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TWO DEGREE OF FREEDOM CONTROLLER DESIGN FOR AN ILL-CONDITIONED PLANT USING μ-SYNTHESIS

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Abstract

The structured singular value framework is applied to a distillation benchmark problem formulated for the 1991 CDC [6]. A two degree of freedom controller, which satisfies all control objectives of the CDC problem, is designed using μ-synthesis. The design methodology is presented and special attention is paid to approximation of given control objectives into frequency dependent weights.

1 Introduction

The purpose of this paper is to demonstrate, by an example, how the structured singular value (SSV, μ) framework [4] may be used to design a robust controller for a given control problem, defined by an uncertain model and control objectives that cannot be directly incorporated in the μ-framework. In particular, we consider how to approximate the given problem into a μ-problem by deriving suitable frequency dependent weights, which define model uncertainty and control objectives in the μ-framework.

The control problem studied in this paper was introduced by Limebeer [6] as a benchmark problem at the 1991 CDC where it formed the basis for a design case study aimed to investigate advantages and disadvantages of various controller design methods for ill-conditioned systems.

The problem originates from Skogestad *et al.* [13] where a simple model of a high purity distillation column was used to demonstrate that ill-conditioned plants are potentially extremely sensitive to model uncertainty. In [13] uncertainty and performance specifications were given as frequency dependent weights, *i.e.* the problem was *defined* to suit the μ-framework and therefore a μ-optimal controller yields the optimal solution to that problem.

However, in the CDC benchmark problem [6] uncertainty is defined in terms of parametric gain and delay uncertainty and the control objectives are a mixture of time domain and frequency domain specifications. These specifications cannot be directly transformed into frequency dependent weights.

The distillation problem in [13] and variants of this problem, like the CDC problem, has been studied by several authors. In three recent studies [7], [3] and [15], two degree of freedom controllers are designed for the CDC problem. All these papers are based on the McFarlane-Glover loop shaping design procedure [10], where uncertainties are modelled as $\mathcal{H}_\infty$-bounded perturbations in the normalized coprime factors of the plant. To obtain the desired performance, [7] use a reference model design approach, [3] use the Hadamard weighted $\mathcal{H}_\infty$-Frobenius formulation from [2], while [15] use the method of inequalities [16] where the performance requirements are explicitly expressed as a set of algebraic inequalities.

The two degree of freedom design in this paper differs from [7], [3] and [15] in that we use μ-synthesis for our design. With this method uncertainty is modelled as linear fractional uncertainty and performance is specified as in a standard $\mathcal{H}_\infty$ control problem. Like [7], we specify some of the control objectives as a model-matching problem.

2 CDC problem definition

The plant model and design specifications for the CDC benchmark problem [6] are presented in this section.

<u>Plant model</u>

The plant is an ill-conditioned distillation column, modelled by

$$\hat{G}(s) = \frac{1}{75s+1}\begin{bmatrix} 0.878 & -0.864 \\ 1.082 & -1.096 \end{bmatrix}\begin{bmatrix} k_1 e^{-\theta_1 s} & 0 \\ 0 & k_2 e^{-\theta_2 s} \end{bmatrix} \quad (1)$$

$$k_i \in [0.8 \;\; 1.2] \;\; ; \;\; \theta_i \in [0 \;\; 1] \;\; ; \;\; i = 1,2 \quad (2)$$

In physical terms this means 20% relative gain uncertainty and up to 1 min delay in each input channel. The set of possible plants defined by Eq.1-2 is in the following denoted Π.

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Design specifications

Specifications **S1** to **S4** should be fulfilled for *every* plant $\hat{G} \in \Pi$:

S1 Closed loop stability.

S2 For a unit step demand in channel 1 at $t = 0$ the plant outputs y_1 (tracking) and y_2 (interaction) should satisfy:

- $y_1(t) \geq 0.9$ for all $t \geq 30$ min.
- $y_1(t) \leq 1.1$ for all t
- $0.99 \leq y_1(\infty) \leq 1.01$
- $y_2(t) \leq 0.5$ for all t
- $-0.01 \leq y_2(\infty) \leq 0.01$

Corresponding requirements hold for a unit step demand in channel 2.

