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DB-Control of Distillation Columns

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Abstract

It is often claimed that for distillation columns the steady-state description is much more important than the dynamic description for control purposes. The ultimate counterexample to this misconseption is the DB-configuration which was recently proposed by Finco et al. (1989) and which involves using distillate and bottom flow to control compositions. This control scheme has previously been labeled 'impossible' by most distillation control experts because D and B are not independent at steady state (since $D+B=F$) and the gain matrix is singular. Yet, as shown by Finco et al. for a propane propylene splitter, both with simulations and with implementation , the scheme does actually work. Finco et al. do not provide any explanations for this, but as shown in this paper the main reason is the flow dynamics (liquid lag from the top to the bottom of the column) which makes the model at high frequency quite easy to control.

The results in the paper demonstrate that steady-state data may be entirely misleading for evaluating control performance. This is of course well-known, for example from the Ziegler-Nichols tuning rules which are based on high-frequency behavior only, but often seems to be forgotten when analyzing multivariable systems.

1 Introduction

The most important decision when designing distillation control systems is the selection of an appropriate control configuration. Finco et al. (1989) have recently proposed the DB-configuration which involves using distillate and bottom flow to control compositions as a viable control scheme. An example of such a control scheme is shown in Fig.1. This control scheme has previously been labeled "impossible" or "nonoperable" by most distillation control experts (eg., Perry (1973, p. 22-123), Shinskey (1984, p. 154), Skogestad and Morari (1987c), Häggblom and Waller (1988)). In particular, the relative gain array (RGA) is infinite at steady state. The reason is that at steady state we must have $D+B=F$ and D and B are not independent which makes the gain matrix singular. However, as shown by Finco et al. for a propane-propylene splitter, both with simulations and with implementation , the scheme does actually work.

Imagine there is an increase in mole fraction of light component in the feed, z_F, to the column. To keep the top and bottom compositions constant we know that the control system should increase D/F somewhat to maintain $Fz_F = Dy_D + Bx_B$, that is, $D/F = (z_F - x_B)/(y_D - x_B)$, and also adjust the internal flows (L and V) somewhat to maintain the overall separation. Assume the DB-configuration is used for control. Initially following the increase in z_F all flows are constant, but the bottom and top mole fractions, x_B and y_D, will start increasing slowly. As a response to the improved purity in the top the control system will increase D, and as a response to the more impure bottom product it will decrease B. This is exactly what the control system is supposed to do. But is the control system able to adjust the internal flows correctly? The answer is "yes"; the level control system will adjust L and V in response to changes in D and B made by the composition control system. Note that there are only two degrees of freedom at steady state. Consequently, if y_D and x_B are specified then there is only one possible solution for the flows D, B, L and V, and since the composition control system will keep changing D and B until the specifications are met, also the internal flows will be adjusted correctly.

However, the situation is actually not quite as simple as described above because the DB-configuration would *not* work if there were no flow dynamics and the levels were controlled perfectly. The reason is that in this case the constraint $D + B = F$ would also apply dynamically and D and B could not be adjusted independently. The DB-configuration therefore works only because it is possible dynamically to accumulate mass in the system which makes D and B independent variables from a dynamic point of view. There are four possible sources of accumulation of mass: reboiler holdup, condenser holdup, vapor holdup (pressure variations) and liquid holdup on the trays. Let $dL_T \equiv dL$ and dV_T represent small changes in the liquid and vapor flows in the top of the column, and let dL_B and $dV_B \equiv dV$ be changes in the bottom. The following relations between the flows apply dynamically (see Fig. 1)

$$dL_T = c_D(s)(dV_T - dD) \tag{1}$$

$$dV_B = c_B(s)(dL_B - dB) \tag{2}$$

$$dV_T = c_p(s)dV_B \tag{3}$$

$$dL_B = g_L(s)dL_T \tag{4}$$

The last to equations assume constant molar flows and neglect dynamic interactions beween liquid and vapor flows, that is, vapor flow has no effect on liquid holdups. $c_D(s)$ and $c_B(s)$ are given by the level control systems, and $c_p(s)$ is primarily given by the pressure control system. The liquid lag from top to bottom of the column, $g_L(s)$, is a self-regulating effect given by the tray hydraulics. The overall liquid lag from the top to the bottom is assumed to be given by $\theta_L = N\tau_L$ where $\tau_L = (\partial\, M_i/\partial L_i)_V$ is the hydraulic lag on each theoretical tray. With N trays in series we get

$$g_L(s) = 1/(1 + (\theta_L/N)s)^N \tag{5}$$

In this paper we shall use the above equations and furthermore assume $c_D(s) = c_B(s) = c_p(s) = 1$, that is, assume immediate control of levels and pressure. The effect of not assuming perfect level and pressure control is briefly discussed towards the end.

The objective of this paper is to show why and for what columns the DB-configuration works. We shall compare the DB-scheme with the more conventional LV-configuration shown in Fig.2.

Example column. As an example column we shall use column D studied by Skogestad and Morari (1988a). This is a propane-propylene splitter similar to the one used by Finco et al. (1989), but we have assumed the relative volatility α to be constant throughout the column. Columnn data are given in Table 1. Constant molar flows are assumed. τ_L is obtained by linearizing the Francis weir formula. Assuming in addition that half of the liquid is above the weir we get (Skogestad and Morari, 1987a)

$$\theta_L = N\tau_L = N0.33\frac{M_i/F}{L/F} \tag{6}$$

and we obtain for our example column $\tau_L = 0.84$ sec and $\theta_L = 1.54$ min. This is 20 times lower than the approx. 30 min used by Finco et al. Industrial C3-splitters we have studied have had $\theta_L \approx 4$ min. Note that a change in holdup will simply scale all time constants, and these differences therefore do not affect the conclusions in this paper. In a practical situation it may be advantageous with larger holdups - at least for disturbance rejection - because the effect of analyzer dead times is then less important.

2 Modelling

Detailed modelling of the column including flow dynamics yields a set of differential equations with two states for each tray. Typically these states are chosen to be the mole fraction of one component and the holdup. Because of the large number of trays the full order model for the example column has 222 states. This makes numerical calculations extremely time consuming. Because of this and to make the exposition clearer we choose to use the simple two time-constant model presented by Skogestad and Morari (1988a). This model was derived assuming the flow and compostion dynamics to be decoupled. In reality, the flow dynamics do affect the composition dynamics and the model may be somewhat in error. In particular, the time constant τ_2 may be larger than that given by Skogestad and Morari (1988a). However, as we shall see the final results derived in this paper do not depend on τ_2.

