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Improved independent design of robust decentralized controllers.

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Abstract

The procedure for independent design of robust decentralized controllers proposed by Skogestad and Morari [10] is improved by requiring the controller to be a decentralized Internal Model Control (IMC) type controller. It is shown how to find bounds on the magnitude of the IMC filter time constants such that robust stability or performance is guaranteed. This allows the use of real perturbation blocks for modeling the uncertainty associated with the controllers. In contrast, Skogestad and Morari [10] found bounds on the sensitivity functions and complementary sensitivity functions for the individual loops, and therefore had to use complex perturbation blocks.

The concept of Robust Decentralized Detunability is introduced. If a system is Robust Decentralized Detunable, any subset of the loops can be detuned independently and to an arbitrary degree without endangering robust stability. A simple test for Robust Decentralized Detunability is developed for systems controller by a decentralized IMC controller.

1 Introduction

Decentralized control remains popular in the chemical process industry, despite developments of advanced controller synthesis procedures leading to full multivariable controllers. Some of the reasons for the continued popularity of decentralized control are:

1. Decentralized controllers are easy to implement.
2. They are easy for operators to understand.
3. The operators can be allowed to retune the controllers to take account of changing process conditions (as a result of 2 above).
4. Some measurements or manipulated variables may fail. Tolerance of such failures are more easily incorporated into the design of decentralized controllers than full controllers.
5. The control system can be brought gradually into service during process startup and taken gradually out of service during shutdown.

Standard controller synthesis algorithms (e.g. H_2 or H_∞ synthesis) lead to full controllers, and cannot handle requirements for controllers with a specified structure, and alternative approaches therefore have to be used for designing decentralized controllers. In this work we consider independent design of robust decentralized controllers, introduced by Skogestad and Morari [10]. New results on independent design are presented which represent improvements over the existing design procedure. Throughout this work we will use the structured singular value (see below) as the measure of control quality.

2 Notation

In this paper, $G(s)$ will denote the plant, which is assumed to be of dimension $n \times n$. $\tilde{G}(s)$ denotes the matrix consisting of the diagonal elements of $G(s)$, and $g_{ij}(s)$ is the ij'th element of $G(s)$. The reference signal (setpoint) is denoted r, manipulated inputs are denoted u and outputs are denoted y. Throughout this work, all controllers are assumed to be completely decentralized. The decentralized conventional feedback controller is denoted $C(s)$, with i'th diagonal element $c_i(s)$. Likewise, the decentralized IMC controller is denoted Q, with i'th diagonal element $q_i(s)$. $C(s)$ and $Q(s)$ are related by

$$C(s) = Q(s)(I - \tilde{G}(s)Q(s))^{-1} \quad (1)$$

$S(s) = (I + G(s)C(s))^{-1}$ is the sensitivity function and $H(s) = I - S(s) = G(s)C(s)(I + G(s)C(s))^{-1}$ is the complementary sensitivity function. The sensitivity functions and complementary sensitivity functions for the individual loops are collected in the diagonal matrices $\tilde{S}(s) = (I - \tilde{G}(s)C(s))^{-1}$ and $\tilde{H}(s) = \tilde{G}(s)C(s)(I - \tilde{G}(s)C(s))^{-1}$. Note that the diagonal elements of $\tilde{S}(s)$ and $\tilde{H}(s)$ do not equal the diagonal elements of $S(s)$ and $H(s)$, respectively. $\tilde{s}_i$ and $\tilde{h}_i$ are the i'th element on the diagonal of $\tilde{S}$ and $\tilde{H}$, respectively.

3 Robust control and the structured singular value.

The realization that no model is a perfect representation of the system it is describing points to the requirement that the control system stability and performance should be little affected by the uncertainties of the model. In this paper we use the structured singular value, μ, introduced by Doyle [2], as a measure of the robustness of feedback systems. Within the μ framework, one accepts that it is the impossible to find a perfect model, and instead require information about the structure, location and estimates of the magnitude of the model uncertainties.

