

Chapter 3

Dynamics and Control of Unstable Distillation Columns

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Abstract

The paper addresses dynamics and control of distillation columns operated at open-loop unstable operating points. The fact that ideal two-product distillation columns may have multiple steady-states and right half plane poles has only recently been recognized. This paper discusses in detail the dynamics and control implications. It is shown that with reflux and boilup as independent variables the operating points become unstable if the internal flows are sufficiently large. An open-loop unstable operating point may be stabilized by use of one-point control, i.e., feedback control of a column composition or temperature. If the control is not sufficiently tight, the column may go into a stable limit cycle. It is shown that the open-loop right half plane pole may worsen the control performance when both product compositions are under feedback control. Finally, it is shown that with distillate flow and boilup as independent variables the operating points may become unstable if the level control is not sufficiently tight. The column may also in this case go into a stable limit cycle.

3.1 Introduction

Distillation is undoubtedly the most studied unit operation in the process control literature. However, in all previous studies the column dynamics have been assumed to be asymptotically stable (with level and pressure loops closed). The main reason is that most authors have considered dynamic models with constant molar flows (neglected energy balance) and in addition assumed the inputs (e.g., reflux and boilup) to be given on a molar basis.

For the case of molar inputs there exists several papers on uniqueness and asymptotic stability of the operating points in homogeneous distillation. Most papers treat the constant molar flow case (neglected energy balance), e.g., Lapidus and Amundson, 1950; Acrivos and Amundson, 1955; Rosenbrock 1960, 1962. Doherty and Perkins (1982) provide a review of results published in this area, and conclude that, for constant molar flows, multiplicity and instability is impossible for single-staged "columns" and any multistage column separating a binary mixture. Sridhar and Lucia (1989) include the energy balance in the model and conclude under certain assumptions that also in this general case binary distillation columns will exhibit unique and stable solutions. They do, however, only study a limited set of specifications, namely $Q_D Q_B$ and LB .

However, in a recent paper Jacobsen and Skogestad (1991) report two kinds of multiplicity which may occur in distillation. 1) Jacobsen and Skogestad (1991) argue that columns under operation only in rare cases have all the manipulated inputs on a molar basis. For instance, fixing the valve position will normally correspond closely to fixing the geometric average of mass and volumetric flow-rate. As they show, the transformation from mass- or volume flows to molar flows is nonlinear due to the composition dependence and may in some cases become singular, even for the binary case with constant molar flows. A singularity in the input transformation will imply that there exist multiple solutions in terms of outputs (compositions) for a given set of inputs (flows). One of the solutions will be unstable. 2) In addition, Jacobsen and Skogestad show that when the energy balance is included in the model, even molar inputs may yield multiple solutions. Both types of multiplicity and instability may be experienced in industrial columns operated with inputs on a mass- or volume basis.

Jacobsen and Skogestad (1991) treat the multiplicity only from a steady-state point of view. In this paper we study the dynamics of columns with multiple solutions, and consider the implications of open-loop instability for feedback control. The last point is important as previous studies on distillation control have assumed open-loop stability, and we investigate whether the achievable closed-loop performance is influenced by the open-loop instability.

We start the paper with a brief summary of the previous results on steady-state multiplicity caused by singularity in the input transformation. We prove the instability for this case and consider the overall dynamics of columns with right half plane (RHP) poles. Finally, we consider whether any fundamental new control problems are introduced by the multiplicity and instability.

We limit ourselves to discuss mainly one control configuration (set of specifications), namely the case where mass reflux L_w and molar boilup V (V is closely related to the heat input Q_B) are used as independent variables. This is the most widespread configuration in industry, and is the configuration for which multiplicity and instability is most likely

to occur (Jacobsen and Skogestad, 1991). With this configuration the product flows (D_w and B_w) are used to control the condenser and reboiler levels (M_D and M_B). We here assume perfect level control, but this assumption is not important since the composition response is only weakly dependent on the level control when reflux and boilup are used as independent variables. At the end of the paper we consider the case with distillate flow D_w and boilup V as independent inputs, and show that with this configuration the responses are strongly dependent on the level control; the operating points may even become unstable if the level control is not sufficiently tight.

In most of the paper we consider the simplest case with constant molar flows, i.e., we neglect the energy balance. Towards the end of the paper we consider the instability that may be caused by including the energy balance.

3.2 Results on Steady-State Multiplicity in Ideal Distillation

We give here a brief review of the results on multiplicity caused by singularity in the input transformation presented in Jacobsen and Skogestad (1991).

Consider the two-product distillation column in Figure 3.1. If the feed to the column is given there are at least four flows that may be specified: reflux L , boilup V , distillate D and bottoms flow B . However, for a given column there are only two degrees of freedom at steady-state, that is, only two of these flows may be specified independently. A specific choice of two independent variables is denoted a "configuration".

Jacobsen and Skogestad (1991) provide an example of steady-state multiplicity in a column separating a mixture of methanol and n-propanol. The column has mass reflux and molar boilup as independent variables, i.e., L_wV -configuration. Data for the column are given in Table 3.1. Note that the energy balance is excluded, i.e., constant molar flows are assumed. Some steady-state solutions are given in Table 3.2, and we see that for a specification of mass reflux $L_w = 50.0$ kg/min and molar boilup $V = 2.0$ kmol/min we have three possible solutions *II*, *III* and *IV* in terms of compositions. The multiplicity is graphically illustrated in Figure 3.2.

The observed multiplicity is caused by the transformation between the actual flow-rates (mass) and the molar flow-rates which determine separation. For a binary mixture the transformation between mass reflux, L_w , and molar reflux, L , is given by

$$L = L_w/M; \quad M = y_D M_1 + (1 - y_D) M_2 \quad (3.1)$$

Here M_i denotes the molecular weight of the individual components. One might expect the molar reflux to increase monotonically with the mass reflux, that is, $(\partial L/\partial L_w)_V > 0$. However, because M is a function of composition, y_D , and thereby of L_w , this might not be the case. Assuming molar boilup V fixed and differentiating $L_w = LM$ on both sides with respect to L yields

$$\left(\frac{\partial L_w}{\partial L}\right)_V = M + L(M_1 - M_2) \left(\frac{\partial y_D}{\partial L}\right)_V \quad (3.2)$$

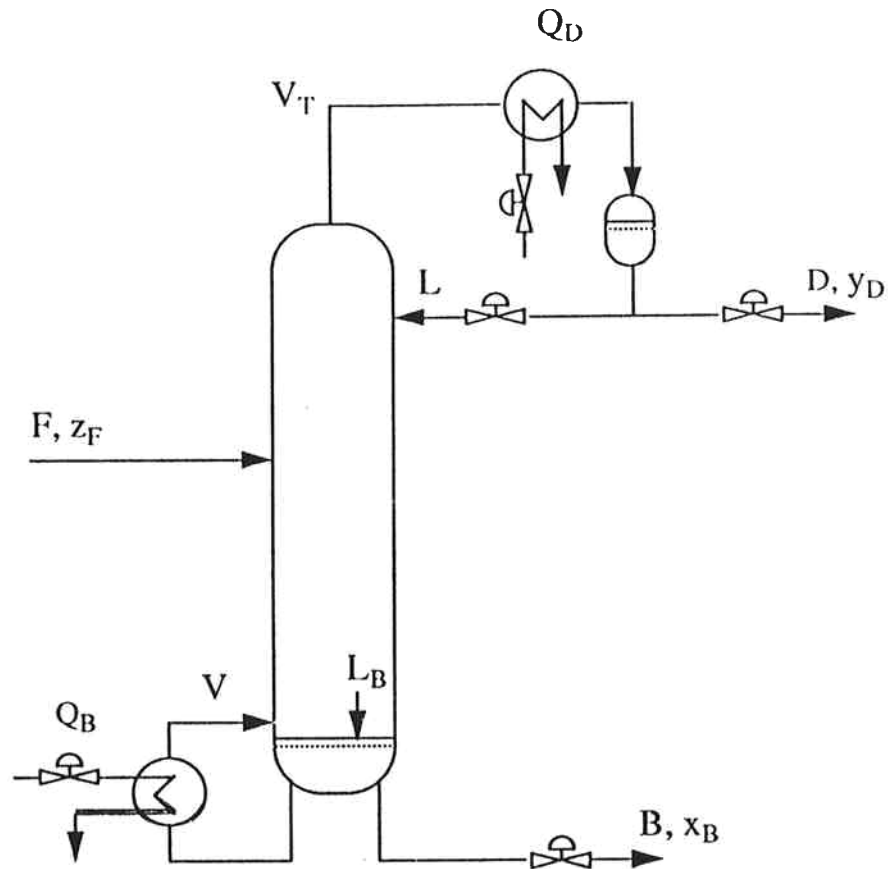


Figure 3.1: Two product distillation column.

z_F	F	α	N	N_F	M_1	M_2
0.50	1	3.55	8	4	32.04	60.10

- Feed is saturated liquid.
- Total condenser with saturated reflux.
- Liquid holdups are $M_{Li}/F = 0.5min$
- L_wV -configuration: $M_D/F = M_B/F = 0.5min$
- D_wV -configuration: $M_D/F = M_B/F = 5.0min$
- Flow dynamics included in dynamic simulations.
- Constant pressure.

Table 3.1: Data for Methanol-Propanol Column.

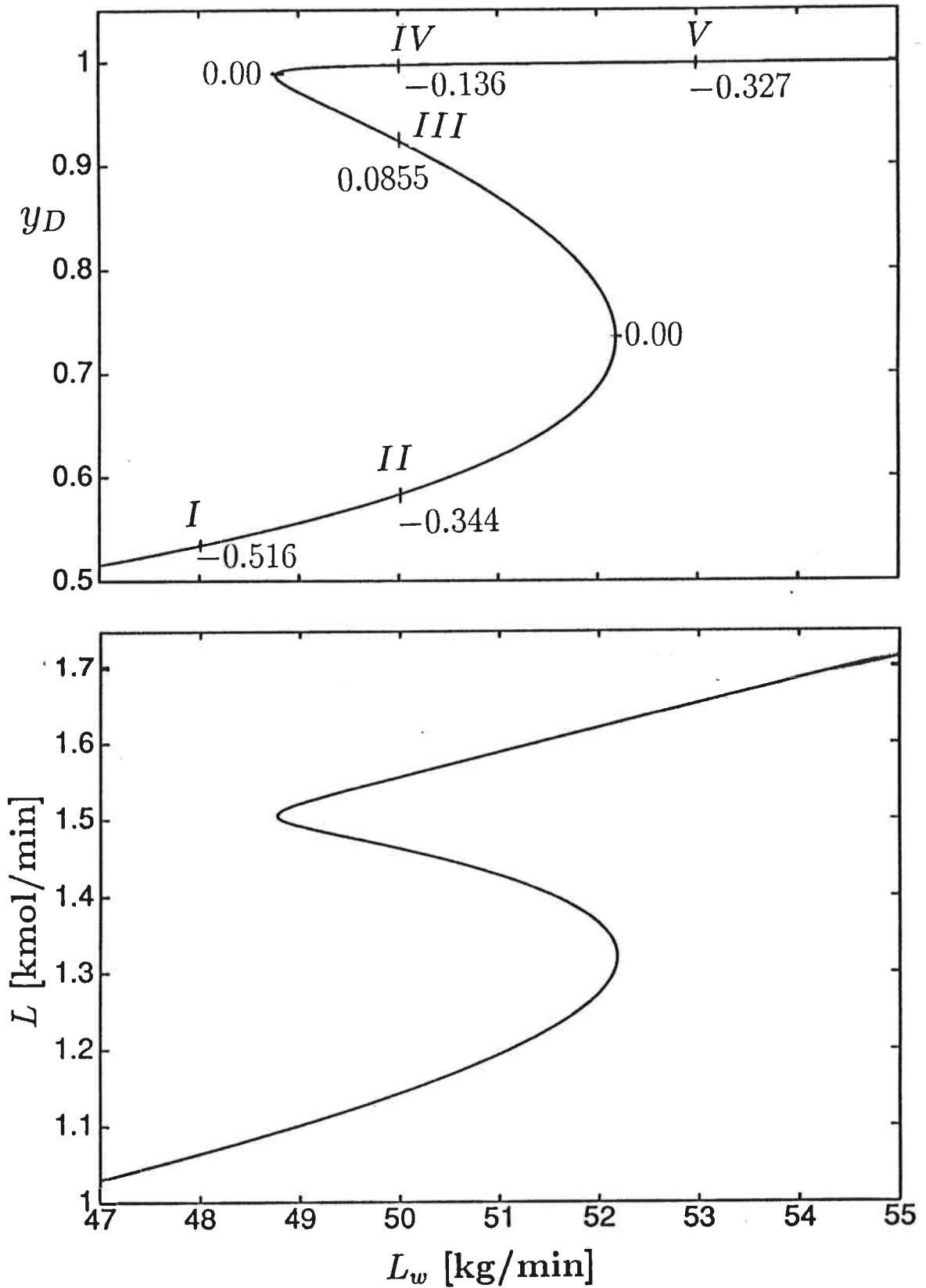


Figure 3.2: Methanol-propanol column with constant molar flows: Multiple steady-states for $L_w V$ -configuration. Mass reflux L_w is varied while molar boilup V is fixed at 2.0 kmol/min. On the upper plot the corresponding maximum eigenvalue is shown at some of the steady-state solutions.

	L [kmol/min]	D [kmol/min]	L_w [kg/min]	y_D	x_B
I	1.064	0.936	48.00	0.534	3.10e-3
II	1.143	0.857	50.00	0.584	3.50e-3
III	1.463	0.537	50.00	0.9237	7.80e-3
IV	1.555	0.445	50.00	0.9969	0.104
V	1.650	0.350	53.00	0.9984	0.233

• Constant molar flows.

Table 3.2: Steady-state solutions for methanol-propanol column with $V=2.0$ kmol/min and L_w in the range 48 to 53 kg/min.

