we see that, as expected from the analysis, the gain related to the small time constant τ_2 is much smaller than the gain related to the dominant time constant τ_1.

5. Low-Order Models of Distillation Columns

The model features of distillation columns are similar to the ones discussed above for the simplified heat-exchanger model. In this section we discuss the problem of obtaining low-order linear dynamic models for high-purity distillation columns.

High-purity distillation columns operating with reflux L and boilup V as independent variables (see Figure 6) may be strongly ill-conditioned. Furthermore, it is well-known that the individual open-loop responses may be

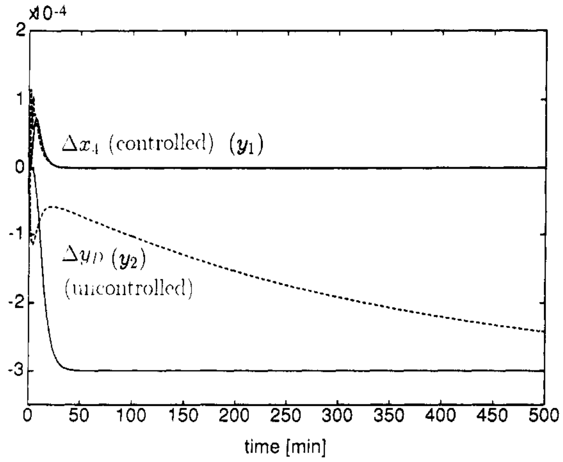

Figure 5. Wahl and Harriot column with one loop closed. Response in y_D and x_4 to a disturbance in feed composition with x_4 controlled by reflux. Dashed lines: simulations with low-order model given by Wahl and Harriot (1970). Solid lines: simulations with full linear model.

well approximated using only one dominant time constant. This has been shown both from plant data (McNeill and Sachs, 1969) and in several theoretical papers (e.g., Davidson, 1956; Moczek et al., 1965; Wahl and Harriot, 1970; Kim and Friedly, 1974; Skogestad and Morari, 1987). Due to this, first-order models are commonly used in the distillation control literature.

We shall first discuss an example from the literature that demonstrates that such simplified models may yield poor results, particularly when one-point feedback control is applied. We shall then consider another example of a high-purity column, and use this as an illustration of the difficulty involved in deriving consistent low-order models for distillation columns.

Inconsistent Distillation Models from the Literature. Wahl and Harriot (1970) used a simple low-order model to study the behavior of a high-purity column under one-point control. Their low-order model is somewhat more complicated than the pure first-order transfer-function matrix given in (2), but the minimal realization of their model contains two time constants equal to 365 min, while the full model only has one time constant at 365 min.

The dashed lines in Figure 5 show the response in top composition y_D (y_2) of the Wahl and Harriot low-order model to a step change in feed composition with the composition on plate 4 (y_1) under feedback control. The controller tuning (PI controller) used here is somewhat different than the one used by Wahl and Harriot, but the responses resemble closely the ones shown in Wahl and Harriot (1970) (actually Wahl and Harriot have the wrong sign on the change in top composition), i.e., a fast response in the composition on plate 4 (y_1) with a slow settling toward steady state for the uncontrolled top composition (y_2). The slow settling in y_2 is noticed by Wahl and Harriot, but they assume it to be a property of the process. However, the slow settling to steady state is simply a result of a modeling error; that is, the model has an excessive slow pole. This is seen from the solid lines in Figure 5 which show the responses obtained using the full linear model. The full model yields a fast response in both compositions.

Also several other authors (e.g., Skogestad et al., 1990a; Sandelin et al., 1991) have used inconsistent models for studies of partial feedback control in distillation. This may be seen from their figures by observing the slow settling in the uncontrolled output. Indeed, as we discuss below, obtaining simple and consistent low-order models for high-purity distillation columns is a challenging problem.

