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Dynamics and Controllability of Heat Exchanger Networks

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1 Abstract

The dynamic and control behaviour of a heat exchanger network should also be taken into account when designing the network. In this paper we present various dynamical models of single heat exchangers, and we present our first results on the dynamics and controllability of heat exchanger networks. We demonstrate that inverse responses can occur when streams are mixed.

2 Introduction

During the last decade there have been a large number of papers dealing with *steady-state* optimal design of heat exchanger networks. This has resulted in simple engineering techniques such as the pinch method by Linnhoff and Hindmarsh, (1983), as well as reliable software based on mathematical programming (e.g. Papoulias and Grossmann, 1983 and Floudas et.al., 1986). Most of the work in this field has been focused on designing the optimal network for a given set of cold and hot streams, and a given set of utilities. A few authors (e.g. Townsend and Morari, 1984) have looked at the resiliency problem where the inlet temperatures and the flow rates of the streams are allowed to vary within certain bounds, and the aim is to design a network *structure* which has optimal energy recovery under all conditions.

However, very little work has been done on the dynamic behaviour of heat exchanger networks. Linnhoff and Kotjabasakis (1986) introduced the concept of "downstream paths" to describe *structurally* how disturbances propagate in a network, but the analysis was done from a steady-state and not from a dynamic point of view. Georgiou and Floudas (1989) present results along the same lines. Daoutidis and Kravaris (1991) include some element of dynamics by considering the order of the response , but this structural approach is generally not too useful in practice since also the numerical values are important.

Dynamic heat exchanger models are either distributed or lumped. For the distributed model one usually assumes plug flow of the cold and hot streams with no axial mixing. In the lumped approach one considers the heat exchanger as a series of mixing tanks. Depending on the application either approach may be the most suitable, but we shall only use lumped models in this paper. Gaddis and Schlünder (1979) suggested a lumped multicell model for steady-state simulation of industrial multipass shell and tube heat exchangers. Roppo and Ganic (1983) and Correa and Marchetti (1987) used this approach for dynamic simulation of heat exchangers.

3 Dynamics

Plug flow model. Most steady-state calculations consider the heat exchangers in terms of countercurrent plug flow. Analytical transfer functions can be derived, but when both streams are modelled as plug flows the expressions become complicated and involve the transcendental operator (exp); see Gilles (1974).

However, the assumption of plug flow is questionable in most cases. In a traditional shell and tube exchanger, it may be reasonable for the tube side where backmixing is minimal. However, on the shell side there usually are baffles which give a considerable mixing effect. Furthermore, heat exchangers with multiple passes (eg., 1-2 exchanger) are frequently used. In these cases models based on some mixing tank approach are more suitable.

Single mixing tank model. To model a stream (ie., one side of the heat exchanger) as one mixing tank is attractive because of the simple expressions that result. It will be a reasonably good approximation if

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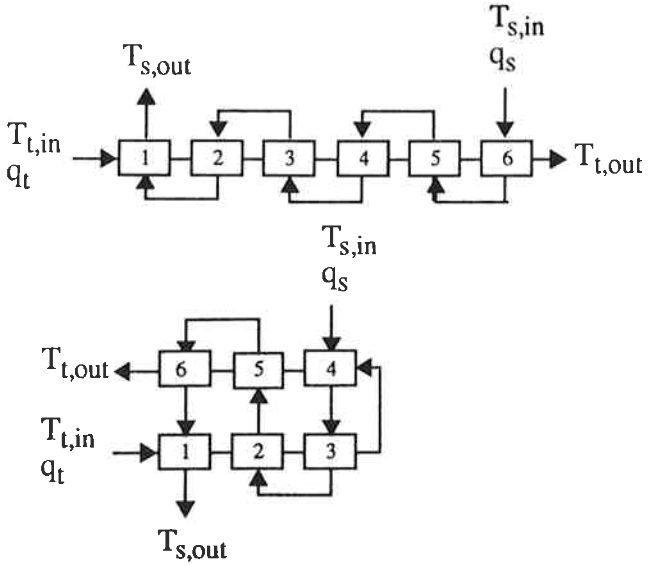