For $M_1 < M_2$, which is usually the case (the most volatile component has the smallest molecular weight), the second term on the right hand side of (3.2) will be negative and the total differential may take either sign. The transformation from L_w to L will be singular when $(\partial L_w / \partial L)_V = 0$. A singular point will correspond to a bifurcation point, and the number of solutions changes from one to three. Jacobsen and Skogestad (1991) state that solutions with $(\partial L_w / \partial L)_V < 0$ (middle branch in Figure 3.2) correspond to unstable operating points, but they do not prove this rigorously.

3.3 Open-Loop Dynamics and Instability for $L_w V$ -configuration

The maximum eigenvalue in selected operating points of the methanol-propanol column with constant molar flows and the $L_w V$ -configuration are shown in Figure 3.2. From the figure we observe that the eigenvalues at the upper and lower branches are negative, implying stability, while those at the intermediate branch (negative slope) are positive, implying instability of the operating points. Note that the unstable operating points only have a single eigenvalue in the right half plane. The eigenvalues at the singular points are zero as expected since they correspond to bifurcation points. The open-loop instability at the intermediate branch is illustrated by the nonlinear simulations in Figure 3.3 which shows the responses in top composition y_D to small changes in mass-reflux L_w (keeping boilup V fixed) at the unstable operating point *III*. The simulations indicate that the two stable solutions *II* and *IV* have equally large regions of attraction as seen from the unstable solution *III*. The purpose of the rest of this section is to prove the observed instability at the intermediate branch and to compare the dynamics of columns with mass or volume inputs with those found for models with molar inputs.

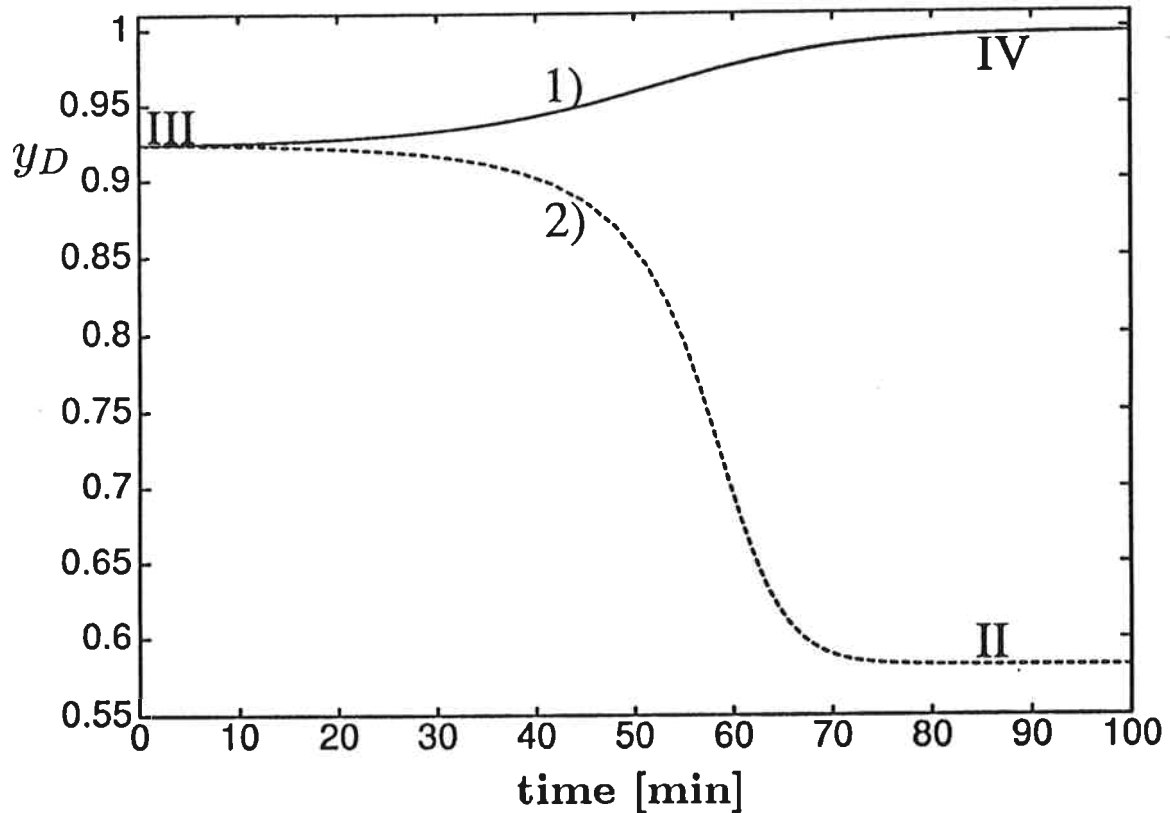


Figure 3.3: Nonlinear open-loop simulation of methanol-propanol column at unstable operating point *III* in Figure 3.1. 1) Small increase in mass reflux L_w . 2) Small decrease in mass reflux L_w . Boilup $V = 2.0$ kmol/min. Constant molar flows.

3.3.1 Conditions for instability

One-stage column Consider the simple column in Figure 3.4 with one theoretical stage (the reboiler) and a total condenser. Of course, such a column will never be operated in practice because the reflux is simply wasting energy and has no effect on separation. However, we start by analyzing this column due to the simplicity of the dynamic model. As Jacobsen and Skogestad (1991) show, even such a simple column with ideal thermodynamics may have multiple steady-state solutions. Assume binary separation, liquid feed, constant holdup in the reboiler (M_L) and negligible holdup in the condenser. The dynamic model of the column becomes:

$$M_L \frac{dx_B}{dt} = Fz_F - Dy_D - Bx_B \quad (3.3)$$

We have $D = V - L$ and $D + B = F$ and with L and V as independent variables we get

$$M_L \frac{dx_B}{dt} = F(z_F - x_B) + L(y_D - x_B) + V(x_B - y_D) \quad (3.4)$$

Linearization, Laplace transformation and introduction of deviation variables assuming F , z_F and V constant yields

$$sM_L \Delta x_B(s) = -D \Delta y_D(s) - B \Delta x_B(s) + (y_D - x_B) \Delta L(s) \quad (3.5)$$

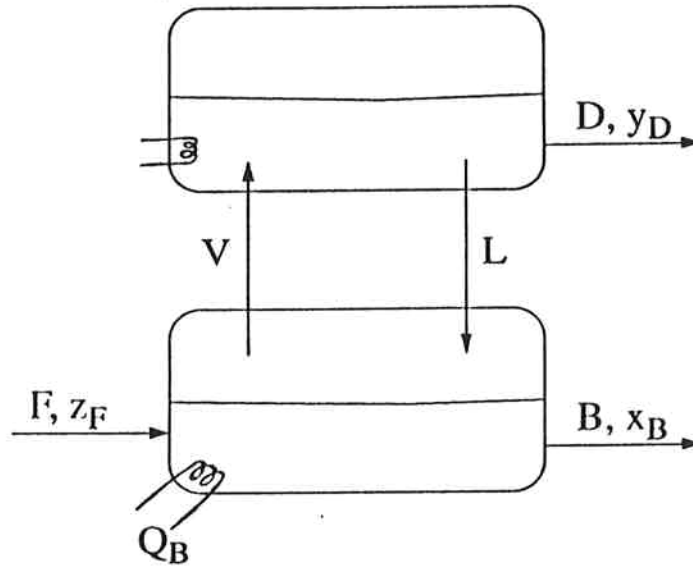


Figure 3.4: One-stage column with total condenser.

Assuming constant relative volatility α yields the following relation between $\Delta y_D(s)$ and $\Delta x_B(s)$

$$\frac{\Delta y_D(s)}{\Delta x_B(s)} = \frac{\alpha}{(1 + (\alpha - 1)x_B)^2} = K(x_B) \quad (3.6)$$

Equation (3.5) then becomes

$$\Delta x_B(s) = \frac{y_D - x_B}{M_L s + a} \Delta L(s) \quad (3.7)$$

where

$$a = KD + B \quad (3.8)$$

As all terms in (3.8) are positive, the pole $-a/M_L$ is always negative, implying that all operating points are stable when molar reflux L and molar boilup V are independent variables.

Now consider mass reflux L_w as an input instead of molar reflux $L = L_w/M$. By linearization we obtain for binary separations

$$\Delta L = \frac{1}{M} \Delta L_w + L \frac{M_2 - M_1}{M} K \Delta x_B \quad (3.9)$$

Substituting (3.9) into (3.7) we obtain the following transfer-function between liquid composition, $\Delta x_B(s)$, and mass reflux $\Delta L_w(s)$:

$$\Delta x_B(s) = \frac{y_D - x_B}{M_L s + a_w} \frac{\Delta L_w(s)}{M} \quad (3.10)$$

where

$$a_w = KD + B - (y_D - x_B) \frac{M_2 - M_1}{M} KL \quad (3.11)$$

The operating point is unstable for $a_w < 0$. At steady-state (3.6), (3.7) and (3.8) yield

$$\left(\frac{\partial y_D}{\partial L}\right)_V = \frac{y_D - x_B}{D + B/K} \quad (3.12)$$

and from (3.2) we find that instability ($a_w < 0$) is equivalent to a negative slope for $(dL_w/dL)_V$.

Multistage column Although the dynamic model of a multistage distillation column is of high order, it is well known that the overall composition dynamics in distillation columns may be well approximated by a first order response (e.g., Davidson, 1956, Skogestad and Morari, 1987). This implies that we may approximate the transfer-function from molar reflux L to top composition y_D with

$$\left(\frac{\partial y_D}{\partial L}\right)_V(s) \approx \frac{k_{y_D L}^{LV}}{1 + \tau_1 s} \quad (3.13)$$

where $k_{y_D L}^{LV}$ ($= (\partial y_D / \partial L)_V$ in (3.2)) is the steady-state gain and τ_1 is the dominant time-constant (we assume stability, $\tau_1 > 0$, which always holds for the case of constant molar flows). We now want to derive the transfer function $(\partial y_D / \partial L_w)_V(s)$. With V constant the total differential of y_D may be written

$$dy_D(s) = \left(\frac{\partial y_D}{\partial L}\right)_V(s) dL(s) \quad (3.14)$$

Here $L = L_w/M$ is a function of both L_w and y_D and we get

$$dL(s) = \left(\frac{\partial L}{\partial L_w}\right)_{y_D}(s) dL_w(s) + \left(\frac{\partial L}{\partial y_D}\right)_{L_w}(s) dy_D(s) \quad (3.15)$$

Combining (3.14) and (3.15) yields:

$$\left(\frac{\partial y_D}{\partial L_w}\right)_V(s) = \frac{\left(\frac{\partial L}{\partial L_w}\right)_{y_D} \left(\frac{\partial y_D}{\partial L}\right)_V(s)}{1 - \left(\frac{\partial L}{\partial y_D}\right)_{L_w} \left(\frac{\partial y_D}{\partial L}\right)_V(s)} \quad (3.16)$$

The Laplace variable s has been deleted for $(\partial L / \partial L_w)_{y_D}$ and $(\partial L / \partial y_D)_{L_w}$ since the relationship $L = L_w/M$ is purely static. For a binary mixture

$$\left(\frac{\partial L}{\partial L_w}\right)_{y_D} = y_D M_1 + (1 - y_D) M_2 \quad (3.17)$$

$$\left(\frac{\partial L}{\partial y_D}\right)_{L_w} = \frac{L_w(M_2 - M_1)}{(y_D M_1 + (1 - y_D) M_2)^2} \quad (3.18)$$

From (3.16) we now find that the dominant pole is given by

$$\lambda_{max} = -\frac{1}{\tau_1} \left(1 - \frac{k_{y_D L}^{LV} L (M_2 - M_1)}{y_D M_1 + (1 - y_D) M_2}\right) \quad (3.19)$$

The pole will be in the right half plane when

$$\frac{k_{yDL}^{LV}L(M_2 - M_1)}{y_D M_1 + (1 - y_D)M_2} > 1 \quad (3.20)$$

This is exactly the same criterion as Jacobsen and Skogestad (1991) found for a negative slope between mass and molar reflux. Thus, a sufficient condition for instability for the L_wV -configuration is

$$\left(\frac{\partial L}{\partial L_w}\right)_V < 0 \quad (3.21)$$

This means that solution branches with a negative slope between L and L_w represent unstable solutions, provided the column is stable on a molar basis. This result is in accordance with numerical results.

In the general case with more complex dynamics, (3.13) may be replaced by

$$\left(\frac{\partial y_D}{\partial L}\right)_V(s) = \frac{k_{yDL}^{LV}(1 + b_1s + b_2s^2 + \dots + b_{n-1}s^{n-1})}{1 + a_1s + a_2s^2 + \dots + a_ns^n} \quad (3.22)$$

This follows since the composition dynamics generally have a pole excess of one¹ (e.g., Skogestad and Morari, 1988). We assume "molar" stability, i.e., all a_i 's in (3.22) are positive. We may now use the Routh-Hurwitz stability criterion (the coefficients in the pole polynomial of (3.16) should have different signs) to conclude that in the general case (3.20) and (3.21) are sufficient conditions for instability.

Equation (3.19) gives an approximate way of calculating the dominating pole for the L_wV -configuration from data computed for molar inputs. Figure 3.5 shows a comparison of this approximation with the maximum eigenvalue computed from the full linear model with mass reflux. We see that the (3.19) yields a correct value of zero λ_{max} at the singular points, and a fairly good approximation in the whole.

3.3.2 Effect of operating conditions on stability

Jacobsen and Skogestad (1991) provide analytical results on when a negative slope between mass and molar reflux, i.e., instability according to (3.21), is most likely. They show that a negative slope is most likely with large internal flows (i.e., large L and V) and with intermediate purities in the top (i.e., intermediate L for given V). This corresponds to having L and k_{yDL}^{LV} large, and according to (3.20) this is the case for which instability is most likely. Note that the analytical treatment in Jacobsen and Skogestad (1991) was based on ideal separation with constant relative volatility and constant molar flows. The same assumptions apply to the discussion below.