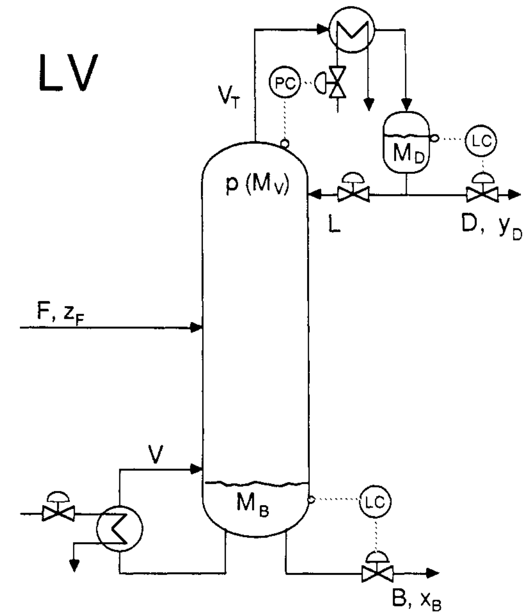

Figure 6. Two-product distillation column with reflux L and boilup V as independent variables.

Table 2. Steady-State Data for Distillation Column (See Also Figure 6)[a]

z_F	α	N	N_F	$1-y_D$	x_B	D/F	L/F	V/F	M_i/F (min)	τ_L (min)
0.5	1.5	40	21	0.01	0.01	0.500	2.706	3.206	0.5	0.063

[a] Feed F is liquid.

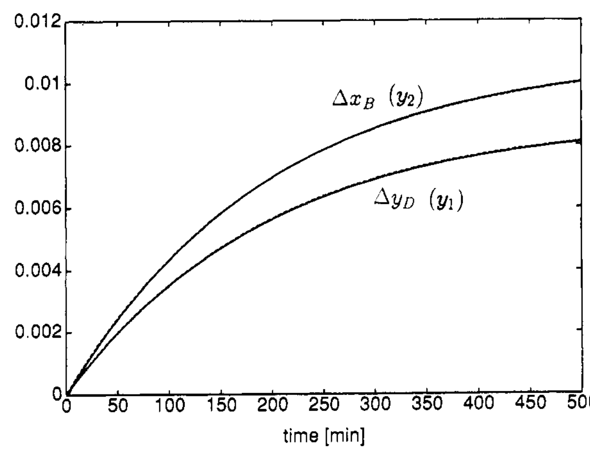

Figure 7. Open-loop responses of distillation column in Table 2 to 1% step change in reflux L. Solid lines: responses of full linear model. Dashed lines: responses of fitted low-order model N1 (11).

Development of Consistent Distillation Models. The discussion in this section is based on a case study of a relatively high-purity distillation column. Data for the column (denoted "column A" in Skogestad and Morari (1987, 1988)) are given in Table 2. We will consider reflux L and boilup V as independent variables (LV configuration). The model is ill-conditioned at steady state where the condition number is 142 and the RGA is 35.5.

Open-loop responses in product compositions y_D and x_B to step changes in reflux L (keeping boilup V fixed) using a full linear model with 82 states are shown in Figure 7. The states are the mole fraction of light component and the total holdup on each stage. Note that liquid flow dynamics, which were neglected in Skogestad and Morari

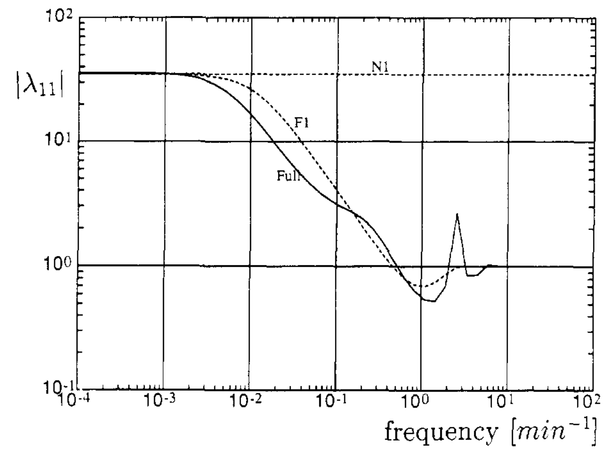

Figure 8. RGA as a function of frequency for distillation column example. N1: fitted low-order model without flow dynamics (11). F1: fitted low-order model with flow dynamics included (13). Full: full 82th-order linear model.

(1987, 1988) are included in this "full" model. It is critical to include flow dynamics if the model is used for "two-point" control studies where both top and bottom compositions are controlled (e.g., Skogestad and Lundström, 1990). The reason is that the flow dynamics provide for varying holdup of liquid and thereby introduce a liquid lag which decouples the high-frequency responses of the top and bottom parts of the column. For the example column the liquid lag from the top to the bottom of the column is about $\theta_L = 2.5$ min.

