

# A SYSTEMATIC APPROACH TO DISTILLATION COLUMN CONTROL

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This paper presents a systematic approach to distillation column control. The main emphasis is on the steps which precede the actual controller design, namely the modelling of the column and the selection of the control configuration. By control configuration in this context we mean the two independent variables used for composition control (for example,  $L$  and  $V$ ,  $D$  and  $V$ , or  $\frac{L}{D}$  and  $\frac{V}{B}$ ). The steps preceding the controller design are generally the most important, yet, they are not usually discussed in any detail in the academic literature. The goal of this paper is to fill in this apparent gap.

## 1. INTRODUCTION

Three separate steps are involved when designing composition control systems for distillation columns:

1. Modelling the column.
2. Choice of the control configuration.
3. Controller design/implementation.

This paper is aimed at discussing Steps 1 and 2 which should precede the controller design. In fact, the controller design itself is probably the least important step in spite of the attention it is given in the literature. Let us consider Step 2. Most two-product distillation columns (Fig.1) may be described as a  $5 \times 5$  plant (Shinskey, 1984). In theory we could imagine designing the "optimal"  $5 \times 5$  controller for the column. While this would certainly be of theoretical interest, it is very unlikely that such a controller would ever be implemented in practice. In order to make the control system failure tolerant and easier to understand and tune, simpler control structures are used. More specifically, we will not use all the five flows  $L$ ,  $V$ ,  $D$ ,  $B$  and  $V_T$  for composition control, but only two independent combinations. Step 2 therefore involves answering the following question: Which two independent variables should be "manipulated" for composition control? Each choice of manipulated inputs corresponds to a specific control *configuration*. For example, the traditional industrial choice of  $L$  and  $V$  for composition control ("indirect material balance") is referred to as the LV configuration. The DV-configuration ("direct material balance") also has many proponents in the literature. More recently the ratio schemes (eg., the  $\frac{L}{D}$   $\frac{V}{B}$ -configuration) have become

increasingly popular.

The distillation columns in Table 1 are used as examples. In all examples we assume binary separation, constant molar flows, constant relative volatility, liquid feed, 100% tray efficiency and total condenser. For the dynamic simulations we assume in addition constant liquid and vapor holdup (ie., instantaneous flow responses) and equal holdup on all trays inside the column. From the linearized model  $\dot{x} = Ax + Bu + Ed$ , where the vector  $x$  denotes the tray compositions,  $u = (dL, dV)^T$  is the input vector, and  $d = (dF, dz_F)^T$  is the disturbance vector, we derive the steady-state gain matrices  $G_{LV} = -CA^{-1}B$  and  $G_d = -CA^{-1}E$  given in Table 2.

In the paper we present simple analytical models for the steady-state and dynamic behavior of distillation columns. We stress that these are by no means intended to replace nonlinear simulations. The objective is to gain insight into the dynamic behavior of distillation columns. For feedback control an accurate model of the plant is usually not needed. Rather, a simple model (preferably a linear model) which includes the factors most important for feedback control is desired. Pinpointing these factors is most easily accomplished with simple analytical models.

Throughout the paper we will make use of the condition number,  $\gamma(G)$ , of the plant, and its Relative Gain Array (RGA). These quantities are both easily computed from the transfer function model of the plant,  $G(s)$ . The condition number is defined as the ratio between the plant's maximum and minimum gain at any given frequency

$$\gamma(G(j\omega)) = \bar{\sigma}(G(j\omega)) / \underline{\sigma}(G(j\omega)) \quad (1)$$

Here  $\bar{\sigma}(G)$  and  $\underline{\sigma}(G)$  are the maximum and minimum singular values of the plant (Klema and Laub, 1980). The RGA is defined by ( $\times$  denotes element-by-element multiplication)

$$RGA = \{\lambda_{ij}\} = G \times (G^{-1})^T, \quad (2 \times 2 \text{ plants}) : \lambda_{11} = \left(1 - \frac{g_{12}g_{21}}{g_{11}g_{22}}\right)^{-1} \quad (2)$$

Shinskey (1984) uses the steady-state RGA-elements as his main tool for choosing the control configuration. Ill-conditioned plants with a large value of  $\gamma(G)$  or with large RGA-elements are generally believed to be difficult to control because of their sensitivity to model uncertainty. In most cases large RGA-elements, rather than a large value of  $\gamma(G)$ , imply control problems (Skogestad and Morari, 1986). It also turns out that the sum of the absolute values in the RGA ( $\|RGA\|_1 = \sum \lambda_{ij}$ ) (which is independent of scaling) is very close in magnitude to the value of  $\gamma(G)$  obtained when it is minimized over all input and output scalings. Therefore,  $\|RGA\|_1$  will be used as a measure of ill-conditionedness rather than  $\gamma(G)$ .

