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BYPASS SELECTION FOR CONTROL OF HEAT EXCHANGER NETWORKS

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Abstract

Decisions made during steady-state process design may put severe limitations on the achievable control performance or controllability of chemical plants. We show how different controllability measures may be used to evaluate controllability and select bypasses and appropriate pairings of heat exchanger networks. Flow rate dependence of heat transfer coefficients is included in the model. This is found to have a significant effect on control. Most of our results confirm good engineering practice: Prefer designs and bypass selections where all critical targets are controlled by either utility streams or bypasses with a direct effect. Consider bypasses over several exchangers in series as such total bypasses may enhance the controllability or the flexibility without having to install more control loops. Avoid bypass selections with two or more downstream paths to one critical target and designs where both output streams of one heat exchanger are critical targets.

Keywords

Process control; Controllability; Heat exchanger networks; Control configuration selection

1 Introduction

During the last decade there have been a large number of papers dealing with *steady-state* optimal design of heat exchanger networks (HEN). However, in practice input temperatures, flowrates, overall heat transfer coefficients etc. vary and we need degrees of freedom for control and on-line optimization. We will refer to the task of keeping the network outlet temperatures at their target values during a *short* time horizon as the controllability or dynamic resiliency problem. When the time horizon is long, the task will be referred to as the flexibility or static resiliency problem.

Quite a few authors have looked at the latter problem. Colberg and Morari (1988) give a comprehensive summary of the research on synthesis and analysis of flexible HENs. Important results are the definition of the four worst-case corners (max exchanger capacity, max cooling, max heating and max area) of the disturbance set (Marselle *et al*, 1982); the resiliency index (Saboo *et al*, 1985); the flexibility index (Swaney and Grossmann, 1985); sensitivity tables and "downstream paths" (Kotjabasakis and Linnhoff, 1986) and automated solution by mathematical programming (Floudas and Grossmann, 1987).

On the other hand, there is little published on controllability of HENs. Nisenfeld (1973) introduce the use of the relative gain array (at steady-state) to evaluate control of a HEN. Holt and Morari (1984) show that controllability of some HENs can be improved by *increasing* the time delay between the exchangers. Reimann (1986) denotes the ratio between the apparent (pseudo first order) time constant and deadtime of exchangers as the controllability index. He uses this index and static efficiency of single heat exchangers to suggest some guidelines on how to design

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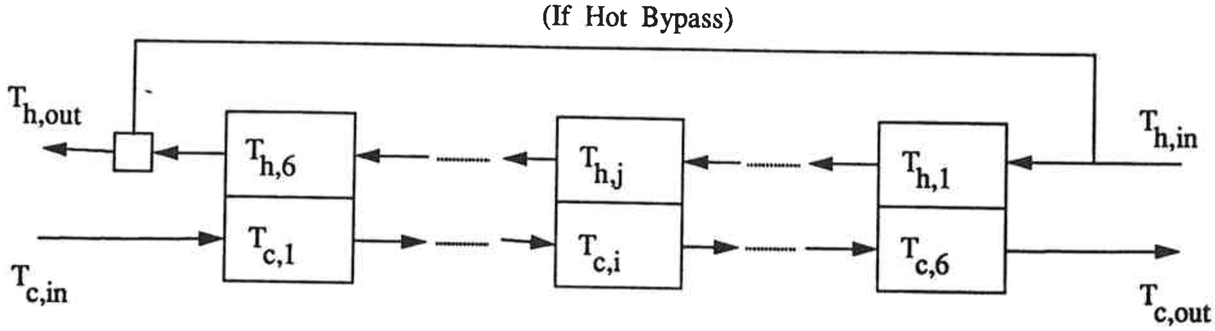

Figure 1: Lumped heat exchanger model used to study controllability of HEN

a network with good controllability. Calandranis and Stephanopoulos (1988) address flexibility and controllability and discuss the dynamics of HEN briefly. They state that bypasses for control purposes should always be placed so that they directly affect the target temperature (controlled output). However, this may not always be possible or desirable, and in this paper we consider all possible bypass locations. Georgiou and Floudas (1990) suggest to use structural analysis (multi-input connectability, structural functional controllability and observability) to select manipulated inputs (bypasses) in HEN. Daoutidis and Kravaris (1991) use structural relative order tables to ensure low relative order between manipulator and target temperature.

We use a lumped model where each side of the exchanger is modeled as 6 mixing-tanks in series. In this "cell-model" heat is transferred from one mixing-tank on the hot side to the corresponding cold as shown in fig. 1. This dynamic model is numerically linearized around the steady-state operating point to get a state-space description of the network as discussed by Wolff *et al* (1991). Wolff *et al* use a pure lumped model and assume the driving force in each cell to be $T_h(i) - T_c(i)$. However, most papers addressing optimization of HENs implicitly assume use of ideal countercurrent heat exchangers, and Reimann (1986) and Jonsson (1990) recommend to use the logarithmic mean temperature difference as driving force in each cell. This represents a hybrid between a lumped and a distributed model. Although one might argue that one never has ideal countercurrent heat exchange, and that a pure lumped model might be better physically, we have chosen to use this hybrid model mainly to get consistency with previous literature.

2 Controllable or dynamic resilient HEN

2.1 Degrees of freedom for control and optimization

A single heat exchanger transfers heat from one stream to another, and has only one degree of freedom, which is the heat duty. During *design* of HEN, necessary area for each exchanger is calculated from the duties. However, during *operation* one has to vary the heat duty in order to meet the specifications, which may be to keep certain temperatures constant. In most of the work on static resiliency it has been assumed that this may be done by manipulating the exchanger area directly. This may be possible in a few cases, for example for flooded condensers, but in most cases one must install bypass streams and manipulate the bypass fractions in order to change the heat duty.

2.2 Bypass placement

In practice it may be necessary to place bypasses for two reasons:

- Flexibility or static resiliency. Each heat exchanger must have sufficient area to maintain the specifications for all possible operating points (static disturbances). In a specific operating point this area may be too large and may be effectively reduced by the use of bypass streams.

- Controllability or dynamic resiliency. In a specific operating point one needs degrees of freedom (bypasses) to get satisfactory control behavior in the presence of dynamic disturbances.

The optimal location for the bypass is generally different depending on whether it is for static or dynamic resiliency. In this paper we mainly address the control aspects and assume that the network is designed and operated at a given operating point. There may then be additional bypasses (or area adjustment) which take care of long-term or static disturbances. These additional bypasses may be used in a hierarchical manner (e.g. using traditional cascade or model predictive control) to reset the control bypass fractions to their nominal values.