Model N1. Despite the high order of the model, the responses in Figure 7 seem to be almost pure first-order with a time constant of approximately $\tau_1 = 194$ min. Fitting each transfer matrix element to a first-order response with time constant $\tau_1 = 194$ min yields a model on the same form as in (2):

$$\begin{pmatrix} d_{yD} \\ d_{xB} \end{pmatrix} = \frac{1}{1+\tau_1 s}\begin{pmatrix} 0.878 & -0.864 \\ 1.082 & -1.096 \end{pmatrix}\begin{pmatrix} dL \\ dV \end{pmatrix} \quad (11)$$

(The "standard" approach would be to fit the responses independently and obtain four slightly different time constants. However, the results would have been very similar.) This model is denoted N1. N means no flow dynamics (since we have made no attempt to fit the initial part of the response where the flow dynamics are important), and 1 indicates one time constant. As seen from Figure 7, this simple model gives an almost perfect fit of the overall open-loop responses of the full model with 82 states. However, as will become clear, the model N1 has two fundamental flaws.

First, an analysis of the full 82 state model reveals that it has only a single pole at $-1/\tau_1$, so the model N1 is inconsistent in that it contains two poles at $-1/\tau_1$. From the previous section it seems clear that this inconsistency will give a poor prediction of the behavior of the full plant under one-point control, and this is indeed confirmed by simulations (not shown).

Second, it has been shown before (Skogestad and Lundström, 1990; Skogestad et al., 1990a–c) that the type model (11) is poor also for the case with both compositions under feedback control ("two-point control"). The main reason is that the flow dynamics are not included such that the directionality of the process, in particular at intermediate and high frequencies, is poorly predicted. To see this, consider Figure 8, which shows the RGA plotted as a function of frequency for both the full model with 82 states and the fitted model N1. Model N1 has $\lambda_{11} = 35.5$ over all frequencies, that is, strong directional dependence at all frequencies. On the other hand, the RGA for the full model breaks off at intermediate frequencies and becomes unity at approximately frequency $1/\theta_L$ where θ_L is the liquid lag from top to bottom. The real process is therefore only weakly directionally dependent at high frequencies.

We now want to study how to develop an improved simple low-order model of a distillation column which (1) does not contain excessive slow poles, and (2) has correct directions at intermediate and high frequencies. However, let us first consider the low-order models most commonly presented in the distillation literature.

Standard Low-Order Models. Most low-order models of distillation columns presented in the literature are obtained by fitting responses to a model of the type first order plus dead time, that is, with elements

$$g_{ij}(s) = \frac{k_{ij}e^{-\theta_{ij}s}}{1+\tau_{ij}s} \quad (12)$$

If the delays were only associated with the inputs (e.g., valves) and/or outputs (e.g., measurements), the sum of delays would be equal in the diagonal and off-diagonal elements. However, a study of models reported reveals that most of them have a larger sum of delays in the off-diagonal elements than in the diagonal elements (e.g., Wood and Berry, 1973; Hammarström et al., 1982; Waller et al., 1988). This seems reasonable and is probably a result of the flow dynamics. For example, it takes time for the reflux L to affect x_B and the 2,1 element, $g_{21}(s)$, of the transfer matrix should contain an additional lag. Most authors use pure dead times to represent this, while the flow dynamics physically is a high-order lag.

Model F1. Let us now try to improve the simplified model N1 (11) by including explicitly the flow dynamics which yield a liquid lag (delay) from the top to the bottom of the column of $\theta_L = 2.5$ min. We do this by simply "adding" a lag term $g_L(s)$ to the off-diagonal 2,1 element as discussed above, and obtain model F1 (F denotes flow dynamics)

$$\begin{pmatrix} dy_D \\ dx_B \end{pmatrix} = \frac{1}{1+\tau_1 s}\begin{pmatrix} k_{11} & k_{12} \\ k_{21}g_L(s) & k_{22} \end{pmatrix}\begin{pmatrix} dL \\ dV \end{pmatrix} \quad (13)$$

where in our case (time is in minutes)

$$\tau_1 = 194; \quad \mathbf{K} = \begin{pmatrix} 0.878 & -0.864 \\ 1.082 & -1.096 \end{pmatrix}; \quad g_L(s) = \frac{1}{(1+0.5s)^5}$$

For our column with 39 stages (plus reboiler and total condenser), the best representation of the liquid lag would be to use $g_L(s) = (1 + \theta_L/N)^N$ with $N = 39$. However, we here use a fifth-order approximation to obtain a model of reasonably low order.