LV-configuration. The simple two time-constant model of the LV configuration including liquid flow dynamics becomes

$$G^{LV}(s) = \begin{pmatrix} \frac{g_{11}^{LV}}{1+\tau_1 s} & \left(\frac{g_{11}^{LV}+g_{12}^{LV}}{1+\tau_2 s} - \frac{g_{11}^{LV}}{1+\tau_1 s}\right) \\ \frac{g_{21}^{LV}}{1+\tau_1 s} g_L(s) & \left(\frac{g_{21}^{LV}+g_{22}^{LV}}{1+\tau_2 s} - \frac{g_{21}^{LV}}{1+\tau_1 s}\right) \end{pmatrix} \tag{7}$$

where g_{ij}^{LV} denote the steady state gains for the LV-configuration.

Models for other configurations. Models for other configurations are easily obtained from eq. (7) by choosing two independent variables u_1 and u_2 for composition control and expressing dL and dV in terms of these variables using equations (1) to (4). This gives

$$\begin{pmatrix} dL \\ dV \end{pmatrix} = M_{LV}^{u_1 u_2}(s) \begin{pmatrix} du_1 \\ du_2 \end{pmatrix} \tag{8}$$

and the linear model for the new configuration

$$\begin{pmatrix} dy_D \\ dx_B \end{pmatrix} = G^{u_1 u_2}(s) \begin{pmatrix} du_1 \\ du_2 \end{pmatrix} \tag{9}$$

becomes

$$G^{u_1 u_2}(s) = G^{LV}(s) M_{LV}^{u_1 u_2}(s) \tag{10}$$

DB-configuration. With the assumption $c_D = c_B = c_p = 1$ we obtain $dV = g_L(s)dL - dB$ where $dL = dV - dD$. From these two equations we obtain

$$M_{LV}^{DB}(s) = \frac{1}{1 - g_L(s)} \begin{pmatrix} -1 & -1 \\ -g_L(s) & -1 \end{pmatrix} \tag{11}$$

The simplified model of the DB-configuration then becomes

$$G^{DB}(s) = -\begin{pmatrix} \frac{g_{11}^{LV}}{1+\tau_1 s} + \frac{g_L(s)}{1-g_L(s)}\frac{g_{11}^{LV}+g_{12}^{LV}}{1+\tau_2 s} & \frac{1}{1-g_L(s)}\frac{g_{11}^{LV}+g_{12}^{LV}}{1+\tau_2 s} \\ \frac{g_L(s)}{1-g_L(s)}\frac{g_{21}^{LV}+g_{22}^{LV}}{1+\tau_2 s} & \frac{g_{21}^{LV}}{1+\tau_1 s} + \frac{1}{1-g_L(s)}\frac{g_{21}^{LV}+g_{22}^{LV}}{1+\tau_2 s} \end{pmatrix} \quad (12)$$

We have written the model in terms of the steady-state gains of the LV-configuration because the steady-state gain elements for the DB-configuration are infinite in magnitude. This is seen from the low- frequency approximations

$$\lim_{s\to 0} g_L(s) = 1 \quad (13)$$

and

$$\frac{1}{1-g_L(s)} \to \frac{1}{N\tau_L s} \to \frac{1}{\theta_L s} \quad \text{for} \quad s \to 0 \quad (14)$$

The physical interpretation is that a decrease in, for example, D with B constant, will immediately yield a corresponding increase in L. This increase will yield a corresponding increase in V, which subsequently will increase L even more, etc. Consequently, the effect is that the internal flows eventually will approach infinity. Mathematically, there is an integrator at low frequency: As $s \to 0$ we have $dL = dV = -(dD + dB)/\theta_L s$.

Also note that the steady state gain matrix of the DB-configuration is singular. One might expect that this implies control problems, but this turns out not to be the case.

Example column. Data for the simple models described above are given in Table 2. The gain data in Table 2 are for scaled (logarithmic) compositions

$$\begin{aligned} \Delta y_D^S &= \Delta y_D/(1-y_D) = \Delta y_D/0.005 \\ \Delta x_B^S &= \Delta x_B/x_B = \Delta x_B/0.10 \end{aligned} \quad (15)$$

The flow dynamics are given by

$$g_L(s) = 1/(1+(1.54/N)s)^N \quad (16)$$

To reduce the order we used $N = 15$ instead of the actual $N = 110$. The magnitude of the transfer matrix elements in LV- and DB-configurations are plotted as a function of frequency in Fig.3. Note that the flow dynamics make the plant models triangular at high frequency.

3 Analysis of the model

Singular values and Relative Gain Array

Singular values and the 1,1-element of the RGA (simply denoted λ in the following) for the simplified model are shown as a function of frequency in Fig.4 and Fig.5. Note the singular values for the DB-configuration. As expected, at low frequency the maximum singular value (maximum gain corresponding to change in internal flows) behaves like an integrator and approaches infinity. It may seem somewhat puzzling that the gain matrix of the DB-configuration approaches singularity at low frequencies even though the minimum singular value is non-zero at steady state. However, note that the maximum singular value approaches infinity such that the ratio between the two singular values is infinity.

The RGA is plotted because it is known to be a good indicator of how easy it is to control the plant. In particular, a plant is difficult to control if the RGA-elements are large at frequencies corresponding to the desired closed-loop bandwidth (Skogestad and Morari, 1987b). For our example a reasonable closed-loop bandwidth is about 0.1 min^{-1}.

The RGA element λ for the LV-configuration starts out at a steady state value of 59, and then falls off with a -1-slope on the log-log-plot and crosses one at the frequency $\omega_1 \approx 1/\theta_L = 0.6\ \text{min}^{-1}$. λ for the DB-configuration is infinite at steady state. It falls off with a -1 slope and the low-frequency asymptote crosses one at $\omega_1 \approx 0.001\ \text{min}^{-1}$. Thus, although the RGA for the DB-configuration is much worse (higher) than for the LV-configuration at low frequency, it is significantly better (closer to one) in the frequency range important for feedback control, that is, in the frequency range from about 0.01 to 1 min^{-1}. The observation of Finco et al. (1989) that the DB-configuration gave better control performance than the LV-configuration is therefore not surprising from the RGA-values. In fact, from the RGA-values the column seems to be quite simple to control using the DB-configuration.

Analytical treatment

The 1,1-element of the RGA is defined as

$$\lambda(s) = \left(1 - \frac{g_{12}(s)g_{21}(s)}{g_{11}(s)g_{22}(s)}\right)^{-1} \tag{17}$$

As one measure of how easy a column is to control we shall consider the frequency, ω_1, where $|\lambda|$ approaches one (Skogestad and Morari, 1987b).