Figure 1: Multicell model of single and double pass heat exchanger

at least one of the following conditions is met:

- the driving forces in the exchanger are large compared to the temperature change of the stream.
- the stream in question has a much larger heat capacity than the other such that it's temperature change is small
- the geometry of the heat exchanger allow for considerable backmixing
- the stream is recirculated or mechanically stirred

Multiple mixing tanks (multicell model). For most industrial fluid-fluid heat exchangers without phase transition, none of these criteria are met, and the multicell model of Gaddis and Schlünder (1979) is attractive. Its performance will lie between plug flow and single mixing tank model, and if the cells are in series it will approach plug flow as the number of cells is increased. Both single and double pass exchangers can be modelled, see Fig.1. Assuming constant fluid properties, equal cell geometry and neglecting flow and wall dynamics, the heat balance around one cell for the tube side gives:

$$\tau_t \frac{dT_t(i)}{dt} = T_t(i-1) - T_t(i) + \alpha_t[T_s(i) - T_t(i)] \quad (1)$$

where $\alpha_t = \frac{UA_{cell}}{\rho c_{p_t} q_t}$ and $\tau_t = \frac{V_{cell}}{q_t}$. With the aid of the shell side trajectory introduced and defined by Correa and Marchetti (1987), the shell side is accordingly:

$$\tau_s \frac{dT_s(L(j))}{dt} = T_s(L(j-1)) - T_s(L(j)) - \alpha_s[T_s(L(j)) - T_t(L(j))] \quad (2)$$

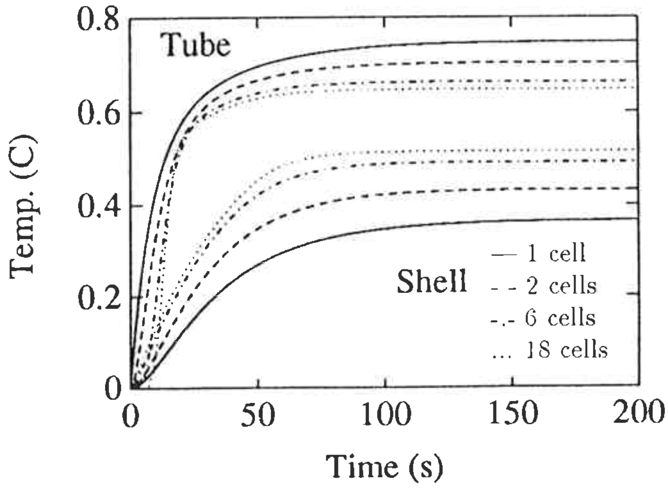

Figure 2: Time response to step in inlet tube side temperature of countercurrent heat exchanger. Comparison of numerical models with 1, 2, 6 and 18 cells.

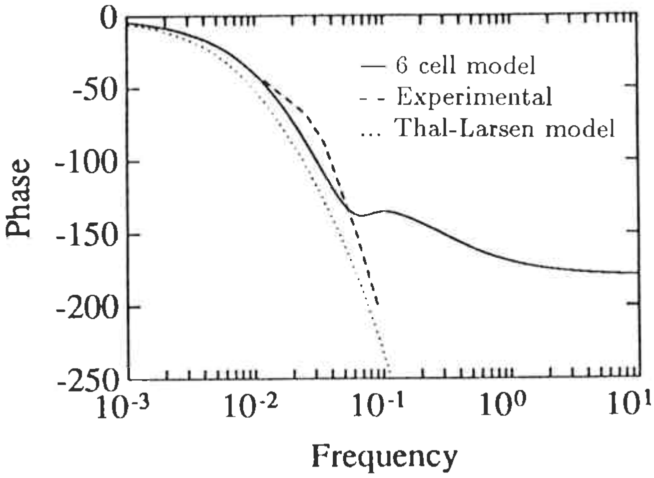

Figure 3: Frequency response from tube side flow rate to shell side temperature of countercurrent heat exchanger. Comparison of experimental data, Thal-Larsen empirical model and numerical model 6 cells on each side. The steady state gain has been normalised to 1 in all cases.