## 2. MODELLING THE COLUMN

### 2.1 Steady-State Behavior

High-purity columns are known to be ill-conditioned at steady-state (see 2.1.2. below). However, for control purposes the steady-state is not as important as the behavior around cross-over (the frequency equal to the inverse of the closed-loop time constant). For most columns the condition number ( $\gamma(G)$ ) and  $\|RGA\|_1$  are smaller at higher frequencies (see below). Still, since the steady-state behavior generally reflects what happens at higher frequency, a good steady-state model is very useful. The linearized-steady state behavior in terms of  $u_1$  and  $u_2$  as manipulated variables for composition control is

$$\begin{pmatrix} dy_D \\ dx_B \end{pmatrix} = G_{u_1 u_2} \begin{pmatrix} du_1 \\ du_2 \end{pmatrix}, \quad G_{u_1 u_2} = \begin{bmatrix} \left(\frac{\partial y_D}{\partial u_1}\right)_{u_2} & \left(\frac{\partial y_D}{\partial u_2}\right)_{u_1} \\ \left(\frac{\partial x_B}{\partial u_1}\right)_{u_2} & \left(\frac{\partial x_B}{\partial u_2}\right)_{u_1} \end{bmatrix} \quad (3)$$

We will derive expressions for estimating  $G_{u_1 u_2}$  and show that the steady-state matrices for various configurations are closely related. For example, we can always write

$$G_{u_1 u_2} = G_{LV}M \quad (4)$$

The matrix  $M$  usually does not change much with operating conditions (while  $G_{LV}$  and  $G_{u_1 u_2}$  may change a lot).

### 2.1.1 Obtaining the Gain Matrix

Three alternative ways to find  $G_{u_1 u_2}$  are

1. Experimentally (not recommended) or numerically based on simulations.
2. Using analytic expressions.
3. From the gain matrix for another configuration (Eq. (4)).

Plant data may be used to adjust the number of theoretical trays in the simulation model, but we do not recommend using plant data for obtaining the gains. Simulation is generally the preferred approach. However, in this paper one goal is to obtain insight into how the gains vary by deriving simple analytical expressions. In particular, it is easy to derive simple consistency-relationships which may be used to correct data obtained by Method 1.

### 2.1.2 Internal and External Flows

The key to understanding the steady-state behavior of distillation columns is to understand the difference between external flows (products  $B$  and  $D$ ) and internal flows (“reflux”,  $L$  and  $V$ ) and how they affect composition (Rosenbrock, 1962). Changing the external flows will usually have a large effect on the product compositions and both compositions will change in the same “direction”. For example, an increase in  $B$  will make both products richer in the light component. This corresponds to making one product purer and the other product less pure. On the other hand, if the external flows are fixed, and only the internal flows  $L$  and  $V$  are changed, the effect on the compositions is much smaller and the compositions will change in opposite directions.

At steady state the effect of the external flows on the product compositions is found using  $\frac{D}{B} = \frac{z_F - x_B}{y_D - z_F}$  (5). This exact expression can be derived from an overall material balance for any component. Differentiating (5) with  $y_D$  constant yields

$$\left(\frac{dx_B}{x_B}\right)_{y_D} = -\frac{y_D - z_F}{x_B} d\left(\frac{D}{B}\right) \quad (6)$$

We see that the relative change in  $x_B$  is extremely sensitive to changes in  $D/B$  if  $x_B$  is small. Similar arguments apply to  $(1 - y_D)$ . Distillation columns with both products of high purity are always sensitive to changes in the external flows, and are therefore inherently ill-conditioned. They can only to a limited degree be made less ill-conditioned by changing the manipulated variables for composition control. (On the other hand, the RGA-values are strongly influenced by the choice of control configuration).

### 2.1.3. The Separation Factor $S$

In order to gain insight we need simple analytical expressions for the gain matrix which capture the essential part of the behavior. Changes in the external flows lead to large changes in the compositions, and it would be useful to find a function of the compositions which does not change very much in such cases. With such a function in hand we would be able to derive the main effect of the external flows on the compositions without having to know further details about the column. The separation factor ( $S$ ) turns out to be such a function (Shinsky, 1984).

$$S = \frac{y_D(1 - x_B)}{(1 - y_D)x_B} \quad (7)$$

For example, consider column A and C (Table 1). This is actually the same column, but operated at two different values of  $D/F$ .  $1 - y_D$  increases by a factor of 10 and  $x_B$  decreases

by a factor of 5. However, the change in  $S$  is "only" a factor of 2 (from 9990 to 4990). [The change in  $\ln S$ , which enters our formulas for the gains below, is from 9.19 to 8.51]. For  $S$  constant we derive

$$d \ln S = \frac{d(1 - y_D)}{y_D(1 - y_D)} + \frac{dx_B}{x_B(1 - x_B)} = 0 \quad (S \text{ constant}) \quad (8)$$

Since  $y_D$  and  $1 - x_B$  are close to 1, this means that the relative change of impurities in the top ( $1 - y_D$ ) and bottom ( $x_B$ ) of the column will be nearly equal, but with different signs. The assumption of constant  $S$  is obviously not appropriate in all cases, and in particular it is not if we consider changes in the internal flows. However, another advantage of using  $S$  is that simple shortcut models may often be expressed in terms of  $S$ . For constant relative volatility and infinite reflux Fenske's exact relationship applies,  $S = \alpha^N$  (9). Jafarey et al. (1979) derived an approximation for finite reflux which also has been adopted by Shinskey (1984). For liquid feed this expression is  $S = \alpha^N \left( \frac{1}{1 + D/Lz_F} \right)^{N/2}$  (10). A slightly different expression is derived by not introducing the approximation  $z_F = D/F$  used by Jafarey et al. :

$$S = \alpha^N \left( \frac{1}{1 + F/L} \right)^{N/2} \quad (\text{feed liquid}) \quad (11)$$