LV-configuration. For the LV-configuration it is easily shown that this is at about

$$\omega_1^{LV} = 1/\theta_L \tag{18}$$

for all columns, see Skogestad et al. (1989). This is not surprising since $1/\theta_L$ is the frequency at which the system becomes decoupled (recall Fig. 3).

DB-configuration. From Fig. 5 it seems that ω_1^{DB} may be obtained as the frequency where the low-frequency asymptote of $|\lambda^{DB}(\omega)|$ crosses one. The same holds for a number of other example columns (Skogestad et al., 1989). Consider low frequencies where $\frac{1+\tau_2 s}{1+\tau_1 s} \approx 1$ and derive from eq. (12)

$$\lambda^{DB}(s) = \frac{[g_L(s)g_{12}^{DV} - (1-g_L(s))g_{11}^{DV}][g_{22}^{DV} + (1-g_L(s))g_{21}^{DV}]}{(1-g_L(s))[g_L(s)g_{12}^{DV}g_{21}^{DV} - g_{11}^{DV}(g_{22}^{DV} + (1-g_L(s))g_{21}^{DV})]} \tag{19}$$

Here we have introduced the steady state gains g_{ij}^{DV} for the DV- configuration: For constant molar flows and perfect control of condenser level $dL = dV - dD$ and we have

$g_{11}^{DV} = -g_{11}^{LV}$; $g_{12}^{DV} = g_{11}^{LV} + g_{12}^{LV}$; $g_{21}^{DV} = -g_{21}^{LV}$; $g_{22}^{DV} = g_{21}^{LV} + g_{22}^{LV}$. The low frequency asymptote ($s \to 0$) of eq. 19 is

$$\lambda^{DB}(s \to 0) = \lambda^{DV}(0) \frac{-g_{12}^{DV}}{g_{11}^{DV}} \frac{1}{\theta_L s} \tag{20}$$

Here we have for constant molar flows and for binary columns with reasonably high purity (Skogestad and Morari, 1987a)

$$\lambda^{DV}(0) \approx 1/(1 + \frac{Bx_B}{D(1-y_D)}) \tag{21}$$

$$-\frac{g_{12}^{DV}}{g_{11}^{DV}} = -\frac{(\partial y_D/\partial V)_D}{(\partial y_D/\partial D)_V} \approx Bx_B \left(\frac{\partial \ln S}{\partial V}\right)_D \tag{22}$$

where S is the separation factor. This yields the following approximation:

$$\omega_1^{DB} = \frac{1}{1/Bx_B + 1/D(1-y_D)} \left(\frac{\partial \ln S}{\partial V}\right)_D \frac{1}{\theta_L} \tag{23}$$

To evaluate this expression further we need to know how the separation factor depends on changes in internal flows. The crude column model

$$S = \alpha^N \frac{(L/V)_T^{N_T}}{(L/V)_B^{N_B}} \tag{24}$$

with $N_T = N_B = N/2$ yields for liquid feeds (Skogestad and Morari, 1987a)

$$\left(\frac{\partial \ln S}{\partial V}\right)_D \approx \frac{N}{2L(L/F+1)} \tag{25}$$

Skogestad and Morari (1987a) found eq. (25) to be reasonably accurate for most columns. For well-designed columns with liquid feed and relative volatility α less than about 2 we have (Skogestad and Morari, 1988a)

$$\frac{N}{2} \approx N_{min} = \frac{\ln S}{\ln \alpha} \approx \frac{\ln S}{\alpha - 1} \approx \ln S(L/F) \tag{26}$$

where lnS is in the range 4 to 10 for most columns. Substituting into (23) yields

$$\omega_1^{DB} \approx \frac{1}{1/Bx_B + 1/D(1-y_D)} \frac{\ln S}{L_B} \cdot \omega_1^{LV} \tag{27}$$

The term multiplying ω_1^{LV} is much less than one for high-purity columns and for columns with large reflux. Thus, although the RGA for the DB- configuration is much worse (higher) than for the LV-configuration at low frequency, it is significantly better (closer to one) in the frequency range important for feedback control, that is, in the frequency range from about 0.01 to 1 min^{-1}.

τ_1 is the dominant time constant for the composition dynamics, and it is of interest to find a relationship between ω_1^{DB} and $1/\tau_1$. Using the approximation (Skogestad and Morari, 1988a)

$$\tau_1 = \frac{NM_i}{\ln S \quad (D(1-y_D) + Bx_B)} \tag{28}$$

and θ_L from eq. (6), we derive from eq. (27)

$$\omega_1^{DB} \approx \frac{1}{r}\frac{1}{\tau_1} \leq \frac{1}{\tau_1} \tag{29}$$

where $r = 0.33(2 + \frac{Bx_B}{D(1-y_D)} + \frac{D(1-y_D)}{Bx_B})$ is always larger than 1. Since τ_1 is often very large, this confirms that the DB- configuration becomes decoupled and easy to control at a very low frequency.

Example column. For the example column eq. (25) gives $(\partial \ln S/\partial V)_D = 0.46$ (the exact value is 0.36) and we derive from eq. (23) that $\omega_1^{DB} = 0.0028 \cdot 0.46/\theta_L = 0.0013/\theta_L = 0.0009 \text{ min}^{-1}$. This is very close to the observed value of 0.001 min^{-1} in Fig. 5. The observed ratio between $1/\tau_1$ and ω_1^{DB} is $1/154 \cdot 0.001 = 6.5$. This compares nicely with the estimated value $r = 4.9$.

Disturbance rejection.

The main reason for applying feedback to distillation columns is to counteract the effect of disturbances. The RGA says nothing about disturbances. The most important disturbances are feed composition and rate, reflux and boilup.

To evaluate the effect of disturbances consider the "open-loop" model, that is, with the two variables for composition control kept constant. For disturbances in reflux and boilup the effect on compositions is zero for the DB-configuration if level control is assumed perfect. For the LV-configuration it is given by the open-loop gains in Fig. 3. For disturbances in feed composition there is no difference between the configurations if we assume constant molar flows such that compositions do not affect flows. For disturbances in feed rate the DB-configuration is more sensitive than the LV- configuration, at least at low frequency. In Fig. 6 the gain for the effect of a feed flow disturbance on compositions is diplayed as a function of frequency. At steady state the effect of this disturbance on compositions is infinite for the DB-configuration. However, the difference between the LV- and DB-configuration is quite small at high frequencies. For comparison Fig. 6 also shows the feed gain for the (L/D)(V/B)-configuration. Even though this configuration is insensitive at steady state, the initial response is similar to that of the other configurations.