To study the effect of operating conditions on stability for the L_wV -configuration, consider different values of V , and for each value let L vary between $L_{min} = V - F$ ($B = 0$) and $L_{max} = V$ ($D = 0$) (Note that these variations in L and V are purely static). On the basis of the analytical results in Jacobsen and Skogestad (1991) we get three different cases of operation depending on the size of the internal flows:

¹For the case with a total condenser there will be a pole excess of two in (3.22).

3.3. OPEN-LOOP DYNAMICS AND INSTABILITY FOR L_wV -CONFIGURATION⁴⁷

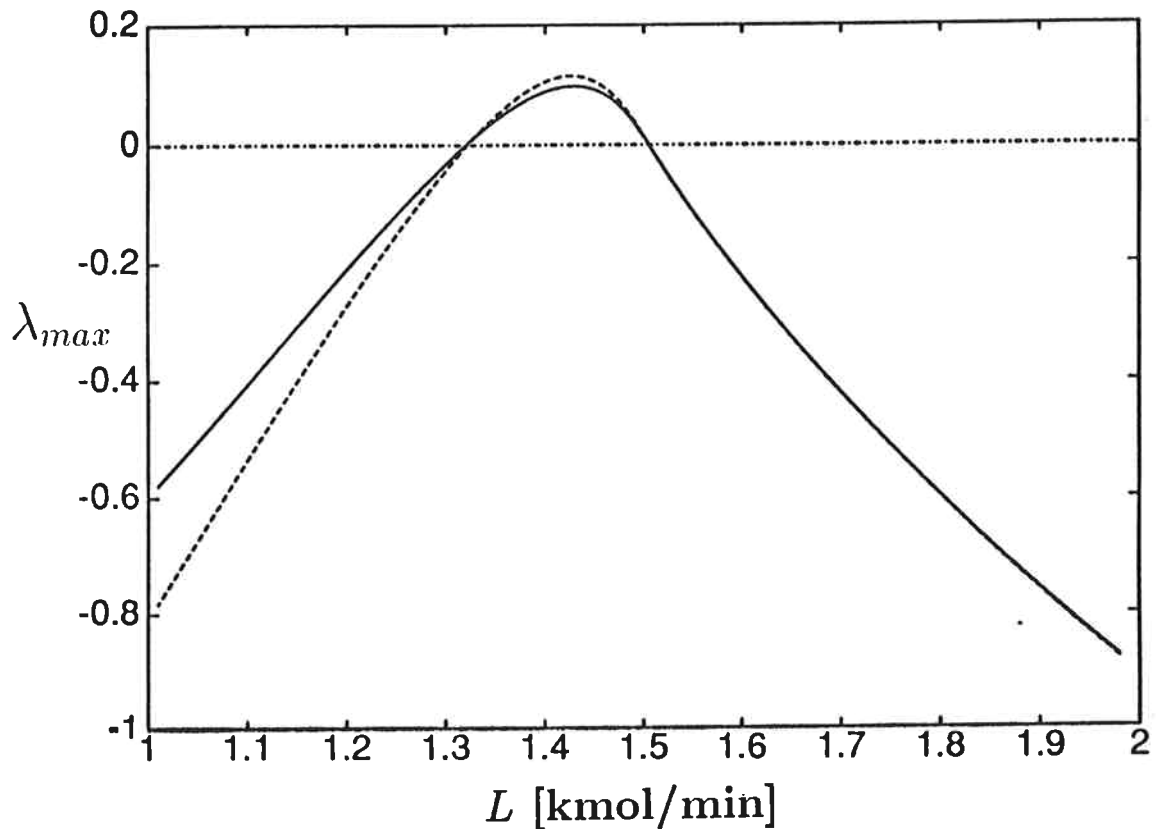


Figure 3.5: Maximum eigenvalue λ_{max} as a function of molar reflux L for methanol-propanol column with L_wV -configuration and $V = 2.0$ kmol/min. Solid line: exact. Dashed line: Computed from (3.19). Constant molar flows.

- A. *Internal flows low (V small):* The largest pole (λ_{max}) starts out in the LHP for $L = L_{min}$ and moves toward the right as L is increased. However, it does not reach the imaginary axis before it begins to move back. This implies that we have uniqueness with all solutions stable.
- B. *Internal flows intermediate:* The pole λ_{max} starts out in the LHP at L_{min} and moves towards the imaginary axis as L is increased. At some value of L the pole crosses the imaginary axis and moves into the RHP. For some intermediate value of L the pole begins to move back and crosses the imaginary axis again (this time from right to left). In this case we have three solutions, one of which is unstable.
- C. *Internal flows high (V large):* The large value of L_{min} implies that λ_{max} starts out in the RHP and moves further towards the right with increasing L . For some intermediate value of L the pole starts to move back but it never crosses the imaginary axis before $L = L_{max}$. In this case we have uniqueness and all solutions unstable.

The three different regimes of operation are illustrated in Figure 3.6 for the one-stage column studied by Jacobsen and Skogestad (1991). Note that there also may be cases where there are two solutions, of which one is unstable. In this case the third solution would actually correspond to one of the product flows, D or B , being negative.

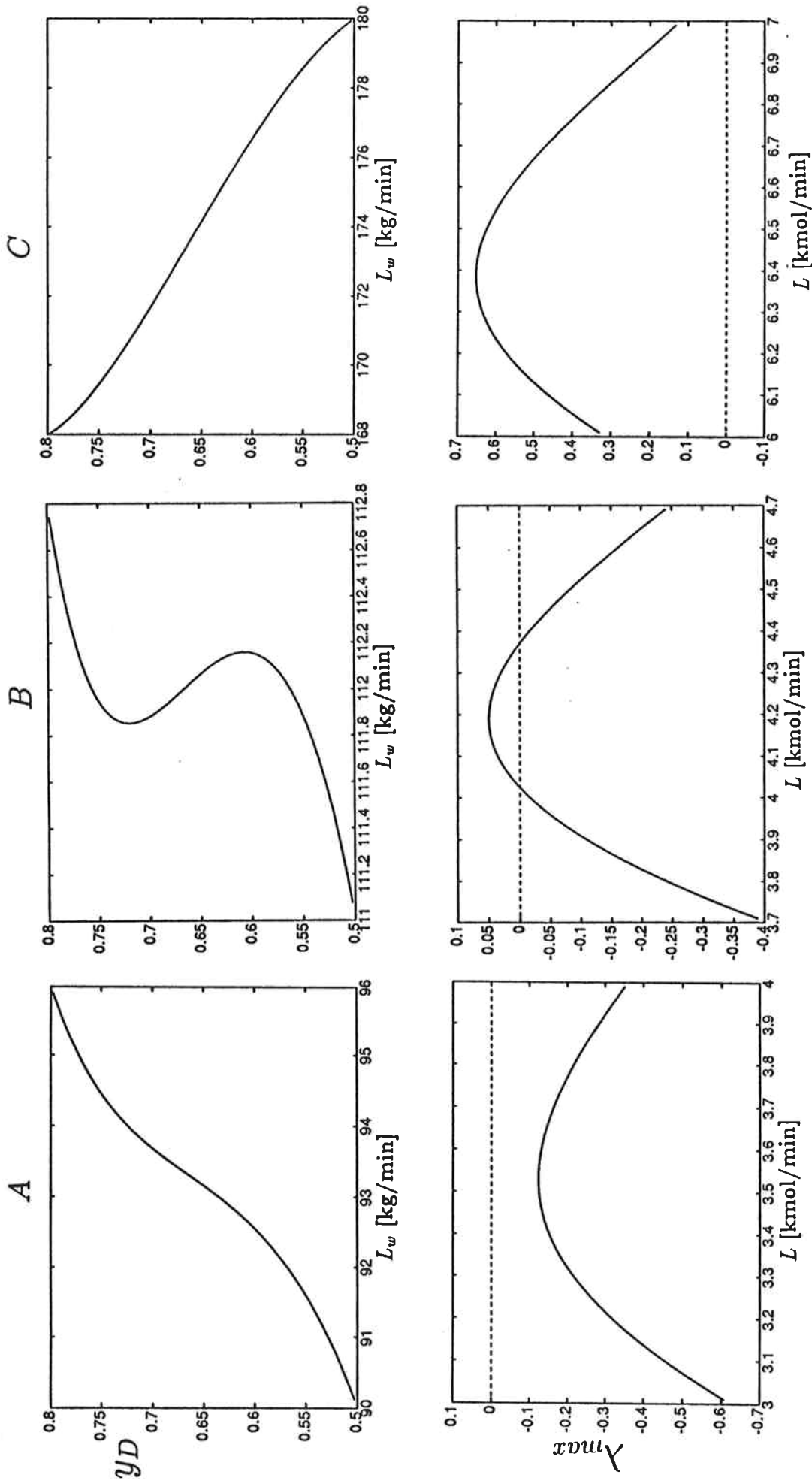


Figure 3.6: Steady-state solutions as a function of mass-reflux L_w for one-stage column with $N = 1$, $F = 1$, $z_F = 0.5$, $M_1 = 20$, $M_2 = 40$ and $\alpha = 4.0$. Lower plot shows corresponding maximum eigenvalue as a function of molar reflux L . A) $V = 4.0$ kmol/min. B) $V = 7.0$ kmol/min. C) $V = 4.7$ kmol/min.

3.3.3 Overall dynamics

The analysis above showed that the dominant (largest) pole, and thereby the stability and low-frequency dynamics, is strongly influenced by the transformation between mass and molar reflux. However, the effect on the high-frequency dynamics is much smaller. Figure 3.7 shows the magnitude and phase of the transfer-functions from L and L_w , respectively, to top composition y_D (keeping boilup V fixed) at operating point *III* of the methanol-propanol column. From the figure we see that the phase of the two transfer-functions differs with 180 degrees at low frequencies as expected since the transfer function from mass-reflux has a RHP pole and negative low-frequency gain. However, the phases approach each other at intermediate frequencies and become identical at high frequencies. The magnitudes of the two systems are almost identical at all frequencies. Note that the magnitude for the $L_w V$ -configuration has been scaled by M . The fact that the magnitudes are similar also at low frequencies is a coincidence for this operating point. The reason is that the dominant poles are similar in magnitude (-0.098 and 0.086). However, the dynamics of the two systems differ in the region where the phases differ. The most important conclusion to draw from Figure 3.7 is that it is mainly the dominant pole that is influenced by the transformation from mass to molar flows. The initial response (high-frequency behavior) is unaffected. Similar results are obtained for the other three transfer-functions of the 2×2 system.

3.4 Feedback Control

3.4.1 Limitations imposed by RHP poles and zeros

As we have seen, columns operating with mass or volume inputs may be open-loop unstable, and will require feedback control (in addition to level and pressure control) for stabilization. From control theory it is well known that unstable poles by themselves do not represent any bandwidth limitations; on the contrary they put a lower limit on allowable bandwidth of the closed-loop system. Problems will therefore only arise if there are bandwidth limitations like right half plane zeros at frequencies comparable to the right half plane pole ("The system goes unstable before we are able to observe what is happening") or if there are constraints ("we can not counteract the instability").

Freudenberg and Looze (1985, 1988) have extended the Bode Sensitivity Integral Theorem for minimum phase systems to systems containing RHP zeros and RHP poles. Here we consider scalar systems, but similar relations are obtained for multivariable systems if one considers the maximum singular value of the sensitivity function $\bar{\sigma}(S)$ instead of $|S|$.

Consider the sensitivity function $S = (1 + gc)^{-1}$ of the closed-loop system. Ideally we want $S = 0$. However, for all real systems $|S(j\omega)| = 1$ at high frequencies. In addition, for an open-loop system with a pole excess of at least two (satisfied for any real system) and a single real RHP pole p the following limitation applies (Freudenberg and Looze, 1985)

$$\int_0^{\infty} \log|S(j\omega)|d\omega = \pi p \quad (3.23)$$

(With no RHP pole ($p = 0$) (3.23) reduces to the theorem of Bode.) From (3.23) we

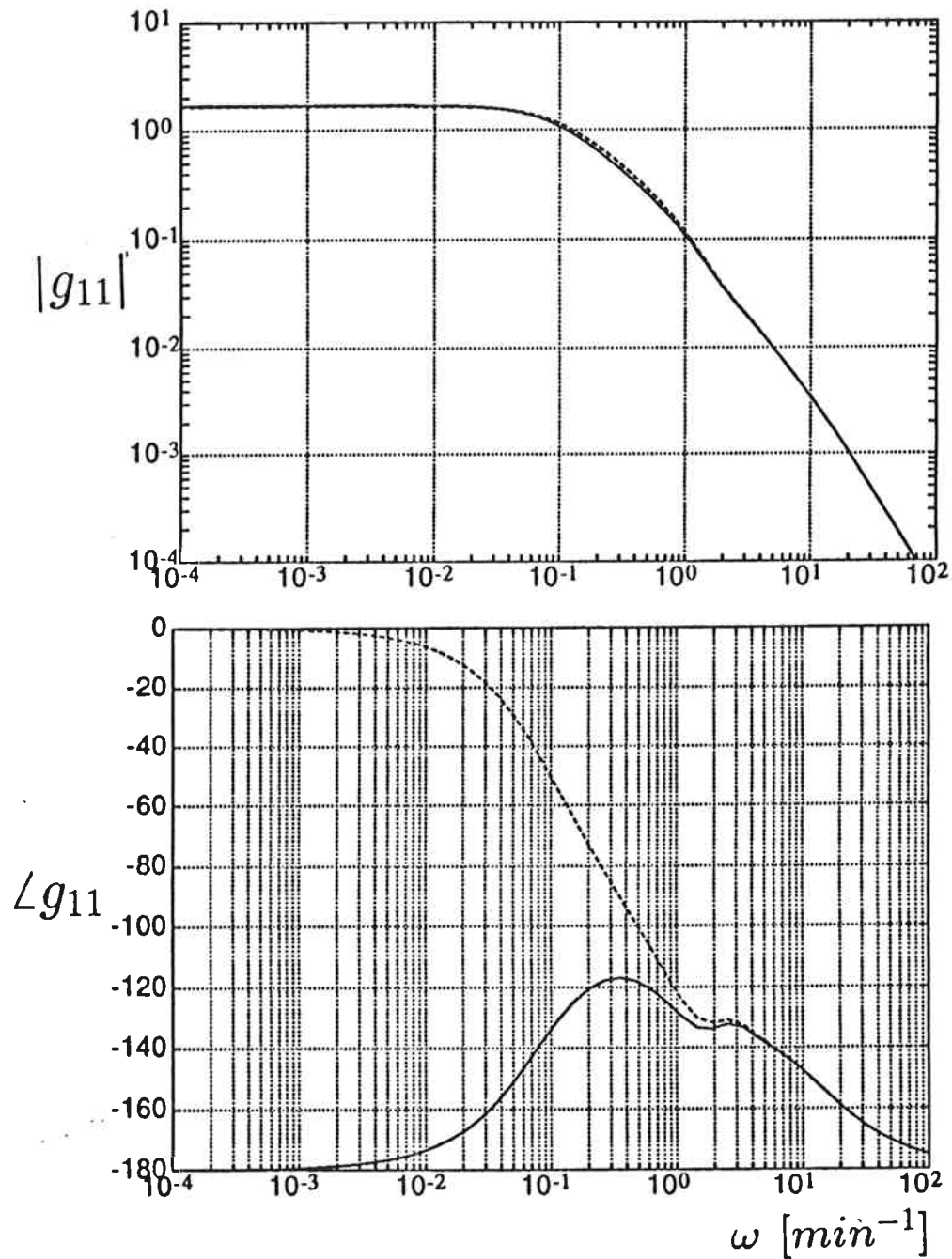


Figure 3.7: Frequency response for transfer-function from reflux to top composition y_D keeping V constant for operating point *III* of methanol-propanol column. Solid line: mass reflux. Dashed line: molar reflux. The magnitude for mass-reflux L_w is scaled by M . Constant molar flows.