with α_s and τ_s defined similarly. This was used for a single pass countercurrent heat exchanger. The dynamic model was numerically linearised around the steady-state operating point. The step responses to a change in inlet temperature are given in Fig. 2. The figure also shows how the responses depend upon the number of cells used in the model. The steady-state gain increases up to 6 cells where it levels off. In Fig. 3 the frequency response for the case with 6 cells is compared to the experimental data of Thal-Larsen (1960) (as reported by Brambilla and Nardini, 1972). It can be seen that the numerical model accurately represents the experimental frequency response. The additional phase lag for the experimental data is probably caused by valve and measurement dynamics.

Most papers addressing optimisation of heat ex-

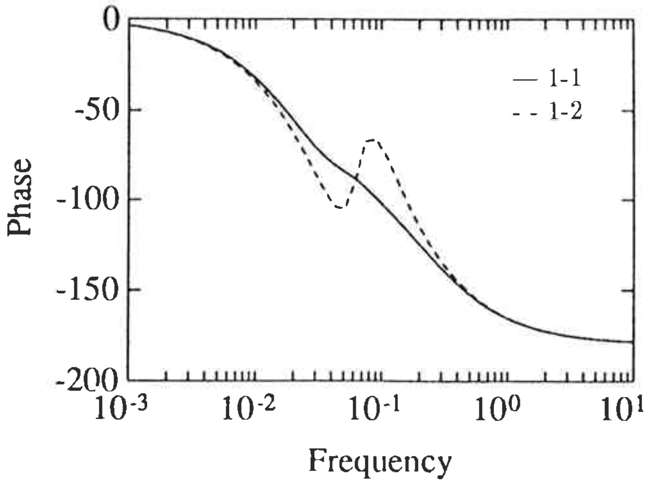

Figure 4: Comparison between 1-1 and 1-2 heat exchangers modelled with 6 cells. Phase response from tube side temperature to shell side temperature.

changer networks implicitly assume use of ideal countercurrent heat exchangers. In terms of a shell and tube heat exchanger this corresponds to an 1-1 (1 shell pass and 1 tube pass) exchanger. However, industrial exchangers frequently have more tube passes than shell passes. This reduces the efficiency of the heat exchanger, but the investment cost is lower. Therefore, 1-2 exchangers are often used if the temperature driving forces are relatively high. In Fig.4 we compare the dynamics of a 1-1 and 1-2 heat exchanger with the same heat transfer area. We see that for this specific response the phase exhibits a dip due to the crossing of flow paths. This delays the response of the 1-2 exchanger.

Heat exchanger networks. Based on the multicell model mentioned above, we have developed a computer program which from on a given network structure automatically generates a dynamic model of the network including a linearised model. The network may also include stream splitting, mixing and bypasses. When the number of baffles and tube passes are unknown we model the heat exchangers with 6 cells and assume countercurrent flow (1-1 exchangers).

4 Control of Heat exchangers

Control of heat exchangers is addressed in some general textbooks on process control, see for example Balchen and Mumme (1988). Hjorthol (1990) addresses controllability of a single heat exchanger, by selecting valve characteristics which counteract the nonlinearity of the dynamic response of the heat exchanger. We will below clarify notation and define the systems to be used in our analysis.

Inputs and outputs. A simple heat exchanger has 4 inputs and 2 outputs. The inputs are the inlet temperatures and Flow rates. Except for utility Flow rates, the inputs are disturbances from a control point of view. If present, the flow rate of a bypass stream is an additional input, and it is usually a manipulated variable from a control point of view.