S3 The maximum singular value of the closed loop transfer function from output disturbances to manipulated inputs should not exceed 50 dB at any frequency, *i.e.* $\bar{\sigma}(K_y\hat{S}) < 50\text{dB} \approx 316,\ \forall\omega$.

S4 The open loop unity gain cross over frequency should be less than 150 rad/min, *i.e.* $\bar{\sigma}(\hat{G}K_y) < 1$ for $\omega > 150$.

Here K_y denotes the feedback part of the controller and $\hat{S} = (I + \hat{G}K_y)^{-1}$ the sensitivity function for the worst case $\hat{G}$.

Note that **S4** in practice is implied by **S1** which in turn is implied by **S2**, so the actual performance requirements are **S2** and **S3**.

3 The μ-framework

This section gives a very brief introduction to μ-analysis and synthesis and define some of the nomenclature used in the rest of the paper. For more details, the interested reader may consult for example [13], [14] and [1].

The $\mathcal{H}_\infty$-norm of a transfer function $M(s)$ is the peak value of the maximum singular value over all frequencies.

$$||M(s)||_\infty \equiv \sup_\omega \bar{\sigma}(M(j\omega)) \qquad (3)$$

The left block diagram in Fig.1 shows the general problem formulation in the μ-framework. It consists of an augmented plant P (including a nominal model and weighting functions), a controller K and a (block-diagonal) perturbation matrix $\Delta_U = \text{diag}\{\Delta_1, \cdots, \Delta_n\}$ representing uncertainty.

Uncertainties are modelled by the perturbations (Δ_i's) and uncertainty weights in P. These weights are chosen such that $||\Delta_U||_\infty \leq 1$ generates the family of possible plants to be considered. In principle Δ_U may contain

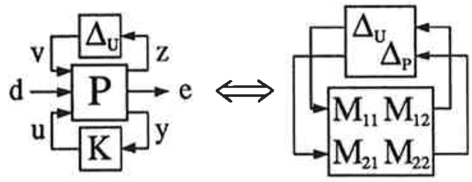

Figure 1: General problem description

both real and complex perturbations, but in this paper only complex perturbations are used.

Performance is specified by weights in P normalizing **d** and **e** such that a closed-loop $\mathcal{H}_\infty$-norm form **d** to **e** less than 1 (for the worst case Δ_U) means that the control objectives are achieved.

The framework in Fig.1 may be used for both one degree of freedom (ODF) and two degree of freedom (TDF) controllers. In the ODF case the controller input **y** is the difference between set-points and measured plant outputs, $\mathbf{y} = r - y_m$, while in the TDF case $\mathbf{y} = [r\ ,\ -y_m]^T$. A TDF controller may be partitioned into two parts

$$K = [K_r\ K_y] = \left[\begin{array}{c|cc} A_K & B_{Kr} & B_{Ky} \\ \hline C_K & D_{Kr} & D_{Ky} \end{array}\right] \qquad (4)$$

where K_y is the feedback part of the controller.

The right block diagram in Fig.1 is used for robustness analysis. M is a function of P and K, and Δ_P ($||\Delta_P||_\infty \leq 1$) is a fictitious "performance perturbation" connecting **e** to **d**. Provided that the closed loop system is nominally stable the condition for Robust Performance (RP) is:

$$RP \quad \Leftrightarrow \quad \mu_{RP} = \sup_\omega \mu_\Delta(M(j\omega)) < 1 \qquad (5)$$

where $\Delta = \text{diag}\{\Delta_U, \Delta_P\}$.

μ is computed frequency-by-frequency through upper and lower bounds. Here we only consider the upper bound

$$\mu_\Delta(M(j\omega)) \leq \inf_{D \in \mathbf{D}} \bar{\sigma}(DMD^{-1}) \qquad (6)$$

where $\mathbf{D} = \{D | D\Delta = \Delta D\}$.