see that we need a frequency range with $|S| > 1$ and that the presence of a RHP-pole increases the area where $|S| > 1$. However, (3.23) does not impose any practical design limitation as the area for $|S| > 1$ may be smoothed out over an arbitrarily large frequency range, and the peak of S may accordingly be made arbitrarily small. Thus, as stated above, the RHP pole will not represent a control limitation if there are no bandwidth constraints present in the system.

However, for an open-loop system with a real RHP pole p and a real RHP zero z , the following restriction applies (Freudenberg and Looze, 1985, 1988)

$$\int_0^{\infty} \log|S(j\omega)|W(z, \omega)d\omega = \pi \log\left|\frac{p+z}{p-z}\right| \quad (3.24)$$

(With no RHP pole ($p = 0$) the integral equals zero). The weight W is given by

$$W(z, \omega) = \frac{2z}{z^2 + \omega^2} \quad (3.25)$$

The form of W (it equals $2/z$ up to $\omega = z$ where it cuts off with a -2 slope.) implies that essentially all the area for $|S| > 1$ has to be at frequencies lower than z , and the sensitivity function must have a peak $|S| > 1$ at $\omega < z$. The peak will have to become increasingly large as the bandwidth frequency (where $|S|$ first reaches 1) approaches z . From (3.24) we also see that as the RHP zero approaches the RHP pole, the peak goes to infinity. This implies that we in general must require

$$p < z \quad (3.26)$$

For the distillation column $p = \lambda_{max}$, and RHP zeros are most likely caused by deadtimes, θ_d , in measurements and actuators. Using a first-order Padé approximation for θ_d results in a RHP zero at $z = 2/\theta_d$. We must then require $\lambda_{max} < 2/\theta_d$, or equivalently $\theta_d < 2/\lambda_{max}$. With a limited structure of the controller, e.g. a PI-controller, we must require a larger distance between the RHP pole and the RHP zero than given by (3.26) in order to stabilize the column.

For a further discussion on the effect of RHP-poles and -zeros on closed-loop performance see Freudenberg and Looze (1985, 1988) and Looze and Freudenberg (1991).

3.4.2 One-point control

Good control of distillation columns usually requires two-point control, i.e., feedback control of both product compositions. However, in order to stabilize an open-loop unstable column one-point control will suffice. This is also the way most industrial columns with composition control are operated. An unstable column operating with the L_wV -configuration may be stabilized by controlling either top or bottom composition, or any other variable related to composition, e.g. a temperature inside the column. The analysis presented above for SISO systems then applies.

For operating point III of the methanol-propanol column the RHP-pole is at $p = 0.086 \text{min}^{-1}$ and we are unable to stabilize the column with a PI-controller when the deadtime exceeds 11 min. ($z \approx 2/\theta_d = 0.182 \text{min}^{-1}$). However, composition measurements

in industrial columns (GC-analysis) may typically have deadtimes up to 30 min., and one should then use faster temperature measurements in order to stabilize the column.

Nonlinear Simulations. Figure 3.8 shows nonlinear simulations of the methanol-propanol column using a single-loop PI-controller² between top composition y_D and mass-reflux L_w with a 1 minute measurement deadtime included. Molar boilup V is kept constant at 2.0 kmol/min. The figure shows the responses to setpoint changes in y_D from operating point II (open-loop stable) to operating point III (open-loop unstable) and then further on to operating point IV (open-loop stable) (see Figure 3.2 and Table 3.2). A logarithmic measurement $Y_D = \ln(1 - y_D)$ was used in the controller as this reduces the nonlinearity of the initial response between different operating points (Skogestad and Morari, 1988). From the figure we see that the controller is able to stabilize the open-loop unstable operating point III with a RHP pole at 0.086 min^{-1} . The simulations also show that the same controller may be used in these three widely differing operating points. The reason is that the initial response (high-frequency dynamics) in terms of logarithmic composition Y_D is similar in all operating points. We would get instability if we used mole fractions, y_D , as is done conventionally. From the plot of mass-reflux L_w against time we see that the steady-state change in the input is zero, showing that the three operating points are multiple solutions.

One should be careful about detuning a controller in an open-loop unstable process as the bandwidth may become lower than the minimum allowable and the operating point becomes closed-loop unstable. This is illustrated in Figure 3.9. where the controller gain has been reduced by a factor of two compared to the simulations in Figure 3.8. Operating point III is now closed-loop unstable, and a small setpoint change makes the system start drifting away. However, this does not imply that the column goes globally unstable in the sense that physical constraints are violated. Since there exists steady-state solutions above and below the unstable solution the column goes into a stable limit cycle. If the controller gain is reduced further the limit cycle will continue, but now with a longer period of each cycle and with higher peaks in composition. There will also exist cases where there are no solutions either above or below the unstable solution. In this case the column is likely to go globally unstable as either the condenser (missing upper branch) or reboiler (missing lower branch) would run dry.

3.4.3 Two-point control

As pointed out above, one-point control is sufficient to stabilize an unstable operating point, but high performance control usually requires control of both product compositions. There exist a large amount of literature on two-point control of distillation columns, but everything is based on open-loop stable models. Here we want to investigate whether the potential instability caused by using mass-flows will affect the achievable closed-loop performance of the column significantly.

In order to compare achievable performance for the stable model with molar reflux and the unstable process with mass reflux we design controllers with optimized performance for both cases. As a design objective we use the structured singular value, μ (see

²Tuned to yield reasonably fast response. Note that Ziegler-Nichols tuning rules resulted in a closed-loop unstable system.

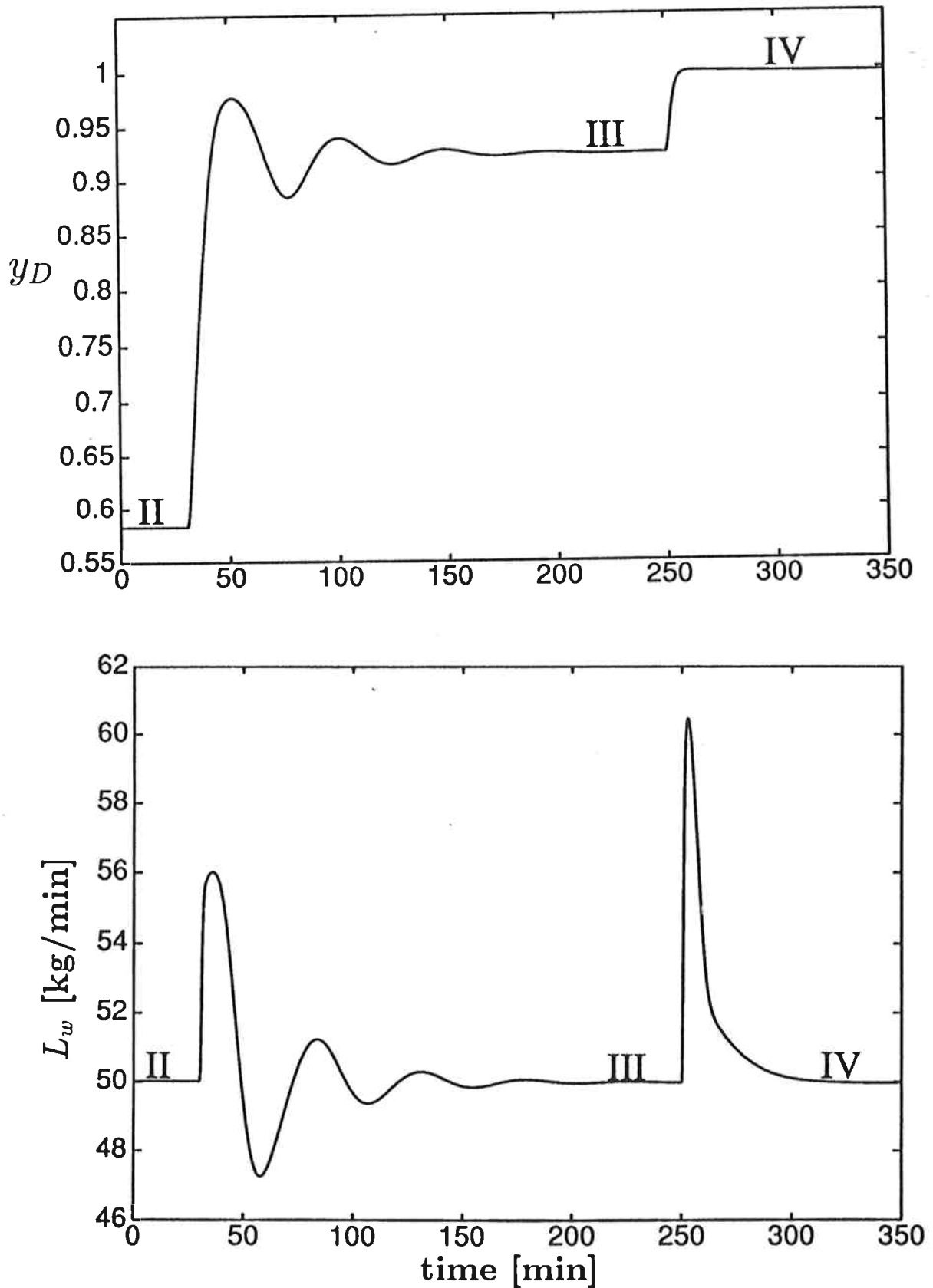


Figure 3.8: Nonlinear simulation of methanol- propanol column with one-point control of top-composition y_D using mass reflux L_w . Setpoint changes from operating point *II* to *III* and from *III* to *IV*. Boilup $V = 2.0$ kmol/min. Controller parameters: $k = 3.0$ and $\tau_I = 11.0$ min. Gain is for logarithmic composition, i.e., $\log(1 - y_D)$. Constant molar flows.

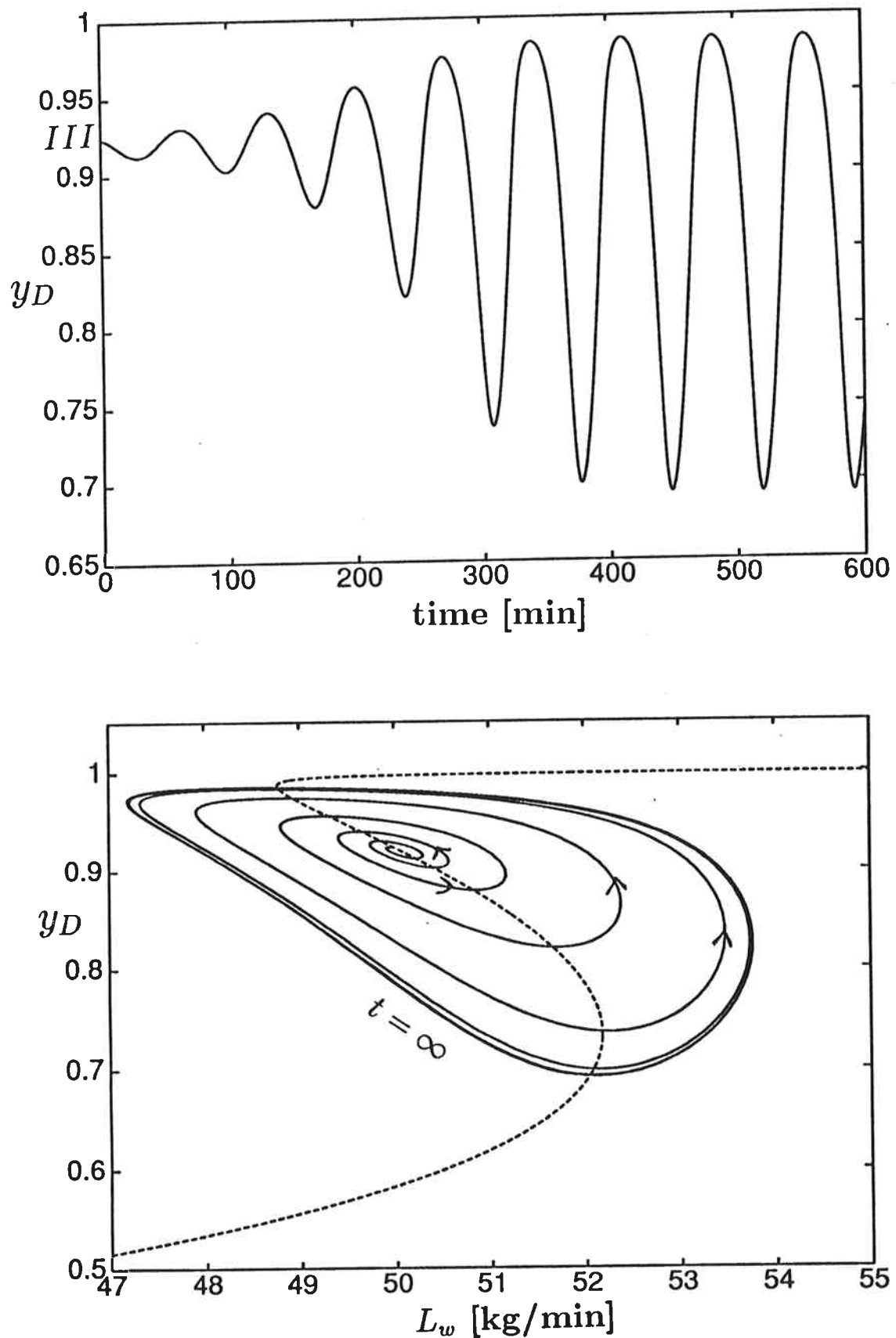


Figure 3.9: Nonlinear simulation of methanol-propanol column with one-point control of top-composition y_D using mass reflux L_w . Controller gain reduced by a factor of 2 compared to Figure 3.8. Upper plot: Time as independent variable. Lower plot: Phase-plane plot. Dashed line shows steady-state solutions. Constant molar flows.