Degrees of freedom for control. A single heat exchanger transfers heat from one stream to another. At best at steady-state it consequently has only one degree of freedom, which is the heat duty in W. During *design* of heat exchanger networks, the optimal heat duties for each exchanger are usually obtained by assuming given target temperatures (final temperatures for the streams exiting the network), and the necessary heat exchanger area is calculated. However, during operation, the heat exchanger area is usually not available as a degree of freedom (there are exceptions, for example, flooded condensers or thermosyphon boilers), so in order to alter and control the duty of an exchanger, one needs additional degrees of freedom ("manipulated variables"). These may be the flow rate or inlet temperature of one of the streams, but these are usually not available as manipulators. Therefore, *bypass streams* are often used as the additional degree of freedom (manipulator). In order to have a sufficient range for control, it is necessary to let the bypass flow rate be a substantial proportion of the total flow rate (i.e, 5-20%).

To obtain the same stream temperatures when a bypass is introduced one needs to increase the area of that heat exchanger. In some cases when the temperature driving forces are small this may be impossible (ie., the area becomes infinite). In such cases, one needs to change some of the internal stream temperatures, but by adding area to some other heat exchanger(s) one may still obtain the same target temperatures. It is interesting to note that none of the presently used design methods for heat exchanger networks seem to take into account the presence of bypass streams.

In summary, most heat exchangers where the duty is controlled have one manipulator (eg., bypass on one side). However, in a few cases one may in order to improve the dynamic response use two manipulators (eg., bypass on one side and flow rate on the other side of the exchanger), but since there is only one degree of freedom at steady-state, in such cases one of them is usually cascaded (eg., the bypass flow) such that it is reset to its original value after some time.

Analysis of model. Consider a state space model of a countercurrent heat exchanger with 3 cells. The state vector is $x = (T_t(1), T_t(2), T_t(3), T_s(1), T_s(2), T_s(3))$

