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DISTURBANCE REJECTION IN DISTILLATION COLUMNS

by

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Abstract

The product compositions in a distillation column depend primarily on the product split D/B, and valueable insight is obtained by considering the effect of various disturbances on D/B. For a given disturbance, say an increase in feed rate F, the effect on D/B is strongly dependent on the chosen control configuration (structure), that is, on the two inputs used for composition control. Configurations where D/B is sensitive to disturbances should be avoided, and this may be used as a valueable tool for selecting column configurations. Presently, the Relative Gain Array (RGA) is the most used tool. The RGA is computed from the process gain matrix and contains no information about disturbances. Yet, for the most commonly used configurations there happens to be a close correlation between large RGA-values and disturbance sensitivity. This correlation gives one explanation for why the RGA actually has proven to be so useful for distillation columns.

1. INTRODUCTION

This paper is concerned with the "open loop" effect of external disturbances on product compositions in a distillation column. By "open loop" we mean that the two input variables chosen for composition control (corresponding to a specific control configuration) are kept constant. As one might expect the effect of disturbances depends strongly on the choice of control configuration. Initially, we shall introduce the concept of control configurations, and also the Relative Gain Array (RGA) which is the main tool used in industry for selecting control configurations.

Control configurations. Consider the two-product distillation column in Fig. 1. There are five valves which may used to manipulate the following five flows: L, V, V_T, D and B. These input flows are used to control the following five outputs: reboiler holdup M_B, condenser holdup M_D, vapor holdup M_V (ie., pressure p) and top and bottom composition (usually expressed by the mole fraction of light component, y_D and x_B). In theory we could imagine designing the "optimal" 5×5 controller for the column. However, in practice, in order to make the control system failure tolerant and easier to understand and tune, simpler control structures are used. Specifically, a control system for the level and pressure control (M_D, M_B, M_V) is designed first. This system has to work in order for the column to operate in a stable manner. There are then two degrees of freedom left for control of top and bottom composition and the issue of control *configuration* selection is to decide which two degrees of freedom (combinations of L, V, D, B, V_T) should be used. The traditional industrial choice is the LV-configuration, ie. to use reflux L and boilup V for composition control. In the literature this configuration is often denoted "energy balance" or "indirect material balance" control. The DV configuration ("direct material balance") also has many proponents in the literature. More recently (eg., Shinskey, 1984) the ratio schemes, such as the $(L/D)(V/B)$-configuration have become increasingly popular. This paper provides support for the ratio schemes in terms of their disturbance rejection capabilities.

Linear models and the RGA. Without the level loops in place the column is described by a 5×5 model expressing the effect of the five input flows on the five controlled outputs. In this paper, we assume the control problem has been decomposed into a separate control system for level control and a 2×2 system for composition control. By assuming the level loops are closed, it is then possible to derive an "open loop" model for the remaining 2×2 plant which has u_1 and u_2 as inputs and y_D and x_B as outputs. To avoid the dependency on how the level loops are tuned we throughout this paper make the simplifying assumption that the level loops are immediate ("perfect level control").

Denote the two combinations of manipulated variables which are used for composition control as u_1 and u_2 (for example, $u_1 = L/D$ and $u_2 = V/B$ for the $(L/D)(V/B)$-configuration). For any configuration we may formulate a linear transfer function model expressing the effect on product compositions of small changes in these inputs and of changes in external disturbances $\mathbf{d} = (d_1, d_2, \dots, d_i, \dots)^T$.

$$\begin{pmatrix} dy_D \\ dx_B \end{pmatrix} = G^{u_1 u_2}(s) \begin{pmatrix} du_1 \\ du_2 \end{pmatrix} + \Sigma_i \mathbf{g}_{di}^{u_1 u_2}(s) dd_i =$$

$$G^{u_1 u_2}(s) \begin{pmatrix} du_1 \\ du_2 \end{pmatrix} + G_d^{u_1 u_2}(s) d\mathbf{d} \quad (1)$$

The Relative Gain Array (RGA) (Bristol, 1966) is easily obtained from the process gain matrix $G^{u_1 u_2}$ for any configuration. Let the 2×2 steady-state gain matrix be $G^{u_1 u_2}(0) = \begin{pmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{pmatrix}$ Then the RGA is defined as

$$RGA = \begin{pmatrix} \lambda_{11} & 1 - \lambda_{11} \\ 1 - \lambda_{11} & \lambda_{11} \end{pmatrix}; \lambda_{11} = \left(1 - \frac{g_{12} g_{21}}{g_{11} g_{22}}\right)^{-1} \quad (2)$$

Use of the RGA for configuration selection. Since its introduction to distillation columns by Shinskey (1967), the RGA has been used increasingly in industry as the main tool for selecting the best control configuration (eg. McAvoy, 1977; Shinskey, 1981; Thurston, 1981; Wang, 1981). In his latest book on distillation column control Shinskey (1984) claims that the use of the RGA combined with simple short-cut models for the column has "moved the design of distillation control systems from what was much of an art to very nearly a science". His rule is to choose a configuration with λ_{11} in the range of about 0.9 to 4. However, even though these recommendation in terms of the RGA may be reasonable, as is confirmed by the acceptance in industry, the reasons given by Shinskey and most other authors are probably not. The RGA was introduced by Bristol as a tool for studying the effect of interactions when single-loop control is applied to a multivariable plant, and from Shinskeys book (1984) the reader is led to believe that this is the reason why the RGA is an effective tool in selecting control configurations for distillation columns. But if this were true the RGA recommendations would be of no use if a multivariable controller (where interactions is not an issue) were chosen. Practical evidence suggests, however, that the RGA is a useful tool also in such cases and even in cases when manual control (u_1 and u_2 nearly constant) is applied. The reasons for why the RGA is a useful tool must therefore be sought also elsewhere.