e.g., Skogestad et.al., 1988). This implies that we may include model uncertainty in the design. We use a relative uncertainty weight in each channel given by

$$w_I(s) = 0.20 \frac{5\theta_d s + 1}{0.5\theta_d s + 1} \quad (3.27)$$

This means that we allow for a deadtime θ_d in addition to 20 % uncertainty in each input. The performance weight used is given by

$$w_P(s) = \frac{1}{P} \frac{\tau_{CL} s + P}{\tau_{CL} s} \quad (3.28)$$

This implies that the worst case peak of $\bar{\sigma}(S)$ should be less than P , and that the closed-loop time-constant should be less than τ_{CL} . We design single-loop controllers³ for different values of θ_d , and for each design we adjust the performance weight until a μ -value of 1 is achieved. This is done by first increasing τ_{CL} and then increasing P if necessary. A μ -value of 1 implies that we can guarantee the specified performance for all plants within the model uncertainty.

We will again consider operating point *III* of the methanol-propanol column with a RHP pole at 0.086 min^{-1} . We use single-loop PID-controllers as this is the preferred control structure in the industry, and also close to optimal (Skogestad and Lundström, 1990). Table 3.3 gives the results for designs with θ_d between 1 and 5 minutes. For a deadtime of 1 min. we see from Table 3.3 that there is only a small difference between the achieved robust performance of the unstable process and the stable model. When the deadtime is increased we must allow for a lower bandwidth as well as a higher peak in the sensitivity function for the open-loop unstable process than for the open-loop stable model. This is as expected from (3.24). For a deadtime of 2 minutes we can only guarantee half the bandwidth for the unstable system compared to the stable model. With a deadtime of 5 minutes (RHP zero $z \approx 0.4 \text{ min}^{-1}$) the response for the *LV*-configuration is poor ($\tau_{CL} = 175 \text{ min.}$ with a maximum peak $P = 3.0$), while the response for the *L_wV*-configuration is unacceptable ($\tau_{CL} = 455 \text{ min.}$ and $P = 6.0$). This implies that when the operating point is open-loop unstable (with the *L_wV*-configuration) and the system in addition has significant deadtime one should consider using a different configuration.

³Note that decouplers will perform poorly on ill-conditioned plants due to uncertainty sensitivity.

θ_d [min]	<i>LV</i>		<i>L_wV</i>	
	P	τ_{CL} [min]	P	τ_{CL} [min]
1.0	2.5	25	2.5	32
2.0	2.5	59	2.5	140
3.0	2.5	100	3.0	227
4.0	2.5	161	5.0	345
5.0	3.0	175	6.0	455

Table 3.3: Robust performance parameters (see (3.28)) obtained for stable *LV*-configuration and unstable *L_wV*-configuration in operating point *III* of the methanol-propanol column. All parameters for minimized $\mu_{RP}=1.00$ using two single-loop PID controllers.

	k_y	τ_{Iy}	τ_{Dy}	k_x	τ_{Ix}	τ_{Dx}
<i>LV</i> -configuration	0.0687	6.55	2.33	0.0680	5.29	0.180
<i>L_wV</i> -configuration	4.064	32.27	2.91	0.0280	4.79	0.209

Table 3.4: Controller parameters for closed-loop simulations in Figure 3.10. (Correspond to last entry in Table 3.3)

Nonlinear Simulations. Figure 3.10 shows responses to setpoint changes in top composition y_D using two single loop PID-controllers for the *LV*-configuration and the *L_wV*-configuration. The simulations include 5 minutes deadtime (using a 1.order Padé approximation) and 20 % input uncertainty. The controller parameters were obtained from the μ -optimal design above, and are given in Table 3.4. The simulations demonstrate the fact that the performance for the case with mass-reflux is clearly worse than for the case with molar reflux. The *L_wV*-configuration has a much larger overshoot as well as a longer settling period.

3.5 Effect of Including the Energy Balance

To this point we have only considered models with constant molar flows, that is, with the energy balance excluded. However, Jacobsen and Skogestad (1991) show that when the energy balance is included in the model, even molar inputs may yield multiplicity and instability in distillation. The multiplicity is in this case caused by interactions between the flows and compositions inside the column. The flows will affect the compositions through the material balance while the compositions will affect the flows through the energy balance.

Figure 3.11 shows steady-state solutions for the methanol-propanol column with the *LV*-configuration and the energy balance included in the model. The enthalpy data used are given in Table 3.5. The maximum eigenvalues in selected operating points are also shown in Figure 3.11 and we see that the solution branch with a negative slope between molar reflux L and top composition y_D corresponds to unstable solutions. Note that the eigenvalues were computed with a static energy balance, that is, the energy dynamics were neglected.

To consider the stability properties for the *LV*-configuration with the energy balance

$$H_i^L = 16.67e^{-1.087x_i}$$

$$H_i^V = 13.49e^{-3.98x_i} + 43.97e^{-0.088x_i}$$

- Reference state: Pure components as liquid at 0°C.
- x_i denotes mole fraction methanol in liquid phase.

Table 3.5: Saturated molar enthalpies (kJ/mol) for methanol-propanol system at a pressure of 1 atm.

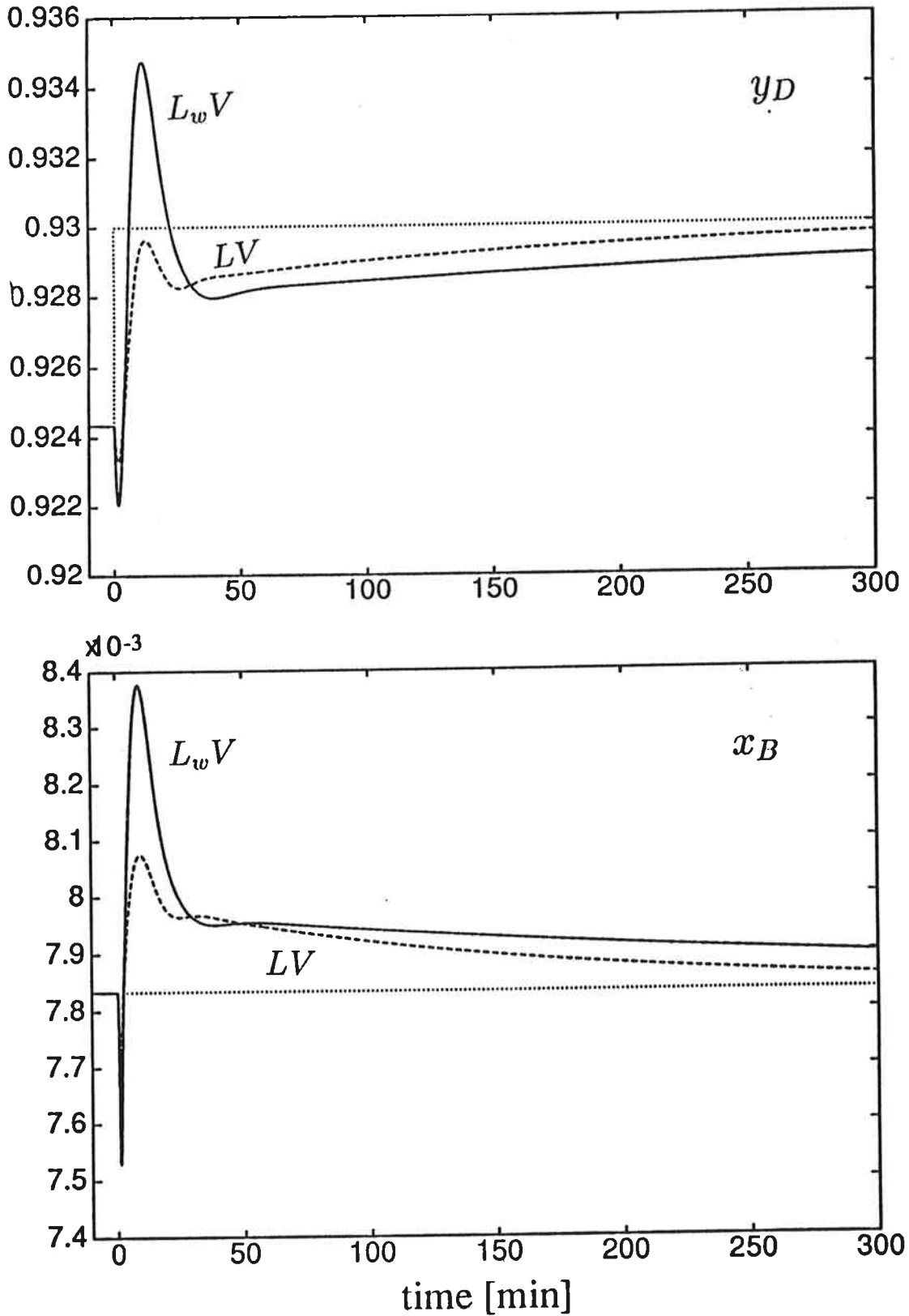


Figure 3.10: Nonlinear simulation of methanol- propanol column at operating point *III* with two-point control using *LV*- and *L_wV*-configuration. Response to a setpoint change in y_D using two single-loop PID controllers. Deadtime $\theta_d = 5$ min. PID-settings from Table 3.4. Constant molar flows.

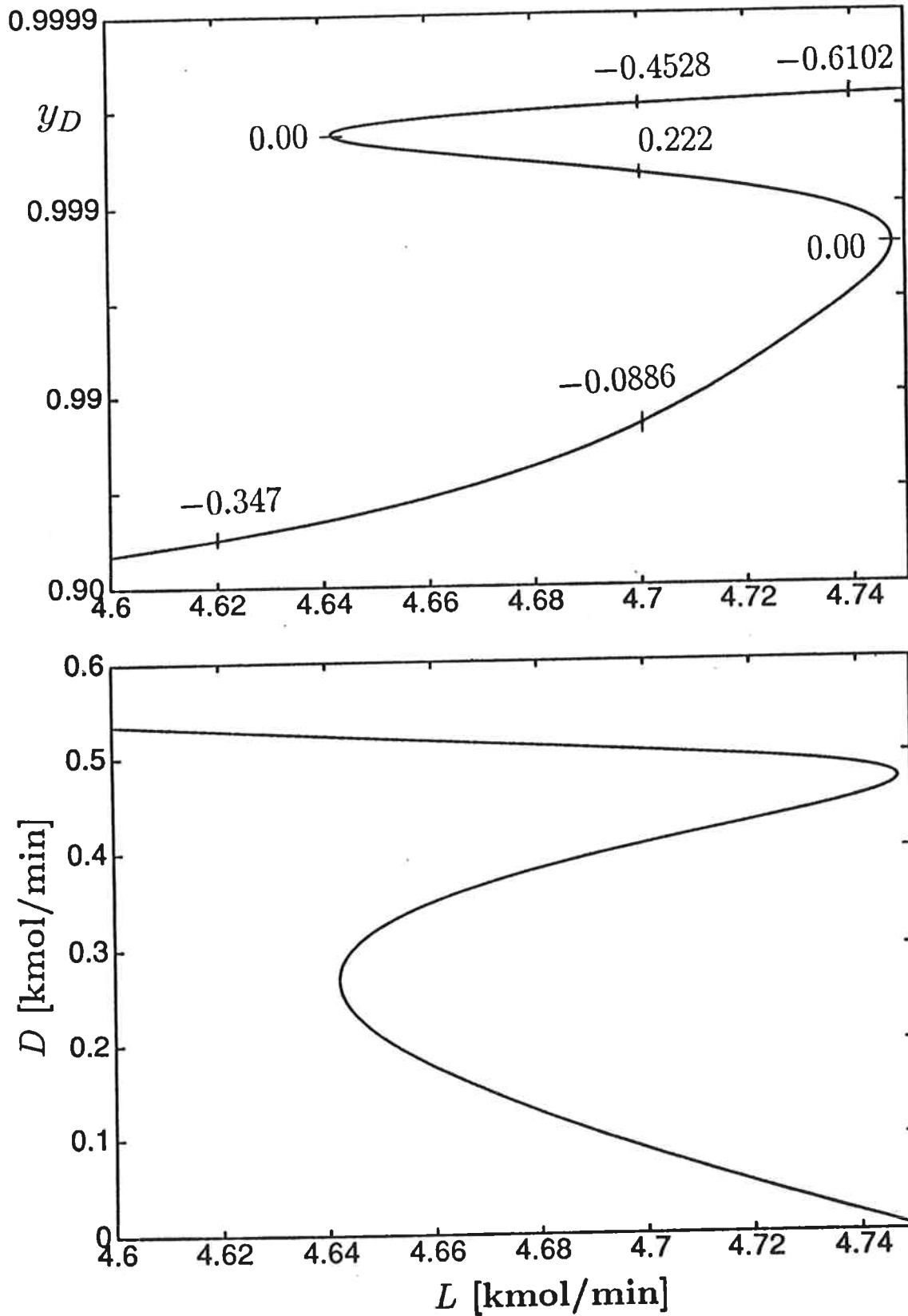


Figure 3.11: Steady-state solutions as a function of molar reflux L for methanol-propanol column with energy balance included. The maximum eigenvalue is shown in selected operating points. Boilup $V = 4.5$ kmol/min.