At present there is no direct method to synthesize a μ-optimal controller, however, μ-synthesis (DK-iteration) which combines μ-analysis and $\mathcal{H}_\infty$-synthesis often yields good results. This iterative procedure was first proposed in [5] and [11]. The idea is to attempt to solve

$$\min_K \inf_{D \in \mathbf{D}} \sup_\omega \bar{\sigma}(DMD^{-1}) \qquad (7)$$

(where M is a function of K) by alternating between minimizing $\sup_\omega \bar{\sigma}(DMD^{-1})$ for either K or D while holding the other fixed. The iteration steps are:

DK1 Scale the interconnection matrix M with a stable and minimum phase rational transfer matrix $D(s)$ with appropriate structure.

DK2 Synthesize an $\mathcal{H}_\infty$-controller for the scaled problem, $\min_K \sup_\omega \bar{\sigma}(DMD^{-1})$.

DK3 Stop to iterate if the performance is satisfactory or if the $\mathcal{H}_\infty$-norm does not decrease, else continue.

DK4 Compute the upper bound on μ (Eq.6) to obtain new D-scales as a function of frequency $D(j\omega)$.

DK5 Fit the magnitude of each element of $D(j\omega)$ to a stable and minimum phase rational transfer function and go to **DK1**.

Each of the minimizations (steps **DK2** and **DK4**) are convex, but joint convexity is not guaranteed.

The $\mathcal{H}_\infty$-controller synthesised in step **DK2** has same number of states as the augmented plant P plus two times the number of states of D, so it is desirable to keep the order of P and the D-scales as low as possible.

4 Design procedure

The CDC problem in section 2 cannot be directly transformed into a μ-problem. The reasons for this are: 1) The gain-delay uncertainty in Eq.1-2 has to be approximated into linear fractional uncertainty (Fig.1); 2) Specification **S2** need to be approximated since it is defined in the time domain; 3) In the μ-framework it is not possible to directly bound the four SISO transfer functions associated with **S2** and the 2×2 transfer function associated with **S3**. Instead these control objectives must be reflected in the $\mathcal{H}_\infty$-norm of the transfer function from **d** to **e** (Fig.1).

The following approach makes it possible to apply μ-synthesis to this kind of a problem:

1 Approximate the given problem into a μ-problem.

2 Synthesize a robust controller for the μ-problem.

3 Verify that the controller satisfies the original specifications (**S1-S4**) for the original set of plants (II).

Step 1 is our major concern in this paper. Several approaches may be used to obtain the μ-problem, however, the following guidelines are general: A) Choose **d** and **e** such that all control objectives are reflected in the $\mathcal{H}_\infty$-norm of the transfer function between these signals. At the same time keep the dimension of **d** and **e** as small as possible. B) Use low order uncertainty and performance weights to keep the order of P and thereby the order of the controller low. C) Use weight parameters with physical meaning, since these parameters are the tuning knobs during the design. Derivation of such weights for the CDC problem is treated in detail in the next section.

Step 2 is fairly straight-forward with DK-iteration using available software (*e.g.* [1]). Experience with this iterative scheme shows that for the first iterations it is best if the controller synthesized in step **DK2** is slightly sub-optimal ($\mathcal{H}_\infty$-norm 5-10% larger than the optimal) and the D-scale fit in step **DK5** are of low order. In subsequent iterations more optimal controllers and higher order D-scales may be used if required. However, it is recommended that also the final controller is slightly sub-optimal since this yields a blend of $\mathcal{H}_\infty$ and $\mathcal{H}_2$ optimality with generally better high frequency roll-off than the optimal $\mathcal{H}_\infty$-controller.

Step 3 is in this paper performed using time simulations with the four extreme combinations of gain uncertainty (Eq.2) and a 1 minute delay (approximated as a second order Padé). In general it is difficult to ensure that the worst case uncertainty is included in a simulation study and μ-analysis may be required to verify that the control objectives are satisfied.