Sensitivity to disturbances. More recently, a numbers of authors has focused their attention on the difference between control configurations when it comes to their open loop disturbance rejection capabilities (Shinskey, 1985; Takamatsu et al., 1987; Skogestad and Morari, 1987a; Waller et al., 1987). The most important disturbances are usually related to the feed; the feed flow rate F, the feed enthalpy expressed in terms of its fraction of liquid q_F, and the feed composition z_F. In addition, there are disturbances on the five manipulated flows; L_d, V_d, D_d, B_d and V_{Td} (the subscript D is used to denote disturbance). Of these last five, disturbances V_d (possibly caused by a disturbance on the heating medium) and V_{Td} (possibly caused by a disturbance on the cooling medium) are ususally most important. Mathematically, we may collect all the disturbances in a vector $\mathbf{d}$ where

$$\mathbf{d} = (F, q_F, z_F, L_d, V_d, D_d, B_d, V_{Td})^T \tag{3}$$

and from (1) we see that their effect on the product composition is given as $G_d^{u_1 u_2} d\mathbf{d}$, or for a single disturbance d (any of the ones in eq. (3)) as $\mathbf{g}_d^{u_1 u_2} dd$. Note that $\mathbf{g}_d^{u_1 u_2}$ is the effect of the disturbance when the manipulated inputs u_1 and u_2 are constant, that is, it represents the effect of the disturbance on compositions when the composition control is "open loop". As the following example demonstrates, the value of $\mathbf{g}_d^{u_1 u_2}$ depends strongly on the choice of control configuration. Configurations with small values of $\mathbf{g}_d$ are preferable because this puts less demand (requires lower gain) on the feedback loops to correct for the effect of disturbances.

Example. Sensitivity to feed rate disturbance. We want to consider how an increase in the liquid feed rate affects compositions for three different configurations. The composition control system is assumed to be open loop, that is, u_1 and u_2 are constant. The difference between the configurations stems from the different way the level loops have to operate in order to keep u_1 and u_2 constant. For example, the initial effect of the feed disturbance is that the reboiler level increases. The level control system will subsequently change some flow in order to keep this level constant. There are three flows that may be used: $B(-), V(-)$ and $L(+)$ (the parenthesis gives the sign of their expected final effect on the level). What flow(s) are used, and the subsequent effect of this change, depends on the the choice of control configuration. The following three cases demonstrate the point:

1. LV-configuration. The composition control system is open loop, which means that L and V are constant. The reboiler level control system therefore has to increase B in response to the increase in F (and with D constant). Because the product compositions generally are very sensitive to changes in the external flows (expressed by D/B) this will lead to a large change in the product compositions y_D and x_B: Assume initially that almost all of the light components in the feed go to the top and all the heavy components go to the bottom. The increase in feed rate increases the amount of light components entering the column, but the amount of feed leaving as top product (D) remains constant. This means that a large amount of light component has to leave the column in the bottom product. The result is therefore a large increase in x_B (bottom product less pure), and, because of a shift in the entire composition profile inside the column, a corresponding increase in y_D (top product purer).

2. LB-configuration. In this case the increase in liquid flow leads to a corresponding increase in the boilup V, which subsequently, by the action of the pressure and condenser level control system, leads to an increase in the top product rate D (with B constant). The effect of the feed flow disturbance on the product compositions is essentially the opposite (similar magnitude, but different sign) of that found for the LV configuration.

3. (L/D)(V/B)-configuration. The initial increase in reboiler level is counteracted by increasing *both* V and B while keeping their ratio V/B constant. The increase in V subsequently leads to an increased condenser level which is counteracted by increasing both L and D while keeping their ratio L/D constant. As things settle, the final (steady state) effect is that all flows are adjusted proportionally and the effect on compositions of the feed rate disturbance is zero.

The example above shows fundamental differences between control configuration with respect to the effect of a feed rate disturbance. To keep the product compositions nearly constant for the LV- or BL- configurations, one would either have to employ feedback control based on composition measurements or use feedforward control based on measuring the feed rate. On the other hand, for the (L/D)(V/B)-configurations the level control system "automatically" takes care of the feed rate disturbance (at least at steady state) with no need for feedback or feedforward control. Similar differences are found for any disturbance that affects the flows in the column and thereby is felt by the level and pressure control systems.

Notation: The notation used to descibe the distillation column is shown in Fig. 1. In gain expressions y denotes an arbitrary composition (usually y_D or x_B), u an arbitrary manipulated flow for composition control, and d an arbitrary disturbance. Subscript d denotes disturbance, subscript m manipulated part of flow (what we "believe" it is), subscript D and T top of column, and subscript B bottom of column. The notation used for matrices and matrix elements mostly follows Häggblom and Waller (1986,1987) and deviates from that used by Skogestad and Morari (1987b) in that superscripts are used to denote the configuration (for example, G^{LV} is the gain matrix for the LV-configuration). Subscripts denote the variables which are varied, for example, $g_{ud}^{LV} = (\partial u/\partial d)_{L,V}$. Superscripts or subscripts are not shown in some cases in order to simplify notation.