Nominal bypass fractions. When evaluating the different examples one must decide on nominal bypass fractions. Preferably this should be done from a rigorous optimization where disturbances and the performance specifications of the controlled outputs are taken into consideration. For simplicity, the different bypass alternatives (manipulated inputs) are linearized with a nominal bypass fraction of 0.1% and scaled to a constant bypass fraction of 10%. Note that most of the controllability measures are independent of the input scaling, and thus not critically dependent on the exact values of the bypass fractions.

2.3 Controllability (dynamic resiliency)

Disturbance range. To be able to assess controllability, the dynamic disturbance range (e.g. variations in supply temperatures and flowrates) must be known or at least estimated. The dynamic disturbance range is the expected variations at *a given operating point.*

Performance specification. In order to assess controllability, performance requirements in terms of the allowed variations of the controlled target temperatures must also be specified. The specification is a rough, but nevertheless *quantitative* expression of how critical the different target temperatures are.

Definition of controllability or dynamic resiliency. Given a HEN design with given nominal bypass fractions, defined disturbance range and performance requirements. *The HEN is controllable or dynamical resilient if the performance specifications can be met for the specified design by use of feedback control.* As it is desirable to have a simple control system without feedforward etc, this is an appropriate definition.

2.4 Number of alternative sets of bypass selections

Suppose that N_{byp} bypasses are to be used as manipulated inputs in a HEN. If N_{hx} is the number of *process* heat exchangers, excluding heaters and coolers, the number of different alternative sets of bypass selections are

$$2^{N_{byp}} \frac{N_{hx}!}{N_{byp}!(N_{hx} - N_{byp})!} \tag{1}$$

Eq. 1 is derived by considering only single bypasses (bypasses over one unit) and only bypass on one side of an exchanger.

In industry, total bypasses (i.e. bypasses over several exchangers in series) are used frequently. There are two good reasons to install total bypasses:

- Reduce process deadtime between manipulator and controlled output.
- Reduce the number of bypasses/control loops in order to reduce the investment cost and simplify operation while maintaining the desired flexibility.

The assumption of single bypasses is implicit in all previous works we know of. The reason for this is probably that these works mainly address flexibility, and neglect the cost of installing bypasses and control loops is neglected. For example, the flexibility of one total bypass over two exchangers in series can always be achieved by installing 2 single bypasses, and the area requirement will be smaller in the latter case.

When total bypasses are allowed the structure of the network must be known to decide the number of alternative sets of bypass selections. Suppose bypasses over one or two exchangers are allowed and that N_{pair} is the number different pairs of exchangers in series (counting also the utility exchangers). If there are j single bypasses, there must be $N_{byp} - j$ number of double bypasses. The number of alternative sets of bypass selections is then

$$\Sigma_{j=0}^{N_{byp}} 2^j \frac{N_{hx}!}{j!(N_{hx}-j)!} \frac{N_{pair}!}{(N_{byp}-j)!(N_{pair}-N_{byp}+j)!} \qquad (2)$$

where the summation is over the number of single bypasses and $N_{pair} \geq N_{byp}$ is assumed. The extension to include total bypasses over three and more exchangers is straigthforward.

Finally, if we want to use decentralized control, N_{byp} will be equal to the number of controlled outputs, and we have $N_{byp}!$ different pairings for each of the alternative bypass sets in Eq. 2.

The rapid growth of this combinatorial problem with number of process exchangers N_{hx} and bypasses N_{byp} is evident, and this makes it difficult to apply techniques which involve searching over all alternatives. Therefore it is desirable to develop simplified methods and to obtain insights in order to be able to formulate simpler "rules".

3 Measures for evaluating controllability

When evaluating whether a set of N_{byp} bypasses may be an appropriate configuration to control the N_{byp} target temperatures ($y's$) we consider the linear model:

$$y(s) = G(s)u(s) + G_d(s)d(s) \qquad (3)$$

where $u(s)$ is the vector of manipulated inputs (bypasses) and $d(s)$ the disturbances.

Controllability measures are used to evaluate the inherent control properties of the process, $G(s)$ and $G_d(s)$ without having to do a controller design. A disadvantage with most measures for analyzing controllability is that they have to be recomputed for each control configuration.

We will use the measures listed below to evaluate controllability or dynamic resilience of HENs. Further justification for their use is given by Hovd and Skogestad (1991).

Scaling. We always assume that $G(s)$ and G_d is scaled so that allowed magnitude of the manipulators ($u's$), disturbances ($d's$) and controlled outputs ($y's$) should vary between 0 and 1 at all frequencies.

Structural controllability. If one row i of $G(s)$ is zero, the set must be discarded because there is no downstream path from any of the manipulators to output i. Alternatively, one could check the rank of the structural matrix corresponding to $G(s)$ as described by Georgiou and Floudas (1990). However, these bypass selections (if any) will be eliminated by the controllability measures, too.

Input constraints. A rough indicator for a *good* configuration is that, for each output y_i, there is one $|g_{ij}| > 1, \omega < \omega_B$ (with the variables scaled as indicated above). This does not take the magnitude of the disturbances or multivariable effects into account, and a better indication is easily derived from the requirement of *perfect disturbance rejection.* For square systems:

$$y(j\omega) = 0 \Rightarrow u(j\omega) = G^{-1}(j\omega)G_d(j\omega) \qquad (4)$$

One should avoid configurations with elements in $|G^{-1}G_d|$ larger than 1. Specifically if $||G^{-1}G_d||_\infty$ (the largest row sum) is greater than 1 in the frequency range important for control, then the nominal bypass fractions (provided they are less than 50%) must be increased to disallow negative bypass flows. If that is impossible, for example due to driving force constraints on the exchangers, the set of bypasses should be discarded.

Bandwidth limitations, RHP-zeros. A right half plan (RHP) transmission zero of the plant transfer function limits the achievable bandwidth regardless of the controller used. When decentralized control is used, one should also avoid RHP zeros in the elements in order to maintain stability of the individual loops. Bypass selections that give no RHP zeros are preferred.

Interactions, use of RGA. The relative gain array (RGA) is used as a measure of interactions in a general sense, and bypasses that minimize interactions are preferred. In particular, one should avoid cases with large RGA-values at frequencies close to the closed-loop bandwidth because such plants are fundamentally difficult to control (irrespective of the controller).

Pairing, use of RGA. We preferably want to control the HEN with decentralized control loops and use the relative gain array (RGA) as function of frequency to the decide the best pairing, i.e. what bypasses should be used to control what target temperatures. We like to pair such that the RGA-value is close to one around the the expected bandwidth of the system. To ensure stability of individual loops and remaining subsystem when one loop fails, pairing on negative steady-state values should be avoided.