Given the μ-problem from step 1 the following iterative design procedure is applied:

i Start with relatively loose performance requirements and check that Nominal Performance ($\mathcal{H}_\infty$-norm less than 1 for $\Delta_U = 0$) is obtainable.

ii DK-iterate until $||DMD^{-1}||_\infty \approx 1$. (The physical interpretation of the weight parameters are most accurate for a $\mathcal{H}_\infty$-norm of about 1.)

iii Check if the control objectives are achieved (step 3 above). If they are not, adjust the weight parameters and go to **ii**. (Since DK-iteration is computationally demanding, it is often a good idea to check the effect of the adjustments by synthesizing a controller using the *old* D-scales, before going to **ii**.)

5 Approximation of the CDC problem

In this section we approximate the benchmark problem into a μ-problem suitable for DK-iteration, *i.e.* we derive uncertainty and performance weights which reasonable well reflects the given uncertainty and design specifications. First an uncertainty model is derived , then performance specifications are derived for both ODF and TDF designs.

5.1 Uncertainty model

The gain-delay uncertainty (Eq.2) is approximated into linear fractional uncertainty by use of a single complex multiplicative perturbation in each input channel. G_p denotes the approximation of $\hat{G}$ in Eq.1.

$$G_p(s) = G(s)(I + \Delta_U(s)W_\Delta(s)) \quad (8)$$

$$G(s) = \frac{1}{75s+1}\begin{bmatrix} 0.878 & -0.864 \\ 1.082 & -1.096 \end{bmatrix} \quad (9)$$

$$\Delta_U(s) = \mathrm{diag}\{\Delta_1, \Delta_2\} \ ; \ ||\Delta_U(s)||_\infty \leq 1 \quad (10)$$

$$W_\Delta(s) = \frac{(1+\frac{k_r}{2})\theta_{max}s + k_r}{\frac{\theta_{max}}{2}s+1} I_{2\times 2} = \frac{1.1s+0.2}{0.5s+1} I_{2\times 2} \quad (11)$$

where $k_r = 0.2$ is the relative gain uncertainty and $\theta_{max} = 1$ is the maximum delay.

This uncertainty model does not quite include all combinations of gain and delay uncertainty in Eq.2, but has the advantage of a low order weight with physical parameters.

If the performance verification (step 3 of the design procedure) shows that this simple uncertainty model does not yield a robust controller for the set of plants Π, then a more rigorous uncertainty model should be used (for example the third order weight presented in [9] which completely covers gain-delay uncertainty). However, it is best to start the design with a simple low order model.

5.2 Performance specifications

5.2.1 One degree of freedom controller

A simple way to approximate **S2** and **S3** into a μ-problem is shown in Fig.2, where K_y is an ODF controller.

The time domain requirements of specification **S2** is approximated by a frequency domain bound (W_{S2}) on the sensitivity function $S_p = (I + G_p K_y)^{-1}$ for the worst case plant G_p. A suitable weight is [8]

$$W_{S2}(s) = \frac{1}{M_S} \frac{\tau_{cl} s + M_S}{\tau_{cl} s + A} I_{2\times 2}. \quad (12)$$

For $||W_{S2} S_p||_\infty < 1$ this weight yields:

1 Steady-state error less than A;

2 Closed-loop bandwidth higher than $\omega_B = 1/\tau_{cl}$;

3 Amplification of high-frequency output disturbances less than a factor M_S.

Specification **S3** yields a simple frequency independent weight

$$W_{S3} = \frac{1}{M_{KS}} I_{2\times 2} \quad (13)$$

For $||W_{S3} K_y S_p||_\infty < 1$ this weight guarantees $\bar{\sigma}(K_y S_p)$ less than M_{KS} for all frequencies.

Note that the formulation in Fig.2 lumps the four SISO requirements of **S2** and the 2×2 requirement of **S3** into a bound on the entire 2×4 transfer function from $\hat{r}$ to $[\hat{e}\ ,\ \hat{u}]^T$. From the relation

$$\max\{\bar{\sigma}(A), \bar{\sigma}(B)\} \leq \bar{\sigma}([A\ B]) \leq \sqrt{2}\max\{\bar{\sigma}(A), \bar{\sigma}(B)\} \quad (14)$$

it is clear that the physical interpretation of the weights results in slightly too 'hard' performance requirements.