2. EVALUATION OF DISTURBANCE GAINS

2.1 Linear models for various configurations

With the assumption of perfect level and pressure control there are two independent inputs (u_1, u_2) left for composition control. The linearized model for the column is written as a total differential of all the independent variables (inputs and disturbances d_i)

$$\begin{pmatrix} dy_D \\ dx_B \end{pmatrix} = G^{u_1u_2}(s)\begin{pmatrix} du_1 \\ du_2 \end{pmatrix} + G_d^{u_1u_2}(s)d\mathbf{d} \qquad (1)$$

where the steady-state gain matrix is

$$G^{u_1u_2} = \begin{pmatrix} (\partial y_D/\partial u_1)_{u_2} & (\partial y_D/\partial u_2)_{u_1} \\ (\partial x_B/\partial u_1)_{u_2} & (\partial x_B/\partial u_2)_{u_1} \end{pmatrix} \qquad (4)$$

and the matrix $G_d^{u_1u_2}$ contains the disturbance gains $(\partial y/\partial d)_{u_1,u_2}$. Choose the LV-configuration as the base configuration and assume a linear model is available for this

$$\begin{pmatrix} dy_D \\ dx_B \end{pmatrix} = G^{LV}(s)\begin{pmatrix} dL \\ dV \end{pmatrix} + G_d^{LV}(s)d\mathbf{d} \qquad (5)$$

Next, use *flow relationships* to express the effect of changes in manipulated flows u_1 and u_2 and disturbances $\mathbf{d}$ on the reflux L and boilup V:

$$\begin{pmatrix} dL \\ dV \end{pmatrix} = M^{u_1u_2}(s)\begin{pmatrix} du_1 \\ du_2 \end{pmatrix} + M_d^{u_1u_2}(s)d\mathbf{d} \qquad (6)$$

Strictly speaking, the matrices M and M_d are transfer matrices, but in this paper we assume the flow dynamics are much faster than the composition dynamics such that we may assume they are constant matrices, ie., use their steady-state value. Introducing (6) into (5) yields the following expressions for the process and disturbance gain matrices for an arbitrary configuration in terms of the base LV-configuration.

$$G^{u_1 u_2}(s) = G^{LV}(s) M^{u_1 u_2} \tag{7}$$

$$G_d^{u_1 u_2}(s) = G_d^{LV}(s) + G^{LV}(s) M_d^{u_1 u_2} \tag{8}$$

Actually, equations (7) and (8) express consistency relations between the transfer function models of various configurations. Similar relationships to (7) have been presented by Häggblom and Waller (1986) and Skogestad and Morari (1987b),and to (8) by Takamatsu et al. (1987) and Waller et al. (1987).

2.2 Obtaining M and M_d from flow relationships

Here we consider only steady state flow relationships. These relate the five manipulated flows (L, V, D, B, V_T) and the feed rate F. Three such relationships result from the steady-state requirement of constant holdups (constant M_D, M_B, M_V). Either one of these may be replaced by the requirement of constant total holdup. We have:
1) Constant total column holdup:

$$F = D + B \tag{9}$$

2) Constant condenser holdup (M_D)

$$V_T = D + L \tag{10}$$

3) Constant vapor holdup (M_V): For the ideal case with

$$\text{constant molar flows}: \quad V_T = V + (1 - q_F) F \tag{11}$$

where q_F is the liquid fraction of the feed. However, for the nonideal case this relationship cannot be written without introducing additional parameters as is dicussed in detail below.

Linear flow relationships. Differentiating (9) and (10) for small changes in the flows yield

$$dF = dB + dD \tag{12}$$

$$dV_T = dD + dL \tag{13}$$

The third relationship is most easily motivated by first considering the case of constant molar flows.

$$\text{constant molar flows}: \quad dV_T = dV + (1 - q_F) dF - F dq_F \tag{14a}$$

The generalization of (14a) to the nonideal case is most easily obtained by expressing the change in V_T as a total differential of all the independent variables

$$dV_T = \epsilon_L dL + (1-\epsilon_V)dV + (1-\hat{q}_F)dF - F dq_F + \epsilon_z dz_F \quad (14b)$$

In deriving (14b) we have used the fact there in addition to the feed are only two independent flows; in this case chosen to be L and V. The quantities $\epsilon_L, \epsilon_V, \epsilon_z$ and $\hat{q}_F$ in (14b) are *defined* by

$$\epsilon_L = \left(\frac{\partial V_T}{\partial L}\right)_V; \qquad (1-\epsilon_V) = \left(\frac{\partial V_T}{\partial V}\right)_L$$

$$(1-\hat{q}_F) = \left(\frac{\partial V_T}{\partial F}\right)_{L,V}; \qquad \epsilon_z = \left(\frac{\partial V_T}{\partial z_F}\right)_{L,V} \quad (15)$$

Note that for the ideal case of constant molar flows ϵ_L, ϵ_V and ϵ_z are all zero and $\hat{q}_F = q_F$. We see that ϵ_L physically represents the fraction of an increase in reflux L which leaves the column as overhead vapor V_T. ϵ_V represents the fraction of an increase in the boilup V which leaves the column as liquid to the reboiler. ϵ_z represents the change in overhead vapor caused by a change in feed composition. The difference between q_F and $\hat{q}_F$ is somewhat subtle and should not cause too much concern. We see that $\hat{q}_F$ represents the fraction of an increase in F which goes to the reboiler, while q_F is a measure of the specific feed enthalpy, defined such that $(\partial L_B/\partial q_F)_{L,V} = F$.

Eliminating dV_T from (13) and (14b) yields

$$dD = -(1-\epsilon_L)dL + (1-\epsilon_V)dV + (1-\hat{q}_F)dF - F dq_F + \epsilon_z dz_F \quad (16)$$

This relationship is usually used instead of (14b) since V_T is normally not used for composition control.