Open-loop disturbance rejection. The frequency-dependent open-loop disturbance gain matrix (G_d) include both the information in the sensitivity tables of Kotjabasakis and Linnhoff (1986) at steady-state or low frequency and the structural relative order tables of Daoutidis and Kravaris (1991) at high frequency.

Closed-loop disturbance rejection. For decentralized control some other measures are even more useful to evaluate disturbance rejection. We assume from now on that the manipulators are numbered after the pairing is decided so that u_1 is used to control y_1 etc. Then the controller matrix C is diagonal with elements c_i.

At low frequency the offsets of the closed loop system may be approximated by:

$$e(s) = y(s) - r(s) \approx -S_{diag}(s)G_{diag}G^{-1}r(s) + S_{diag}(s)G_{diag}G^{-1}G_d(s)d(s) \qquad (5)$$

where G_{diag} consists of the diagonal elements (g_{ii}) of G and S_{diag} is defined as $(I + G_{diag}C)^{-1}$, i.e. has elements $1/(1+g_{ii}c_i)$ (Hovd and Skogestad, 1991). We define the closed-loop disturbance gain (CLDG) as $\Delta = G_{diag}G^{-1}G_d$. The elements are denoted δ_{ik} and represent the apparent disturbance gain from disturbance k to output i when the other loops are closed.

Since G_d and G are scaled the magnitude $|\delta_{ik}|$ at a given frequency directly gives the necessary loop gain $|g_{ii}c_i|$ at this frequency needed to reject this disturbance. The frequency where $|\delta_{ik}(j\omega)|$ crosses 1 gives the minimum bandwidth requirement for this disturbance. It should be less than the bandwidth that can be achieved in practice, which will be limited by time delays, RHP zeros etc.

Set-point tracking, use of PRGA. In a similar manner the performance relative gain array (PRGA) defined as $\Gamma = G_{diag}G^{-1}$ can be used to evaluate set-point tracking of the system. However, in process control disturbance rejection is often the major concern, and since PRGA (Γ) will generally be small when CLDG ($\Delta = \Gamma G_d$) is small, evaluation of set-point tracking can normally be omitted.

4 Examples

Notation: The input configuration with a *hot single* stream bypass on exchanger no 1 and a *cold single* stream bypass on exchanger 2 is denoted *1H2C*. With decentralized control case $1H2C$ is the control configuration using bypass $1H$ to control y_1 and bypass $2C$ to control y_2. Total bypasses are denoted similarly, the input configuration with a *hot double* bypass over exchangers 1 and 3 and a *cold single* bypass around exchanger 1 is denoted 13H1C. All other cases are denoted accordingly.

Data: Overall heat transfer coefficients are calculated from the simplified expression $1/U = 1/h_h + 1/h_c$. The film transfer coefficients dependence on flowrate is given by: $h_h \sim q^{0.6}$ and $h_c \sim q^{0.8}$ (assuming hot fluid on the shell side of the exchangers). Additional data for the examples can be found in the references.

Scalings: For inputs (bypasses): Unit change corresponds to ± 10% bypass fraction. For outputs (temperatures): Unit change corresponds to ± 3^oC. For disturbances in supply temperatures of the streams: Unit change corresponds to ± 10^oC. For disturbances in flowrates of the streams: Unit change corresponds to ± 20%.

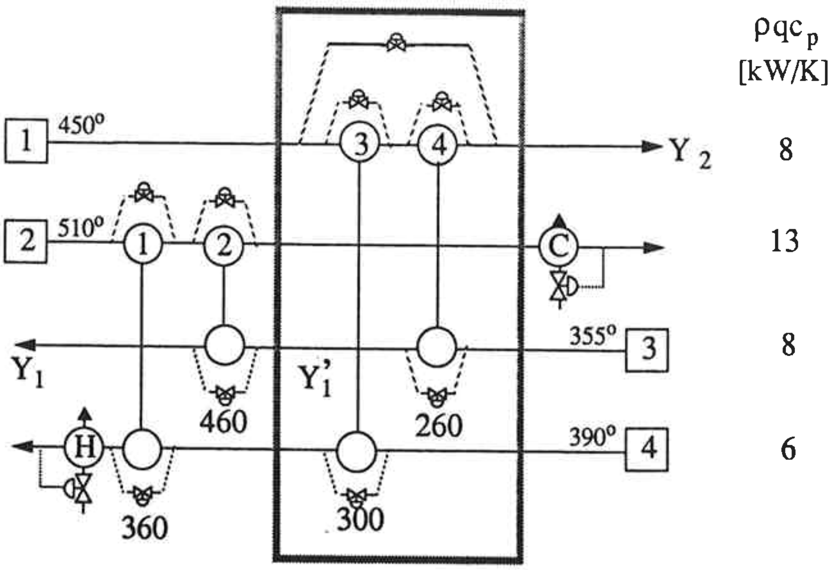

Figure 2: Example HEN design from Townsend and Morari (1984)

4.1 Full network from Townsend & Morari (1984)

Consider the network in Fig. 2 from Townsend and Morari (1984) where temperatures $y_1 = T_{3t}$ and $y_2 = T_{1t}$ are to be controlled by introducing two bypasses. In this example $N_{hx} = 4$ and $N_{byp} = 2$ which gives 24 different sets of single bypasses. As $N_{pair} = 6$ and there are 2 possible triple bypasses, there are 116 different sets if total bypasses is allowed. For simplicity, we will only consider single bypasses in this example.

Analysis of steady-state matrices and input constraints. The steady-state gains from the 8 alternative single bypasses to the outputs are as follows:

$G^{all}(0) =$	$1H$	$1C$	$2H$	$2C$	$3H$	$3C$	$4H$	$4C$
y_1	0.11	0.41	−0.59	−1.05	−0.04	−0.08	−0.17	−0.20
y_2	0	0	0	0	0.16	0.34	0.45	0.50

This linear gain matrix was obtained by linearizing around a steady-state with bypasses around all exchangers, but with a nominal bypass fraction of zero. Note the following:
1) The gain from exchangers 1 and 2 to output y_2 is zero (also dynamically). This eliminates the 4 cases using both these exchangers (e.g. cases $1H2H, 1H2C, 1C2H$ and $1C2C$).
2) Most gains are small compared to one. This signals potential problems with input constraints, but to understand this better we also have to consider the disturbances. The disturbance gain matrix is given by

$G_d(0) =$	T_{1s}	T_{2s}	T_{3s}	T_{4s}	q_1	q_2	q_3	q_4
y_1	0.94	1.22	0.57	0.60	0.98	2.08	−2.20	−1.46
y_2	0.54	0	1.88	0.91	2.03	0	−0.99	−0.68

For example for output y_2, to reject a unit disturbance in q_1 (corresponds to a 20% change in q_1) by use of bypass $4H$ we need a bypass change of $2.03/0.45 = 4.5$. A unit bypass change corresponds to 10% so this corresponds to a bypass fraction of 45%. Note that this is a linear analysis, the necessary bypass is in fact only 32%, so the linear analysis gives a conservative estimate of the necessary bypass in this case.