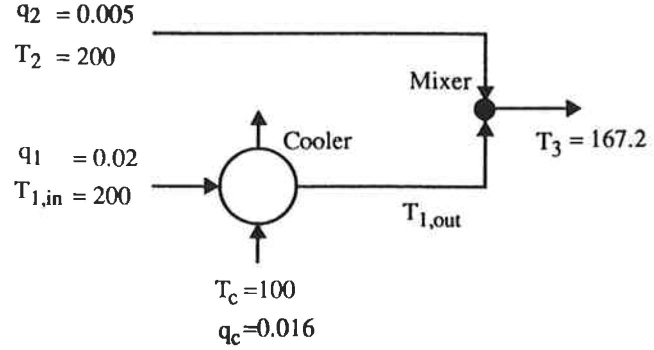

Figure 5: Stream mixing example which displays inverse response

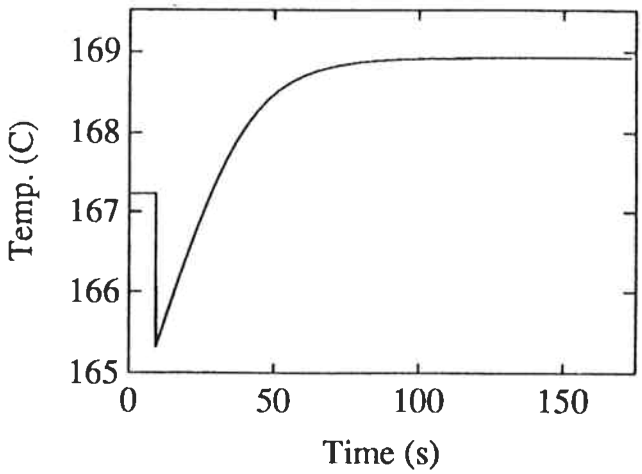

Figure 6: Inverse response due to stream mixing

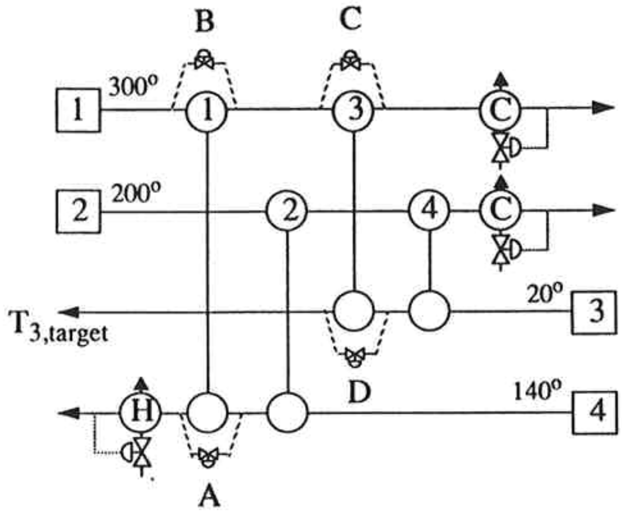

Figure 7: Example network from Kotjabasakis and Linnhoff (1986) used to study effect of bypass location

(see notation in Fig.1). With the assumptions and definitions of τ and α from the previous section and with $\beta_t = \frac{1+\alpha_t}{\tau_t}$ and $\beta_s = \frac{1+\alpha_s}{\tau_s}$. we obtain the following state matrix:

$$A = \begin{pmatrix} -\beta_t & 0 & 0 & \frac{\alpha_t}{\tau_t} & 0 & 0 \\ \frac{1}{\tau_t} & -\beta_t & 0 & 0 & \frac{\alpha_t}{\tau_t} & 0 \\ 0 & \frac{1}{\tau_t} & -\beta_t & 0 & 0 & \frac{\alpha_t}{\tau_t} \\ \frac{\alpha_s}{\tau_s} & 0 & 0 & -\beta_s & \frac{1}{\tau_s} & 0 \\ 0 & \frac{\alpha_s}{\tau_s} & 0 & 0 & -\beta_s & \frac{1}{\tau_s} \\ 0 & 0 & \frac{\alpha_s}{\tau_s} & 0 & 0 & -\beta_s \end{pmatrix}$$

The model is clearly asymptotically stable for any α and τ (use the Gersghorin theorem row-wise for the eigenvalues of A). Complex eigenvalues may be obtained in some cases. The model has no zeroes in the right-half-plane (RHP).

Inverse response caused by mixing. Inverse response may occur for changes in flow rate when two streams are mixed after a heat exchanger, and when the temperature of the combined stream is the output (see Fig. 5). The inverse response is the result of two effects counteracting each other. The immediate effect of a flow increase in stream 1 will be to cool the output stream, since stream 1 is colder than 2. However, after some delay the temperature of stream 1 will increase and make stream 3 warmer. The response for a numerical example is given in fig. 6.

5 Control of Heat exchanger networks

In this paper, we will only consider outlet ("target") temperatures as controlled variables. As mentioned above the most common (and efficient) way to control the outlet temperatures is to install a bypass. Several considerations may determine where to place the bypass; heat exchanger area (it is generally bad to have a bypass on a heat exchanger with a small driving force), material stresses, intermediate target temperatures, special spatial or equipment constraints, cost of piping, etc.. These are all steady-state concerns.

Dynamic considerations when placing bypasses. A bypass is primarily introduced for control reasons, and one should usually assure that the bypass not is located too far away from the controlled variable. Consider the network in Fig.7 (Kotjabasakis and Linnhoff, 1986) where a bypass must be introduced to control temperature $T_{3,target}$. Four possible locations (A, B, C and D) are indicated. Figure 8 shows the Phase-plot of the resulting open-loop transfer function for the four cases.

In practice, the closed-loop bandwidth is usually limited to approximately the frequency where the open-loop phase reaches -180 degrees (-3.14 radians). (In some cases the phase flattens out above -180 de-