For the general case with no assumption regarding the feed state, and without introducing Jafarey's assumption  $N_T = N_B = N/2$ , we derive the following "nice" generalization to finite reflux of Fenske's formula (Jafarey's expression is much more complicated in this case)

$$S = \alpha^N \frac{(L/V)_T^{N_T}}{(L/V)_B^{N_B}} \quad (12)$$

which reduces to (11) if we assume  $N_T = N_B = N/2$  and feed liquid. The subscripts T and B denote the top and bottom of the column. Jafarey et al. (1979) checked the model's validity for estimating N. However, we are interested in using the model to estimate steady-state gains, and in this case it is the *derivatives* of S which are of interest (see Eq. (17) below). The effect on  $S$  of changes in the internal flows predicted from (11) is

$$\left( \frac{\partial \ln S}{\partial L} \right)_D = \left( \frac{\partial \ln S}{\partial V} \right)_D = \frac{N}{2} \frac{1}{L} \frac{F}{L + F} \quad (\text{feed liquid}) \quad (13)$$

which is reasonably accurate as seen from Table 3. However, from Table 4 we see that (11) describes poorly the effect of changes in the external material balance. Fortunately, it turns out that this is of less importance if the model for  $S$  is used to obtain estimates for the steady-state gains because the  $e_\zeta$ -term in (17) ususally dominates in this case. Surprisingly, the estimates obtained from the seemingly similar expressions (10) and (11) are often quite different (Table 4).

#### 2.1.4. Simple Gain Expressions

The total material balance for light component is

$$Fz_F = Dy_D + Bx_B \quad (14)$$

Let  $\zeta$  represent any manipulated input or disturbance. Differentiating (14) with respect to  $\zeta$  gives

$$D \frac{\partial y_D}{\partial \zeta} + B \frac{\partial x_B}{\partial \zeta} = e_\zeta \quad (15)$$

where  $e_\zeta$  is defined as

$$e_\zeta = -(y_D - x_B) \frac{\partial D}{\partial \zeta} + F \frac{z_F}{\partial \zeta} + (z_F - x_B) \frac{\partial F}{\partial \zeta} \quad (16)$$

Note that the value of  $\frac{\partial y_D}{\partial \zeta}$  depends heavily on the particular choice of manipulated variables ( $u_1$  and  $u_2$ ). Sometimes this is shown explicitly by writing, for example,  $(\frac{\partial y_D}{\partial u_1})_{u_2}$ . Analytic expressions for the gains are found by combining (15) and (7). We find for binary mixtures

$$\frac{1}{(1 - y_D)y_D} \frac{\partial y_D}{\partial \zeta} = \frac{1}{I_s} (e_\zeta + Bx_B(1 - x_B) \frac{\partial \ln S}{\partial \zeta}) \quad (17a)$$

$$\frac{1}{(1 - x_B)x_B} \frac{\partial x_B}{\partial \zeta} = \frac{1}{I_s} (e_\zeta - Dy_D(1 - y_D) \frac{\partial \ln S}{\partial \zeta}) \quad (17b)$$

where the "impurity sum" is defined as

$$I_s = Bx_B(1 - x_B) + Dy_D(1 - y_D) \quad (18)$$

For high-purity separations  $I_s \approx Bx_B + D(1 - y_D)$ . Except for the assumptions about binary mixture and two-product column, these expressions are exact and express in a compact form the major contributions to the steady-state gains. The important point is that the first term in (17a) and (17b) involving  $e_\zeta$  is often dominating (unless  $e_\zeta$  is identically zero as for changes in the internal flows). Furthermore, this "material balance" term can be evaluated exactly from the product compositions if constant molar flows are assumed. A good model for  $S$  is obviously needed for changes in the internal flows (with  $D$  and  $B$  constant). Gains for the  $LV$ -configuration obtained with (17) using the model (11) for  $S$  and assuming constant molar flows (Eq. 20) are given in Table 5. Corresponding RGA-values are given in Table 6.

### 2.1.5. Linearized flow relationships

To evaluate the term  $e_\zeta$  in (17) we need to know the effect of changes in  $d\zeta$  on  $dD$  (Eq. 16). In the following we do not include disturbances and consider changes in the manipulated flows only, ie.,  $\zeta$  is  $u_1$  or  $u_2$ . Then

$$e_{u_1} = -(y_D - x_B) \left( \frac{\partial D}{\partial u_1} \right)_{u_2}, \quad e_{u_2} = -(y_D - x_B) \left( \frac{\partial D}{\partial u_2} \right)_{u_1} \quad (19)$$

Constant molar flows. In this case a material balance over the top part of the column yields

$$dD = dV - dL \quad (20)$$

and, for example, with the  $LV$ -configuration we obtain  $e_L = -e_V = -(y_D x_B)$ .