In Fig. 1 we have drawn an example of a feedback system with uncertainty in the inputs and outputs[1], represented by the perturbation blocks Δ_I and Δ_O, respectively. Note that the individual perturbation can be restricted to have a certain structure. For instance, as individual inputs and outputs usually do not affect each other, both Δ_I and

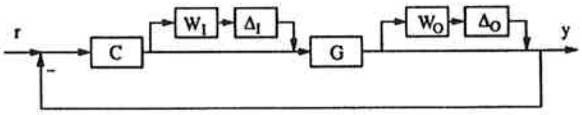

Figure 1: Block diagram for feedback system with uncertainty in the inputs and outputs.

Δ_O are assumed to be diagonal. W_I and W_O are frequency-dependent weights normalizing the maximum magnitude of Δ_I and Δ_O, respectively, to unity.

Any block diagram with uncertainties represented by perturbation blocks

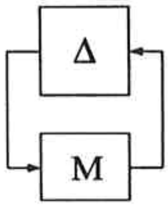

Figure 2: Feedback system rearranged into a perturbation block Δ and an interconnection matrix M.

can be rearranged into the $M - \Delta$ structure of Fig. 2, if external inputs and outputs are neglected. In Fig. 2, Δ is a block diagonal matrix with the perturbation blocks of the original block diagram on the diagonal, and M contains all the other blocks in the block diagram (plant,

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[1] Many other types of uncertainties possible, see [2] for details on how to represent different uncertainties with perturbation blocks

controller, weights). Provided M is stable (the system has Nominal Stability, NS) and Δ is stable (stable perturbation blocks), it follows from the Nyquist stability criterion [2] that the overall system is stable provided $\det(I - M\Delta) \neq 0 \quad \forall\Delta, \forall\omega$. In this case the system is said to have Robust Stability (RS). The structured singular value is defined such that

$$\mu_{\Delta}^{-1} = \min_{\delta}\{\delta | \det(I - M\Delta) = 0 \quad \text{for some } \Delta, \bar{\sigma}(\Delta) \leq \delta\} \qquad (2)$$

If weights are used to normalize the maximum value of the largest singular value of Δ to unity ($\bar{\sigma}(\Delta) = 1$) at all frequencies, like in Fig. 1, the system will remain stable for any allowable perturbation Δ provided $\mu_{\Delta}(M) < 1$.

Doyle [2] showed that performance can be analyzed in the μ framework by considering an equivalent stability problem of larger dimension. We use a performance specification of the type $\bar{\sigma}(W_P S_p) \leq 1 \quad \forall\omega$ where S_p is the worst sensitivity function (S) made possible by the perturbation blocks. This performance specification can be incorporated in the μ framework by closing the loop from outputs to output disturbances with the performance weight W_P and a *full* perturbation block Δ_P. If $\mu_{\Delta}(M) \leq 1$ (after normalizing the magnitude of the perturbation blocks) for the corresponding $M - \Delta$ structure of increased dimension, the system is said to have Robust Performance (RP), as the performance specification is fulfilled for all the possible model uncertainties. Doyle and Chu [3] proposed an algorithm for the synthesis of controllers which minimizes μ, known as $D-K$ iteration. However, $D-K$ iteration results in full controllers, and the problem of synthesizing μ-optimal decentralized controllers has not been solved.

4 Independent design

Independent design of robust decentralized controllers was introduced by Skogestad and Morari [10]. It is based on Theorem 1 in [9], which we state here:

Theorem 1 *Let the μ interconnection matrix M be written as a Linear Fractional Transformation (LFT) of the transfer function matrix T*

$$M = N_{11} + N_{12}T(I - N_{22}T)^{-1}N_{21} \qquad (3)$$

and let k be a given constant. Assume $\mu_{\Delta}(N_{11}) < 1$ and $\det(I - N_{22}T) \neq 0$ then

$$\mu_{\Delta} \leq 1 \qquad (4)$$

if

$$\bar{\sigma}(T) \leq c_T \qquad (5)$$

where c_T solves

$$\mu_{\tilde{\Delta}} \begin{bmatrix} N_{11} & N_{12} \\ c_T N_{21} & c_T N_{22} \end{bmatrix} = 1 \qquad (6)$$

and $\tilde{\Delta} = \mathrm{diag}\{\Delta, T\}$

Proof: See [9].