	L [kmol/min]	D [kmol/min]	y_D	x_B
I	4.60	0.535	0.9324	2.474e-3
II	4.70	0.505	0.9845	6.344e-3
III	4.70	0.406	0.9993	0.1587
IV	4.70	0.0866	0.9997	0.4526

• Energy balance included in model.

Table 3.6: Steady-state solutions for methanol-propanol column with boilup $V=4.5$ kmol/min.

included we utilize the fact that the DV -configuration in all known cases yields unique solutions (Sridhar and Lucia, 1989, Jacobsen and Skogestad, 1991) which are stable under the assumption of perfect level control (see section 3.7 below). The transfer function from molar reflux L to top composition y_D may be written

$$\left(\frac{\partial y_D}{\partial L}\right)_V(s) = \left(\frac{\partial y_D}{\partial D}\right)_V(s) \left(\frac{\partial L}{\partial D}\right)_V^{-1}(s) \quad (3.29)$$

As before we assume that the composition dynamics for the stable case may be approximated by a first-order response, i.e.,

$$\left(\frac{\partial x_i}{\partial D}\right)_V(s) \approx \frac{k_{x_i D}^{DV}}{1 + \tau_1 s} \quad (3.30)$$

A material balance around the condenser yields (assuming perfect level control)

$$L = V_T - D \quad (3.31)$$

where V_T denotes vapor flow to the condenser. For simplicity we neglect changes in the liquid enthalpy with composition, and obtain from an overall static energy balance (saturated liquid feed)

$$V_T = \frac{\Delta H_1^{vap}}{\Delta H_T^{vap}} V \quad (3.32)$$

Here the "heat of vaporization", ΔH_i^{vap} , is the enthalpy of the vapor with each component as pure saturated liquid as reference. (See Appendix for details.) Inserting (3.32) in the material balance (3.31) and differentiating, with ΔH^{vap} a function of composition x only (saturated vapor at constant pressure), yields for a binary mixture

$$\left(\frac{\partial L}{\partial D}\right)_V(s) = -1 + V \frac{\left(\frac{d\Delta H^{vap}}{dx}\right)_{x=x_B} \left(\frac{\partial x_B}{\partial D}\right)_V(s) \Delta H_T^{vap} - \left(\frac{d\Delta H^{vap}}{dx}\right)_{x=x_T} \left(\frac{\partial x_T}{\partial D}\right)_V(s) \Delta H_1^{vap}}{(\Delta H_T^{vap})^2} \quad (3.33)$$

Inserting (3.30) and (3.33) into (3.29) we find that the largest pole of $(\partial y_d/\partial L)_V(s)$ will be

$$\lambda_{max} = -\frac{1}{\tau_1} \left(1 - V \frac{\left(\frac{d\Delta H^{vap}}{dx} \right)_{x=x_B} k_{x_B D}^{DV} \Delta H_T^{vap} - \left(\frac{d\Delta H^{vap}}{dx} \right)_{x=x_T} k_{x_T D}^{DV} \Delta H_1^{vap}}{(\Delta H_T^{vap})^2} \right) \quad (3.34)$$

Comparing (3.34) and (3.33) we see that the pole will be in the right half plane when $(\partial L/\partial D)_V$ is positive. This is in accordance with the results presented in Jacobsen and Skogestad (1991).

From (3.34) we see that the probability of instability for the LV -configuration will increase with internal flows (i.e., V). This is similar to what was found for the instability caused by the input transformation with the L_wV -configuration. If we assume ideal vapor phase, then $d\Delta H^{vap}/dx = \Delta H_L^{vap} - \Delta H_H^{vap}$ which is the difference in heats of vaporization for the light and heavy component at their normal boiling point. Thus, for the normal case where the most volatile component has the smallest heat of vaporization ($d\Delta H^{vap}/dx_i < 0$) instability is most likely when we have $|k_{x_B D}^{DV}|$ large relative to $|k_{x_T D}^{DV}|$, which corresponds to having high purity in the top relative to the bottom. Note that this is different from what was found for the L_wV -configuration where instability was found to be most likely with intermediate purities in the top (unpure relative to bottom.)

The singularity caused by interactions between the material and energy balance corresponds to a bifurcation point with a single pole crossing the imaginary axis, similar to what was found for constant molar flows with the L_wV -configuration. The control problems will therefore be similar to what was discussed in section 3.4.

3.5.1 Combining mass flows and energy balance

Jacobsen and Skogestad (1991) show that when both types of multiplicities are present in the same region of operation, a column operating with the L_wV -configuration may have five different solutions, two of which will be unstable. This is illustrated in Figure 3.12 which shows solutions for the methanol-propanol column with L_w in the range 84 to 91 kg/min and $V = 2.7$ kmol/min. In the same figure is also shown the maximum eigenvalue as a function of top composition y_D . The bifurcation (singular) points at low purities in the top are due to singularities between mass reflux and molar reflux, while those at high purities are due to singularities between molar reflux and top composition.

3.6 Other Bifurcation Parameters

So far we have only considered the inputs, e.g. reflux L_w and boilup V , as potential bifurcation parameters. That is, in all studies we have assumed the other parameters, e.g., feed flow F , feed composition z_F , feed liquid fraction q_F , tray efficiency etc., to be fixed. However, it is clear that these parameters will vary during operation and may, similarly to the inputs, cause the column operation to go from open-loop stable to open-loop unstable.

To illustrate this consider Figure 3.13 which shows steady-state solutions for the methanol-propanol column (assuming constant molar flows) with $L_w = 50.0$ kg/min,

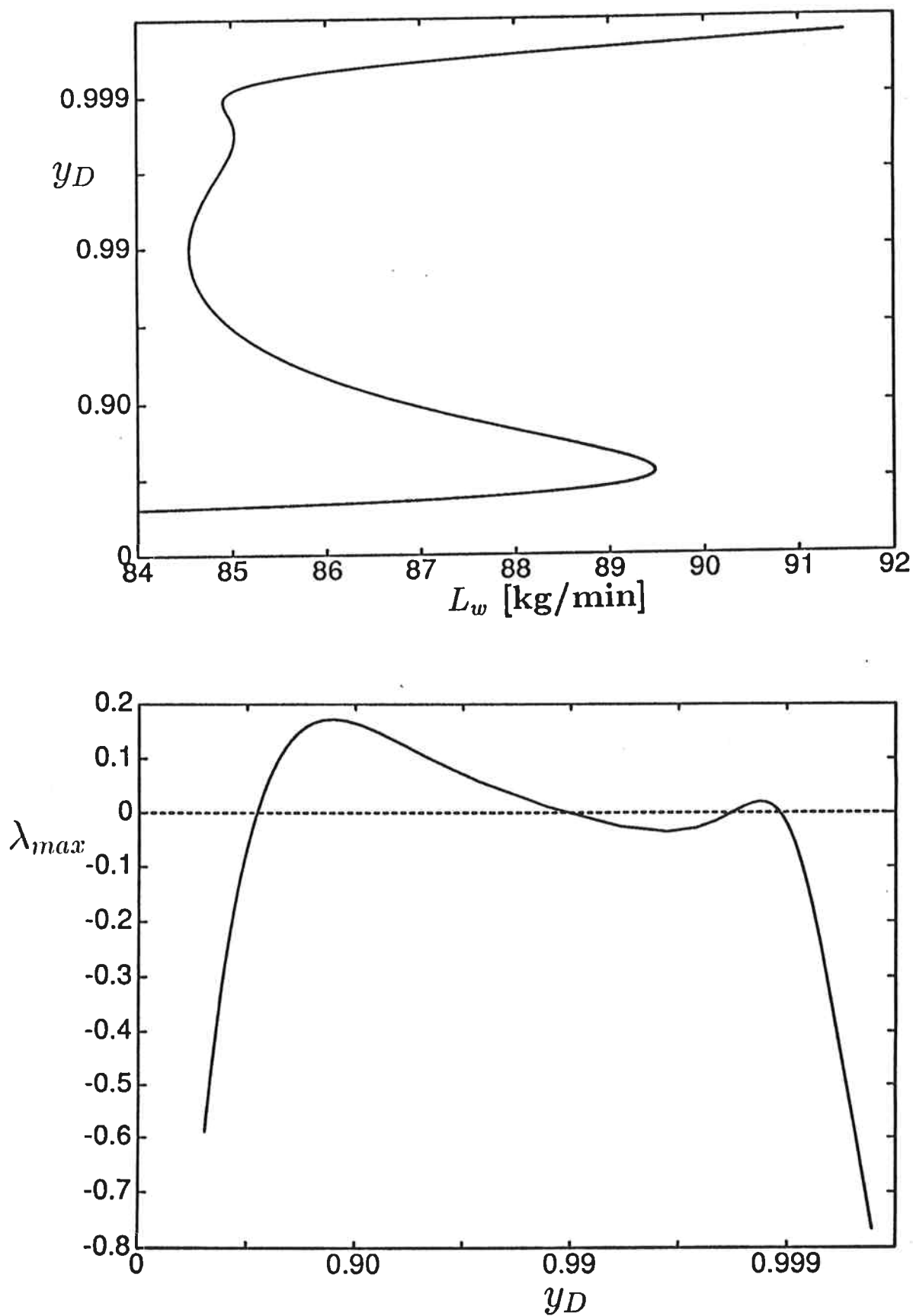


Figure 3.12: Steady-state solutions as a function of mass reflux L_w for methanol-propanol column with energy balance included in model. Lower plot shows the maximum eigenvalue as a function of top composition y_D . Boilup $V = 2.7$ kmol/min.

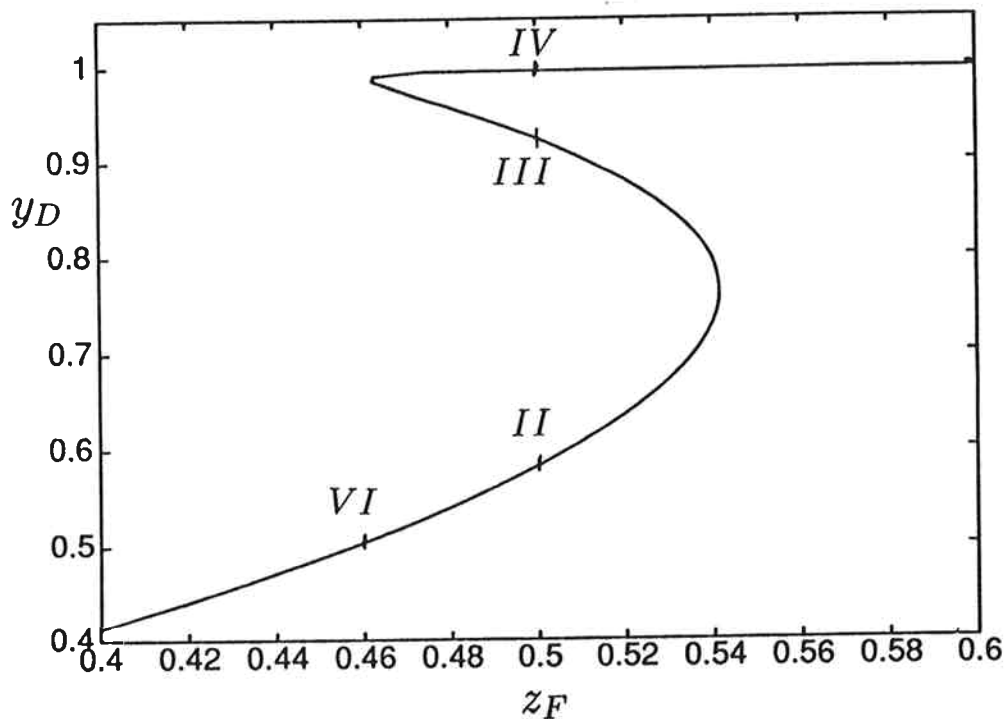


Figure 3.13: Steady-state solutions as a function of feed composition z_F for methanol-propanol column. Reflux $L_w = 50$ kg/min, Boilup $V = 2.0$ kmol/min. Constant molar flows.

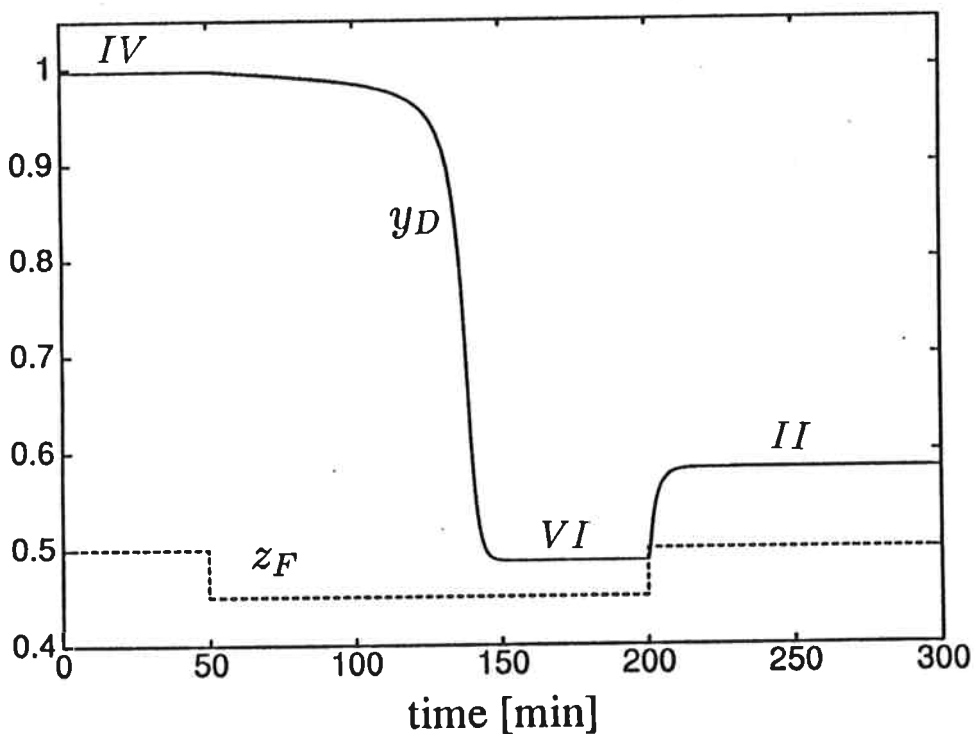


Figure 3.14: Nonlinear open-loop simulation of changes in feed-composition z_F for methanol-propanol column. Reflux $L_w = 50$ kg/min and boilup $V = 2.0$ kmol/min. Roman numbers II–IV refer to Table 3.2 with $z_F = 0.50$. Operating point VI corresponds to $z_F = 0.46$. Constant molar flows.