General case. In the general case we need to introduce two additional parameters in (20)

$$dD = (1 - \epsilon_V)dV - (1 - \epsilon_L)dL \quad (21)$$

The parameters  $\epsilon_L$  and  $\epsilon_V$  are defined by (21). It is easily shown that  $\epsilon_L = (\partial V_T / \partial L)_V$  and physically represents the fraction of an increase in reflux  $L$  which returns to the condenser as overhead vapor. Similarly, it is easily shown that  $\epsilon_V = (\partial L_B / \partial V)_L$  ( $L_B$  is the liquid flow entering the reboiler) and  $\epsilon_V$  therefore represents the fraction of an increase in boilup  $V$  which returns to the reboiler as liquid. In general,  $\epsilon_L$  and  $\epsilon_V$  are most easily obtained

by nonlinear simulations; for the special case of constant molar flows they are identically zero.  $(\partial D/\partial u_1)_{u_2}$  and  $(\partial D/\partial u_2)_{u_1}$  are now easily found for any control configuration by combining (21) with the overall material balance,  $dB = -dD$  (22), and the material balance for the condenser,  $dV_T = dD + dL$  (23).

### 2.1.6. Consistency relationships between column elements

One important consequence of (15) is that it provides an exact consistency relationship between the *column elements* for any steady state gain matrix.

$$D \left( \frac{\partial y_D}{\partial u_1} \right)_{u_2} + B \left( \frac{\partial x_B}{\partial u_1} \right)_{u_2} = -(y_D - x_B) \left( \frac{\partial D}{\partial u_1} \right)_{u_2} \quad (24)$$

( $F$  and  $z_F$  are not influenced by changes in  $u_1$  or  $u_2$ ). The only assumption used when deriving (24) is that the column has two products such that  $dD = -dB$ . For example, for the *LV*- and *DV*- configurations we derive from (24) and (21)

$$\text{LV - configuration : } \begin{cases} Dg_{11} + Bg_{21} = (y_D - x_B)(1 - \epsilon_L) \\ Dg_{12} + Bg_{22} = -(y_D - x_B)(1 - \epsilon_V) \end{cases}$$

$$\text{DV - configuration : } \begin{cases} Dg_{11} + Bg_{21} = -(y_D - x_B) \\ Dg_{12} + Bg_{22} = 0 \end{cases}$$

Here  $G_{u_1 u_2} = \{g_{ij}\}$ . Similar relationships are easily derived for other configurations. Such relationships should always be used to check the gain matrix.

### 2.1.7. Consistency relationships between configurations (The matrix $M$ )

With the flow relationships (21)-(23) we can now easily evaluate the matrix  $M$  in Eq. (4) for any configuration. Writing  $dL$  and  $dV$  as a total differential of the independent variables  $du_1$  and  $du_2$  yields

$$\begin{pmatrix} dL \\ dV \end{pmatrix} = M \begin{pmatrix} du_1 \\ du_2 \end{pmatrix}, \quad M = \begin{bmatrix} \left( \frac{\partial L}{\partial u_1} \right)_{u_2} & \left( \frac{\partial L}{\partial u_2} \right)_{u_1} \\ \left( \frac{\partial V}{\partial u_1} \right)_{u_2} & \left( \frac{\partial V}{\partial u_2} \right)_{u_1} \end{bmatrix} \quad (25)$$

Introducing (25) into Eq. (3),  $\begin{pmatrix} dy_D \\ dx_B \end{pmatrix} = G_{u_1, u_2} \begin{pmatrix} du_1 \\ du_2 \end{pmatrix} = G_{LV} \begin{pmatrix} dL \\ dV \end{pmatrix}$ , yields  $G_{u_1, u_2} = G_{LV}M$  (Eq. 2). For any choice of  $u_1$  and  $u_2$ ,  $M$  is easily obtained using (21) - (23) above. Some examples are given in Table 7.

### 2.1.8. Estimates of the RGA

Using the gain expressions in 2.1.4 the following approximation may be derived for the 1-1-element in the RGA for various configurations:

$$\lambda_{11}(G_{LD}) \approx 1 - \lambda_{11}(G_{DV}) \approx \frac{1}{1 + \frac{D(1-y_D)}{Bx_B}} \quad (\text{Shinsky, 1967}) \quad (26)$$

$$\lambda_{11}(G_{LV}) \approx \frac{1}{Bx_B + D(1-y_D)} \frac{1}{(\partial \ln S / \partial L)_D} \quad (27)$$

$$\lambda_{11}(G_{\frac{L}{B} \frac{V}{B}}) \approx \frac{\lambda_{11}(G_{LV})}{1 + \frac{L}{D} + \frac{V}{B}} \quad (28)$$

$\lambda_{11}$  for the LD- and DV-configuration is always less than one (this is the case even for high-purity columns which generally are ill conditioned with large values of  $\gamma(G)$ ) and these

configurations are therefore not sensitive input uncertainty (Skogestad and Morari, 1986).  $\lambda_{11}$  for the LV-configuration is large if both products are of high purity which will result in a system which is sensitive to uncertainty.  $\lambda_{11}$  for the  $\frac{L}{V} \frac{D}{B}$ -configuration may be significantly smaller than for the LV-configuration if the reflux is large.