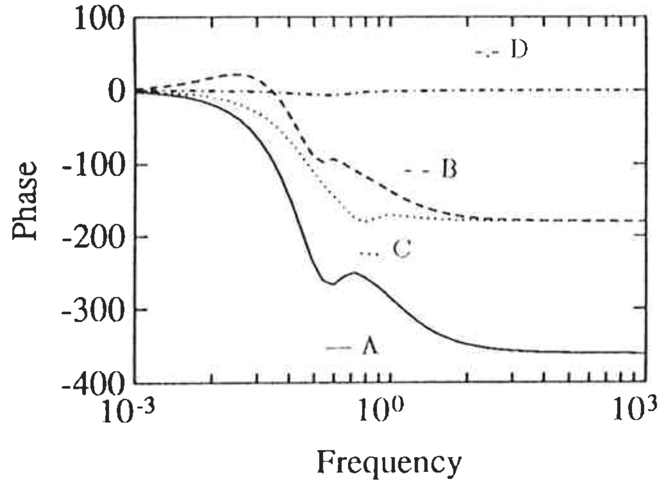

Figure 8: Phase-plot of the 4 alternative choices for bypass location

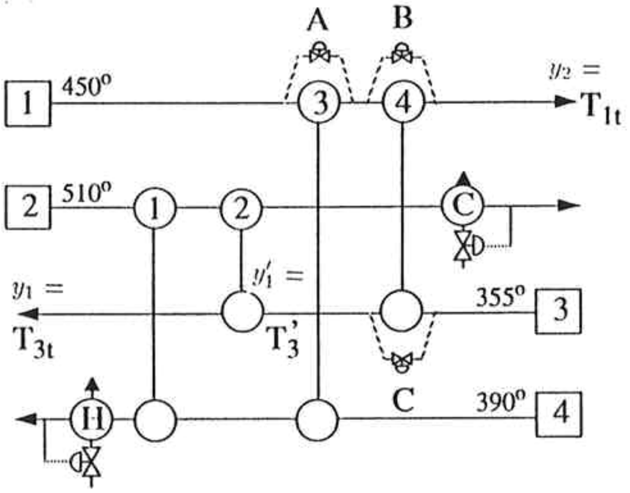

Figure 9: Network from Townsend and Morari (1984) used to study effect of alternative bypass location and pairings

grees, in this case consider the frequency where the asymptote would cross -180 degrees). This frequency will correspond closely to the apparent dead time on the time response. The phase plot indicates that from a feedback control point of view, design D is the best, followed by designs B, C and A. This order correspond to the increasing distance of the bypass from the output. As might be expected this "controllability" selection is different from the "flexibility" selection from Kotjabasakis and Linnhoff (which was shown to be design B).

Interaction and pairing considerations when placing bypasses. When there are several bypasses, the issue of interactions and pairing is also important. Consider the network in Fig. 9 taken from Townsend and Morari (1984) where temperatures T_{1t} and T_{3t} are to be controlled by introducing two bypasses. Possible locations for the bypasses are indicated as A, B and C in the figure.

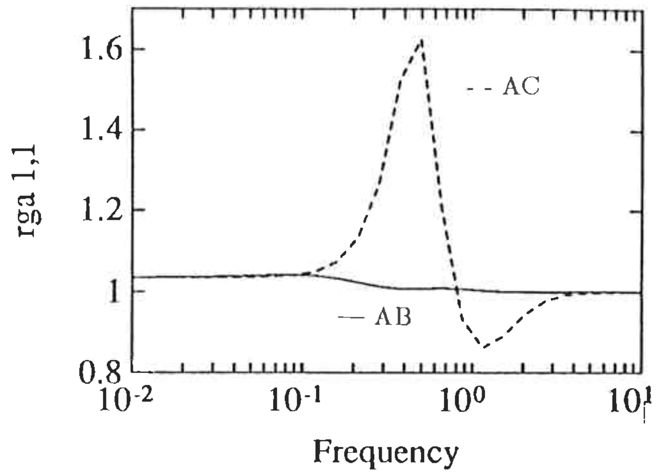

Figure 10: Relative gain array of the alternative pair AB and AC of manipulated variables

Consider the following two combinations: **1)** Case AB where the two manipulators (inputs) are bypass A and B. **2)** Case AC where the inputs are bypass A and C.

