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DYNAMIC MODELS FOR HEAT EXCHANGERS AND HEAT EXCHANGER NETWORKS‡

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ABSTRACT

Dynamic models are needed to assess controllability of heat exchangers and heat exchanger networks. A simple model is desirable, but all important model features must be included. We discriminate between important and less important model features by order of magnitude argumentation, comparison of controllability measures and dynamic simulations. Important model features for single heat exchangers are model order, wall capacitance and fluid compressibility, whereas flow configuration and temperature driving force have only a small effect on the dynamics. The most important model feature for heat exchanger *networks* is residence time of the connecting pipes. Simpler models may fail to identify inherent control limitations such as zeros in the right half plane.

INTRODUCTION

The steady-state energy balance for heat exchangers is

$$w^h(T_i^h - T_o^h) = w^c(T_o^c - T_i^c) = UA\Delta T_{hx} \tag{1}$$

where superscript h means hot side, c cold; $w = FM_w c_p$ heat capacity flowrate and $U = h^h * h^c/(h^h + h^c)$ overall heat transfer coefficient. The overall temperature driving force of the heat exchanger, ΔT_{hx}, depend on the flow configuration. By defining the static heat exchanger effectiveness $P = (T_o^c - T_i^c)/(T_i^h - T_i^c)$ and heat capacity flow ratio $R = w^c/w^h = (T_i^h - T_o^h)/(T_o^c - T_i^c)$ the steady-state transfer function between inlet and outlet temperatures may be expressed as:

$$\begin{bmatrix} T_o^h \\ T_o^c \end{bmatrix} = \begin{bmatrix} 1 - RP & RP \\ P & 1 - P \end{bmatrix} \begin{bmatrix} T_i^h \\ T_i^c \end{bmatrix} \tag{2}$$

where P is a function of flow configuration, number of transfer units ($N_{TU} = (UA)/w^c$) and R only. Both P and RP are physically bounded between zero and unity, and they are usually below 0.8. R and N_{TU} are often between 0.2 and 5 except for reboilers and condensers.

The following model features will be evaluated: 1) heat transfer coefficients, 2) model order, 3) temperature driving force, 4) wall capacitance, 5) flow configuration, 6) fluid compressibility and 7) pipe residence time. Primarily, we are interested in prediction of control limitations such as apparent dead-time and right half plane (RHP) zeros. We have chosen to use lumped heat exchanger models, where each stream is modelled as a series of mixed tanks, because of mathematical simplicity as well as physical resemblance to shell-and-tube exchangers with baffles. Lumped models also match experimental frequency response data well. Furthermore, when a lumped compartment or

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cell model is used, distributed model behaviour may be achieved by using a large number of cells or using the logarithmic mean temperature difference as the temperature driving force (Reimann, 1986; Rinard and Nieto, 1990). The latter represents a hybrid between a lumped and a distributed model. Lumped models are preferred to empirical models to get a rational transfer function and to be able to simulate different flow configurations. Furthermore, low-order heat exchanger models may become "ill-consistent" in closed-loop (Jacobsen and Skogestad, 1993). Fig. 3 is a schematic representation of a 1-2 heat exchanger with 12 cells. In addition to ideal mixing tank assumptions we assume negligible heat loss, constant heat capacity, and that exchanger area A and volume V are equally distributed over the N cells. For liquid exchangers fluid densities are assumed constant and pressure drop is neglected. For gas exchangers densities are computed from ideal gas law and flowrates from pressure drop.

The dynamic characteristics for heat exchangers are determined by flow configuration and time constants related to three holdups of energy: 1) hot side, 2) wall, 3) cold side. They may be derived as (respectively):

$$\tau_F^h = \frac{V^h \rho^h}{F^h M_w^h} \qquad \tau_k = \frac{(V^w \rho^w c_p^w)}{(h^h + h^c)A} \qquad \tau_F^c = \frac{V^c \rho^c}{F^c M_w^c}$$

where τ_F is the residence time. Typical values are $0.5 < \tau_F < 60$ seconds and $0.1 < \tau_k < 30$ seconds. Differences in heat transfer coefficient of single- and multiphase fluids, density of gases and liquids and area density (m^2/m^3) of different heat exchangers are the main reasons for the large ranges. In most of our examples we use assume a liquid heat exchanger with equal heat capacity flowrates ($R = 1$) and heat tranfer coefficients ($h^h = h^c$) where the time constants are $\tau_F^h = \tau_F^c = 32$ seconds and $\tau_k = 14$ seconds.