T is generally some important transfer function which depends on the controller. Skogestad and Morari [10] uses Thm. 1 to find bounds on the sensitivity function and complementary sensitivity functions for the individual loops (i.e. $T = \tilde{S}$ and $T = \tilde{H}$ are used). The bounds on $\tilde{S}$ and $\tilde{H}$ can be combined over different frequency ranges. Thus, if either the bound on $\tilde{S}$ or the bound on $\tilde{H}$ is fulfilled for all loops at all frequencies, then $\mu_{\Delta}(M) \leq 1$ is achieved.

In this method one treats the transfer functions (T) as uncertainty, and thereafter finds bounds on the magnitude of this fictitious uncertainty which guarantees that $\mu_{\Delta}(M) \leq 1$. Thereafter, one is faced with finding controllers such that the bounds on the transfer functions are fulfilled. It is therefore important for the success of independent design that T introduces as little additional uncertainty as possible. It turns out that choosing $T = \tilde{S}$ and $T = \tilde{H}$ are not ideal for this purpose.

4.1 Example 1.

Consider Example 1 in Chiu and Arkun [1]:

$$G(s) = \begin{bmatrix} \frac{1.66}{39s+1} & \frac{-1.74e^{-2s}}{4.4s+1} \\ \frac{0.34e^{-s}}{8.9s+1} & \frac{1.4e^{-s}}{3.8s+1} \end{bmatrix} \qquad (7)$$

There is independent input uncertainty with input uncertainty weight $W_I(s) = 0.07 I_2$, and the performance requirement is given by the performance weight $W_p(s) = 0.25\frac{7s+1}{7s}I_2$

Chiu and Arkun [1] attempted independent design for this example, using $T = \tilde{S}$ and $T = \tilde{H}$, but were unable to find a controller which fulfilled the resulting bounds. In [1] it was therefore claimed that independent design can not be performed for this example. We will however demonstrate below that independent design can be performed for this example, within the framework of Internal Model Control.

5 Independent design with decentralized IMC controllers.

Here we shall select T not as a transfer function, but rather as a parametrization of the tuning constant in the controller. We use the Internal Model Control (IMC) technique [5] to parametrize the individual controller elements. The relationship between the elements q_i of the IMC controller and the elements c_i of the conventional controller is given by

$$c_i = q_i(1 - g_{ii}c_i)^{-1} \qquad (8)$$

In the IMC design procedure [7], q_i has the form

$$q_i = \hat{g}_{ii}^{-1} f_i \qquad (9)$$

where $\hat{g}_{ii}$ is the minimum phase part of g_{ii}, and f_i is a low pass filter used to make q_i realizable and to detune the system for robustness. In order to simplify the exposition, we will assume the plant G to be open loop stable, and use a low pass filter of the form

$$f_i = \frac{1}{(\epsilon_i s + 1)^{n_f}} \qquad (10)$$

That is, the f_i is taken to be a low pass filter of order n_f, consisting of n_f *identical* first order low pass filters in series. For details on IMC design, and on filter form for unstable systems, the reader is referred to Morari and Zafiriou [7].

5.1 Choice of T for independent design

After fixing n_f, the only thing which remains uncertain in the IMC technique is the value of ϵ_i. To fulfill performance requirements at low frequencies, the closed loop system must be sufficiently fast, which means that the filter time constant ϵ must be smaller than a certain value. On the other hand, the closed loop system must be sufficiently detuned to avoid robustness problems at higher frequencies, thus requiring ϵ to be larger than a certain value, meaning that $1/\epsilon$ must be smaller than some value. We will therefore use Thm. 1 to find bounds on ϵ and $e_i \stackrel{\text{def}}{=} 1/\epsilon$ which can be combined over different frequency ranges. Since ϵ is a positive real constant and $\tilde{S}$ and $\tilde{H}$ take complex values, we will thereby make the uncertainty description for independent design much less conservative.