We see from the RGA plot in Figure 8 that we now obtain a much better fit of the directionality of the process. The model F1 has been studied by Skogestad et al. (1990a) and Jacobsen et al. (1991), and they concluded that it was a reasonably good model when both outputs are controlled simultaneously ("two-point" control). However, we find that including the flow dynamics does not correct the fundamental error of excessive slow poles, and the model will be poor for studies of partially controlled distillation columns. This is seen from Figure 9, which shows the response of the model under one-point control. Top composition y_D is controlled by reflux L using a PI controller while x_B is left uncontrolled. As seen from curve F1, the model yields an incorrect slow settling toward

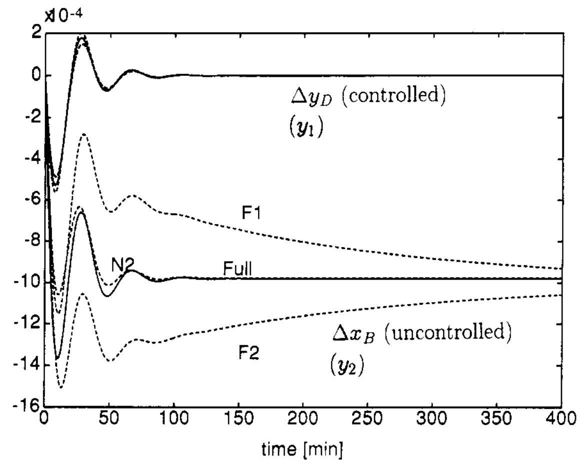

Figure 9. Dynamic response of distillation column with one loop closed. Response to a 1% increase in boilup. Top composition y_D controlled by reflux L using a PI controller. Full: full 82th-order linear model. F1: fitted low-order model with flow dynamics included (13). N2: two-time-constant model (14). F2: two-time-constant model (14) with flow dynamics included.

steady state for the uncontrolled output x_B. As model F1 is similar to the standard low-order model most commonly used in the literature for fitting data, it follows that most simple models presented in the literature are inconsistent in that they contain excessive slow poles.

Model N2. Skogestad and Morari (1988) studied the case *without* flow dynamics and suggest to use a two-time-constant model with the dominant time constant τ_1 for the high-gain direction ($dL = -dV$) and a smaller time-constant τ_2 for the low-gain direction ($dL = dV$) (model N2)

$$\begin{pmatrix} dy_D \\ dx_B \end{pmatrix} = \begin{pmatrix} \frac{k_{11}}{1+\tau_1 s} & \frac{k_{11}+k_{12}}{1+\tau_2 s} - \frac{k_{11}}{1+\tau_1 s} \\ \frac{k_{21}}{1+\tau_1 s} & \frac{k_{21}+k_{22}}{1+\tau_2 s} - \frac{k_{21}}{1+\tau_1 s} \end{pmatrix} \begin{pmatrix} dL \\ dV \end{pmatrix} \quad (14)$$

For our example column $\tau_1 = 194$ min and $\tau_2 = 15$ min. The minimal realization of this model contains only one "slow" pole at $-1/\tau_1$ and is thus consistent in this respect, and it also gives an excellent fit of a full 41th-order model which results from neglecting the flow dynamics. The N2 model also agrees well with the 82th-order full model with flow dynamics for the case of one-point control. This is seen from the curve N2 in Figure 9. However, model N2 (14) does not include flow dynamics and may therefore be relatively poor for two-point control studies (Skogestad and Lundström, 1990; Jacobsen et al., 1991).