## 2.2. Dynamic Behavior

The external and internal flows often have drastically different effects on the steady-state behavior, and it is not surprising that they also in such cases give very different dynamic responses. The time constant for changes in the external flows (the dominant time constant,  $\tau_1$ ) is generally significantly larger than the time constant for changes in the internal flows ( $\tau_2$ ). In particular, this is the case for columns with both products of about equal purity. The response to changes in internal flows is therefore faster than the response to changes in the external flows. The consequence is that the condition number of the plant ( $\gamma(G)$ ) is often much lower at high frequencies than at steady-state, and the column may not be as difficult to control as we may expect from studying the steady-state alone. Below, we will first discuss simple methods for estimating the dominant time constant ( $\tau_1$ ) and then present a simple model which explicitly takes the difference between internal and external flows into account.

### 2.2.1. The Dominant Time Constant

The dynamics of most distillation columns are dominated by a first order response with time constant  $\tau_1$ . This dominant time constant is nearly the same, regardless of where a disturbance is introduced or where composition is measured. This is well known both from plant measurements (McNeill and Sachs, 1969) and from theoretical studies (Moczek et al., 1963). Furthermore,  $\tau_1$  is generally close to the time constant ( $\tau_{1e}$ ) corresponding to the smallest eigenvalue of the linearized model. Because of nonlinearity the numerical value of  $\tau_1$  often varies drastically with operating conditions (Fig. 2). Levy et al. (1969) showed that the dominant slowest model primarily involves composition effects and is nearly unaffected by flow dynamics. This leads to the conclusion that the dominating dynamics can be captured by considering the composition dynamics only.

A simple approximation for  $\tau_1$  is obtained by considering the total holdup of each component in the column. By assuming that all trays have the same response, this directly leads to a first order model, and the dominating time constant can be estimated. Moczek et al. (1963) used this idea to introduce the “inventory time constant” for a column going from one steady state (subscript 0) to another (subscript f)

$$\tau_{1c} = \frac{\Delta(\sum M_i x_i)}{D_f \Delta y_D + B_f \Delta x_B} = \frac{\text{“change in holdup of one component (mol)”}}{\text{“imbalance in supply of this component (mol/s)”}} \quad (29)$$

Here  $\Delta$  represents the change, for example,  $\Delta D = D_f - D_0$ . Note that (29) does not apply to changes in the internal flows ( $\Delta D = \Delta B = 0$ ) because the denominator in (29),  $D_f \Delta y_D + B_f \Delta x_B = \Delta(Fz_F) - y_{D0} \Delta D - x_{B0} \Delta B$ , is zero in this case. In spite of the excellent agreement often found between  $\tau_{1c}$  (29) and  $\tau_1$  from the actual response (Moczek et al., 1963) this expression does not seem to have found widespread use. Table 8 compares  $\tau_{1c}$  (29) for the case  $\Delta \rightarrow 0$  with  $\tau_{1e}$ . The agreement is amazing for columns with both products of high purity.

From Table 8 we see that very large time constants are found for small perturbations to columns with both products of high purity. The reason is that the compositions inside the column may change significantly (large change in component holdup,  $\sum M_i x_i$ ), while the product compositions may almost be unchanged (resulting in a small value of  $D \Delta y_D + B \Delta x_B$ ).

$\tau_{1c}$  may be evaluated easily (and accurately) using a steady-state simulation program if the total holdup ( $M_i$ ) on each stage is known. (Two simulations are needed). However, to

gain insight, the following analytical shortcut formula, which is based on (29),  $S$  constant, and a linearized plant is useful:

$$\tau_{1c} = \tau_{1cI} + \tau_{1cD} + \tau_{1cB} \approx \frac{M_I}{I_s \ln S} + \frac{M_D(1 - y_D)}{I_s} + \frac{M_B x_B}{I_s} \quad (30)$$

Here  $M_I = \sum M_i$  is the total holdup *inside* the column and  $M_D$  and  $M_B$  are the condenser and reboiler holdups (in kmol).  $I_s$  is defined in (18). The time constant given by (30) is for the linearized plant and may therefore be misleading for large perturbations. From Table 8 we see that the estimate (30) is excellent for columns with high reflux and with both products of high purity (large values of  $\lambda_{11}(G_{LV})$ ). However, the main value of (30) is the insight it gives into the dynamic behavior: Since  $\ln S$  usually does not change much with operating conditions, the value of  $\tau_{1c}$  is mainly determined by  $I_s$ , which again is determined by the composition of the least pure product:

$$I_s \approx \begin{cases} D(1 - y_D) & \text{if distillate least pure} \\ Bx_B & \text{if bottoms least pure} \end{cases} \quad (31)$$

$\tau_1$  reaches its maximum value approximately when both products have equal purity (Fig. 2). Case studies using linearized models and assuming equal purities are often presented in the academic literature, and the reported values for the time constants are therefore often misleading. For example, this assumption is used by Wahl and Harriot (1971); the figure they present for estimating time constants is therefore of very limited value. In practice, for such columns, any disturbance or input to the column will take the column to a new steady state where one of the products is less pure, and the actual time constant will be smaller than found by linear analysis.