Obtaining the matrices M and M_d. The flow matrices $M^{u_1 u_2}$ and $M_d^{u_1 u_2}$ defined in (6) are obtained by writing dL and dV in terms of the chosen manipulated variables (u_1 and u_2) and the disturbances as shown in eq. (6). All the information needed for this is contained in the flow relations (12),(13) and (16) derived above, and all that is required is therefore some algebraic manipulations. The matrices M and M_d are sometimes written without the superscript $u_1 u_2$ to simplify notation. For quick reference the matrix M is given in Table 1 for some configurations. The disturbance flow matrix is obtained from

$$M_d = -M\tilde{M}_d \quad (17)$$

where $\tilde{M}_d = \{\tilde{\mathbf{m}}_{d1}, \tilde{\mathbf{m}}_{d2}, \ldots\}$ may be obtained from Table 2.

2.3 Special case: Feed rate disturbance

Feed flow disturbances are often the most important disturbances. Fortunately, the *steady-state* gain for this disturbance can be obtained directly from the gain matrix without any need for a separate simulation. We make use of the fact that the effect a change in feed rate (an extensive quanity) on the product compositions (an intensive quantity) at steady state is perfectly rejected by increasing all other flows proportionally. For example, consider the LV-configuration. The effect on composition of a feed flow disturbance is

$$dy = g_{yL} dL + g_{yV} dV + g_{yF} dF$$

he effect of this disturbance is counteracted by choosing $dL/L^* = dV/V^* = dF/F^*$. Consequently

$$0 = g_{yL}(L^*/F^*)dF + g_{yV}(V^*/F^*)dF + g_{yF} dF$$

and we have $g_{yF} = -g_{yL}(L^*/F^*) - g_{yV}(V^*/F^*)$, or on matrix form

$$\mathbf{g}_F^{LV} = -G^{LV} \begin{pmatrix} L^*/F^* \\ V^*/F^* \end{pmatrix} \tag{18}$$

Similar expression apply to other configurations. However, one must be careful about the difference between cases where the manipulated input u is an extensive quantity (eg., u is L or V) and when it is an intensive quantity (ie., u is a ratio, eg., L/D or V/B). In the former case a disturbance in F is rejected by choosing $du/u^* = dF/F^*$, in the latter by choosing $du = 0$. The insight that the effect of a feed rate change can be obtained directly from the process gains (eq. 18) seems to be largely unknown, as no such relationship was found in the literature.

It is also possible to obtain $\hat{q}_F = (\partial L_B/\partial F)_{L,V}$ without making a change in F. Consider (14b) which expresses the total differential of V_T. Integrate (14b) from zero flow to actual conditions with all intensive properties (q_F, z_F, y_D, x_B, etc.) constant. We get

$$V_T = \epsilon_L L + (1 - \epsilon_V)V + (1 - \hat{q}_F)F \tag{19}$$

and $\hat{q}_F$ is obtained from (19) when ϵ_L and ϵ_V are known. (19) may be interpreted as the generalization of (11) to the nonideal case. As a curiosity note that differentiating (19) and subtracting (14b) yields the "Gibbs-Duhem equation for a distillation column":

$$L d\epsilon_L - V d\epsilon_V + F(dq_F - d\hat{q}_F) - \epsilon_z dz_F = 0 \tag{20}$$

2.4 Procedure for obtaining process and disturbance gains

The following is a step-by-step procedure for obtaining gains for various configurations. A nonlinear simulation package with rigorous thermodynamics such as PROCESS or ASPEN should be used. The minimum number of simulations that must be performed is five: One base run, two to find the effect of changing the manipulated flows (any choice will do, eg. L and V or D and V), and one each for the disturbances in q_F and z_F.

1. Run base simulation.
2. Obtain gain matrix G^{LV} and parameters ϵ_L and ϵ_V numerically by making small changes in L and V from their base values.
3. Obtain disturbance gain vectors $\mathbf{g}_z^{LV}$ and $\mathbf{g}_q^{LV}$ and the parameter ϵ_z numerically by making small changes in z_F and q_F (feed enthalpy) from their base values.
4. $\mathbf{g}_d^{LV}$ for disturbances in L_d and V_d are given by $\{\mathbf{g}_{L_d}^{LV}\mathbf{g}_{V_d}^{LV}\} = G^{LV}$.
5. $\mathbf{g}_{D_d}^{LV}$ and $\mathbf{g}_{B_d}^{LV}$ for disturbances in D_d and B_d are both zero.
6. Disturbance in F. $\mathbf{g}_F^{LV}$ from eq. (18); $\hat{q}_F$ from eq. (19).
7. All data for the LV-configuration are now known. Convert to any other configuration as follows:
8. Obtain flow matrices $M^{u_1u_2}$ and $\tilde{\mathbf{M}}_d^{u_1u_2}$ from Table 1 and 2.
9. The gain elements are subsequently obtained from the base LV-configuration by

$$G^{u_1u_2} = G^{LV}M^{u_1u_2} \tag{7}$$

$$G_d^{u_1u_2} = G_d^{LV} - G^{LV}M^{u_1u_2}\tilde{M}_d^{u_1u_2} \tag{21}$$

For cases when the LV-configuration yields large RGA-values, the determination of G^{LV} using Step 2 is not reliable. In such cases it is recommended to first obtain the gains for the LD-configuration which always has the RGA-elements between 0 and 1. The modified Step 2 then becomes

2'. Obtain gain matrix G^{LD} and parameters ϵ_L and ϵ_V numerically by making small changes in L and D from their base values (Note from (16) that $\left(\frac{\partial V}{\partial D}\right)_L = \frac{1}{1-\epsilon_V}$ and $\left(\frac{\partial V}{\partial L}\right)_D = \frac{1-\epsilon_L}{1-\epsilon_V}$). Obtain M^{LD} from Table 1 and compute $G^{LV} = G^{LD}\left(M^{LD}\right)^{-1}$.