At each frequency point, we will have to solve Eq. (6) iteratively. To bypass the problem of having to find a new realization of Q for each value of ϵ and e considered, we choose to work with frequency responses. Although $\hat{g}_{ii}^{-1}$ in Eq. (9) will normally not be realizable, its frequency response is easily calculated. As a result of the choice of working with frequency responses, we will have to check *à posteriori* for the (internal) stability of the μ interconnection matrix M for one choice of ϵ within the bounds found.

We refer the readers to [9] or [7] for details on how to find the LFT's needed for Thm. 1. We will here only elaborate on how to express f_i as an LFT of *the uncertainty associated with* ϵ or e.

5.1.1 First order low pass filters.

Consider first the case $n_f = 1$. The objective is to find the allowable ranges for ϵ_i and $e_i = 1/\epsilon_i$ that at each frequency guarantee $\mu(M) \leq 1$. Since we do not allow negative values for ϵ_i we should not write $|\epsilon_i| \leq c_\epsilon$. Instead write

$$\epsilon_i = \epsilon_i^*(1 + \Delta_\epsilon) \qquad |\Delta_\epsilon| \leq 1 \qquad (11)$$

$$e_i = e_i^*(1 + \Delta_e) \qquad |\Delta_e| \leq 1 \qquad (12)$$

and iterate in Thm. 1 for ϵ_i^* and e_i^* until $c_\epsilon = 1$ or $c_e = 1$. We then get the allowable ranges to be $0 \leq \epsilon_i \leq 2\epsilon_i^*$ and $0 \leq e_i \leq 2e_i^*$. Note that all quantities, including Δ_ϵ and Δ_e are real. In order to use Thm. 1 we now ned to write f_i as an LFT of Δ_ϵ and Δ_e. We then get

$$N_{\epsilon_i} = \frac{1}{\epsilon^* s + 1} \begin{bmatrix} 1 & -1 \\ \epsilon^* s & -\epsilon^* s \end{bmatrix} \qquad (13)$$

$$N_{e_i} = \frac{1}{e^* + s} \begin{bmatrix} e^* & e^* \\ s & -e^* \end{bmatrix} \qquad (14)$$

5.1.2 Higher order low pass filters.

In IMC design, one will often use filters of order higher than one. We therefore need to be able to express the higher order filters as LFT's of Δ_ϵ and Δ_e. For this we can use the rules for series interconnection of linear dynamical systems. First note that $G(s) = C(sI - A)^{-1}B + D$ may be written as an LFT of $\frac{1}{s}I$, with

$$N_{11} = D; \quad N_{12} = C; \quad N_{21} = B; \quad N_{22} = A$$

The formulae for series interconnection $G = G_1 G_2$ of dynamical systems $G_1(s) = C_1(sI - A_1)^{-1}B_1 + D_1$ and $G_2(s) = C_2(sI - A_2)^{-1}B_2 + D_2$ are (e.g. [6]):

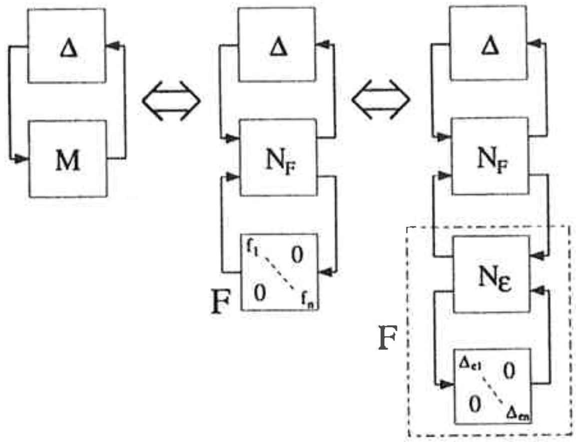

Figure 3: The interconnection matrix M expressed as an LFT of the IMC controller Q and as an LFT of the "uncertainty" associated with the filter time constants.