$V = 2.0$ kmol/min and feed composition z_F in the range 0.40 to 0.60. From the figure we see that there are multiple solutions for z_F in the range 0.46 to 0.54. This implies that disturbances in the feed composition may cause the column to go through a singular point and thereby "jump" to another solution branch. This is illustrated in Figure 3.14 which shows the response in top composition y_D to a change in feed composition z_F from 0.50 (operating point *IV* in Table 3.2) to 0.46. The figure illustrates how the top composition "jumps" to the lower solution branch and settles in operating point *VI*. When the feed composition returns to $z_F = 0.50$ the solution remains on the lower branch and settles in operating point *II*.

3.7 Instability with the D_wV -Configuration

We have so far only considered using reflux and boilup as independent variables, e.g., the L_wV -configuration. This is also the most widespread configuration in industry. However, there are many different configurations that may be used. For instance, changing condenser level control from using distillate D_w to using reflux L_w results in the D_wV -configuration. For all the examples we have studied this configuration yields a unique steady-state solution in terms of compositions. Furthermore, we have assumed perfect level control, in which the operating point is found to be asymptotically stable. However, here we show that without the assumption of perfect level control the operating point may become unstable also with the D_wV -configuration. We start by considering an example and will then explain the results thereof using analytical results.

Example. We will again consider the methanol-propanol column in Table 3.1. The holdups in the reboiler and condenser are increased to $M_D/F = M_B/F = 5.0$ min. We consider the case with constant molar flows, and use distillate flow D_w and boilup V as independent inputs, i.e., D_wV -configuration. With this configuration the condenser level is controlled by reflux L_w and the reboiler level is controlled by bottoms flow B_w . The nominal operating point we consider has $D_w=18.36$ kg/min and $V=2.0$ kmol/min. For these specifications we obtain $y_D=0.9237$ and $x_B=0.0078$, and the steady-state is unique. Note that the operating point corresponds to solution *III* for the L_wV -configuration in

	y_D	x_B	D	λ_{max}
I	0.5207	2.024e-3	0.9601	-0.5323
II	0.9673	6.756e-3	0.5135	7.327e-2
III	0.9978	7.251e-2	0.4620	-1.744e-2
IV	0.99855	0.1236	0.4302	1.7512e-2
V	0.99908	0.2321	0.3493	-5.608e-2

• Energy balance included in model.

Table 3.7: Steady-state solutions for methanol-propanol column with $L_w=85$ kg/min and $V = 2.7$ mol/min.

Table 3.2 and Figure 3.2, that is, the operating point is unstable with reflux and boilup as independent variables.

We now consider the stability of the of the operating point for different gains K_{M_D} in the condenser level controller. A pure proportional controller is used, i.e., $dL_w(s) = K_{M_D} dM_{D_w}(s)$. We assume perfect level control in the reboiler. Figure 3.15a shows the response in top composition to a small increase in D_w , keeping V constant, with level control gain $K_{M_D}=0.10$. We see that the response is stable and slightly oscillatory. Figure 3.15a also shows the phase plot of top composition y_D against mass bottoms flow B_w , and we see that the steady-state is a stable spiral attractor. Figure 3.15b shows the corresponding response with $K_{M_D}=0.05$. The response is now more oscillatory, but the steady-state is still a stable spiral attractor. With K_{M_D} reduced to 0.03 the operating point becomes an unstable spiral as seen from Figure 3.15c. However, the response settles into a stable periodic solution, that is, a stable limit cycle.

The fact that the steady-state changes from a stable spiral to an unstable spiral as the level control gain is reduced implies that a pair of complex conjugate eigenvalues cross the imaginary axis. This may be seen from Figure 3.16a which shows the largest eigenvalues as a function of level control gain K_{M_D} , i.e., the root locus. We see that as the gain is reduced below a value of 0.043, the eigenvalues cross the imaginary axis, and the operating point becomes unstable. The fact that a stable limit cycle appears as the steady-state becomes unstable, implies that the system undergoes a dynamic bifurcation known as the Hopf bifurcation.

3.7.1 Analytical treatment

To understand why the steady-state for the D_wV -configuration becomes unstable, consider the transfer function $(\partial y_D / \partial D_w)_V(s)$ which may be written

$$\left(\frac{\partial y_D}{\partial D_w}\right)_V(s) = \left(\frac{\partial y_D}{\partial L_w}\right)_V(s) \left(\frac{\partial L_w}{\partial D_w}\right)_V(s) \quad (3.35)$$

Here the transfer function $(\partial y_D / \partial L_w)_V(s)$ expresses the effect of reflux on top composition with the L_wV -configuration, and we have seen that it may be unstable with a single RHP pole. For simplicity we consider only the largest pole in the transfer function

$$\left(\frac{\partial y_D}{\partial L_w}\right)_V(s) = \frac{k^{L_wV}}{s - a} \quad (3.36)$$

Here a denotes the maximum eigenvalue for the L_wV -configuration. The transfer function $(\partial L_w / \partial D_w)_V(s)$ may be computed from a material balance around the condenser

$$dL_w(s) = \frac{K_{M_D}}{s} (dV_{T_w}(s) - dL_w(s) - dD_w(s)) \quad (3.37)$$

Differentiation of (3.37) yields

$$\left(\frac{\partial L_w}{\partial D_w}\right)_V(s) = \frac{K_{M_D}}{K_{M_D} + s} (V_T(M_1 - M_2) \left(\frac{\partial y_T}{\partial D_w}\right)_V(s) - 1) \quad (3.38)$$

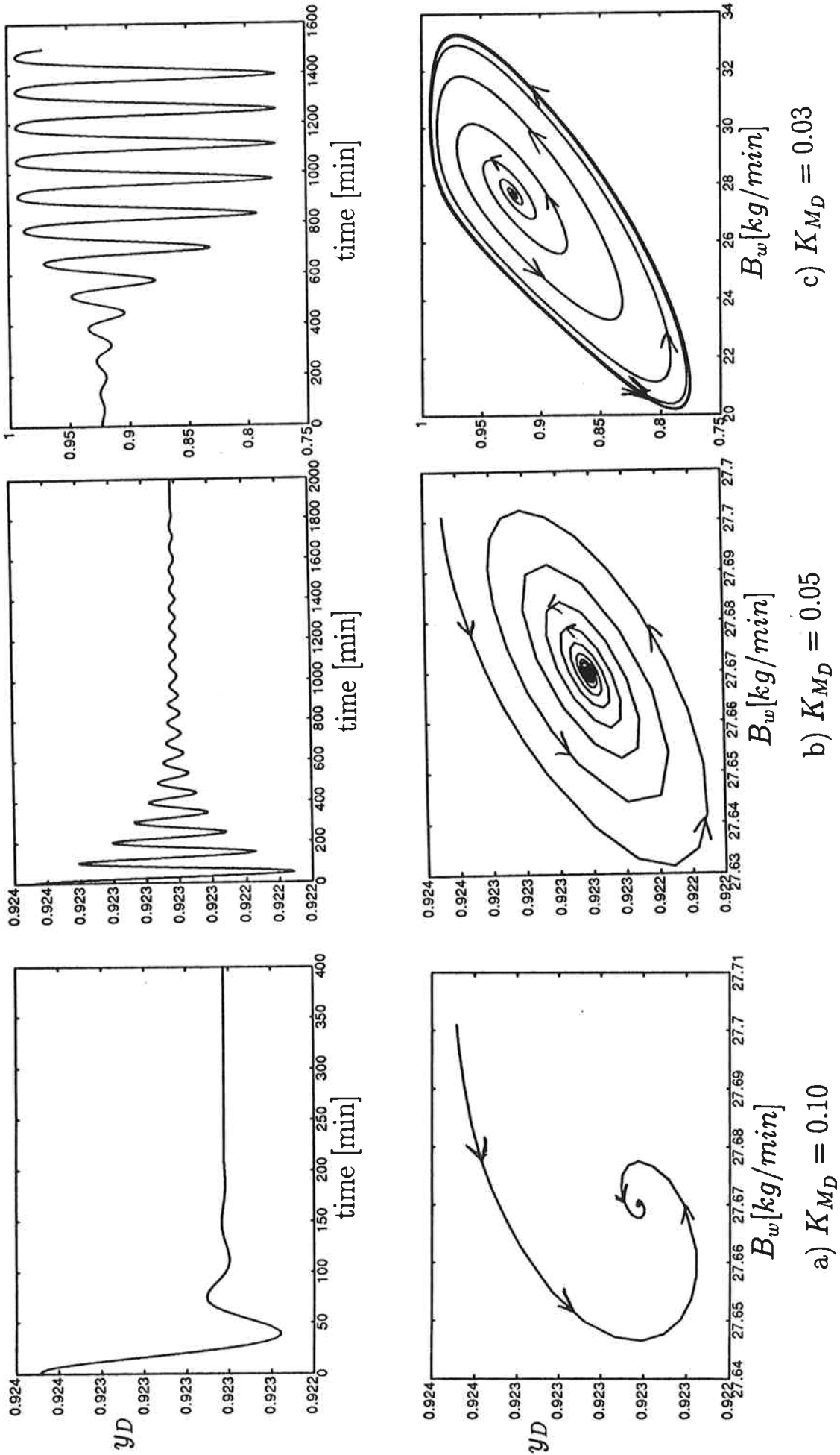


Figure 3.15: Nonlinear open-loop simulation of methanol-propanol column with D_wV -configuration. Responses to small increase in D_w with different gains K_{M_D} in condenser level controller. a) $K_{M_D}=0.10$. b) $K_{M_D}=0.05$ c) $K_{M_D}=0.03$. Upper plot: time as independent variable. Lower plot: Phase-plane plot. Boilup $V=2.0$ kmol/min. Constant molar flows.

Here y_T denotes the composition of V_T . We assume negligible condenser holdup so that $(\partial y_T / \partial D_w)_V(s) = (\partial y_D / \partial D_w)_V(s)$. Inserting (3.36) and (3.38) into (3.35) yields

$$\left(\frac{\partial y_D}{\partial D_w}\right)_V(s) = \frac{K_{M_D} k^{L_w V}}{s^2 + (K_{M_D} - a)s - K_{M_D}(a + k^{L_w V} V_T(M_1 - M_2))} \quad (3.39)$$

The poles of the transfer function (3.39) become

$$\lambda_{1,2} = -\frac{1}{2}(K_{M_D} - a) \pm \frac{1}{2}\sqrt{(K_{M_D} - a)^2 + 4K_{M_D}(a + k^{L_w V} V_T(M_1 - M_2))} \quad (3.40)$$

Figure 3.16b shows the root locus for the example computed using equation (3.40), and we see that the simple expression (3.40) yields a reasonable prediction of the behavior of the full model in Figure 3.16a.

Let us now use (3.40) to consider the stability of the $D_w V$ -configuration for the two cases when the pole a of the $L_w V$ -configuration is in the LHP and RHP, respectively:

1) Stable $L_w V$ -configuration, $a < 0$: In this case the first term in (3.40) is negative for all values of $K_{M_D} > 0$. Furthermore, the second term under the root in (3.40) is negative and the root will be real with a value less than $(K_{M_D} - a)$ or it will be imaginary. This implies that both eigenvalues in (3.40) are in the LHP, that is, the $D_w V$ -configuration is stable for all values of $K_{M_D} > 0$.

2) Unstable $L_w V$ -configuration, $a > 0$: In this case we have that the first term in (3.40) is positive if $K_{M_D} < a$, that is, at least one of the eigenvalues in (3.40) are in the RHP with $K_{M_D} < a$. The size of K_{M_D} will determine whether the root in (3.40) is imaginary. For $K_{M_D} = a$, i.e., the bifurcation point, we have that the root is imaginary if $k^{L_w V} V_T(M_1 - M_2) < -a$, which is the case in all examples we have studied.

We conclude from the above analysis that a prerequisite for instability with the $D_w V$ -configuration is that the operating point is unstable with the $L_w V$ -configuration. This is not too surprising as the level control for the $D_w V$ -configuration may be viewed as a feedback effect on the $L_w V$ -configuration. If the feedback control is not tight enough, we are not able to stabilize the column, which is similar to what we found for the case of one-point control with the $L_w V$ -configuration. With a gain $K_{M_D} = 0$, i.e., no condenser level control, we see from (3.40) that there will be a RHP pole at a (in addition to a pole at 0), and we effectively have the stability properties of the $L_w V$ -configuration. This may also be seen from the root locus in Figure 3.16a for $K_{M_D} = 0$.

In our example we find that the $L_w V$ -configuration is unstable with a pole $a = 0.047$ (with $M_D/F = M_B/F = 5 \text{ min.}$) and from (3.40) we predict instability for the $D_w V$ -configuration with $K_{M_D} < 0.047$. From the full model we find that instability occurs for $K_{M_D} < 0.043$. The deviation in predicted and computed value is explained by our assumptions of first-order response in (3.36) and negligible condenser holdup in the analytical treatment.