MODEL FEATURES FOR SINGLE HEAT EXCHANGERS

Heat transfer coefficients

Usually heat transfer coefficients are based on a distributed heat exchanger model (i.e., U_{lm}). Heat transfer coefficients used in lumped models (U_N) must be increased to give the same overall effectiveness, larger increases needed for fewer cells. By combining effectiveness-expressions for one single mixing tank (Stevens *et al*, 1957) with equations for series of countercurrent heat exchangers (Domingos, 1969), the necessary heat transfer coefficient may be computed from the number of lumped cells. For equal heat capacity flowrates the relationship is simple:

$$N = \frac{N_{TU,lm}}{1 - U_{lm}/U} \qquad N_{TU,lm} = \frac{U_{lm} A}{w} \tag{3}$$

where N is the number of cells in the lumped heat exchanger model. When different model features yield different temperature driving forces, we adjust the heat transfer coefficients to get the same static thermal effectiveness. Interestingly, this will tend to distribute the temperature driving forces more equally among the exchangers in the network, and thus remove a weakness of conceptual designs based on logarithmic temperature driving forces.

From steady-state considerations the heat transfer coefficients dependence on flowrate should be included because the effects are significant. This will also affect dynamics by increasing the speed of response to flowrate variations. The effect is illustrated in Table 1 where the asymptotic phase shifts for different model features are compared. Note that the phase shift from cold flowrate is reduced when flow-dependent heat transfer coefficients are introduced. This makes it more desirable to include wall capacity.

Number of mixing tanks (model order)

A one-cell model will fail to predict the apparent dead-time of countercurrent heat exchangers and must be rejected from dynamic considerations. However, steady-state arguments may also be used to reject the *pure lumped* one-cell model. Consider a heat exchanger with equal heat capacity flowrates ($R = 1$) where it is assumed that the inlet temperatures are manipulated inputs (for example by manipulating bypasses of upstream heat exchangers on each stream) and the outlet temperatures are the controlled outputs. Even with infinite heat transfer coefficients, the thermal

Transfer function to T_o^h from:	T_i^h	T_i^c	F^h	F^c
Lumped; h const.; w/o wall cap.; incompr.	$-N\pi/2$	$-\pi$	$-\pi/2$	$-\pi$
Lumped; h flow-dep.; w/o wall cap.; incompr.	$-N\pi/2$	$-\pi$	$-\pi/2$	$-\pi/2$
Hybrid; h const; w/o wall cap.; incompr.	$-N\pi/2$	$-\pi/2$	$-\pi/2$	$-\pi$
Hybrid; h flow-dep.; w/o wall cap.; incompr.	$-N\pi/2$	$-\pi/2$	$-\pi/2$	$-\pi/2$
Lumped; h flow-dep.; w/ wall cap.; incompr.	$-N\pi/2$	$-3\pi/2$	$-\pi/2$	$-\pi$

Table 1: Different model assumptions and model features give different asymptotic phase shifts of the frequency response.

	Min. from steady-state	Min. from dynamics	Max.
Pure lumped model	N_{TU}	2	$(N_B+1)N_P$
Hybrid model	1	3	$(N_B+1)N_P$

Table 2: Recommended number of cells.

effectiveness P cannot exceed 0.5, and the transfer matrix (Eq. 1) has full rank for all parameter combinations. For two or more cells the effectiveness may become 0.5 which makes the system singular. This occurs when the outlet temperatures become equal (Reimann, 1986; Mathisen and Skogestad, 1992).

From Eq. 3 it is clear that the number of cells N must be greater than the number of transfer units N_{TU} (U will approach infinity as N approaches $N_{TU,lm}$). This simple steady-state consideration seems to have been overlooked by previous authors (e.g., Papastratos *et al*, 1992). However, to be able to predict the apparent dead-time with good accuracy even more cells are usually necessary, see Fig. 1. For shell and tube heat exchangers, the number of cells is usually recommended to be one above the number of baffles (N_B) times the number of tube passes (N_P), which seems intuitively attractive.

Temperature driving force (hybrid or pure lumped model)

The hybrid model is attractive because heat transfer coefficients are often based on a distributed model and only one cell is necessary to match any steady-state thermal effectiveness. A comparison of the overall phase shift of the lumped and the hybrid model is given in Table 1. Note that the phase shift from cold inlet temperature and flowrate is reduced when using a hybrid model, which favors the pure lumped model compared to the hybrid model. The recommended numbers of cells are summarized in Table 2.