Example. Column A has  $x_B = 1 - y_D = 0.01$ ,  $\ln S = 9.19$  and  $I_s = x_B y_D = 0.0099$ . With  $M_i/F = M_D/F = M_B/F = 1$  min we get from (30),  $\tau_{1c} = \frac{39}{0.0099 \cdot 9.19} + 1.0 + 1.0 = 430$  min. Assume there is a change in operating conditions ( $D/F$  changes from 0.5 to 0.555) such that  $x_B = 0.1$  and  $1 - y_D = 0.002$  (column C in Table 1). Then  $\ln S = 8.51$  is almost unchanged but  $I_s = 0.0510$  increases about five times. We find  $\tau_{1c} = \frac{39}{0.051 \cdot 8.51} + 1.96 + 0.04 = 92$  min. Thus, the nominal value of the time constant (430 min) holds only for very small perturbations from the nominal steady-state.

### 2.2.2. Simplified Dynamic Model

The reflux ( $L$ ) and boilup ( $V$ ) are the only flows which immediately affect the product compositions. The effect of the product flows ( $D$  and  $B$ ) depends heavily on how the level loops are tuned. In order to avoid this dependency on the level loops, we prefer to write the dynamic distillation model in terms of  $L$  and  $V$ . However, to model explicitly the difference in dynamic behavior between internal and external flows, we will first neglect the effect of flow dynamics and level loops. Let  $\{g_{ij}\}$  in the following denote the elements of  $G_{LV}$ . Then at steady state we have

$$\begin{pmatrix} dy_D \\ dx_B \end{pmatrix} = \begin{pmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{pmatrix} \begin{pmatrix} dL \\ dV \end{pmatrix} \quad (32)$$

Written in terms of  $D$  and  $V$  as manipulated variables (assuming constant molar flows):

$$\begin{pmatrix} dy_D \\ dx_B \end{pmatrix} = \begin{pmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{pmatrix} \begin{pmatrix} dV - dD \\ dV \end{pmatrix} = \begin{pmatrix} g_{11} + g_{12} & -g_{11} \\ g_{21} + g_{22} & -g_{21} \end{pmatrix} \begin{pmatrix} dV \\ dD \end{pmatrix} \quad (33)$$



Next, assume that the effect of changes in external flows ( $D$ ) is given by a first-order response with time constant  $\tau_1$ , and the effect of changes in internal flows (change  $V$  keeping  $D$  constant) has time constant  $\tau_2$ . Then

$$\begin{pmatrix} dy_D \\ dx_B \end{pmatrix} = \begin{pmatrix} g_{11} + g_{12} & -g_{11} \\ g_{21} + g_{22} & -g_{21} \end{pmatrix} \begin{pmatrix} dV/(1 + \tau_2 s) \\ dD/(1 + \tau_1 s) \end{pmatrix} \quad (34)$$

Switching back to  $L$  and  $V$  as manipulated input (still assuming immediate flow responses such that  $dD = dV - dL$ ):

$$dy_D = \frac{g_{11}}{1 + \tau_1 s} dL + \left( \frac{g_{11} + g_{12}}{1 + \tau_2 s} - \frac{g_{11}}{1 + \tau_1 s} \right) dV \quad (35a)$$

$$dx_B = \frac{g_{21}}{1 + \tau_1 s} dL + \left( \frac{g_{21} + g_{22}}{1 + \tau_2 s} - \frac{g_{21}}{1 + \tau_1 s} \right) dV \quad (35b)$$

$\tau_1$  may be estimated using (29) or (30).  $\tau_2$  may be estimated by matching the RGA-values at high frequency (Skogestad and Morari, 1987d). We find

$$\frac{\tau_2}{\tau_1} \approx \frac{L}{F} \left( \frac{g_{12}}{g_{11}} - \frac{g_{22}}{g_{21}} \right) + \left( 1 + \frac{g_{12}}{g_{11}} \right) \quad (36)$$

$\tau_2$  may also be obtained from simulations (without flow dynamics) of changes in the internal flows. In most cases it will be very difficult to obtain  $\tau_2$  using plant data, since it is almost impossible, in practice, to avoid changes in the external flows (because of disturbance in feed rate, boilup, etc.).

Example. Consider again column A. Neglecting flow dynamics (i.e., assume constant holdup ( $M_i$ ) on all stages) results in a dynamic model with 41 states. The effect of a small change in external and internal flows is shown in Fig. 3 for the case with  $M_i/F = 0.5$  min, on all stages (including reboiler and condenser). The responses are almost identical to those found using the simple linear model (35) with  $\tau_1 = 194$  min and  $\tau_2 = 15$  min (The estimates found using (30) and (36) are  $\tau_1 = 215$  min and  $\tau_2 = 18$  min):

$$\begin{pmatrix} dy_D \\ dx_B \end{pmatrix} = \frac{1}{1 + 194s} \begin{pmatrix} 0.878 & -0.864 \frac{1+12.1s}{1+15s} \\ 1.082 & -1.096 \frac{1+17.3s}{1+15s} \end{pmatrix} \begin{pmatrix} dL \\ dV \end{pmatrix} \quad (37)$$

In Fig. 4  $\|RGA\|_1$  is plotted as a function of frequency. The RGA-elements are large at low frequency ( $\lambda_{11} \approx 35$  at steady-state), but much smaller at high frequency ( $|\lambda_{11}| \approx 3.5$  at  $\omega = 0.1$ ). The simple model (37) captures this behavior very well. The column is therefore not as difficult to control as might be expected from the steady-state data.