Model F2. To improve the model N2, one may try to "add" flow dynamics $g_L(s)$ to (14) as we did for model N1 above.

$$\begin{pmatrix} dy_D \\ dx_B \end{pmatrix} = \begin{pmatrix} \frac{k_{11}}{1+\tau_1 s} & \frac{k_{11}+k_{12}}{1+\tau_2 s} - \frac{k_{11}}{1+\tau_1 s} \\ \frac{k_{21}}{1+\tau_1 s} g_L(s) & \frac{k_{21}+k_{22}}{1+\tau_2 s} - \frac{k_{21}}{1+\tau_1 s} \end{pmatrix} \begin{pmatrix} dL \\ dV \end{pmatrix} \quad (15)$$

This model, denoted F2, is used by Skogestad and Lundström (1990) and Skogestad et al. (1990b,c) and yields good results for two-point-control studies. However, adding the lag term $g_L(s)$ to the 2,1 element of (14) does again result in a minimal realization with two slow poles at $-1/\tau_1$, and therefore the model is poor for one-point-control studies. This is seen from curve F2 in Figure 9 where we again get an incorrect slow settling towards the new steady-state for the uncontrolled output.

Conclusion Analytic Low-Order Models. We have not been able to obtain a simple "analytic" (in the meaning that all parameters have physical significance) low-order model for high-purity columns which is consistent in terms of the number of slow poles (and is then useful for one-point-control studies) and at the same time includes flow dynamics (and is then useful for two-point-control studies). In addition, it seems difficult to include the effect of disturbances in F and z_F in a consistent manner (see discussion below).

Model Reduction. On the other hand, by mathematical model reduction of the full linear model it is possible for this example to obtain low-order models with only two states which are good for both one- and two-point-control studies (Jacobsen et al., 1991). For instance, applying the optimal Hankel approximation without balancing (Safonov et al., 1987) to reduce the full 82nd-order linear model (with flow dynamics included) to a 2nd-order model yields the model

$$\begin{pmatrix} dy_D \\ dx_B \end{pmatrix} = \frac{1}{(1.61s+1)(194s+1)} \times \begin{pmatrix} 0.871(2.17s+1) & -0.861(0.721s+1) \\ 1.089(0.45s+1) & -1.101(3.48s+1) \end{pmatrix} \begin{pmatrix} dL \\ dV \end{pmatrix} \quad (16)$$

Note that the small time constant of 1.61 min is much smaller than the value $\tau_2 = 15$ min used in model 14 which applies when flow dynamics are neglected. Model 16 has a minimal realization with only one slow pole at $-1/\tau_1$ and is a good model for both one- and two-point-control studies. However, the model structure resulting from mathematical model reduction is not "analytic" in the sense that it is parametrized in terms of physical parameters. For instance, in model 16 the decoupling at intermediate and high frequencies, physically caused by the flow dynamics, is described by letting the zeros in the off-diagonal elements be smaller than in the diagonal elements.

Proposed Low-Order Structure for Distillation Columns. Based on model 16, one possible model structure for obtaining consistent low-order models is the following:

$$G(s) = \frac{1}{(\tau_1 s+1)(\tau_2 s+1)} \begin{pmatrix} k_{11}(z_{11}s+1) & k_{12}(z_{12}s+1) \\ k_{21}(z_{21}s+1) & k_{22}(z_{22}s+1) \end{pmatrix} \quad (17)$$

There are 10 parameters in this model, but only 8 of these are independent since the model should only have two states; that is, we must have two multivariable zeros at $-1/\tau_1$ and $-1/\tau_2$, respectively. [Alternatively, we may use a state space formulation, $\mathbf{G}(s) = \mathbf{C}(s\boldsymbol{I} - \mathbf{A})^{-1}\mathbf{B}$ with two states. This formulation also has eight independent parameters; consider for example a canonical form with $\mathbf{C} = \mathbf{I}$ and where each of the matrices $\mathbf{A}$ and $\mathbf{B}$ contains four parameters.] We now discuss how the parameters in (17) may be obtained.