$$A = \begin{bmatrix} A_1 & 0 \\ B_2C_1 & A_2 \end{bmatrix}; \quad B = \begin{bmatrix} B_1 \\ B_2D_1 \end{bmatrix}$$
$$C = \begin{bmatrix} D_2C_1 & C_2 \end{bmatrix}; \quad D = [D_2D_1]$$

The formulae for series interconnection of dynamical systems can be used directly to express an n_f'th order low pass filter as LFT's of $\mathrm{diag}\{\Delta_{\epsilon 1}, \cdots, \Delta_{\epsilon n_f}\}$ and $\mathrm{diag}\{\Delta_{e1}, \cdots, \Delta_{en_f}\}$. As we will normally use the same time constant for all first order factors of the n_f'th order filter, we will have $\Delta_{\epsilon 1} = \Delta_{\epsilon 2} = \cdots = \Delta_{\epsilon n_f}$ and $\Delta_{e1} = \Delta_{e2} = \cdots = \Delta_{en_f}$, and we have repeated scalar, real "uncertainty" associated with the filter in each IMC controller element.

5.1.3 The overall low pass filter.

Above we have shown how to express an individual filter element f_i (being a low pass filter of order n_f) as an LFT of the real "uncertainty" in the filter time constant in that filter element. The LFT for the overall IMC filter $F = \mathrm{diag}\{f_i\}$ is then just a simple diagonal augmentation of the corresponding blocks of the LFT for the individual filter elements. For example, let $N_{11}^{f_i}$ denote the N_{11} block for the LFT of element i. The block N_{11} for the LFT of the overall IMC filter will then be given by $N_{11} = \mathrm{diag}\{N_{11}^{f_i}\}$.

Note that although we have repeated scalar "uncertainties" for each individual filter element, the filter time constants may differ in different filter elements, and the "uncertainties" in different filter elements are therefore independent. For a plant of dimension $n \times n$ we therefore end up with n repeated scalar uncertainty blocks for the IMC filter, each of these blocks being repeated n_f times[2].

5.2 Example 1 continued.

Consider again Example 1 studied above. For this problem we choose a second order low pass filter in each element of the decentralized IMC controller. Since we have a 2×2 system, this will add two real, repeated scalar perturbations, each repeated twice. Solving Eq. (6), we obtain

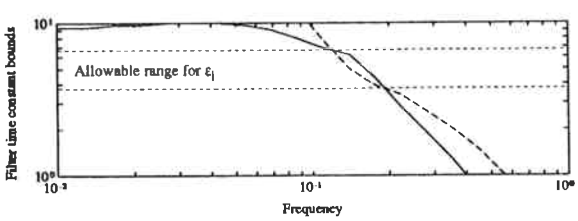

Figure 4: Filter time constant bounds for Example 1. Solid: upper bound. Dashed: lower bound.

the results in Fig. 4. We see that values of ϵ between 3.7 and 6.6 are at all frequencies either below the upper bound or above the lower bound. Choosing $\epsilon = 5$ for both loops, it is easily verified that the system is nominally (internally) stable. We have thus completed an independent design for this example.

5.3 Independent design procedure.

With the preliminaries above, we can now propose an independent design algorithm:

[2]One may use low pass filter of different orders in the different filter elements, in which case the value of n_f will differ for different filter elements.