The inputs (bypasses) needed for perfect control, $G^{-1}G_d$, indicate that even for the best case $2C4H$ we need a bypass fraction of about 40%, see fig. 3. Note that using flow dependent film transfer coefficients favors control as $g_{ij}(s)$ increase whereas $g_{dik}(s)$ decrease. The combined reduction on the required input for perfect control ($G^{-1}G_d$) is large (typically about 40%).

Bandwidth limitations: RHP-transmission zeros. The are 2 *opposing effects* from exchanger 3 to output y_1, one through exchangers 4 and 2 with positive gain and one through exchangers

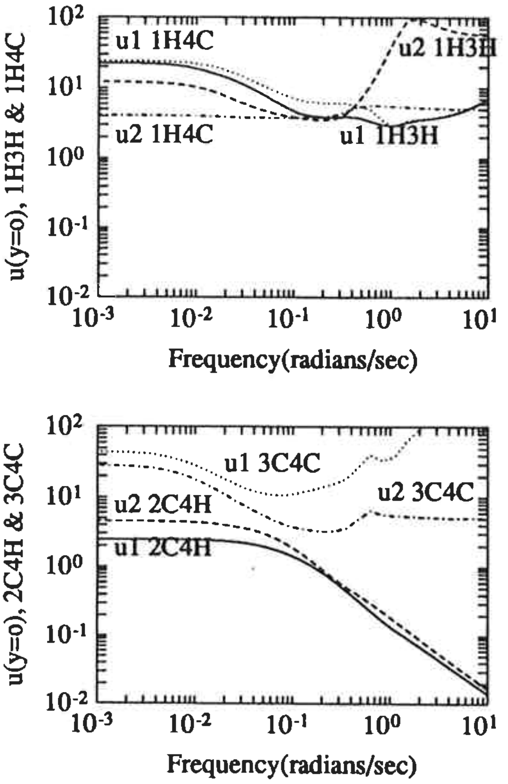

Figure 3: Required manipulation for perfect control, $G^{-1}G_d$, for different cases. Example from Townsend & Morari (1984).

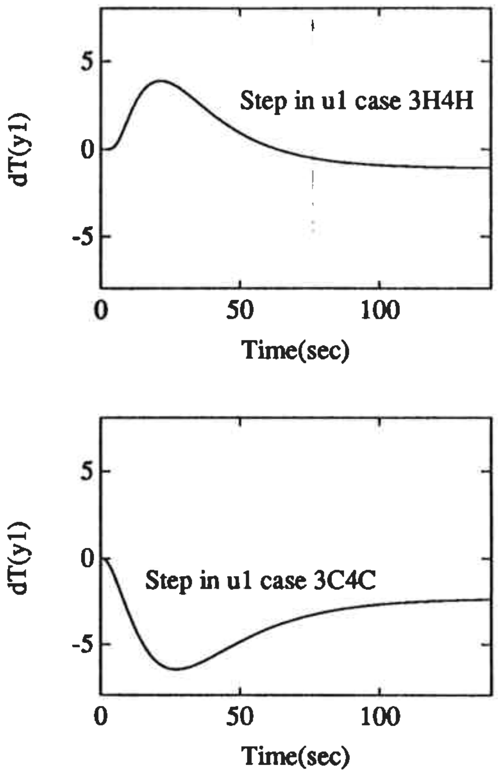

Figure 4: Time simulation (linear model) of step in u_1 for cases $3H4H$ and $3C4C$. Example from Townsend & Morari (1984).

1 and 2 with negative gain. Such opposing effects always exist when there are downstream paths from both sides of a bypassed exchanger to an output. For certain parameter values the opposing effects yield a RHP-zero in the elements which limit the achievable bandwidth and the controllability of the HEN for decentralized control. For this example cases $3H4H, 3H4C, 3C4H$ and $3C4C$ give a RHP-zero for g_{11} *and a multivariable RHP-zero.* For cases $3C4H$ and $3C4C$ the multivariable RHP-zero is significant ($0.004 rad/sec$) and these cases ought to be disregarded. Actually, with our dynamic model case $3H4H$ will give an inverse response. A time simulation of a step in bypass on exchanger 3 for cases $3H4H$ and $3C4C$ is shown in fig. 4.

Interaction, pairing: RGA. The 1,1-element of the RGA (λ_{11}) is 1.0 at all frequencies for all cases with the first bypass on either exchanger 1 or 2 and the other bypass on exchanger 3 or 4. This can be seen directly from the network structure, since there is no downstream path from exchangers 1 and 2 to output y_2. As exchangers 1 and 2 can only be used to control output y_1, the best pairing for decentralized control is obvious for all these cases. Consider in the following the 4 cases with bypasses on exchangers 3 and 4 as the 2 manipulated variables. In this case it is not easy to decide the appropriate pairing. Bode-plot of λ for different choices of manipulated variables is shown in fig.5. Pairing exchanger 4 to output y_2 give λ equal to 1.0 at high frequency in all cases, but *negative λ at steady-state* as could be expected from the RHP-transmission zeros. Only case $3H4C$ with reversed pairing (i.e. $4C3H$) seems to be acceptable for decentralized control. From this example it is clear that even in simple cases it may not be obvious how to select bypasses and appropriate pairings, and the conclusion will depend on the operating point. For illustration, in a previous paper (Wolff *et al*, 1991), we considered the same network structure, but used (due to an error) larger heat capacity flowrates. In that case the RGA values were not negative at steady state, and we concluded that pairing u_1 to y_1 was acceptable in all cases.