The steady-state gains k_{ij} and the dominant time constant τ_1 are physically motivated and may be obtained directly from specific experiments. However, some care must be exercised when obtaining the gains since it is difficult to observe the low-gain direction of the plant (Skogestad and Morari, 1988; Andersen et al., 1989), and one may for high-purity columns with large RGA values easily get the wrong sign of the RGA (and the determinant) of the steady-state gain matrix, $\mathbf{K}$. The model will then be useless for feedback control purposes. This may be corrected by performing separate experiments for changes in internal flows [e.g., by using the DV configuration as suggested by Skogestad and Morari (1988) Alsop and Edgar (1990), Andersen and Kümmel (1991), and Kuong and McGregor (1991)], or by adjusting the steady-state gains to match an estimated steady-state RGA value as suggested by Jacobsen et al. (1991), or by using a "perturbed model" (Kapoor et al., 1986) based on steady state where the RGA elements are smaller. The basis for the two last suggestions is that the steady-state behavior is not of primary importance for feedback control.

The remaining five parameters, τ_2 and the four zeros (of which only three are independent), may be obtained based on fitting time responses for the compositions. It is also advisable to obtain separate data for the flow dynamics so as to get the correct decoupling at intermediate and high frequencies.

6. Discussion

Identification. The general literature on identification has so far not treated multivariable issues in much detail. In the process control literature some discussion is given by Andersen et al. (1991). There do not seem to exist identification algorithms that are well suited to identify, on the basis of "realistic" (noisy) data generated from the model in eq 1, a model that is consistent in terms of the number of slow poles. In particular, this is the case if the model structure and order is unknown. Indeed, we have proposed (Jacobsen and Skogestad, 1993) to use the heat exchanger described by eq 1 as a benchmark problem for identification.

The identification problems are mainly associated with the weak direction. The 2 × 2 processes studied in this paper contain only one slow pole, and since a rank 2 model requires at least two states, we need to identify at least one more time constant. For the heat-exchanger example the small time constant τ_2 appears in the weak (low-gain) direction, and may in theory be identified by doing a simultaneous step increase in hot and cold flow of the same magnitude, i.e., applying an input vector $[1\ 1]^T$. However, due to input errors, this is difficult to realize in practice, and only small errors will yield changes in the high-gain-input direction and make it difficult to observe the time constant τ_2.

If the model structure is known, such that one explicitly models the "weak" direction, the problem becomes much simpler. For the heat-exchanger example, the exact model structure is given by eqs 26 and 27 in the Appendix. We note, for example, that the effect of disturbance 2 on output 1 is $\alpha/(\tau s + 1)(\tau s + 1 + 2\alpha)$, and it should be possible to estimate the parameters τ and α from a single open-loop experiment. The entire matrices $\mathbf{G}_d(s)$ and $\mathbf{G}(s)$ may then be obtained.

Number of Slow Modes. Although all the processes studied in this paper contain only a single slow pole, there are of course many processes that contain several slow poles. When doing open-loop identification, it is necessary to know in advance how many slow poles the process actually contains. In a well-designed high-purity distillation column there will usually only be one slow pole, while an overdesigned high-purity column with a pinch in the composition profile usually will have two slow poles. The reason is that the pinch tends to decouple the column into two sections with relatively little interaction, resulting in one slow pole related to each of the two column sections. An open-loop blackbox identification method will not be able to discriminate between the two cases, and some physical knowledge needs to be added. Closed-loop identification, on the other hand, is likely to reveal the fact that there indeed is a single dominant time constant in the process. Thus, for ill-conditioned processes where physical knowledge is lacking, one should apply some closed-loop identification method when obtaining low-order models.

Disturbance Modeling. The linear process model is often written

$$y(s) = \mathbf{G}(s)\, u(s) + \mathbf{G}_d(s)\, d(s) \tag{18}$$

In this paper we have only discussed obtaining low-order models, $\mathbf{G}(s)$, for the effect of inputs. However, the main purpose of process control is usually to reject the effect of disturbances, d, entering the process, so a disturbance model, $\mathbf{G}_d(s)$, is usually also required. The common approach is to identify the disturbance model $\mathbf{G}_d(s)$ independently of $\mathbf{G}(s)$ (e.g., Shunta and Luyben, 1972; Waller et al., 1988), as is suggested from the model form in (18). However, in reality, these models are usually coupled and share the same states. For the processes studied in this paper the single dominating pole seen in $\mathbf{G}(s)$ will also appear in $\mathbf{G}_d(s)$ and will also dominate the open-loop responses to disturbances. If the fitted disturbance model $\mathbf{G}_d(s)$ is inconsistent with the input model $\mathbf{G}(s)$, then these inconsistencies may appear under feedback control. For instance, for our distillation example using the low-order model F1 (13) with time constant τ_1, we find using two-point control that the responses to *setpoint changes* are reasonably correct compared to the full model. However, with a simple first-order model with the same time constant τ_1 for disturbances in feed flow rate F, we find that the simulated response to a *disturbance* in F is erroneous due to inconsistent slow poles left in the disturbance model.