In summary, the DB-configuration is also better than the LV- configuration when disturbances are taken into account (because it is insensitive to disturbances in reflux and boilup). However, it is probably not quite as good as the (L/D)(V/B)-configuration.

Simulations.

Simulations of the LV- and DB-configurations with a full-order nonlinear model with 222 states are shown in Fig. 7. The controllers were obtained based on the simplified models and the PI-parameters were tuned to optimize robust performance (optimize worst-case response with model error) with the same weights as used by Skogestad and

Morari (1988b). The tuning parameters are given in Table 3. The simulation results are similar to the ones obtained by Finco et al. and confirm that the DB-configuration is better for two-point control than the LV-configuration.

4 Discussion

The results in this paper are based on a simplified linear model. However, other results (Skogestad et al., 1989) demonstrate that the use of a full-order linear model (with $2N$ states) yield very similar results. The results were also derived assuming perfect control of level and pressure. With regards to interactions as measured in terms of the RGA, these will become less for the DB-configuration when levels and pressure are not perfectly controlled. The reason is that the time for the mass to recycle within the column is increased and D and B become more independent. With perfect level control this recycle time is about θ_L. Assume the transfer functions for pressure and level control are given by

$$c_D(s) = \frac{1}{1+\tau_D s}; \quad c_B(s) = \frac{1}{1+\tau_B s}; \quad c_p(s) = \frac{1}{1+\tau_p s} \tag{30}$$

(These simple transfer functions will be obtained with pure proportional controllers). In this case the total time for mass to recycle is about $\theta = \theta_L + \tau_D + \tau_B + \tau_p$, and it is probably not too surprising that the results in terms of the low-frequency asymptotes of the RGA for the DB-configuration given above, eg. Eq. 23, are unchanged but with θ_L replaced by θ. Note that although the DB-configuration is improved with respect to interactions by introducing lags τ_D and τ_B, the response to disturbances may be worse because changes in D and B will not immediately affect L abd V. For the LV-configuration the introduction of lags τ_D and τ_B will have almost no effect on neither RGA nor disturbance rejection.

The DB-configuration has previously been labelled "impossible" because the inputs D and B are not independent at steady state. This conclusion is partly correct for one-point composition control, but we have shown that for two-point control the DB-configuration is actually quite easy to control at high frequency, and will perform better than the LV-configuration for most columns, in particular for columns with high purity products and/or large reflux.

However, as pointed out by Finco et al. (1989) the DB-configuration lacks integrety, as it will not work satisfactory if one of the inputs D or B for some reason is no longer used for feedback control. The reason is that this will lock the product split D/B and good control of either composition is impossible. Since most industrial columns are operated with one-point composition control (that is, one input in manual), this is probably the reason why the DB-configuration has not been applied in industry, and it is also the reason why industrial practicioners have advised strongly against it (eg., McCune and Gallier, 1973). For two-point control the integrety problems may be handled by use of some override control scheme (Finco et al., 1989).

The analysis of the DB-configuration demonstrates that steady state data may be entirely misleading for the evaluation of control performance. For feedback control, it is the initial response, corresponding to the time constant of the closed-loop system, which is of primary importance. This is of course well-known, for example from the Ziegler-Nichols tuning rules which are based on high-frequency behavior only, but it often seems to be forgotten when analyzing multivariable systems. The analysis in this paper provides further evidence that the RGA in the crossover frequency range is an excellent indicator of potential control difficulties while its steady state value is not informative (Skogestad and Morari, 1987b).

It is of course possible in some cases to make conclusions about control behavior from steady state data. For example, integral control gives instability if the determinant of the steady state gain matrix changes sign during operation (Grosdidier et al., 1985). This is a generalization of the requirement of negative feedback for stability of single-loop systems under integral control. The DB-configuration has a singular gain matrix at steady state, but a sign change is not possible and therefore integral control is feasible.

In the paper we have considered composition control only. However, an important practical consideration is often level control. For columns with large reflux, good level control is probably the most important reason for not using the LV-configuration. It is almost impossible to control the level with the small product streams. In such cases level control is much better with the DB- or (L/D)(V/B)-configurations.

The main alternative to the DB-configuration is probably the (L/D)(V/B)-configuration which, in addition, is good for one-point control. However, it requires all flows L,D,V and B to be measured or estimated which makes implementation more difficult. The main advantage of the DB configuration is that it is very easy to implement. This advantage has to be weighed against its lack of integrety.

5 Conclusion

As one measure of how easy a column is to control we considered the frequency, ω_1, where the 1,1-element of the RGA, λ, approaches one. For the LV-configuration this is at about $\omega_1^{LV} = 1/\theta_L$ where θ_L is the overall liquid lag from the condenser to the reboiler. For the DB-configuration the RGA is *infinite* at low frequency, but decreases with a -1 slope on a log-log plot. Analytical approximations for the case with perfect level and pressure control yield $\omega_1^{DB} \approx \frac{1}{1/Bx_B + 1/D(1-y_D)} \frac{\ln S}{L_B} \cdot \omega_1^{LV}$ where S is the separation factor. The term multiplying ω_1^{LV} is much less than one for high-purity columns and for columns with large reflux. Thus, although the RGA for the DB- configuration is much worse (higher) than for the LV-configuration at low frequency, it is significantly better (closer to one) in the frequency range important for feedback control, that is, in the frequency range from about 0.01 to 1 min^{-1}.

The control behavior of the DB-configuration is therefore an execellent example of how misleading steady-state arguments may be for evaluating control performance. For feedback control the plant behavior in the frequency region corresponding to the closed-loop bandwidth is of primary importance, and the steady state is useful only if it reflects this behavior.

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NOMENCLATURE (also see Fig. 1) .

g_{ij} - steady state gains for column
$L = L_T$ - reflux flow rate (kmol/min)
L_B - liquid flow rate into reboiler (kmol/min)
M_i - liquid holdup on theoretical tray no. i (kmol)
N - no. of theoretical trays in column
RGA - Relative Gain Array, elements are λ_{ij}
$S = \frac{y_D(1-x_B)}{(1-y_D)x_B}$ - separation factor
$V = V_B$ - boilup from reboiler (kmol/min)
V_T - vapor flow rate on top tray(kmol/min)
x_B - mole fraction of light component in bottom product
y_D - mole fraction of light component in distillate (top product)
z_F - mole fraction of light component in feed

Greek symbols

$\alpha = \frac{y_i/x_i}{(1-y_i)/(1-x_i)}$ - relative volatility
$\lambda_{11}(s) = (1 - \frac{g_{12}(s)g_{21}(s)}{g_{11}(s)g_{22}(s)})^{-1}$ - 1,1-element in RGA.
ω - frequency ($\min^{-1}$)
$\bar{\sigma}(G), \underline{\sigma}(G)$ - maximum and minimum singular values
τ_1 - dominant time constant for external flows (min)
τ_2 - time consant for internal flows (min)
$\tau_L = (\partial M_i/\partial L)_V$ - hydraulic time constant (min)
$\theta_L = N\tau_L$ - overall lag for liquid response (min)

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Table 1. Data for distillation column example.

z_F	α	N	N_F	$1-y_D$	x_B	D/F	L/F
0.65	1.12	110	39	0.005	0.10	0.614	11.862

Feed is liquid.
Constant molar flows.
Holdup on each theoretical tray; $M_i/F = 0.5$ min

Table 2. Data used in simple model of distillation column.