Disturbance rejection. To discriminate between the remaining 16 cases where output y_1 is controlled by a bypass on exchanger 1 or 2 and output y_2 is controlled by a bypass on exchanger 3 or 4, the closed loop disturbance gain may be helpful. The worst disturbances to reject was found to be temperature disturbance on stream 3 on output y_2 and flowrate disturbance on stream

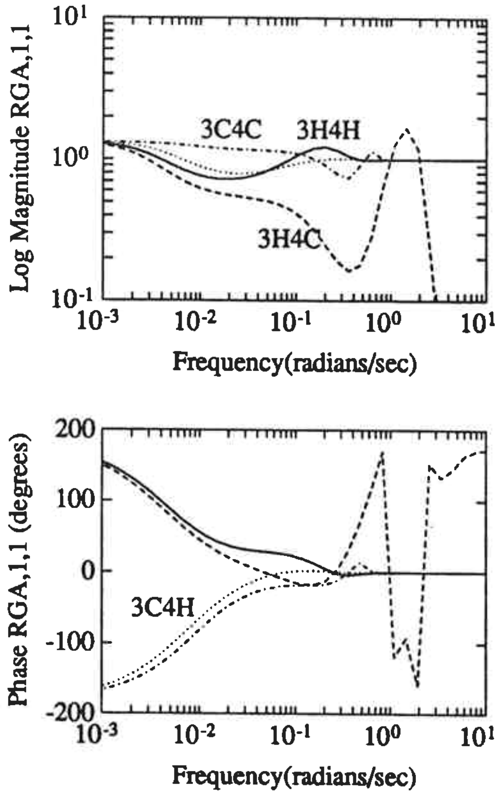

Figure 5: RGA,1,1 for cases $3H4H$, $3H4C$, $3C4H$ and $3C4C$. Example from Townsend & Morari (1984).

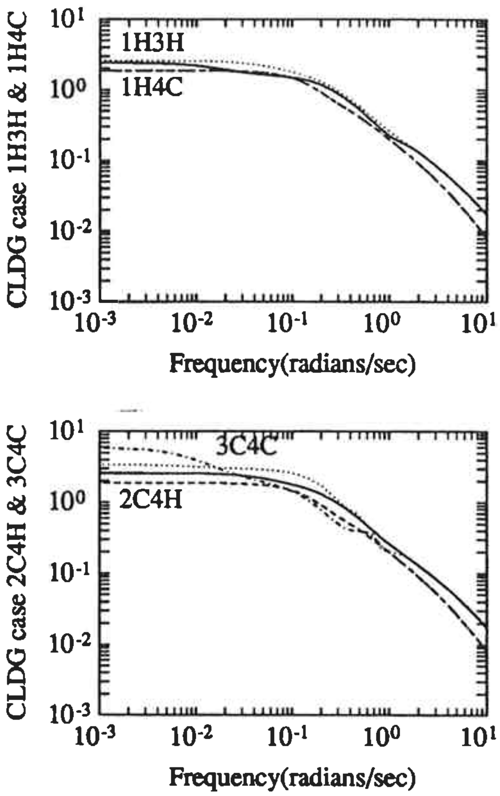

Figure 6: Selected (worst case) elements of CLDG = $G_{diag}G^{-1}G_d$ for different cases. Example from Townsend & Morari (1984).

3 on output y_1. The most important information from the CLDG-plot is the frequency were the curves cross 1.0. For all cases and both loops the necessary bandwidth is $\approx 0.2 rad/sec$, see fig. 6. For case $2C4H$ with direct effects from both inputs to the corresponding outputs, the speed of the response will be about 0.05 to 0.5 rad/sec, i.e. about the required. For all other cases the speed of response will be slower. For example, for case $2H4H$ there is no direct effect from $2H$ to output y_1 and the response will be slowed down by exchanger 2. The effect will however not be very large because bypass $1H$ affect the hot end of exchanger 2 fast.

4.2 Part of network from Townsend & Morari

Suppose that the cold outlet of exchanger 4 of the network considered above was to be controlled, for example to avoid fouling or corrosion. The 2 outputs, y_1' and y_2 from exchanger 4 must be controlled by adjusting the duty of exchangers 3 and 4. This reduced problem (inside the frame in fig. 2) has 4 control configurations using single bypasses exclusively (e.g. $3H4H, 3H4C, 3C4H, 3C4C$) and 4 control configurations with a total bypass around exchangers 3 and 4 (e.g. $34H3H, 34H3C, 34H4H, 34H4C$). Each configuration has 2 alternative pairings.

The steady-state gains from the 5 alternative manipulated variables to the outputs are:

$$G^{all}(0) = \begin{array}{cccccc} & 3H & 3C & 4H & 4C & 34H \\ y_1' & 0.21 & 0.44 & -0.45 & -0.50 & -0.94 \\ y_2 & 0.16 & 0.34 & 0.45 & 0.50 & 1.32 \end{array}$$

The steady-state disturbance transfer function gains are

$$G_d(0) = \begin{array}{ccccccccc} & T_{1s} & T_{2s} & T_{3s} & T_{4s} & q_1 & q_2 & q_3 & q_4 \\ y_1' & 0.71 & 0 & 1.45 & 1.18 & 1.88 & 0 & -1.18 & -0.88 \\ y_2 & 0.54 & 0 & 1.88 & 0.91 & 2.03 & 0 & -0.99 & -0.68 \end{array}$$

A controllability analysis similar to the one used above yields the following conclusions [1]:

[1] The complete analysis with figures may be obtained from the authors by request.

- Pairing exchanger 3 to output y_1' is preferred (from frequency-dependent RGA, at steady-state they are equal)
- Bypass $3C$ is better than $3H$ at steady-state whereas $3H$ is better than $3C$ dynamically (from input constraints, $G^{-1}G_d$, RGA and CLDG are similar).
- Bypass $4H$ is better than $4C$ with single bypasses. (from RHP-zero and frequency-dependent RGA and CLDG, at steady-state they are equal and similar, respectively) whereas bypass $4C$ is better than $4H$ with total bypass (from input constraints, $G^{-1}G_d$ and CLDG).
- Bypass $4H$ is similar to bypass $34H$ when the other bypass is $3H$ or $3C$.

Thus, with decentralized control, either $3H$ or $3C$ should be paired to y_1 and either $4H$ or $34H$ to y_2. The trade-off between $3H$ and $3C$ depend on the controller to be used.

5 Summary

5.1 Proposed stepwise procedure

We have looked at the problem of selecting bypasses and appropriate pairings for decentralized control and evaluation of controllability or dynamic resilience of HENs. We suggest the following stepwise procedure (as all matrices are assumed to be scaled, "large" means greater than unity):

1. G: Discard bypass set if one row i of G is zero. (No downstream from any input to y_i)
2. $G^{-1}G_d$: Discard set if large within interesting frequency
3. RHP-zeros: Discard set if significant RHP-zero exist
4. Interactions, RGA: Discard set if negative at steady-state *or* large within expected bandwidth
5. Decentralized control, CLDG: Used to check results above and design controllers (gives expected bandwidth)
6. Decoupler: Consider decoupler if RGA $\neq I$ or PRGA is large

If the HEN design passes tests 1, 2 and 3 for some bypass set, the design is feasible. A frequency dependent singular value decomposition (SVD) of G(s) may provide additional insight into the strong and weak "directions" of the process.