Thus, care needs to be taken also when identifying the disturbance model of an ill-conditioned process. One approach is to use a state space form as the basis for parameter fitting, that is, use

$$\dot{x} = \mathbf{A}x + \mathbf{B}u + \mathbf{E}d; \quad y = \mathbf{C}x$$

This approach will ensure that the models $\mathbf{G} = \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B}$ and $\mathbf{G}_d = \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{E}$ share the same states.

Practical Implications of Inconsistency. The inconsistency we present in this paper represents a fundamental model error. However, one may question whether it has any important practical implications. First, in the case of partial control the inconsistency mainly has implications for the uncontrolled output and does not affect the tuning of the control loop significantly if one considers the response in the controlled output only. Second, we concluded above that some models which are inconsistent, e.g., model $F2$ for the distillation column, yield good results when used for tuning two-point controllers based on setpoint changes only.

However, we argue here that there are many practical situations where the inconsistency presented may yield misleading results. Consider first the case with partial control. In industrial practice partial control is commonly employed and in many situations the control is applied to an output which is not of primary importance. As an example, consider the control of distillation columns. Few industrial columns have feedback control of the primary outputs, i.e., the product compositions, due to significantly delayed or even lack of composition measurements. Instead, a temperature on a plate inside the column is controlled in order to reduce sensitivity to disturbances entering the column. Thus, one applies feedback control to an output which is of little importance in order to improve the dynamic characteristics of some uncontrolled outputs which are of primary importance. If an inconsistent model is used to study the closed-loop behavior, in this case one will end up with erroneous conclusions regarding the effect of the feedback controller on the primary outputs (see, e.g., Wahl and Harriot, 1970).

In addition, as discussed above, if a two-point controller is tuned for disturbance rejection based on input and disturbance models that are inconsistent, the resulting controller is likely to yield a poor performance when applied to the process.

7. Conclusions

The open-loop responses of ill-conditioned processes often take the form of almost pure first-order dynamics. The responses are dominated by a single slow pole resulting from interactions in the process. The open-loop dynamics of such processes are seemingly well approximated by a low-order model containing only the dominant time constant. However, the model will contain an excessive number of slow poles and is therefore physically inconsistent.

The inconsistency results in a poor prediction of the process behavior under partial feedback control. It is sufficient to close one feedback loop to move the single slow pole of the process, and thus make all outputs respond quickly. However, for models with excessive slow poles, at least one slow pole will be left when one feedback loop is closed, causing erroneous slow responses in the uncontrolled outputs.

We have found that it is difficult to define a physically motivated low-order model structure for high-purity distillation columns which contains the flow dynamics and is consistent in terms of the number of slow poles. Obtaining consistent low-order dynamic models of high-purity distillation columns is therefore an open research problem.

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Nomenclature

A = heat-transfer area (m^2)
$\mathbf{A}$ = Jacobian state matrix
c_P = heat capacity (kJ/(°C kg))
D = distillate flow (kmol/min)
d = process disturbance vector
F = feed flow (kmol/min)
$\mathbf{G}(s)$ = process transfer matrix for effect of inputs u
$\mathbf{G}_d(s)$ = process transfer matrix for effect of disturbances d
$g_{ij}(s)$ = transfer matrix element i, j
k_{ij} = steady-state process gains
$\mathbf{I}$ = identity matrix
K = controller gain
L = reflux rate (kmol/min)
N = number of theoretical trays
N_F = feed tray
q_C = cold inlet flow (m^3/min)
q_H = hot inlet flow (m^3/min)
T_C = cold outlet temperature (°C)
T_H = hot outlet temperature (°C)
U = heat-transfer coefficient (kJ/(m^2 °C min))
u_i = process input i (deviation variable)
V = boilup rate (kmol/min)
V_C = liquid volume cold side (m^3)
V_H = liquid volume hot side (m^3)
x_B = bottoms composition
$Y = k_{12}k_{21}/k_{11}k_{22}$ = interaction measure
y_D = distillate composition
y_i = process output i (deviation variable)
z_F = feed composition