Based on **S2**, **S3** and the results in [13] we use $A = 0.01$, $\tau_{cl} = 20$, $M_S = 2$ and $M_{KS} = \sqrt{2} * 316$. A few iterations shows that the ODF controller probably not will yield the required performance, so we focus on the TDF design.

5.2.2 Two degree of freedom controller

The block diagram in Fig.3 defines the μ-approximation of the CDC problem we will use for the TDF design.

Specification **S2** is here approximated by a 'model-matching problem'. Compared to a weight on the sensitivity function, this allows a more direct way to deal with both the set-point tracking and interaction specifications of **S2**. The reference model $T_{yr,id}$ defines the *ideal* transfer function from set-points to plant outputs, and the signal weights W_r and W_e are used to penalize the difference between desired and actual response.

The set-point tracking should ideally be decoupled and the response and overshoot requirements are the same for both channels. To keep the order of $T_{yr,id}$ small, while at the same time have the freedom to allow for some overshoot in the ideal response, we use a second order reference model in each channel

$$T_{yr,id} = \frac{1}{\tau_{id}^2 s^2 + 2\zeta_{id}\tau_{id} s + 1} I_{2\times 2} \quad (15)$$

The weights W_n and W_u in Fig.3 are used to obtain specification **S3** by bounding $||K_y S_p||_\infty$. Note that even without **S3** it is necessary to include the noise $\hat{n}$ or another signal (non-zero for $\omega \to \infty$) between G and K to obtain a proper controller, since G is strictly proper ($\lim_{\omega\to\infty} \bar{\sigma}(G(j\omega)) = 0$).

Figure 3 gives

$$\begin{bmatrix} \hat{e} \\ \hat{u} \end{bmatrix} = \begin{bmatrix} W_e N_{11} W_r & W_e N_{12} W_n \\ W_u N_{21} W_r & -W_u N_{22} W_n \end{bmatrix} \begin{bmatrix} \hat{r} \\ \hat{n} \end{bmatrix} \quad (16)$$

where $N_{11} = S_p G_p K_r - T_{yr,id} = T_{yr,p} - T_{yr,id}$; $N_{12} = T_{yr,p}$; $N_{21} = (I + K_y G_p)^{-1} K_r$ and $N_{22} = K_y S_p$. For simplicity, we use diagonal weights with the same weight in both channels ($W_i = w_i * I_{2\times 2}$, $i = e, u, r, n$), *i.e.* $W_e W_r$ forms a bound on N_{11} and $W_u W_n$ forms a bound on N_{22}.

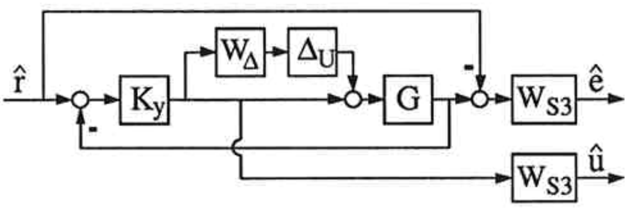

Figure 2: Block diagram for one degree of freedom controller.

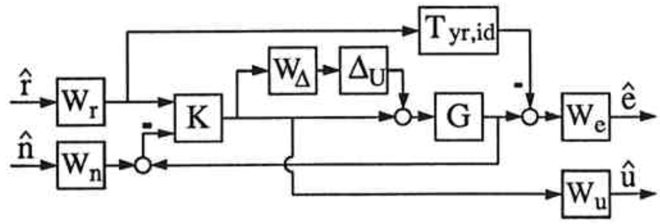

Figure 3: Block diagram for two degree of freedom controller.

Now, W_{S2} and W_{S3} from the ODF-design are used as a starting point to find appropriate signal weights W_r, W_n, W_e and W_u. The combined weight $W_u W_n$ should be similar to W_{S3} and $W_e W_r$ may be choosen similar to W_{S2} to obtain a reasonable bound on the mismatch between actual and ideal response. However, the off-diagonal elements in Eq.16, $W_e N_{12} W_n$ and $W_u N_{21} W_r$ also have to be considered when selecting the signal weights, since it is the $\mathcal{H}_\infty$-norm of the entire transfer function that is minimized by the controller. This demonstrates that the weights have to be selected with some care in order to avoid impossible performance specifications.