5.2 Design for controllability

So far our work with different HEN example problems (Mathisen *et al*, 1991, 1992) indicate that the following controllability "rules" may be recommended:

Selection between different HEN designs:

- Avoid designs with both output temperatures of one exchanger as controlled outputs
- Avoid designs with more critical targets than exchangers on one side of the pinch.

Selection of manipulated inputs (placement of bypasses):

- Prefer bypasses on exchangers with a large effect on only one output.
- Avoid bypasses exchangers with 2 (or more) downstreams paths to outputs.
- Prefer bypasses with a direct (dynamic) effect on an output. **If not possible**:

  Prefer bypasses on the opposite stream *or* the upstream exch. of the opposite stream.

  Avoid bypasses on the upstream exchanger of the controlled stream (or further away).

Nomenclature

$C(s)$ - Diagonal controller transfer function matrix; $c_i(s)$ - Controller element for output i
$d(s)$ - Vector of disturbances.
$e(s) = y(s) - r(s)$ - Vector of output errors
$G^{all}(s)$ - Augmented process transfer function matrix with all possible inputs
$G(s)$ - Process transfer function matrix; $g_{ij}(s)$ - ij'th element of $G(s)$
$G_d(s)$ - Disturbance transfer function matrix; $g_{dik}(s)$ - ik'th element of $G_d(s)$
h - Film transfer coefficient $[W/m^2, K]$
N_{byp} - No of bypasses in HEN (Dec. control $\Rightarrow N_{byp}$ = No of controlled outputs)
N_{hx} - No of process exchangers in HEN
N_{pair} - No of different pairs of units in series in HEN
q - Volume flowrate $[m^3/s]$
$r(s)$ - Reference signal (set-point) for outputs
$S(s)$ - Sensitivity function $S = (I + GC)^{-1}$
T - temperature $[K]$ or $[°C]$
U - Overall heat transfer coefficient $= h_h h_c/(h_h + h_c)$
$u(s)$ - Vector of manipulated inputs.
$y(s)$ - vector of outputs
$\Delta(s)$ - Closed loop disturbance gain matrix; $\delta_{ik}(s)$ - ij'th element of $\Delta(s)$
$\Gamma(s)$ - Performance relative gain matrix
$\lambda_{ij}(s)$ - ij'th element of Relative Gain Array matrix
ω (ω_B) - Frequency (Closed loop bandwidth)

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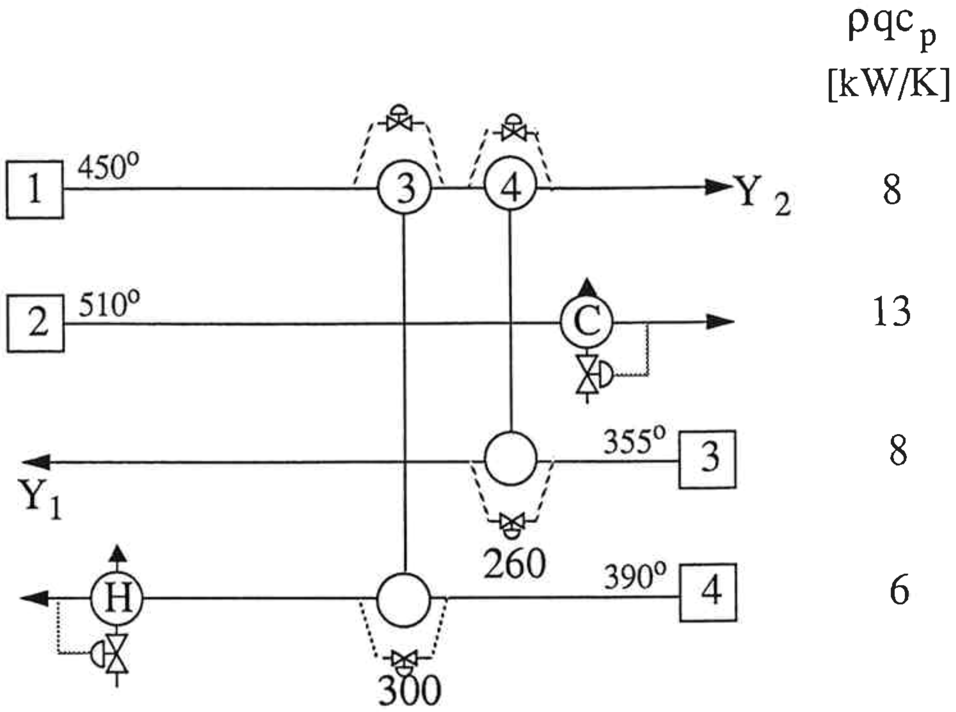

Figure 7: Part of network from Townsend and Morari (1984)

Appendix

Controllability analysis of Part of network from Townsend & Morari

The reduced problem, which is shown in fig. 7, has 4 control configurations using single bypasses exclusively (e.g. $3H4H, 3H4C, 3C4H, 3C4C$) and 4 control configurations with a total bypass around exchangers 3 and 4 (e.g. $34H3H, 34H3C, 34H4H, 34H4C$). Each configuration has 2 alternative pairings.

Analysis of steady-state matrices and input constraints. The steady-state gains from the 5 alternative manipulated variables to the outputs are:

$$G^{all}(0) = \begin{array}{c|ccccc} & 3H & 3C & 4H & 4C & 34H \\ y_1' & 0.21 & 0.44 & -0.45 & -0.50 & -0.94 \\ y_2 & 0.16 & 0.34 & 0.45 & 0.50 & 1.32 \end{array}$$

Note that the gains from bypass $3C$ are twice as large as those from bypass $3H$. The gains from the double bypass $34H$ are by far the largest. The magnitude of these gains will however be a bit reduced when a second bypass around either exchanger 3 or 4 is present.