Greek Symbols

α = relative volatility
γ = condition number
λ_{11} = 1,1 element of RGA
τ_1 = dominant (largest) process time constant (min)
τ_2 = smaller process time constant (min)
τ_{CL} = closed-loop time constant (min)
τ_L = lag in liquid response for individual tray (min)
θ_L = lag in liquid response from top to bottom of column (min)

Subscripts

s = setpoint change

Appendix. Simple Model of Heat Exchanger

Consider a very simplified heat exchanger with one mixing tank on each side as shown in Figure 1. Assume constant volumes, V, on each side, and constant values of ρ and c_P. A heat balance for the cold and hot side then yields

$$\tau_c \frac{dT_C}{dt} = \frac{q_C}{q_C^*}(T_{Ci} - T_C) + \alpha_C(T_H - T_C) \tag{19}$$

$$\tau_H \frac{dT_H}{dt} = \frac{q_H}{q_H^*}(T_{Hi} - T_H) - \alpha_H(T_H - T_C) \tag{20}$$

where q^* denotes the nominal (steady-state) flow, and

$$\tau_C = \frac{V_C}{q_C^*};\quad \alpha_C = \frac{UA}{\rho_C q_C^* c_{PC}} \tag{21}$$

$$\tau_H = \frac{V_H}{q_H^*};\quad \alpha_H = \frac{UA}{\rho_H q_H^* c_{PH}} \tag{22}$$

Linearizing the model assuming UA and thus α constant (independent of flow and temperature), introducing deviation variables, and taking Laplace transforms yields

$$\tau_C s T_C(s) = T_{Ci}(s) - T_C(s) + (T_{Ci}^* - T_C^*)\frac{q_C(s)}{q_C^*} + \alpha_C(T_H(s) - T_C(s)) \tag{23}$$

$$\tau_H s T_H(s) = T_{Hi}(s) - T_H(s) + (T_{Hi}^* - T_H^*)\frac{q_H(s)}{q_H^*} - \alpha_H(T_H(s) - T_C(s)) \tag{24}$$

where the superscript * denotes steady-state values. In this paper we consider a symmetric case with $\tau_C = \tau_H = \tau$, $\alpha_C = \alpha_H = \alpha$ and $q_C^* = q_H^* = q^*$. Rearranging yields

$$y = \mathbf{G}(s)u + \mathbf{G}_d(s)d;\quad y = \begin{pmatrix} T_C(s) \\ T_H(s) \end{pmatrix},\quad u = \begin{pmatrix} q_C(s) \\ q_H(s) \end{pmatrix},\quad d = \begin{pmatrix} T_{Ci}(s) \\ T_{Hi}(s) \end{pmatrix} \tag{25}$$

where

$$\mathbf{G}_d(s) = \frac{1}{(\tau s + 1)(\tau s + 1 + 2\alpha)} \begin{pmatrix} \tau s + 1 + \alpha & \alpha \\ \alpha & \tau s + 1 + \alpha \end{pmatrix} \tag{26}$$

and

$$\mathbf{G}(s) = \mathbf{G}_d(s)\begin{pmatrix} -c & 0 \\ 0 & c \end{pmatrix}; \quad c = \frac{T_{Hi}^* - T_H^*}{q^*} = -\frac{T_{Ci}^* - T_C^*}{q^*} \tag{27}$$

Inserting the numerical values $\tau = 100$ (min), $\alpha = 20$, $q^* = 0.01$ (m^3/min) (see data in Table 1) finally yields

$$\mathbf{G}_d(s) = \frac{0.02439}{(100s + 1)(2.439s + 1)}\begin{pmatrix} 21(1 + 4.76s) & 20 \\ 20 & 21(1 + 4.76s) \end{pmatrix} \tag{28}$$

$$\mathbf{G}(s) = \mathbf{G}_d(s)\begin{pmatrix} -3659 & 0 \\ 0 & 3659 \end{pmatrix} \tag{29}$$

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