The above procedure is very well suited for computerization. Note that if the outputs are measured in terms of mole fractions then the exact component material balance

$$Fz_F = Dy_D + Bx_B \tag{22}$$

provides consistency relationships between the steady state values of the column elements in $G^{u_1u_2}$ and between the column elements in $\mathbf{g}_d^{u_1u_2}$ (Häggblom and Waller, 1986, 1987;

Skogestad and Morari, 1987b). These consistency relationships are obtained by differentiating (22) with respect to the input u or disturbance d and applying the flow relationships (12),(13) and (16). These consistency relationships should always be used as constraints when obtaining gains numerically.

2.5 Disturbance gains. Summary

This section has shown how to evaluate process and disturbance gains for various configurations. We are therefore in a position to evaluate the disturbance gains $\mathbf{g}_d^{u_1 u_2}$ (column vectors of $G_d^{u_1 u_2}$) for any configuration and any disturbance. As noted in the introduction we will prefer configurations with small values of $\mathbf{g}_d^{u_1 u_2}$ because this means that the level loops (with the composition control loops open, ie., u_1 and u_2 constant) help counteract the disturbances, and there is less need to apply feedback control based on composition measurements. However, even though we in this Section have presented methods for how to obtain $g_d^{u_1 u_2}$ *numerically*, these methods do not provide much general insight into which configurations should be preferred. Such insight it easier obtained from the approach in the next Section where the effect of disturbances on D/B for various configurations is evaluated.

3. EFFECT OF DISTURBANCES ON D/B

3.1 Most senstive direction

The basis for the results in this Section is the observation that, irrespective of the control configuration, the product compositions are mainly affected by the external material balance, ie. by changes in D/B (Rosenbrock, 1962; Shinskey, 1984). The effect of changes in *internal* flows, ie., increasing L and V simultaneously with D/B constant, is generally much smaller. This leads to the conclusion that disturbances with a large effect on D/B will lead to large upsets in composition and should be avoided.

A more mathematical way of showing this is as follows. The overall separation in a distillation column between components i and j is given in terms of the separation factor $S = \frac{y_{Di}/y_{Dj}}{x_{Bi}/x_{Bj}}$. For a binary separation S becomes

$$S = \frac{y_D(1 - x_B)}{(1 - y_D)x_B} \tag{23}$$

Shinskey (1984) has made the observation that the variation in S is small for most columns. For example, at infinite reflux and constant relative volatility we have $S = \alpha^N$ (Fenskes equation) and we see that S is fixed even though y_D and x_B individually may show large variations. The main effect on product compositions of any change in the column may then be obtained by assuming S constant. Differentiating the component balance (22) assuming S *constant* yields (Skogestad and Morari, 1987b)

$$\frac{dy_d}{(1-y_D)y_D} = \frac{dx_B}{(1-x_B)x_B} = \frac{1}{I_S}\left(F dz_F - \frac{B^2}{F}(y_D - x_B) d\frac{D}{B}\right) \tag{24}$$

where the impurity sum is defined as

$$I_S = Bx_B(1-x_B) + Dy_D(1-y_D) \tag{25}$$

3.2 Effect of disturbances on D/B.

From eq. (24) wee see that, with the exception of z_F, disturbances mainly affect product compositions because of their effect on D/B. The difference between configurations with repect to their ability to reject disturbances with the level loops can therefore be evaluated by evaluating the quantity $\frac{B^2}{F}\left(\partial(D/B)/\partial d\right)_{u_1,u_2}$ in (24). The procedure for obtaining this quantity from the flow relationships in Section 2.2 is given by Skogestad and Morari (1987a). The results for various configurations are summarized in Table 3 (this is a generalization to the nonideal case of the corresponding table in Skogestad and Morari (1987a)). Table 3 is a useful tool for selecting control configurations.

3.3 Configurations with same disturbance sensitivity.

Note from Table 3 that different configurations may have identical properties when it comes to their open loop disturbance rejection. For example, keeping L/V and V constant is equivalent to keeping L and V constant. Thus, the LV- and $(L/V)V$-configurations have identical open-loop sensitivies to disturbances and have identical entries in Table 3.

3.4 Relationship to RGA.

There is a close correlation between the effect of disturbances in V, L and q_F and the RGA. Define the net disturbance flow v_d from the reboiler to the condenser as (assume z_F constant)

$$v_d = V_d - L_d - F^* q_F \tag{26}$$

and consider the *relative* effect on compositions of a *relative* change in v_d (relative to the maximum liquid flow in the column, L_B), ie., consider the following quantity

$$R_d \stackrel{\text{def}}{=} \frac{L_B}{y_D(1-y_D)}\left(\frac{\partial y_D}{\partial v_d}\right)_{u_1,u_2} = \frac{L_B}{x_B(1-x_B)}\left(\frac{\partial x_B}{\partial v_d}\right)_{u_1,u_2} \tag{27}$$

From (24) which applies when S is assumed constant we obtain

$$R_d = -\frac{L_B(y_d - x_B)}{I_S}\frac{B^2}{F}\left(\frac{\partial(B/D)}{\partial v_d}\right)_{u_1,u_2} \quad (28)$$

where the last term is given in Table 3 for various configurations. We will now demonstate the close correlation between R_d and the RGA.