The steady-state disturbance transfer function gains are

$$G_d(0) = \begin{array}{c|cccccccc} & T_{1s} & T_{2s} & T_{3s} & T_{4s} & q_1 & q_2 & q_3 & q_4 \\ y_1' & 0.71 & 0 & 1.45 & 1.18 & 1.88 & 0 & -1.18 & -0.88 \\ y_2 & 0.54 & 0 & 1.88 & 0.91 & 2.03 & 0 & -0.99 & -0.68 \end{array}$$

Note that 1) Disturbance of stream 3 is the most difficult temperature disturbance to reject because stream 3 immediately affect the hot outlet of exchanger 4. 2) Flow disturbance of stream 1 is more difficult to reject than flow disturbance of stream 3 because stream 1 goes through exchanger 3 before reaching exchanger 4, altering the inlet temperature of exchanger 4, too. It is important to beware of the fundamental difference between temperature and flowrate disturbances, temperature disturbances are dampened through exchangers in series whereas the temperature effect of flowrate disturbances are enforced. The steady-state gains from $G(s)$

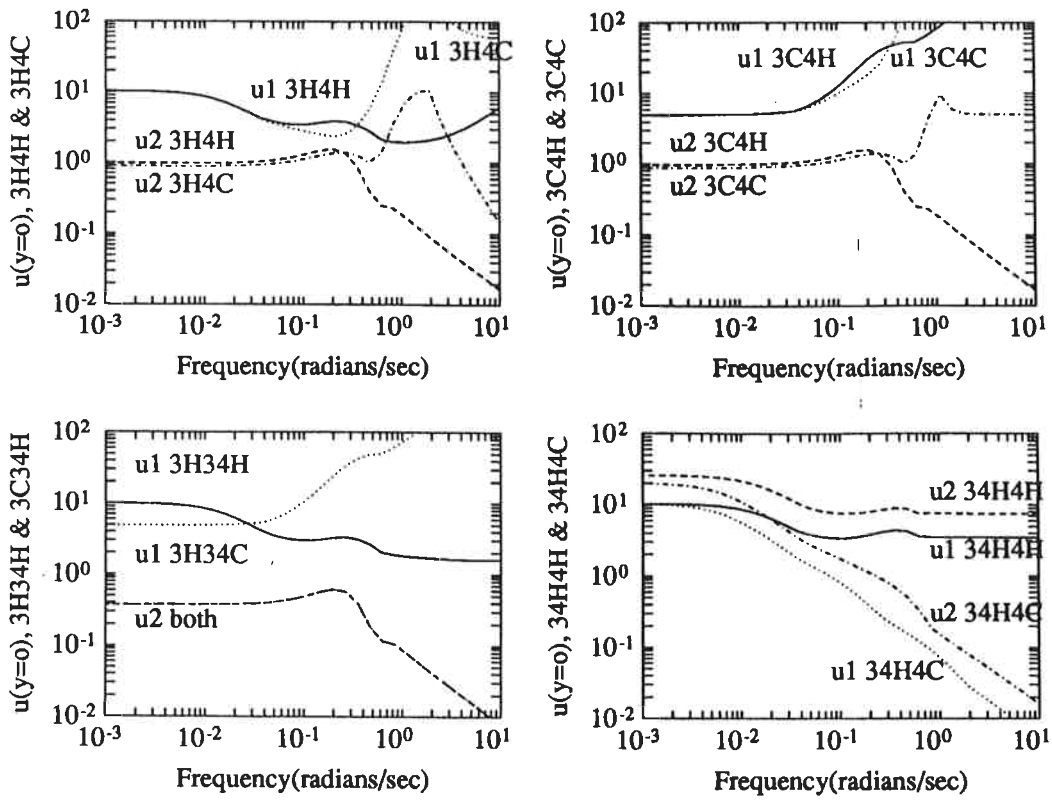

Figure 8: Required manipulations for perfect control $u_{y=0}$ for the 8 cases. Part of example from Townsend & Morari (1984).

The magnitude of the inputs (bypasses) needed for perfect control is given by row sums of $G^{-1}G_d$. The 8 alternative cases, are plotted in fig. 8.

Among the cases with single bypasses only, $3C4C$ is best at steady-state, whereas case $3H4C$ seems to be best at higher frequencies. Among the cases with a total bypass around exchangers 3 and 4, case $3C34H$ is best at steady-state, whereas case $3H34H$ is best at higher frequencies. Note that including a total bypass cannot remedy the problems with constraints. The reason for this is that there are only 2 independent duties in any case.

Bandwidth limitations: RHP zeros. Cases or control configurations $3H4C$ and $3C4C$ result in a system with a multivariable RHP-zero at $0.14 rad/sec$. This is a limitation of the control performance.

Interaction, pairing: RGA. The 1,1-element of the RGA (λ_{11}) for control configurations $3H4H, 3H4C, 3C4H, 3C4C, 3H34H, 3C34H, 4H34H$ and $34H4C$ are shown in fig. 9. At steady-state λ_{11} is the same for the 4 single bypass cases. This is because we control both streams out of a exchanger which has only one degree of freedom. The value of $\lambda_{11}(0)$ is 0.56 illustrating the interaction between the loops. λ_{11} for the cases $3H4H$ and $3C4H$ (solid line) is the same at all frequencies. The reason for this is that both bypass $3H$ and $3C$ must affect which is where the other manipulator is placed. λ_{11} for the cases $4H34H$ and $34H4C$ is considerably higher than for the single bypass cases. Here both bypasses ends immediately after exchanger 4, and the strong competition between the loops gives higher λ_{11}. The steady-state value is 3.35.

When λ_{11} is used to decide the appropriate pairing for decentralized control, we get some surprising and interesting results. The selected pairings is indicated in fig. 9. For all single bypass cases we should use exchanger 3 to control y_1'. When $4C$ is used λ_{11} changes sign at $\omega \approx 0.5$ requiring a lower bandwidth. The pairings are opposite of what is used in industry, at least for cases $3H4C$ and $3C4C$.

For the total bypass cases with a single bypass around exchanger 3 the pairing is analogous, i.e. exchanger 3 should be used to control output y_1'. This as one would expect as the total bypass have a direct effect on output y_2. When a single bypass around exchanger 4 is used together with the total bypass, λ_{11} indicate that exchanger 4 should be used to control output y_1'.