Note that without the “correction terms”  $\frac{1+12.1s}{1+15s}$  and  $\frac{1+17.3s}{1+15s}$  the plant (37) would have the same condition number (and RGA) at all frequencies. Each transfer matrix element is often appreciated by a first-order lag with time delay ( $ke^{-\theta s}/1 + \tau s$ ). It is clear that unless special care is taken, it is extremely unlikely that such a model will capture the change in condition number with frequency.

We know that the time constant of 194 min used in (37) only holds for small perturbations from the nominal steady-state. Similarly, the steady-state gains vary a lot with operating conditions. However, there turns out to be a strong correlation between these such that the column behavior at higher frequencies (“initial response”) changes much less. This is illustrated by the plots of  $\|RGA\|_1$  for columns A and C in Fig. 4. Since the model

(37) correctly predicts the behavior at high frequency it is useful also for large perturbations from the nominal steady state (Skogestad and Morari, 1987cd).

### 3. CHOICE OF CONTROL CONFIGURATION

The most important step when designing a composition control system is probably to choose which two independent combinations of  $L, V, D$  and  $B$  to use for composition control, yet this issue is seldom addressed in the academic literature. Some guidelines are given in the industrial literature, in particular by Shinskey (1984), who uses the RGA for the various configurations as an important tool. However, although his conclusions may be correct, his interpretations are probably not: From Shinskey's book the reader is led to believe that the RGA is useful because it provides a measure of interactions when using a decentralized controller. His rule (though he does not express it explicitly) is to choose a configuration with  $\lambda_{11}$  in the range of about 0.9 to 4 (Shinskey, 1984, Table 5.2). If  $\lambda_{11}$  were used only as an interaction measure this recommendation would not make any sense; in this case  $\lambda_{11}$  should be chosen to be as close to one as possible and  $\lambda_{11} = 0.67$  would be almost equivalent to  $\lambda_{11} = 2$  (both have  $|\frac{g_{12}g_{21}}{g_{11}g_{22}}| = 0.5$ ). Consequently, Shinskey's use of the RGA is a way of categorizing his experience on distillation columns, rather than expressing the effect of interactions. In fact, his rules also seem to apply when a multivariable controller is used. His recommendations regarding the RGA should therefore only be used for distillation columns. In another paper (Skogestad and Morari, 1987a) we treat the subject of distillation configuration selection in detail and provide justification for Shinskey's RGA-rules. We do not have space to discuss these issues in detail in the present paper, and we will only focus the attention on two points which may be of particular interest:

1. A linear analysis shows, for example, that if a multivariable controller is used, then there is not much difference in the expected performance of the  $\frac{L}{V}$ - and  $LV$ -configurations. On the other hand, the  $\frac{L}{D}$ -configuration may have entirely different properties.
2. Table 9 gives the effect on  $D/B$  for various disturbances and demonstrates fundamental differences in the "open-loop" properties of various configurations.

#### 3.1. Linear interpretation of ratio schemes

Ratio control systems have been used in the industry for at least forty years (Rademaker et al. (1975), p. 445). Yet, almost no discussion is found in the literature on why such schemes may be beneficial. The simplest justification for using ratios as inputs follows from steady-state considerations: To keep the compositions constant, the ratio  $\frac{L}{V}$  inside the column (slope of the operating line on the McCabe-Thiele diagram) should be constant. Intuitively, it seems that some disturbances may be counteracted by keeping this ratio constant. However, this is somewhat misleading since the effect of using a given ratio depends entirely on which second manipulated variable is chosen for composition control. For example, we can easily show using an analysis similar to the one given below for the  $\frac{L}{D}\frac{V}{B}$ -configuration, that there is almost no difference between the  $\frac{L}{V}$ - and  $LV$ -configurations if a multivariable controller is used. We adopt the following convention: Writing the ratio between  $L$  and  $V$  as  $\frac{L}{V}$  means that  $L$  is manipulated to change  $\frac{L}{V}$ , and writing  $\frac{V}{L}$  means that  $V$  is manipulated to change the ratio.