Wall capacitance

The time constants for energy holdup of the fluids and the wall are related:

$$\tau_k = \frac{(V^w \rho^w c_p^w)}{(h^h + h^c)A} = C\frac{V^w \rho^w c_p^w}{V^c \rho^c c_p^c}\frac{\tau_F^c}{N_{TU}} \qquad C\varepsilon\,[0.25, 1] \tag{4}$$

For liquid heat exchangers the ratio $(V^w \rho^w c_p^w)/(V^c \rho^c c_p^c) > 1$ may well be larger than one, and this indicate that the commonly used assumption that wall capacitance may be neglected for liquid exchangers is not valid. For gas exchangers the ratio is at least an order of magnitude larger due to the lower fluid density. Thus, the wall capacitance is expected to completely dominate the dynamics of gas exchangers, and this is in accordance with previous results. Note that the time constants for energy holdup are related through the number of transfer units N_{TU}, revealing a close connection between steady-state and dynamic behaviour of heat exchangers.

A numerical comparison with and without wall capacitance is shown in Fig. 2. The time simulation confirms that there is a considerable delay in the response when the wall capacitance is included.

Flow configuration

Countercurrent 1-1 heat exchangers with one tube and one shell pass are almost invariably assumed during conceptual design. Due to mechanical, maintenance or pressure drop considerations heat exchangers with two tube passes per shell pass (1-2 exchangers) are more common in practice. Thus, we will compare dynamic behaviour of 1-1 and 1-2 exchangers, and have selected heat transfer coefficients and the number of transfer units to give the same steady-state thermal effectiveness.

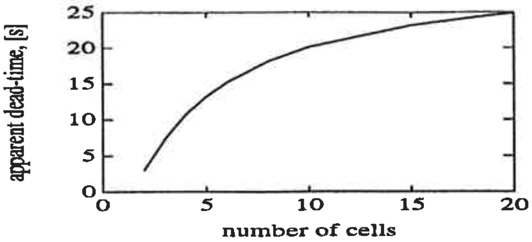

Fig. 1: Apparent dead-time from hot inlet to hot outlet temperature as function of number of cells. 1-1 exchanger with thermal effectiveness P = 0.5.

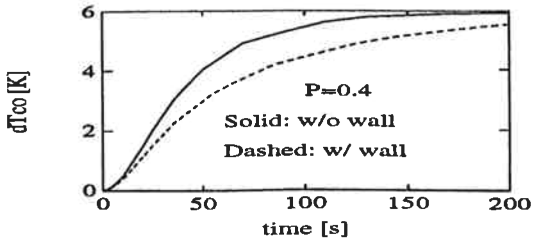

Fig. 2: Cold temperature response to 10 K hot inlet temperature increase of 1-1 heat exchanger with and without wall capacitance.

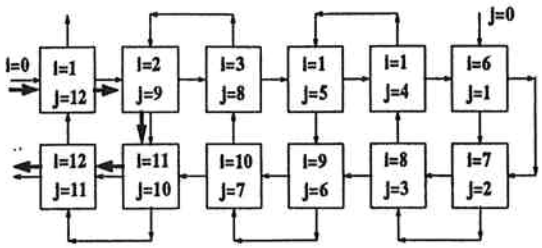

Fig. 3: Cell model of 1-2 heat exchanger with short-cut from inlet to outlet temperature of tube fluid.

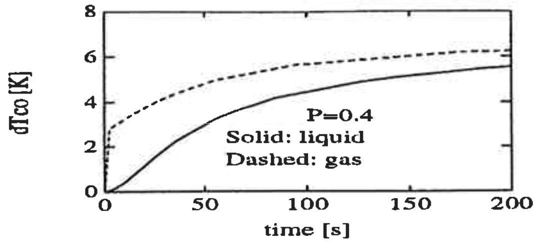

Fig. 4: Cold temperature response to 10 K cold inlet temperature increase of liquid and gas 1-1 exchangers.

Interestingly, the dynamic response from 1-2 exchangers may become different from 1-1 exchangers because of the short-cut via the opposite stream, see Fig. 3, and this appears as a dip in the phase response (Wolff *et al*, 1991). The temperature responses for 1-1 and 1-2 exchangers with the first tube pass countercurrent were found to be quite similar. The short-cut path including conduction from cold to hot and back is relatively slow, which agrees with our conclusion that wall capacitance is important for the overall dynamics. Thus, discrimination between single and multiple tube pass exchangers is not critical for controllability assessment.