1. Find the matrices N_ϵ, expressing the μ interconnection matrix M as an LFT of Δ_ϵ, and the matrix N_e, expressing M as an LFT of Δ_e. N_ϵ will depend on the value of ϵ^*, and N_e will depend on the value of e^*.
2. We get
$$\mu(M) \leq 1$$
if
$$0 \leq \epsilon_i \leq 2\epsilon^* \quad \forall i \tag{15}$$
where ϵ^* solves
$$\mu(N_\epsilon) = 1 \tag{16}$$
Similarly, let e^* solve $\mu(N_e) = 1$, giving the bound
$$0 \leq e_i \leq 2e^* \quad \forall i \quad \Leftrightarrow 1/(2e^*) \leq \epsilon_i \quad \forall i \tag{17}$$
3. From 2 and Thm. 1 we know that $\mu(M) < 1$ for the range of values of ϵ which at *all* frequencies is *either* within the range of values in Eq. (15) *or* within the range of values in Eq. (17).
4. Choose a value of ϵ within the range of values found in point 3, and verify the stability of M for this choice of ϵ. [3]

If we are successful in points 3 and 4, the controller design is completed. Since we have real perturbations, point 2 requires Real μ calculations [11], which is still a research topic. However, the existing Real μ software has proved to be acceptable in many cases.

5.4 More examples

5.4.1 Example 2.

Here we consider Example 2 in [1], in which it is claimed that independent design cannot be used to design a robust controller for this example.

$$G(s) = \begin{bmatrix} \frac{0.66}{6.7s+1} & \frac{-0.61}{8.4s+1} & \frac{-0.005}{9.06s+1} \\ \frac{1.11}{3.25s+1} & \frac{-2.36}{5s+1} & \frac{-0.01}{7.09s+1} \\ \frac{34.7}{8.15s+1} & \frac{46.2}{10.9s+1} & \frac{0.87(11.61s+1)}{(3.89s+1)(18.8s+1)} \end{bmatrix} \tag{18}$$

In this example only robust stability is considered, with independent, multiplicative input uncertainty with uncertainty weight $W_I(s) = 0.13\frac{5s+1}{0.25s+1}$. As for example 1, a second order low pass filter is used in each diagonal element of the IMC controller. This will add three real, repeated scalar perturbations, each repeated twice. From point 2 in the

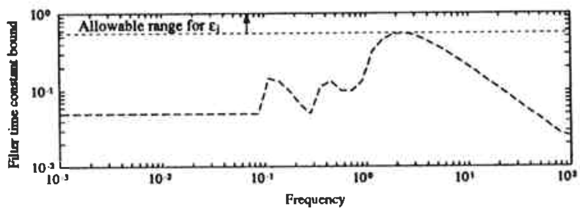

Figure 5: Lower bound on filter time constant bounds for Example 2.

independent design procedure we obtain the results in Fig. 5. From Fig. 5 we see that any value of ϵ larger than 0.55 will be acceptable. Choosing $\epsilon = 1$ for all loops, we find that the system is stable. We thus find that the system will be robustly stable for any value of $\epsilon_i > 0.55$. In general we want ϵ to be small for a faster nominal response.

For both example 1 and example 2, Chiu and Arkun [1] were unable to perform an independent design, using the procedure of Skogestad and Morari [10]. This demonstrates the importance of introducing as little conservatism as possible in the description of the uncertainty associated with the controllers when performing an independent design.

[3]For any value of ϵ within the range found in point 3, the map under the Nyquist D-contour of $\det(I - M\Delta)$ will encircle the origin the same number of times. Thus, if M is found to be unstable in point 4, it is "robustly unstable".

5.4.2 Robust decentralized detunability

Definition 1 *A closed loop system is said to be Robust Decentralized Detunable if each controller element can be detuned independently by an arbitrary amount without endangering robust stability.*

In the IMC framework, controllers are detuned by increasing the filter time constants. We have thus found for example 2 above that the loops can be detuned independently of each other, without endangering robust stability, provided all loops have $\epsilon_i > 0.55$. Thus the closed loop system in example 2 with $\epsilon_i > 0.55$ in all loops is found to be robust decentralized detunable according to Definition 1[4]. After removing the performance requirement from example 1 and redoing the calculations for robust stability, we find that it is robust decentralized detunable provided $\epsilon_i > 0.16$ for both loops.