LV-configuration. We find from (28) and Table 3

$$R_d = \frac{L_B(y_D - x_B)}{I_S} \quad (29)$$

Consequently, the relative effect of a disturbance in v_d on compositions is large when $I_S \approx D(1-y_D)+Bx_B$ is small, ie. when both product are pure. Next, consider the RGA. For the LV-configuration the following approximation applies (Skogestad and Morari, 1987b): $\lambda_{11}^{LV} \approx 1/(I_S(\partial \ln S/\partial L)_D)$. Here $(\partial \ln S/\partial L)_D \approx \ln S/L_B$ and we derive

$$R_d \approx \lambda_{11}^{LV} \ln S \quad (30)$$

(where $\ln S$ is in the order 3 to 15 for almost all separations) and, since we want R_d small, the LV-configuration should clarely be avoided if RGA-elements are large.

Other configurations. RGA-expressions are given by Skogestad and Morari(198b) also for some other configurations. Comparing these with R_d yields the following conclusions:

- LV: RGA and R_d both go as $1/I_S$
- $(L/D)(V/B)$: RGA and R_d both reduced by a factor $r = 1+L'/D+V'/B$ compared to the LV-configuration.
- Other ratio configurations : Similar effect as for $(L/D)(V/B)$-configuration.
- DX, BX: RGA is always in the range 0-1 and R_d is zero.

Also note that RGA is not changed if we consider $D/(L+D)$ instead of L/D as an input (because the RGA is scaling invariant), and neither does this change affect R_d.

Even though the correlation between the RGA and R_d is astounding there are som exceptions. For example, the LV and $(L/V)V$-configurations have the same values for R_d, but the RGA is different (often significantly smaller in the latter case). The DV and $(D/V)V$-configurations also have the same value for R_D, but the RGA is different. In fact, the $(D/V)V$ and $(L/V)V$-configurations usually have almost identical RGA-values, but their open-loop disturbance sensitivities to v_d are entirely different (Table 3).

Nevertheless, the overall conclusion is that there is close correlation between RGA-values and R_d for most of the important configurations. One important reason for the usefulness of the RGA with respect to distillation column control is therefore that it yields the column's sensitivity to some of the most important disturbances.

4. DISCUSSION AND CONCLUSION

The paper has shown how to evaluate the disturbance gains for various configurations. However, more insight is obtained by considering the effect of various disturbances on D/B (Table 3). Configurations with small entries in Table 3 are preferable because they are less sensitive to disturbances. Table 3 is therefore a valueable tool for selecting column configurations. We have already introduced the RGA which presently is the mostly used tool. The RGA is computed from the process gain matrix and contains no information about disturbances. Yet, for the most commonly used configurations there happens to be a close correlation between large RGA-values and large entries in Table 3. This correlation gives one explanation for why the RGA actually has proven to be so useful for distillation columns.

Use of consistency relationships. Actually, as discussed by Hä ggblom and Waller (1987), it is possible in some cases to obtain the parameters ϵ_L and ϵ_V (and thereby the matrix M) directly from G^{LV} by employing consistency relationships. This is the case if the outputs are measured in terms of mole fractions. In this case the exact steady state component balance (32) may be used to obtain a consistency relationship between the column elements in a given gain matrix. For example, the column elements in G^{LV} must in this case satisfy $Dg_{y_D L} + Bg_{x_B L} = (y_D - x_B)(1-\epsilon_L)$ and $Dg_{y_D V} + Bg_{x_B V} = -(y_D - x_B)(1-\epsilon_V)$ (Skogestad and Morari, 1987b) and we may therefore obtain ϵ_L and ϵ_V from G^{LV} Similarly, the parameters $\hat{q}_F$ and ϵ_z may be obtained from consistency relationships on the matrix G_d. However, the use of the consistency relationships to obtain the "non-ideal" parameters $\epsilon_L, \epsilon_V, \epsilon_z$ etc. is not recommended because of potential numerical difficulties. In practice, we should use the consistency relationships as constraints to *derive* consistent gains, rather than to *extract* data from the gains.

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Appendix. Derivation of Table 1 and 2.

We have to be careful about one thing which we until now have only trested lightly in order to avoid cunfusion: There may be *disturbances* on flows which are used as manipulated variables for composition control (the flows used only for level control are fixed by the requirement of constant holdups and disturbances on these have no effect). Assume reflux L is used for composition control. Then the actual value of L is not the same as what we believe it is (L_m), but rather $L = L_m + L_d$ where L_d represents the (usually) unknown disturbance. A change in L may then result from either our manipulation dL_m or from a disturbance dL_d, ie. the total change is $dL = dL_m + dL_d$. In general write

$$dL = dL_m + dL_d; \quad dV = dV_m + dV_d; \quad dD = dD_m + dD_d;$$

$$dB = dB_m + dB_d; \quad dV_T = dV_{Tm} + dV_{Td} \qquad (A1)$$

If we assume that L_d, V_d, etc. are included in the vector $\mathbf{d}$, then to be correct dL, dV, du_1 and du_2 in equations (1),(5) and (6) should be replaced by dL_m, dV_m, du_{1m} and du_{2m} (note that the equations are correct as is if $\mathbf{d}$ only includes disturbances in F, q_F and z_F) . For example, with all disturbances included in $\mathbf{d}$, (6) should read

$$\begin{pmatrix} dL_m \\ dV_m \end{pmatrix} = M^{u_1 u_2} \begin{pmatrix} du_{1m} \\ du_{2m} \end{pmatrix} + M_d^{u_1 u_2} d\mathbf{d} \qquad (A2)$$

The subscript m will be included when it is necessary for clarity, but is otherwise deleted. Also note that equations (12)-(16) represent the total flows and are correct as is without subscripts.