We may choose one of the signal weights arbitrary and then shape the other signals relatively to the arbitrary weight. Let $W_r = I$ at all frequencies. This yields N_{11} bounded by $W_e = W_{S2}$ and N_{21} bounded by W_u. At low frequencies $N_{21} \approx N_{22}$[1] so let $W_u = W_{S3}$. Next consider how to choose W_n such that $W_u N_{22} W_n$ reflects **S3** and $W_e N_{12} W_n$ does not limit the performance of the overall system. At low frequencies $N_{12} \approx I$, so W_n has to be smaller than W_e^{-1} in this frequency range. At higher frequencies W_n is chosen such that $W_u W_n$ becomes an active bound on N_{22}. One way to obtain this is to use

$$W_r(s) = I_{2\times 2} \tag{17}$$

$$W_e(s) = \frac{1}{M_S}\frac{\tau_{cl}s + M_S}{\tau_{cl}s + A} I_{2\times 2} \tag{18}$$

$$W_u(s) = \frac{1}{M_{KS}} I_{2\times 2} \tag{19}$$

$$W_n(s) = \frac{\tau_{cl}s + A}{\tau_{cl}s + M_T} I_{2\times 2} \tag{20}$$

M_T in Eq.20 is a bound on the low frequency peak value of N_{12} (the complementary sensitivity function). This parameter is used to adjust the frequency where $W_u W_n$ becomes an active bound on N_{22}.

The performance weights derived above have several parameters, however, it is easy to find reasonable numerical values for these parameters since they all have some physical meaning. In fact, most of the numerical values are almost directly obtained from the specifications in section 2.

6 TDF controller design

Δ_U in Eq.10 is structured, however, it can be shown that the TDF problem (as defined in section 5) belongs to a class of problems where an unstructured Δ_U may be used without introducing conservativeness (Hovd *et al.*, 1993). In the following we use an unstructured Δ_U which gives $D(s) = \text{diag}\{d(s), d(s), 1, 1, 1, 1\}$.

Initially $d(s)$ was set to 0.01, obtained from a natural physical scaling ('logarithmic compositions' [12]). This

[1] At low frequencies $(I + GK_y)^{-1}GK_r \approx I \Rightarrow K_y \approx K_r \Rightarrow N_{21} = (I + K_y G)^{-1} K_r \approx K_y(I + GK_y)^{-1} = N_{22}$

Table 1: Final weight parameters and D-scales

Weight parameters						
τ_{id}	ζ_{id}	τ_{cl}	A	M_S	M_T	M_{KS}
8.0	0.71	9.5	10^{-4}	3.5	2.0	630

$$D(s) = \text{diag}\{d(s), d(s), I_{4\times 4}\}$$

$$d(s) = 0.00299\frac{(s + 5.70)}{(s + 0.0144)}\frac{(s^2 + 2*0.6645*0.112s + 0.112^2)}{(s^2 + 2*0.622*0.568s + 0.568^2)}$$

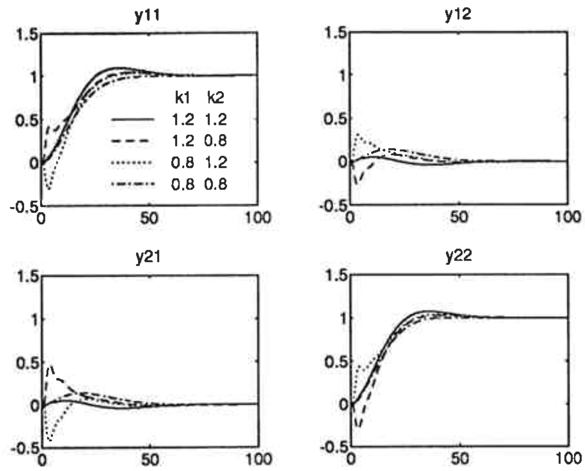

Figure 4: Time responses with plant-model mismatch. y_{ij} shows response in output i for step change of set-point j at $t = 0$. All responses with 1 min. delay (2nd order Padé).

simple scaling substantially reduces the number of iterations required to obtain 'good' D-scales.