Steady state gains (scaled compositions) (eq. 15):

$$G^{DV}(0) = \begin{pmatrix} -24.585 & 0.385 \\ -21.270 & -0.030 \end{pmatrix}$$

$$G^{LV}(0) = \begin{pmatrix} 24.585 & -24.2 \\ 21.270 & -21.3 \end{pmatrix}$$

Time constants: τ_1=154 min; τ_2=30 min; θ_L=1.54 min.

Table 3. μ-optimal PI-settings; $C(s) = k\frac{1+\tau_I s}{\tau_I s}$. Gains k_y and k_x are for scaled compositions (eq.15).

Configuration	μ_{RP}	τ_{Iy} min	τ_{Ix} min	k_y	k_x
LV	1.090	15.1	1.46	3.96	0.47
DB	0.744	40.0	24.5	1.61	1.26

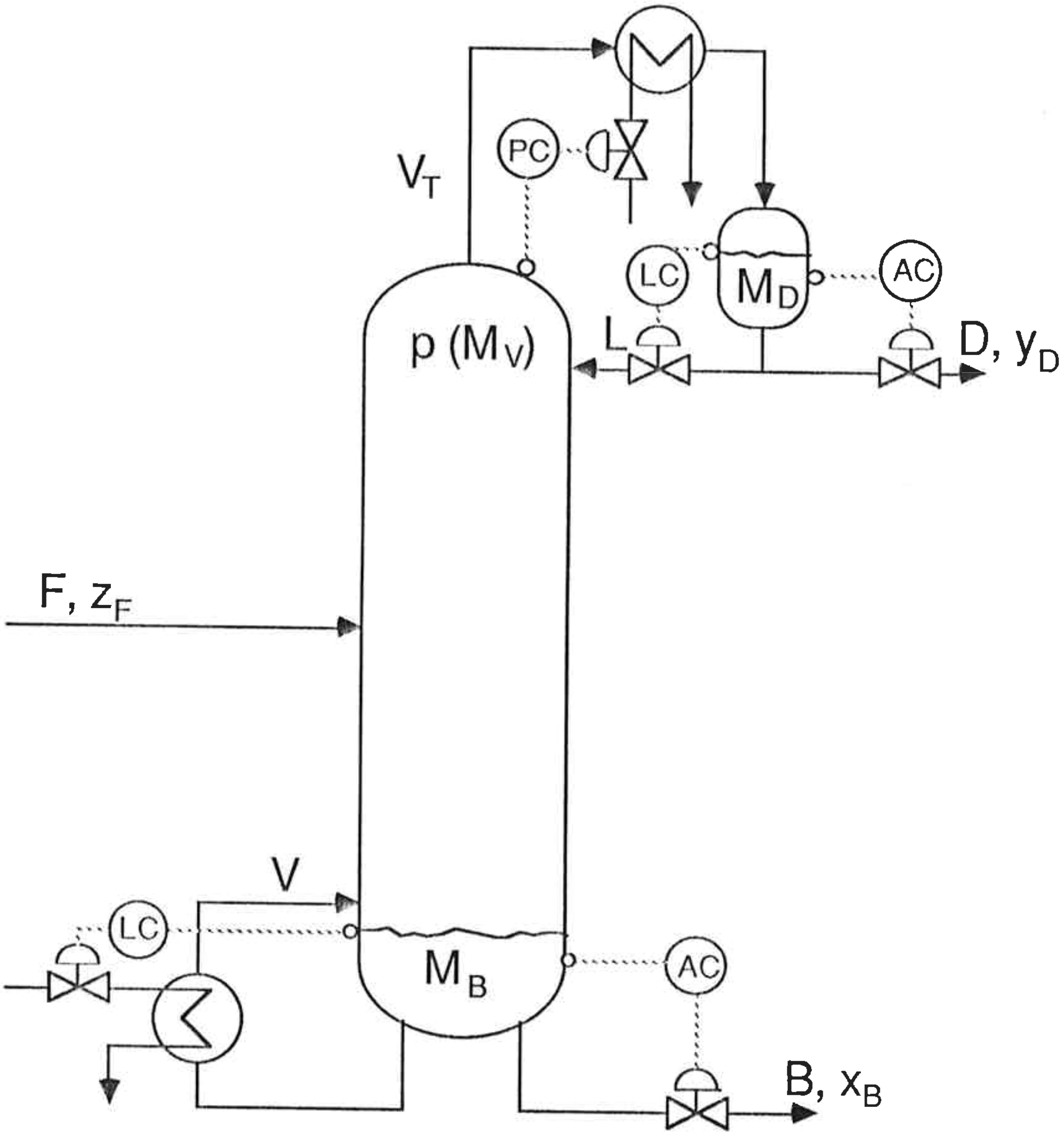

Figure 1. Two product distillation column with DB-configuration.