The simplest way to obtain M and M_d is usually by first finding the matrices M^{-1} and $\tilde{M}_d = -M^{-1} M_d$. To this effect rewrite (A2) as follows:

$$\begin{pmatrix} du_{1m} \\ du_{2m} \end{pmatrix} = M^{-1} \begin{pmatrix} dL_m \\ dV_m \end{pmatrix} + \tilde{M}_d d\mathbf{d} \qquad (A3)$$

Obtaining M. M^{-1} expresses the effect of dL_m and dV_M on du_{1m} and du_{2m} when the are disturbances are constant,e., $dL = dL_m$, $dV = dV_m$, etc. . The matrix M^{-1} is obtained by the following procedure:

1. Express $du_{1m} = du$ and $du_{2m} = du$ in terms of $dL_m, dV_m, dD_m, dB_m, dV_{Tm}$.
2. Eliminate $dD_m = dD$, $dB = dB_m$ and $dV_T = dV_{Tm}$ using (12), (13) and (16) (with no disturbances, ie. $dF = dq_F = dz_F = 0$).
3. The equations are now on the form (A3) which directly yields the matrix M^{-1}.

The resulting matrix M is shown in Table 1 for some configurations. Note that if the order u_1, u_2 is interchanged, then the columns of the matrix M should be interchanged.

Obtaining $\tilde{M}_d$. The matrix $\tilde{M}_d$ consists of one column vector $\tilde{\mathbf{m}}_d$ for each disturbance d. We have

$$\tilde{M}_d = \{\tilde{\mathbf{m}}_{d1}, \tilde{\mathbf{m}}_{d2}, \ldots\} \quad \text{where} \quad \tilde{\mathbf{m}}_d = \begin{pmatrix} (\partial u_{1m}/\partial d)_{L_m,V_m} \\ (\partial u_{2m}/\partial d)_{L_m,V_m} \end{pmatrix}$$

The elements $(\partial u_{im}/\partial d)_{L_m,V_m}$ (which are independent of the choice for the other manipulated input) are shown in Table 2 for some inputs u_{im}. They are obtained by the following procedure:

4. Express du_{im} in terms of $dL_m, dV_m, dD_m, dB_m, dV_{Tm}$.
5. Set $dL_m = dV_m = 0$ and eliminate dB_m, dD_m and dV_{Tm} using (A1).
6. Eliminate dD, dB and dV_T using (12), (13) and (16).
7. We now have du_{im} as a function of all disturbances with constant L_m and V_m, and may directly obtain $(\partial u_{im}/\partial d)_{L_m,V_m}$ for each disturbance.

The vector $\tilde{\mathbf{m}}_d$ for a given configuration is obtained by combining the rows in Table 2 that correspond to the two manipulated inputs.

Example. Obtaining M *and* $\tilde{M}_D$ *for* $\frac{L}{D}\frac{V}{B}$ *- configuration.*

1. Note that we assume constant disturbances. Then

$$du_{1m} = du_1 = d\frac{L}{D} = \frac{1}{D}dL - \frac{L}{D^2}dD$$

$$du_{2m} = du_2 = d\frac{V}{B} = \frac{1}{B}dV - \frac{V}{B^2}dB$$

where the coefficients multiplying the differentials represent steady state values, that is, $1/D$ should read $1/D^*$, etc. where superscipt $*$ denotes steady-state value. To be strict we should actually use D^*_m instead of D^*, but in most cases we assume there are no disturbances at steady state and we have $D^* = D^*_m$.

2. Eliminate dD and dB using (12) and (16) (no disturbances)

$$d\frac{L}{D} = \left(\frac{1}{D} + \frac{L}{D^2}(1-\epsilon_L)\right)dL - \frac{L}{D^2}(1-\epsilon_V)dV$$

$$d\frac{V}{B} = -\frac{V}{B^2}(1-\epsilon_L)dL + \left(\frac{1}{B} + \frac{V}{B^2}(1-\epsilon_V)\right)dV$$

The matrix M^{-1} is then simply

$$M^{-1} = \begin{pmatrix} \frac{1}{D}\left(1 + \frac{L}{D}(1-\epsilon_L)\right) & -\frac{L}{D^2}(1-\epsilon_V) \\ -\frac{V}{B^2}(1-\epsilon_L) & \frac{1}{B}\left(1+\frac{V}{B}(1-\epsilon_V)\right) \end{pmatrix}$$

and we obtain $M^{(L/D)(V/B)}$ as shown in Table 1.

4. Here there are disturbances and we must keep the subscript m:

$$du_{1m} = d\left(\frac{L}{D}\right)_m = \frac{1}{D}dL_m - \frac{L}{D^2}dD_m$$

$$du_{2m} = d\left(\frac{V}{B}\right)_m = \frac{1}{B}dV_m - \frac{V}{B^2}dB_m$$

5. Set $dL_m = dV_m = 0$ and eliminate dB_m, dD_m and dV_{Tm} using (A1).

$$d\left(\frac{L}{D}\right)_m = -\frac{L}{D^2}(dD - dD_d)$$

$$d\left(\frac{V}{B}\right)_m = -\frac{V}{B^2}(dB - dB_d)$$

6. Eliminate dD and dB using (12) and (16).

$$\left(d\frac{L}{D}\right)_m = -\frac{L}{D^2}(-(1-\epsilon_L)dL_d + (1-\epsilon_V)dV_d + (1-\hat{q}_F)dF - Fdq_F + \epsilon_z dz_F - dD_d)$$

$$\left(d\frac{V}{B}\right)_m = -\frac{V}{B^2}(dF + (1-\epsilon_L)dL - (1-\epsilon_V)dV - (1-\hat{q}_F)dF + Fdq_F - \epsilon_z dz_F - dB_d)$$

7. Collecting terms yields the entries for L/D and V/B as inputs as shown in Table 2. For example, for a disturbance in F we find the elements in $\tilde{m}_F$ for the $(L/D)(V/B)$-configuration to be $-(L/D^2)(1-\hat{q}_F)$ and $-(V/B^2)\hat{q}_F$.