The initial weight parameters were choosen to: 1) Yield an ideal response which satisfies **S2** with some margin without too large overshoot ($\tau_{id} = 8, \zeta_{id} = 0.71$); 2) Require a close fit to the ideal response at low frequencies ($A = 10^{-4}$) and a looser fit at high frequencies ($\tau_{cl} = 10, M_S = 3$); 3) Yield a loose requirement on $K_y S_p$ to be increased if required ($M_T = 3, M_{KS} = 630$ (56dB)).

Only two DK-iterations was needed to get $\mu_{RP} < 1$, however, the performance with respect to **S2** and **S3** was not quite achieved. M_S, M_T and τ_{cl} was adjusted to 3.5, 2.0 and 9.5, respectively. After two more DK-iterations a controller which satisfies **S1-S4** was obtained. The controller has 24 states, yields a closed loop $\mathcal{H}_\infty$-norm of 1.015 and may be synthesized using the final weights and D-scales given in Table 1.

The performance of the TDF controller is demonstrated in Fig.4 where time responses for the four extreme combinations of uncertainty are shown. The simulation results are also summerized in Table 2 and are seen to satisfy spesification **S2**. The maximum peak of $\bar{\sigma}(K_y\hat{S}) = 306$, which is less than 316 (50 dB), as required in **S3**, and the unit gain cross over frequency,

Table 2: **Control performance with gain uncertainty and second order Padé approximation of a 1 min. delay.** (See also Fig.4)

step	gain unc.		set-point tracking			interaction	
ch.	k_1	k_2	$t = 30$	max	$t = 100$	max	$t = 100$
1	1.2	1.2	1.066	1.092	0.998	0.051	0.001
1	1.2	0.8	0.984	1.036	0.999	0.471	-0.001
1	0.8	1.2	0.969	1.030	1.000	0.426	0.001
1	0.8	0.8	0.906	1.000	1.000	0.138	0.000
2	1.2	1.2	1.052	1.074	0.999	0.051	0.001
2	1.2	0.8	0.987	1.030	1.000	0.265	0.001
2	0.8	1.2	1.002	1.038	0.999	0.310	0.000
2	0.8	0.8	0.950	1.002	1.000	0.138	0.000

$\bar{\sigma}(\hat{G}K_y) = 1$, is at 1 rad/min, well below 150 rad/min, as required in **S4**.

The transfer functions N_{12} and N_{21}, which are not part of the CDC problem, have peak values of 3.4 and 420, respectively.

7 Discussion

There has been some confusion with respect to specifications **S4** of the CDC problem. In [15] both **S3** and **S4** are applied to the closed loop transfer function $K_y\hat{S}$ from output disturbances to plant inputs, *i.e* this transfer functions is gain limited to 316 (50dB) for frequencies below 150 rad/min and to 1 (0dB) for frequencies above 150 rad/min. This objective may also be achieved using the design procedure presented this paper, however it requires more complicated weights W_n and W_u. For the design presented in this paper $\bar{\sigma}(K_y\hat{S}) < 1$ for frequencies above 700 rad/min.

The inability to independently penalize separate elements of the closed loop transfer function complicates the performance weight selection in the μ-framework. The Hadamard weighted approach [3] does not have this problem and will therefore yield better performance with respect to the specifications in the CDC problem, **S1 - S4**. However, for a practical engineering problem the transfer functions N_{12} and N_{21} in Fig.3 *are* of importance, so it seems reasonable to include them into the control problem.

8 Conclusions

μ-synthesis has been successfully applied to a demanding ill-conditioned uncertain problem where uncertainty is defined as parametric gain-delay uncertainty and the design objectives are a mixture of time domain and frequency domain specifications.

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