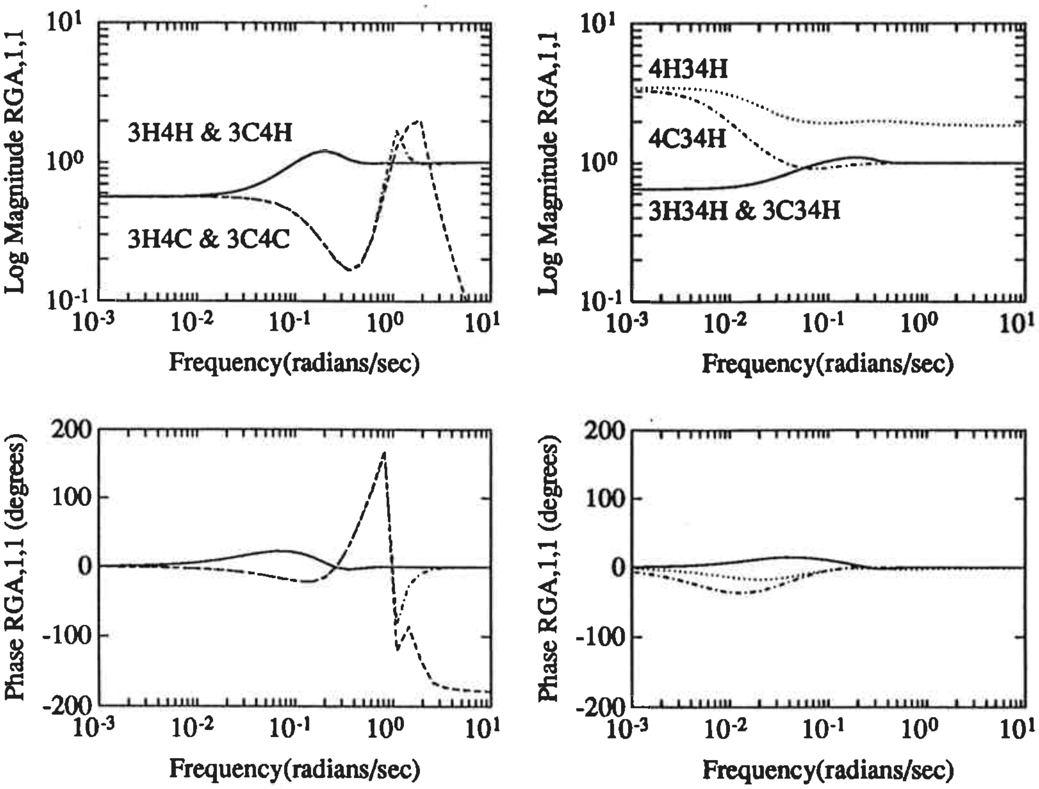

Figure 9: RGA,1,1 for the 8 cases. Part of example from Townsend & Morari (1984).

For case $34H4H$ the interaction between the loops are large even at higher frequencies because both bypasses ends at the same place in the HEN.

In summary, for all cases we should use exchanger 3 to control y_1'. The reason for this is the "deadtime" through exchanger 4 which occur when exchanger 3 is used to control y_2. It should be noted that the pairings might be reversed if the dynamics of the metal tubes between the fluids is slow.

Disturbance rejection with decentralized control CLDG. The closed loop disturbance gains for the disturbance that is most difficult to reject is plotted in fig. 10. For all cases the most difficult disturbances to reject are flowrate disturbance of stream 1 on output y_1'. For cases $4H34H$ and $34H4C$ this disturbance is worst on output y_2, too. For the other cases temperature disturbance on stream 3 is the worst disturbance to reject on output y_2. The magnitude of these elements of the CLDG (i.e. $|\delta_{15}|, |\delta_{25}|$ and $|\delta_{23}|$) are shown in fig. 10. Note the high $|\delta_{15}|$ for cases $4H34H$ and $34H4C$. This is due to the strong interaction (competition) between the loops in these cases. Conclusion: In most cases output y_1' is the most difficult to control decentralized. With single bypasses only bypass $4H$ is a bit better than bypass $4C$, whereas it is much worse with a total bypass.

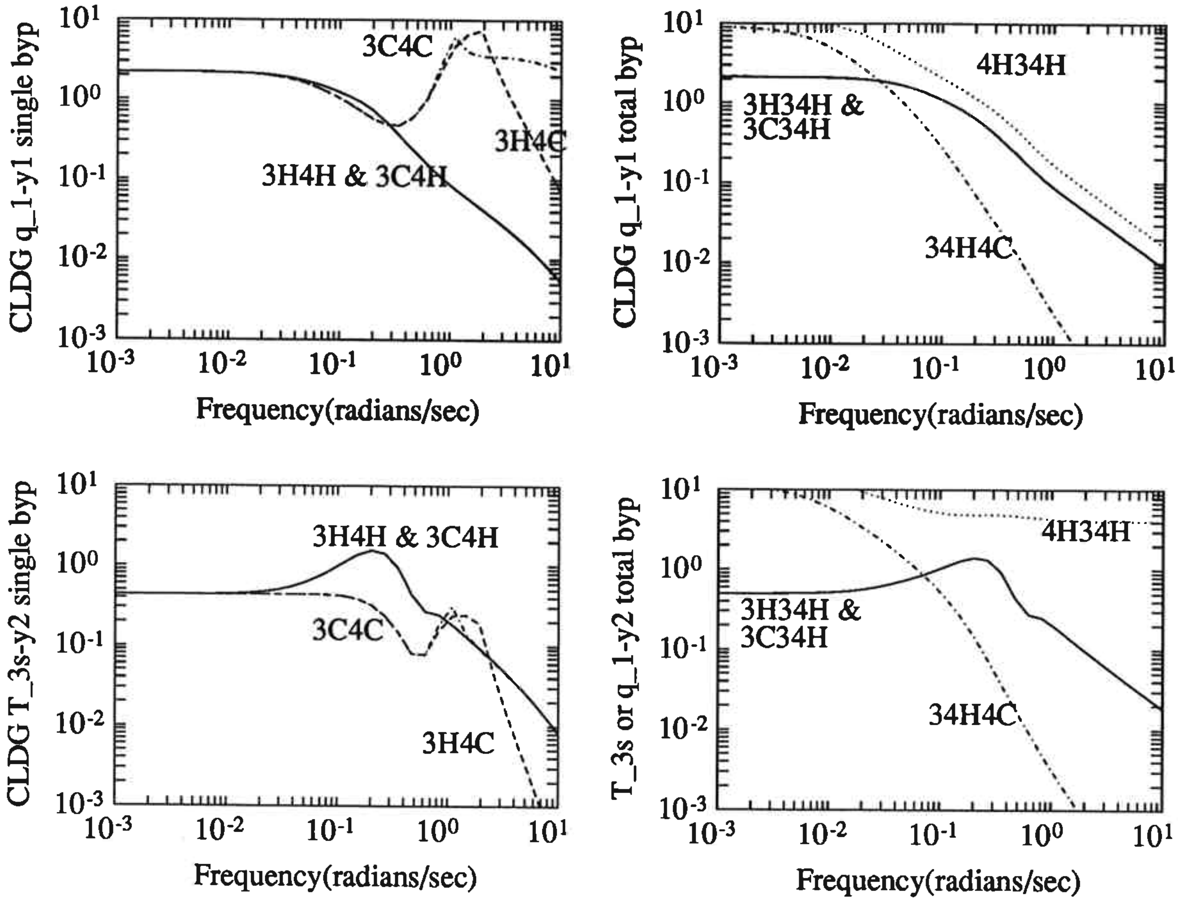

Figure 10: Selected (worst case) elements of the CLDG = $G_{diag}G^{-1}G_d$ the 8 cases. Part of example from Townsend & Morari (1984).