5.4.3 Example 3, with some further notes on nominal stability.

We would like to emphasize point 4 in the Independent design procedure, that nominal stability must be checked explicitly for one value of ϵ within the bounds found. A decentralized IMC controller as parametrized in Eq. (9) will make the individual loops stable, which in many cases will be considered an advantage. However, integral action is inherent in IMC controllers, and integral action and stability of the individual loops is known to be incompatible with stability of the overall system for certain plants. The Niederlinski Index criterion [8] gives a necessary condition for obtaining stability both of the individual loops and the overall system when there is integral action in all channels. The Niederlinski Index criterion has recently been generalized to open loop unstable plants [4]. Let the number of Right Half Plane (RHP) poles in G be n_U (including multiplicities), and the number of RHP poles in $\tilde{G}$ be $\tilde{n}_U$. Note that in general $\tilde{n}_U \neq n_U$. If all the individual loops are stable, a necessary condition for the stability of the overall system is that

$$\text{sign}\{N_I\} = \text{sign}\{\frac{\det G(0)}{\det \tilde{G}(0)}\} = \text{sign}\{(-1)^{-n_U+\tilde{n}_U}\} \tag{19}$$

Thus, before attempting to perform an independent design, one should check that overall stability can be achieved with integral action in all channels and having stable individual loops.

Example 3. Consider the process

$$G(s) = \begin{bmatrix} \frac{5}{20s^2+12s+1} & \frac{8}{20s^2+12s+1} \\ \frac{6}{40s^2+12s+1} & \frac{2}{40s^2+12s+1} \end{bmatrix} \tag{20}$$

with independent actuator uncertainty with uncertainty weight $W_I(s) = 0.2\frac{10s+1}{s+1}I_2$. Since this plant is stable and the Niederlinski Index is neqative, $N_I = -3.8$, we know that we cannot have the individual loops stable and at the same time achieve overall system stability. Nevertheless, we proceed with independent design, and choose third order low pass filters for both loops. We find that point 3 in the independent design procedure indicates that any value of $\epsilon > 4$ (approximately) will give robust stability (figure omitted). Calculating μ for $\epsilon = 5$ for both loops, we do indeed obtain a value of $\mu < 1$ at all frequencies. The reason, which we find in point 4 in the independent design procedure, is that the system is nominally unstable. The μ test merely tells us that this instability is a robust property. For other cases, it may not be this easy to tell *à priori* that the overall system will be unstable with the individual loops stable.

5.5 Conclusions

We have found that:

- The independent design procedure can be made more powerful by considering decentralized IMC controllers only. The result of considering only decentralized IMC controllers with a specified filter structure, is that the set of possible controller designs considered is much smaller than the set of possible controller designs when trying to find bounds on $\tilde{S}$ and $\tilde{H}$. Of course, for problems where independent design based on bounds on $\tilde{S}$ and $\tilde{H}$ is feasible, restricting the choice of controller to one specific structure may actually be a disadvantage.
- We have demonstrated how to find bounds on the IMC filter time constants which guarantee robust stability/performance. The only uncertainty associated with the controller elements is then the uncertainty in the filter time constants. We therefore have real uncertainty, which is less conservative than the complex uncertainty one has to use when trying to find bounds on $\tilde{S}$ and $\tilde{H}$, as suggested by Skogestad and Morari [10].
- Within the independent design framework, one can derive a bound on the IMC filter time constants which ensures that the system is "robust decentralized detunable", that the loops can be detuned by an arbitrary amount, independent of each other, without endangering robust stability. If a system is robustly decentralized detunable, any subset of the loops can be taken out of service without introducing instability.
- A disadvantage of the proposed independent design procedure is that the bounds obtained are common to all the filter elements, and it is not obvious how to take advantage of the possibility of having differing filter time constants in the different filter elements. However, one may of course use constant ratios between the filter time constants in the independent design procedure (e.g. choosing $\epsilon_1 = \epsilon^*$, $\epsilon_2 = 10\epsilon^*$, etc.).

[4] "Decentralized detunability" for a given controller should not be confused with decentralized integral controllability (DIC), which is a property of the plant only.

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