We have shown that the operating points with the $D_w V$ -configuration may become unstable with two complex conjugate eigenvalues crossing the imaginary axis. However, we have seen from the example that a stable limit cycle appears as the steady-state solution goes unstable. The proof of the existence of a stable limit cycle is rather involved for a high-order dynamic model, and is therefore left out here.

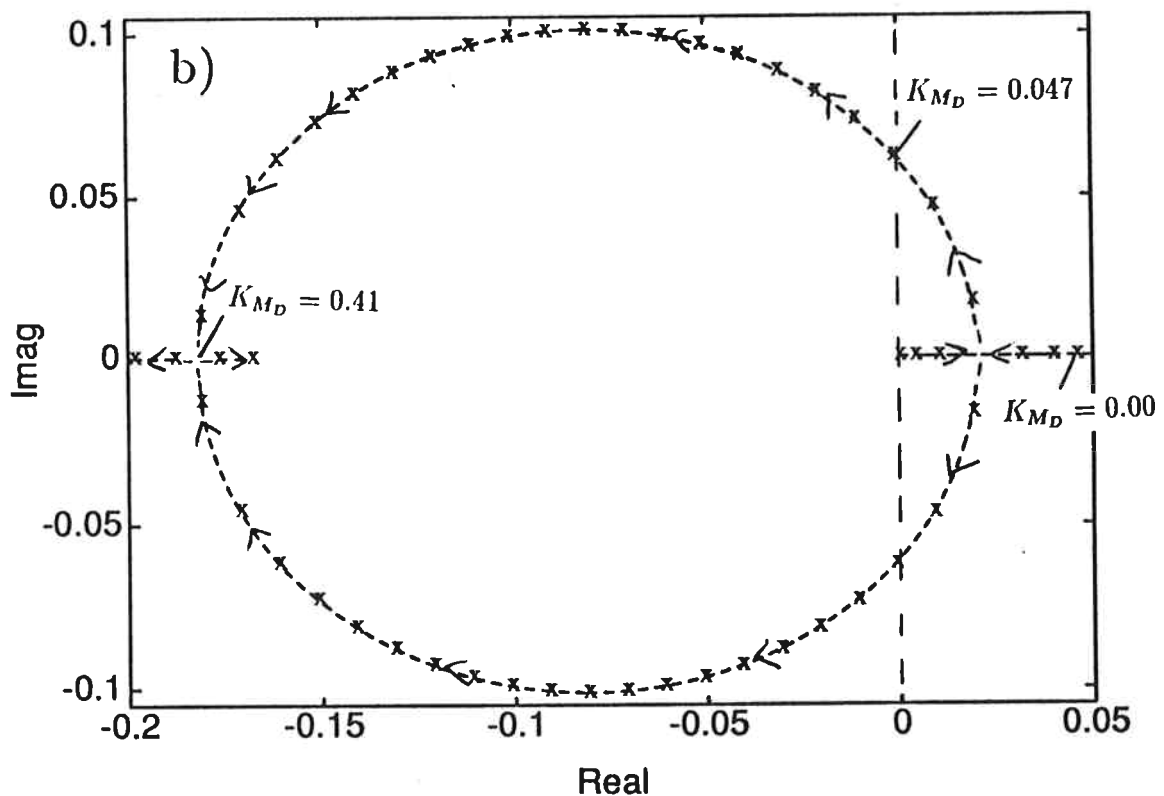
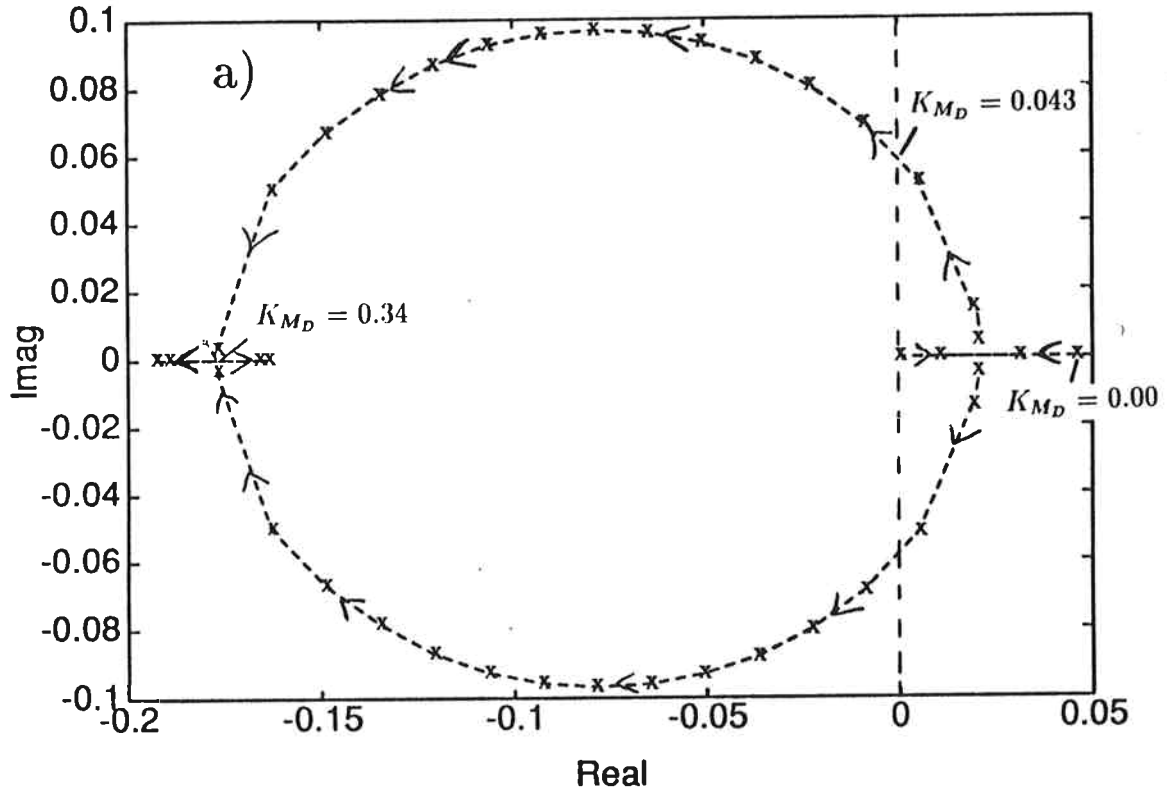


Figure 3.16: Root locus for methanol-propanol column with constant molar flows and D_wV configuration. $D_w=18.36$ kg/min and $V=2.0$ kmol/min. Plot shows maximum eigenvalues as function of gain K_{M_D} in condenser level controller (arrows indicate increasing gain). a) Eigenvalues computed from full model. b) Eigenvalues computed from simple expression (3.40).

3.8 Discussion

Other Configurations. We have in this paper only considered two configurations, namely the L_wV - and D_wV -configuration. We have shown that with the L_wV -configuration, the operating points may be open-loop unstable with a single RHP pole. This instability is linked to the existence of multiple steady-state solutions and is independent of the level control tuning. To stabilize an unstable operating point with the L_wV -configuration, feedback control of a temperature or composition is required. For the D_wV -configuration we have shown that instability may result if the level control is not sufficiently tight. Many other configurations are of course also possible, and the type of instability found for the D_wV -configuration is likely to be found with other configurations. For instance, the L_wB_w -configuration may become unstable with too slow reboiler level control. However, Jacobsen and Skogestad (1991) show that output multiplicity and the corresponding type of instability is unlikely with other flows than reflux and boilup as independent variables. Because relatively tight level control usually is easily achieved, one may in most cases avoid instability with the L_wV -configuration by changing to another configuration. However, note that different configurations will also have different control properties like interactions and disturbance sensitivity (e.g., Skogestad et.al., 1990). The choice of a proper configuration should therefore not only be based on open-loop stability properties.

Detecting open-loop instability experimentally. There are several ways to determine whether a column under operation will be open-loop unstable with reflux and boilup as independent variables.

1. L_wV -configuration with one loop closed:

- Increasing the purity in the top of the column corresponds to decreasing reflux, which is opposite to what one would expect. Similar behavior will be seen in the bottom, i.e., increasing purity corresponds to decreasing boilup.
- Detuning the controller makes the column go into a stable limit cycle.
- Turning the controller off causes the column profile to drift away. However, it may be difficult to distinguish this from a "slow" column with a stable pole close to the imaginary axis.

2. Using other configurations:

- If an increase in distillate flow, i.e., decreasing y_D , results in increased reflux, the operating point will be unstable with the L_wV -configuration. Similarly, if boilup increases when bottoms flow is increased the operating point will be unstable with the L_wV -configuration.

Effect of column design. The methanol-propanol column that we have studied in this paper is not optimally designed for the product compositions of operating point III in Table 3.2. In fact it is doubtful that the column would be unstable when optimally designed as the internal flows then would be significantly smaller. This is probably true for many separations, i.e., an optimally designed column will be open-loop stable. However, few industrial columns are operated close to an optimal operating point. One reason is that the desired compositions will change after the column is built. In addition, many operators prefer to use high internal flows (over-purification) in order to assure that specifications are kept when disturbances enter the column. It is therefore likely that many industrial columns may have problems with open-loop instability. The fact that this has not been

reported previously is probably due to the fact that open-loop instability has been believed to be impossible, and problems have therefore been explained by other means.

3.9 Conclusions

1. Two-product distillation columns operating with reflux and boilup as independent inputs may be open-loop unstable with a single right half plane pole. Two different effects may cause the instability:
 - Possible singularities in the transformation between the actual input units and the molar units which determine separation.
 - Possible singularities between molar flows and compositions due to interactions through the material and energy balance.

In both cases the probability of instability is increased with increased internal flows.

2. An unstable operating point may be stabilized by use of one-point control provided the bandwidth of the controller is sufficiently high, that is, if the measurement delay is sufficiently small. If an operating point becomes closed-loop unstable due to a too low bandwidth, the column may go into a stable limit cycle provided there exists stable solutions above and below the unstable solution.
3. The presence of open-loop instability will worsen the performance of the closed-loop system. This will become more marked as the deadtime in the system is increased.
4. Distillation columns operating with distillate flow and boilup as independent variables may have unstable operating points if the condenser level control is not sufficiently tight. The instability will in this case correspond to a Hopf bifurcation, that is, a pair of complex conjugate eigenvalues cross the imaginary axis and a stable limit cycle appears. A prerequisite for instability with this configuration is that the operating point is unstable with reflux and boilup as independent variables.

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APPENDIX. Simplification of the energy balance.

The static energy balance on a single tray may be written (no external heating or cooling)

$$V_{i-1}H_{i-1}^V + L_{i+1}H_{i+1}^L - V_iH_i^V - L_iH_i^L + F_iH_i^F = 0 \quad (3.41)$$

Here subscript i denotes tray-number (reboiler is tray 1), H^V and H^L denotes vapor and liquid enthalpies respectively and H^F feed enthalpy.

As reference state for enthalpies we select pure components as saturated liquids. Then, under the assumptions of no heat of mixing, equal heat capacities for the components and linear boiling point curve we get $H_i^L = 0$ on all trays (this is a common assumption in distillation which yields "constant molar flows" if we in addition assume the same heat of vaporization for all components.) Also assume that the feed is saturated liquid so that $H_i^F = 0$. With these assumptions the energy balance (3.41) becomes

$$V_i\Delta H_i^{vap} = V_{i-1}\Delta H_{i-1}^{vap} \quad ; \quad \Delta H_i^{vap} = H_i^V \quad (3.42)$$

An overall static energy balance then yields

$$V_T\Delta H_T^{vap} = V_1\Delta H_1^{vap} \quad (3.43)$$

NOMENCLATURE (see also Figure 3.1)

A - State matrix of distillation column

B - bottoms flow (kmol/min)

c - controller

D - distillate flow (kmol/min)

F - feed rate (kmol/min)

g - process transfer function

H^F - enthalpy of feed (kJ/mol)

H^L - saturated liquid enthalpy (kJ/mol)

H^V - saturated vapor enthalpy (kJ/mol)

ΔH^{vap} - ($= H^V - H^L$) heat of vaporization (for mixture)

K_{MD} - condenser level control gain

$k_{y_{u_1}}^{u_1 u_2}$ - steady-state gain from input u_1 to output y with $u_1 u_2$ -configuration.

L - reflux flow rate (kmol/min)

LHP - left half plane

M - molecular weight, usually of top product (kg/kmol)

M_1 - pure component molecular weight of most volatile component (kg/kmol)

M_2 - pure component molecular weight of least volatile component (kg/kmol)

M_L - tray liquid holdup (kmol)

M_D - condenser holdup (kmol)

M_B - reboiler holdup (kmol)

N - no. of theoretical stages in column

N_F - feed stage location (1-reboiler)

P - maximum allowed peak on sensitivity function

p - right half plane pole (min^{-1})

Q_B - heat input to reboiler
 Q_D - heat removal in condenser
 q_F - liquid fraction in feed
 RHP - right half plane
 S - sensitivity function
 V - boilup from reboiler (kmol/min) (determined indirectly by heating Q)
 V_T - vapor flow to condenser (kmol/min)
 x_B - mole fraction of most volatile component in bottom product
 x_i - mole fraction of most volatile component at tray i
 y_D - mole fraction of most volatile component in distillate (top product)
 y_T - mole fraction of most volatile component in V_T .
 z - right half plane zero (min^{-1})
 z_F - mole fraction of most volatile component in feed

Greek symbols

$\alpha = \frac{y_i/x_i}{(1-y_i)/(1-x_i)}$ - relative volatility (binary mixture)
 $\lambda_i(A)$ - i 'th eigenvalue of A .
 $\lambda_{max} = \max_i |\lambda_i(A)|$ - maximum eigenvalue = dominant pole
 μ - structured singular value
 $\bar{\sigma}$ - maximum singular value
 τ_{CL} - required closed-loop time constant (min)
 θ_d - deadtime (min)
 ω - frequency (min^{-1})

Subscripts

w - flow rate in kg/min
 H - least volatile component
 L - most volatile component

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