Figure Captions

Fig. 1. Distillation column with 5 manipulated inputs (L, V, D, B and V_T) and 5 controlled outputs (y_D, x_B, M_D, M_B and p).

$\mathbf{u_1}$	$\mathbf{u_2}$	$\mathbf{M^{u_1 u_2}}$
L	V	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
L	$D(-B)$	$\begin{bmatrix} 1 & 0 \\ \frac{1-\epsilon_L}{1-\epsilon_V} & \frac{1}{1-\epsilon_V} \end{bmatrix}$
$D(-B)$	V	$\begin{bmatrix} -\frac{1}{1-\epsilon_L} & \frac{1-\epsilon_V}{1-\epsilon_L} \\ 0 & 1 \end{bmatrix}$
D	V/B	$\begin{bmatrix} -\frac{1+(1-\epsilon_V)\frac{V}{B}}{1-\epsilon_L} & B\frac{1-\epsilon_V}{1-\epsilon_L} \\ -\frac{V}{B} & B \end{bmatrix}$
L/D	$D(-B)$	$\begin{bmatrix} D & \frac{L}{D} \\ D\frac{1-\epsilon_L}{1-\epsilon_V} & \frac{1+\frac{L}{D}(1-\epsilon_L)}{1-\epsilon_V} \end{bmatrix}$
L/D	V	$\begin{bmatrix} \frac{D}{1+\frac{L}{D}(1-\epsilon_L)} & \frac{\frac{L}{D}(1-\epsilon_V)}{1+\frac{L}{D}(1-\epsilon_L)} \\ 0 & 1 \end{bmatrix}$
L/D	V/B	$\frac{1}{r}\begin{bmatrix} D(1+\frac{V}{B}(1-\epsilon_V)) & B\frac{L}{D}(1-\epsilon_V) \\ D\frac{V}{B}(1-\epsilon_L) & B(1+\frac{L}{D}(1-\epsilon_L)) \end{bmatrix}$

$$r = 1 + \frac{L}{D}(1-\epsilon_L) + \frac{V}{B}(1-\epsilon_V)$$

Table 1. Flow matrix $M^{u_1 u_2}$ for some choices of manipulated inputs (u_1 and u_2) with the LV-configuration as basis. The gain matrix is then $G^{u_1,u_2} = G^{LV} M^{u_1 u_2}$.

Disturbance d

Input				Dist$d$			
u_m	dF	dq_F	dz_F	dL_d	dV_d	dD_d	dB_d
L, V or L/V	0	0	0	0	0	0	0
D	$1-\hat{q}_F$	$-F$	ϵ_z	$-(1-\epsilon_L)$	$(1-\epsilon_V)$	-1	0
B	$\hat{q}_F$	F	$-\epsilon_z$	$(1-\epsilon_L)$	$-(1-\epsilon_V)$	0	-1

L/D	$-L/D^2$ times entries for D
V/B	$-V/B^2$ times entries for B
$D/(L+D)$	$L/(L+D)^2$ times entries for D

Applies to steady state
A disturbance in V_{Td} has no effect for these inputs
The matrix $\tilde{M}_d^{u_1u_2}$ for a given configuration is obtained by combining the corresponding rows in the Table.
Disturbance gains are $G_d^{u_1u_2} = G_d^{LV} - G^{LV} M^{u_1u_2}\tilde{M}_d^{u_1u_2}$

Table 2. $(\partial u_m/\partial d)_{L_m,V_m}$ = Effect of disturbance d on u_m with constant L_m and V_m.

Configuration		Disturbance d		
$u_1 u_2$	dF	dv_d	dD_d	dB_d
$LV, \frac{L}{V}V, \frac{L}{V}L$	1-$\hat{q}_F$-D/F	1	0	0
$DX, \frac{D}{X}X$	$-D/F$	0	1	0
$BX, \frac{B}{X}X$	B/F	0	0	-1
$\frac{D}{B}X$	0	0	B/F	$-D/F$
$\frac{L}{D}V, \frac{D}{L+D}V$	$-\frac{V'/F}{1+L'/D}$	$\frac{1}{1+L'/D}$	$\frac{L'/D}{1+L'/D}$	0
$L\frac{V}{B}, L\frac{B}{V+B}$	$\frac{L'/F}{1+V'/B}$	$\frac{1}{1+V'/B}$	0	$-\frac{V'/B}{1+V'/B}$
$\frac{L}{D}\frac{V}{B}, \frac{D}{L+D}\frac{V}{B}$	0	$\frac{1}{r'}$	$\frac{L'/D}{r'}$	$\frac{-V'/B}{r'}$

Applies to steady state
$dv_d = (1-\epsilon_V)dV_d - (1-\epsilon_L)dL_d - Fdq_F + \epsilon_z dz_F$
$\hat{q}_F$ - fraction of liquid in feed as defined in eq. (15)
X - any other manipulated input u except D, B and $\frac{D}{B}$.
subscript d denotes an additive disturbance on this flow.
$V' = (1-\epsilon_V)V; \quad L' = (1-\epsilon_L)L; \quad r' = 1 + L'/D + V'/B$
The Table contains no schemes with V_T used for composition control. Disturbances in V_T are then perfectly rejected by the pressure and level control system.
z_F has only a minor effect on D/B. Its main effect on compositions is given by (24) and is equal for all configurations.

Table 3. $\frac{B^2}{F}(\frac{\partial D/B}{\partial d})_{u_1,u_2}$ = Linearized effect of flow disturbances on D/B when both composition loops are open.