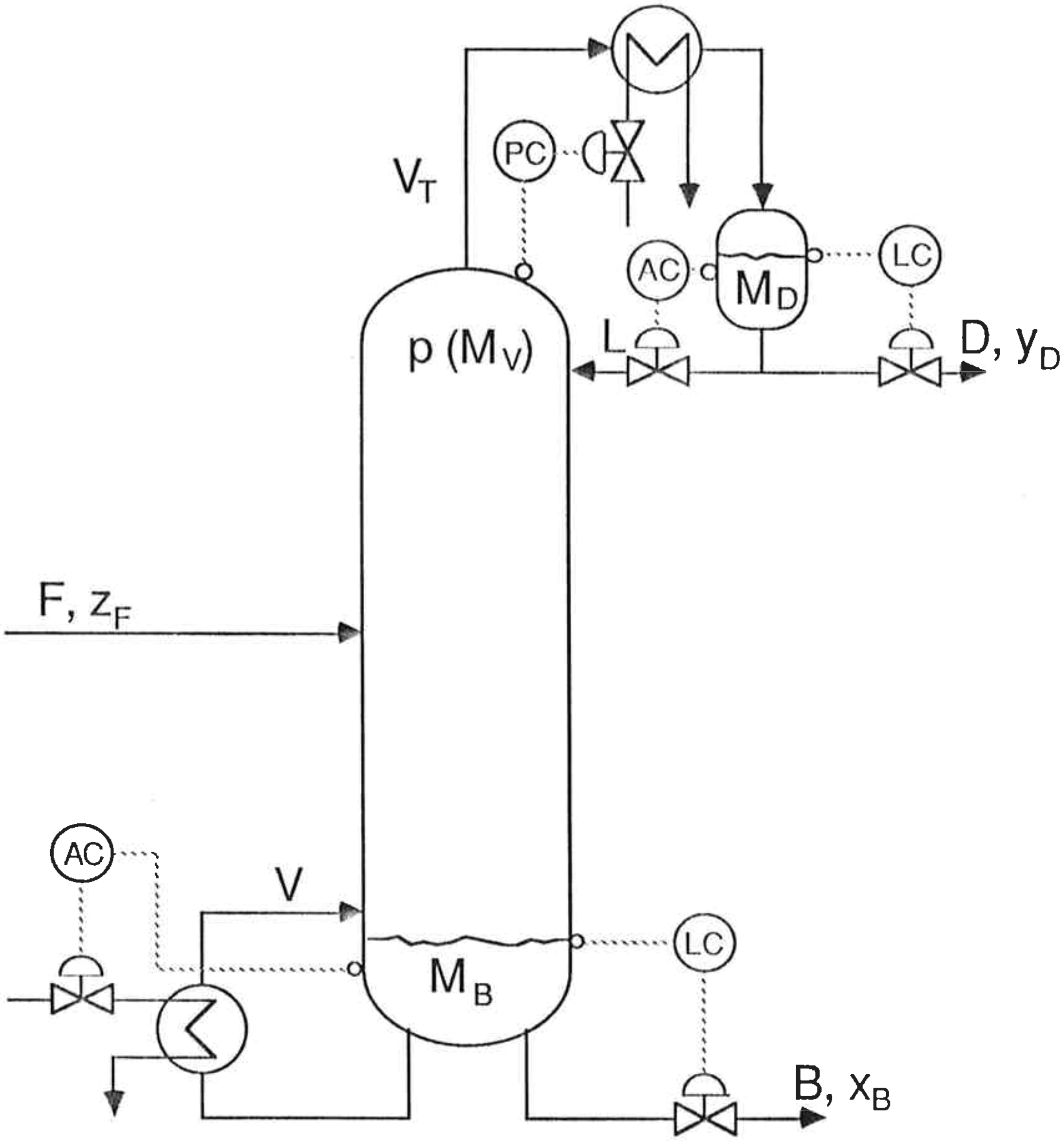

Figure 2. Two product distillation column with LV-configuration.

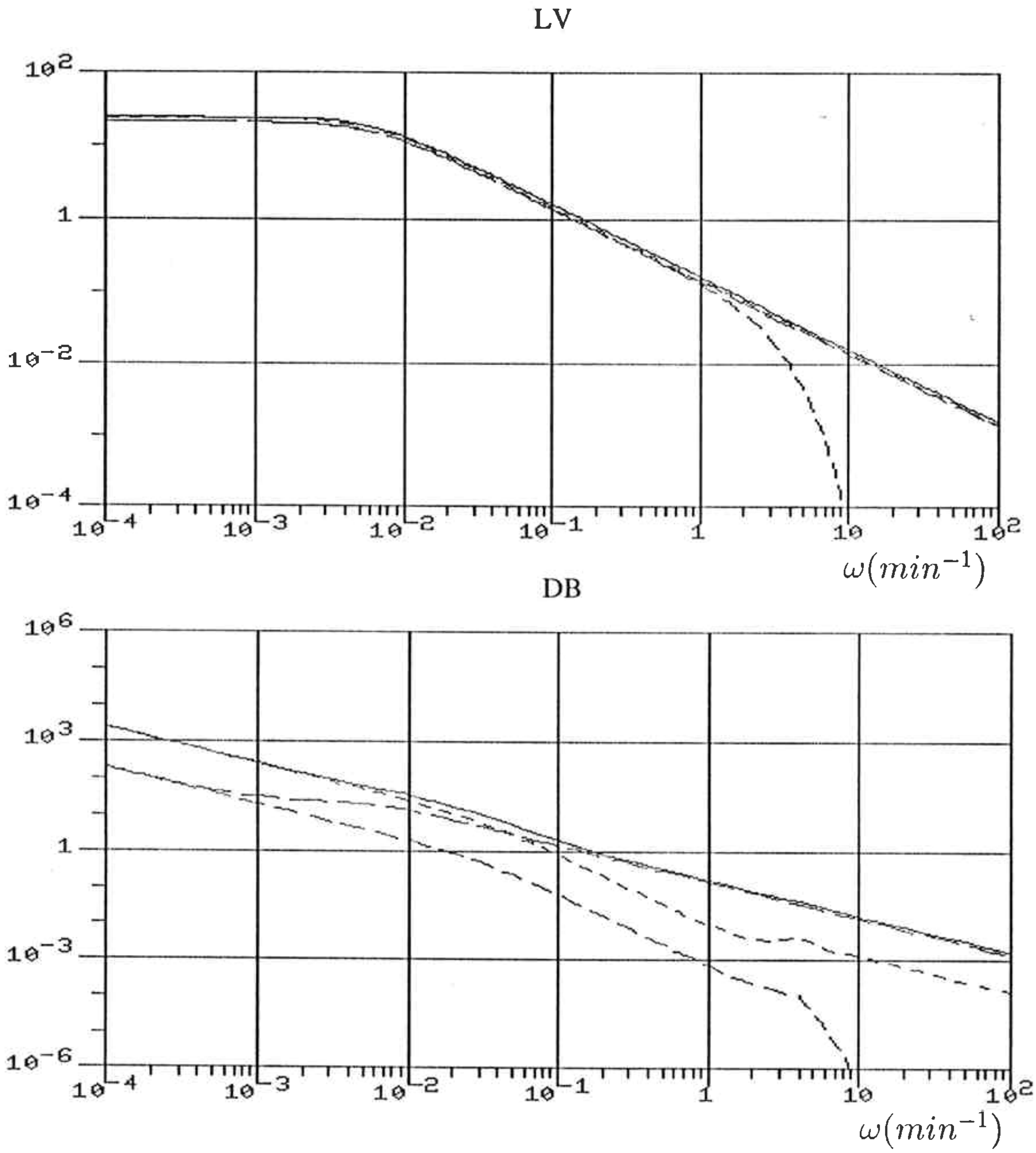

Figure 3. Gain elements as a function of frequency for LV- and DB-configuration.