Fluid compressibility

Dynamics of gas and liquid exchanger have important differences which are mainly due to two factors concerning the density. Firstly, fluid density of gases is much lower than for liquids, which gives higher volumetric flowrates and shorter residence times. Secondly, the heat and mass balances are coupled due to the compressibility. In gas exchangers an inlet temperature changes the flowrate, and therefore has a much faster effect on the outlet temperature on the same side than in a liquid exchanger. An inlet pressure change of gas exchangers does not have an immediate effect on the flow throughout the exchanger as for liquid exchangers due to the compressibility. Typical responses from gas and liquid exchangers are shown in Fig. 4. The initial response for gases is much faster than for liquids, and remains faster when heat exchangers with equal residence times are compared. The gas exchanger response is fast because the inlet temperature increase reduces the cold flowrate. The approach to steady-state is rather slow, mainly due to the considerable wall capacitance. One may conclude that distinction between incompressible and compressible fluids is important for controllability assessment.

MODEL FEATURES FOR HEAT EXCHANGERS NETWORKS

The basic building blocks for simulations of heat exchanger networks are heat exchangers modules, bypasses and pipes. We model the pipes and bypasses using tanks in series. The modules have been implemented in MATLAB/Simulink. This yields a very flexible interface which also allows various control structures to be studied in an efficient manner. Due to space limitations only pipe residence time is discussed here.

Pipe residence times

Pipe residence times should be included because they are essential for correct prediction of the apparent dead-time, and the dead-time in the pipes may exceed the dead-time in the exchangers.

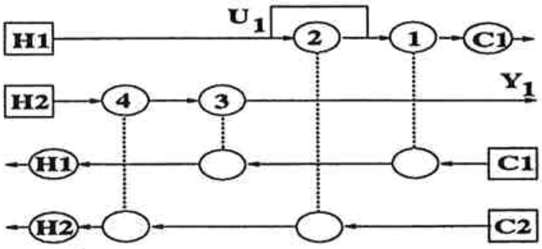

Fig. 5: Heat exchanger network where RHP-zero may exist.

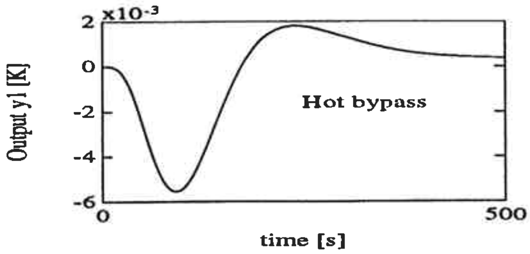

Fig. 6: Inverse response and RHP zero due to pipe residence time.

This will typically occur at low plant load where bypass fractions are often high. Less obvious is the fact that depending on the pipe residence time RHP zeros may or may not occur. Consider the example in Fig. 5. The stream properties are equal for all the streams except for the flowrate of stream $H2$ which is 10% higher than for the other streams. The heat exchanger parameters are equal for all the exchangers, and pipe residence times are neglected. For this system, bypassing the cold side of exchanger 2 results in a system with an inverse response due to a RHP zero, whereas no such control limitation will occur if the hot side is bypassed instead. However, if the residence time in the pipe connecting exchangers 1 and 3 is of the same order of magnitude as the residence times in the heat exchangers, bypassing the hot side also yields an inverse response, see Fig. 6.

NOMENCLATURE

A	Heat exchanger area	$[m^2]$
C	Constant in Eq. 4	$[-]$
c_p	Spec. heat capacity	$[J/kgK]$
F	Molar flowrate	$[kmole/s]$
h	Heat transfer coefficient	$[W/m^2K]$
i	Index (of tube side)	$[-]$
j	Index (of shell/wall side)	$[-]$
M_w	Molar weight	$[kg/kmole]$
N	Number (of cells)	$[-]$
P	Thermal effectiveness	$[-]$
R	Heat capacity rate ratio	$[-]$
T	Temperature	$[K]$
U	Overall heat transfer coeff.	$[W/m^2K]$
V	Volume	$[m^3]$
w	Heat capacity flowrate	$[kW/K]$

Greek

ΔT	Temperature difference $[K]$
ρ	density $[kg/m^3]$
τ	time constant $[s]$

Superscripts

c	cold side/fluid
h	hot side/fluid
w	wall between fluids

Subscripts

B	baffles
F	convection
i	inlet
k	heat transfer
lm	logarithmic mean (hybrid)
o	outlet
P	tube passes
TU	transfer units

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