In both cases the two outputs are $y_1 = T_{3t}$ and $y_2 = T_{1t}$. The steady-state gain matrices from inputs to outputs are as follows:

$$G^{AB} = \begin{pmatrix} -7.37 & -1.41 \\ 4.02 & 22.98 \end{pmatrix}$$

$$G^{AC} = \begin{pmatrix} -7.30 & -1.44 \\ 3.97 & 23.26 \end{pmatrix}$$

The magnitude of the 1,1-element of the relative gain array (λ) for these two cases is shown as a function of frequency in fig.10. In both cases λ is close to 1 (in the range 0.85 to 1.60) at all frequencies, indicating that the pairing using bypass A to control $y_1 = T_3$ is best in both cases.

However, consider case (AB)' and (AC)', where $y_1' = T_3'$ is the target temperature rather than T_3t. In this case the steady-state gain matrices are

$$G^{AB'} = \begin{pmatrix} 10.15 & -22.97 \\ 4.02 & 22.98 \end{pmatrix}$$

$$G^{AC'} = \begin{pmatrix} 10.20 & -23.26 \\ 3.96 & 23.26 \end{pmatrix}$$

We note the following: **1)** From figure 11 we see that this small change in the control objective has introduced interaction in terms of RGA for both configurations choices. **2)** The gain from bypass A to output 1 has changed sign. The reason is that the response from A to $y_1 = T_{3t}$ has two different paths: one through heat exchanger no. 4 (y_1') with positive gain, and one through heat exchanger no's 1 and 2 with negative gain. The fact that we have two opposing effects

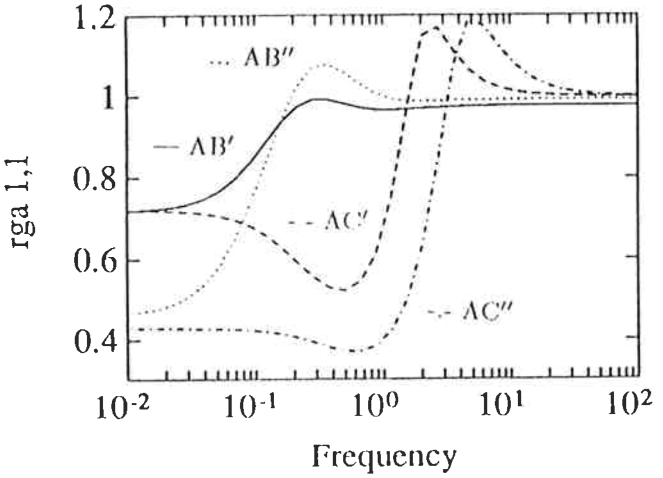

Figure 11: Relative gain array when temperature T_3' is the control variable instead of T_3.

implies that we may get an *inverse* response from bypass A to y_1 in some cases. **3)** λ_{11} is in the range from 0.5-1.2 at all frequencies for case AC'. This means that one should pair output $y_1' = T_3'$ with bypass A This is somewhat surprising since bypass C has a direct effect om T_3'. However, bypass A has a relatively "slow" initial (high frequency) effect on T_{1t}, and this makes it undesirable to pair these variables.

Also, varying flow conditions may change the scene. Increasing the flow of a stream may give even more serious interactions. This is shown in case (AB)'' and (AC)'', where stream 1 is increased with 100 % and $Y_1 = T_3'$.

Placing of heaters and coolers. The utility exchangers are usually placed based on optimal steady-state considerations in terms of heat recovery and total exchanger area. However, the presence of utility heaters and coolers also have a large effect on the controllability. Clearly, control is very simple if a utility exchanger is used to control directly the target temperature. Therefore, from a control point of view, one should use utility exchangers on the most critical target temperatures, or on streams where large disturbances are expected.

Disturbances. Disturbance rejection is usually the primary reason for using a feedback control system, and these should obviously be taken into account when designing the control structure for the network. Clearly, disturbances entering close to the controlled variable are usually most critical.

Note that the frequency-dependent open-loop disturbance gain matrix (G_d) represents an extension of the structural information presented in the downstream-path approach (at steady state) of Linnhoff and Kotjabasakis (1986) and the structural relative order of Daoutidis and Kravaris (1991) (at high frequency).

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