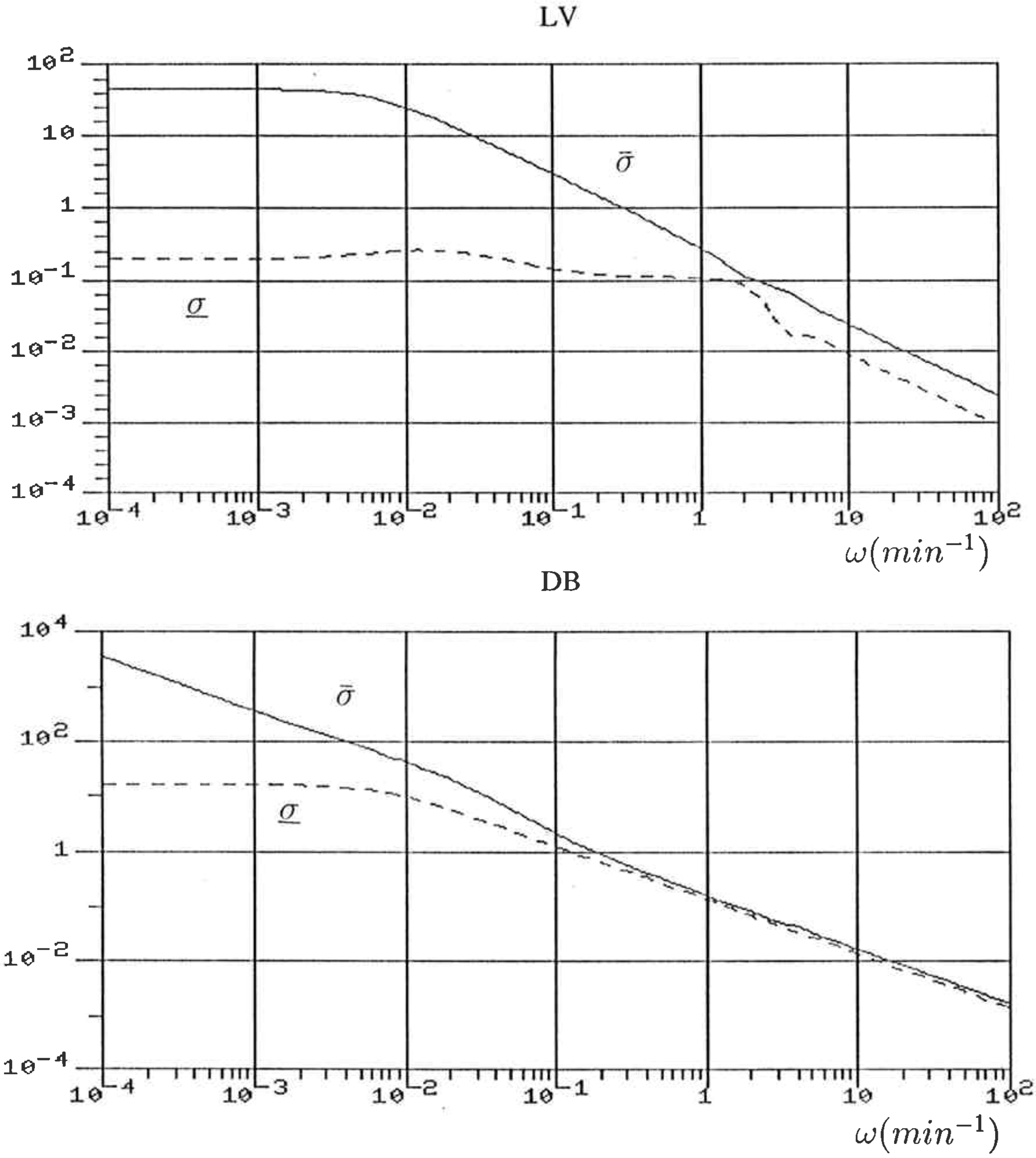

Figure 4. Singular values as a function of frequency for LV- and DB-configuration.

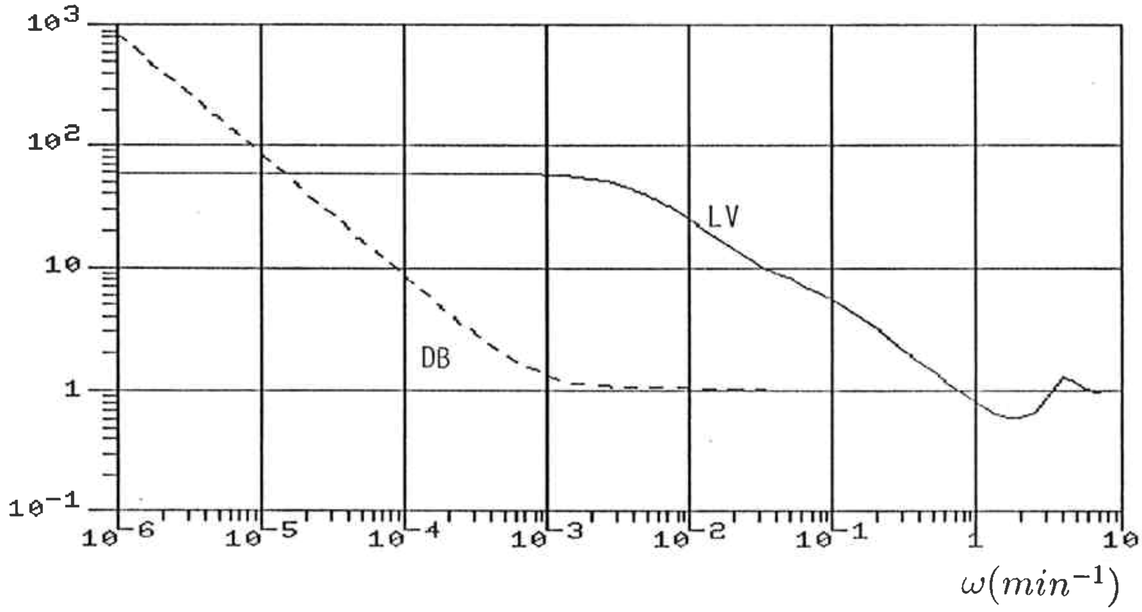

Figure 5. λ as a function of frequency for LV- and DB-configuration.

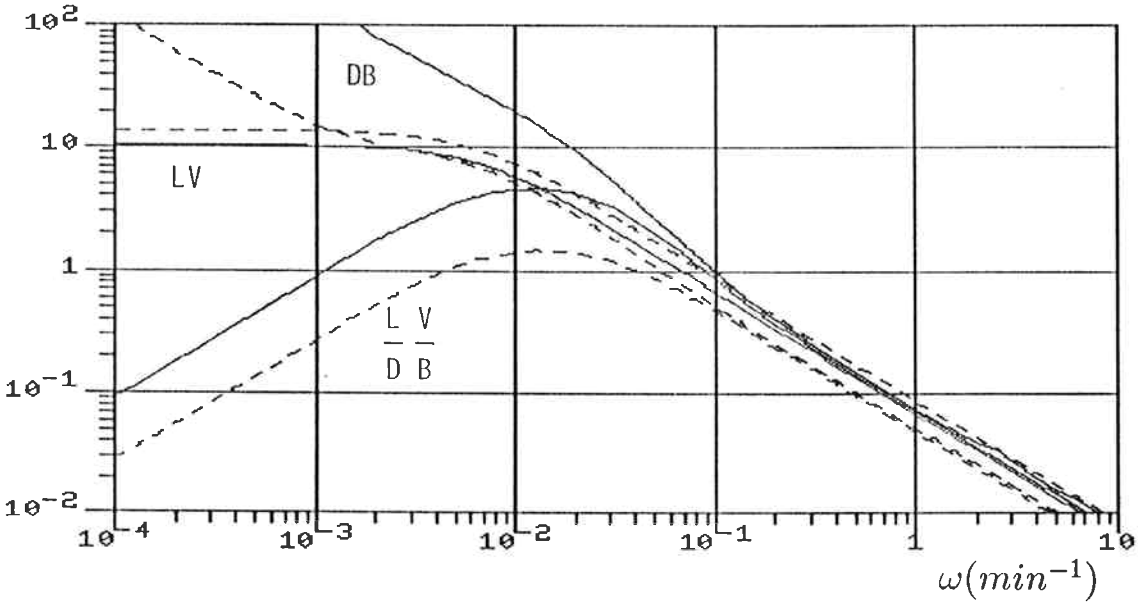

Figure 6. Effect of disturbances in F on compositions using three different configurations: LV, DB and (L/D)(V/B). Solid line: $(\delta y_D^S/\delta F)$ Dotted line:$(\delta x_B^S/\delta F)$

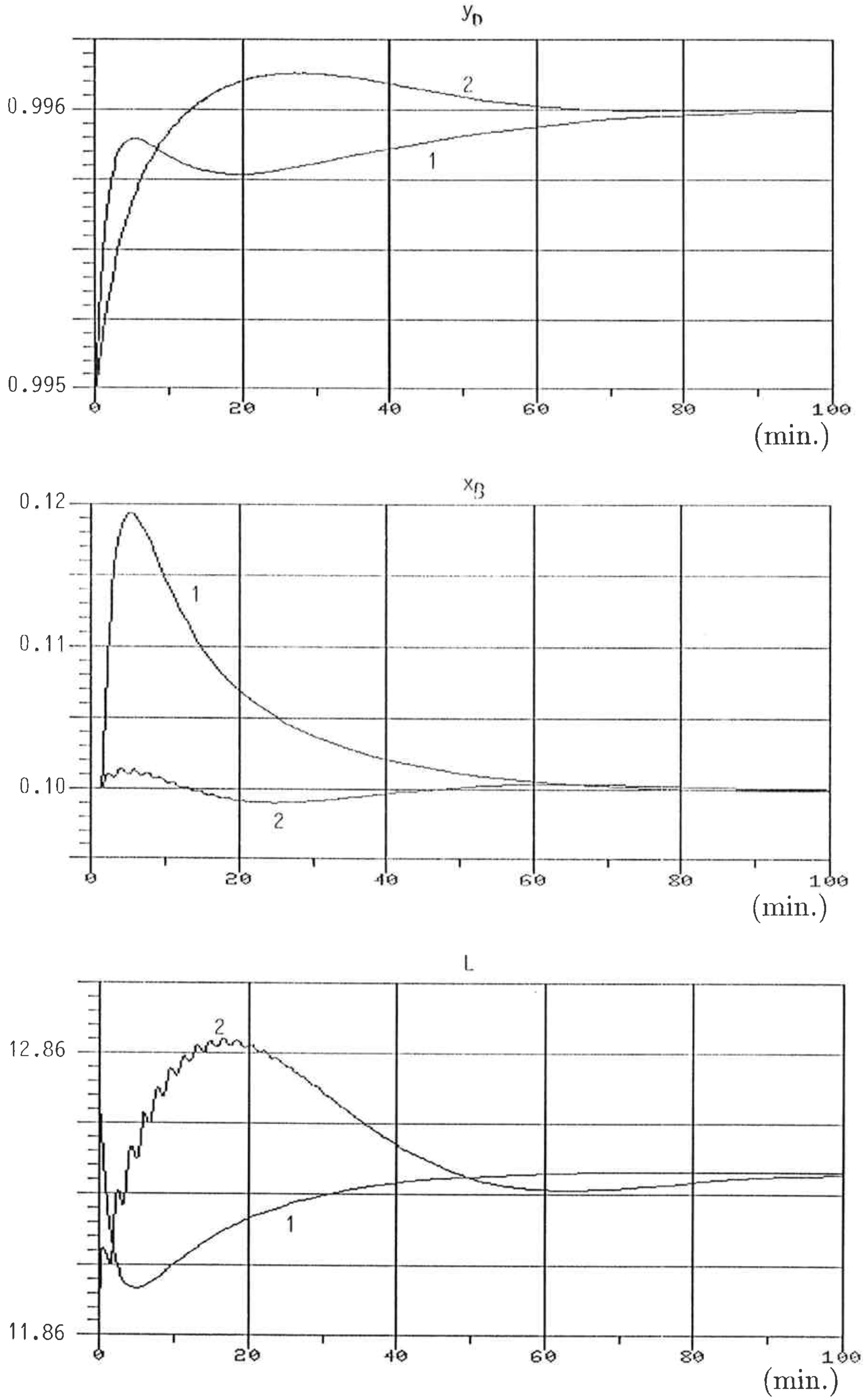

Figure 7. Time responses for y_D, x_B and L for a setpoint change in y_D using **1.**LV-configuration and **2.**DB-configuration.