Exampe.  $\frac{L}{D}\frac{V}{B}$ -Configuration. The  $\frac{L}{D}\frac{V}{B}$ -configuration is claimed by Shinskey (1984) to be applicable over the broadest range of conditions and also Rademaker et al. (1985, p. 463) recommend using ratio schemes. According to the convention introduced above  $dL$  and  $dV$  are manipulated to change the ratios  $L/D$  and  $V/B$ . Linear analysis gives

$$dL = Dd\left[\frac{L}{D}\right] + \frac{L}{D}dD \quad dV = Bd\left[\frac{V}{B}\right] + \frac{V}{B}dB \quad (38)$$

Consequently,  $dL$  and  $dV$  depend on  $d[\frac{L}{D}]$  and  $d[\frac{V}{B}]$  (which are “manipulated” based on the compositions  $y_D$  and  $x_B$ ) and on the flow rate changes  $dD$  and  $dB$ . Because  $L$  and  $V$  are manipulated for composition control, the condenser and reboiler levels should be controlled using  $D$  and  $B$ , i.e.,

$$dD = c_D(s)dM_D, \quad dB = c_B(s)dM_B \quad (39)$$

(The SISO controllers  $c_D(s)$  and  $c_B(s)$  are in many cases simple proportional controllers). Let the composition controller (possibly multivariable) be

$$\begin{pmatrix} d\frac{L}{D} \\ d\frac{V}{B} \end{pmatrix} = \hat{K} \begin{pmatrix} dy_D \\ dx_B \end{pmatrix} \quad (40)$$

and define

$$K = \begin{pmatrix} D & 0 \\ 0 & B \end{pmatrix} \hat{K} \quad (41)$$

The overall controller found by combining (38)-(41) is

$$\begin{bmatrix} dL \\ dV \\ dD \\ dB \end{bmatrix} \begin{bmatrix} K & \frac{L}{D}c_D & 0 \\ 0 & 0 & \frac{V}{B}c_B \\ 0 & 0 & c_D \\ 0 & 0 & 0 & c_B \end{bmatrix} \begin{bmatrix} dy_D \\ dx_B \\ dM_D \\ dM_B \end{bmatrix} \quad (42)$$

We see from (42) that the flow rates  $L$  and  $V$  are manipulated based both on the product compositions ( $y_D$  and  $x_B$ ), and on the levels ( $M_D$  or  $M_B$ ). Furthermore, the two SISO level controllers ( $c_D(s)$  and  $c_B(s)$ ) each manipulate two flow rates, and therefore appear at two places in the transfer matrix for the overall controller. Thus, the use of ratios as manipulated variables introduce in an ad-hoc manner a multivariable control system.

In most cases the major effect of using ratios for composition control is captured by the linear analysis. Ratios do not tend to correct the nonlinear behavior of distillation columns because the manipulated inputs vary only moderately with operating conditions (neglecting startup). On the other hand, the product compositions do often vary significantly with operating conditions, and a significant “linearization” effect may be obtained, for example, by using  $\ln(1 - y_D)$  and  $\ln x_B$  as “redefined” controlled outputs (Skogestad and Morari, 1987cd).

### 3.2. Rejection of Flow Disturbances

Table 9 gives in a compact form the “open-loop” effect (the two manipulated inputs for composition control are constant) of various flow disturbances on  $D/B$  at steady state. Variations in  $D/B$  have a large effect on product compositions and should be avoided. Configurations with small entries in Table 9 are therefore preferable. In particular, this is the case if manual control is used, but it also applies to two-point control. For example, Table 9 shows that the effect of disturbances in  $L$ ,  $V$  and feed enthalpy on  $D/B$  is reduced by a factor of about  $1 + \frac{L}{D} + \frac{V}{B}$  for the  $\frac{L}{D}\frac{V}{B}$ -configuration compared to the  $LV$ -configuration, and in addition the ratio configuration is insensitive to disturbances in the feed rate (this last observation is obvious, since at steady state a disturbance in feed rate ( $F$ ) is perfectly rejected by adjusting the flows keeping all flow ratios constant).

## 4. CONTROLLER DESIGN / IMPLEMENTATION

Given a good model of the plant, and with an appropriate choice of control configuration, the controller design itself should not be too difficult. Firstly, the level control system

has to be designed ("closing the material balance"). This is usually reasonably straightforward once the choice of configuration has been made. Secondly, the  $2 \times 2$  controller (K) for composition control must be designed. The academic literature has discussed for years whether to use "decouplers" as part of the controller. Recent work (Skogestad and Morari, 1986) shows clearly that decouplers should only be used for plants with small RGA-elements. For distillation column control, this implies that decouplers should not be used for the LV-configuration, but may be helpful in improving the response if, for example, the DV-configuration is used. The presence of input uncertainty is important for these conclusions.

An important issue with regard to implementation is composition measurements. Often such measurements are not available for on-line control, or the measurements are delayed. In such cases temperature and pressure measurements at different locations in the column are used to estimate the product compositions. A fairly sophisticated non-linear model may be needed to obtain acceptable estimates. This may be a tray-by-tray model of the column, or correlations between temperature profiles, pressure and composition obtained from simulations and plant measurements. The temperatures may also be used directly by the controller. However, this results in a more complicated controller, and may not give as good results as using a separate "estimator", since the controller is usually restricted to being linear.

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Fig. 1. Distillation column with 5 manipulated inputs ( $L, V, D, B$  and  $V_T$ ) and 5 controlled outputs ( $y_D, x_B, M_D, M_B$  and  $p$ ). At steady state in the case of constant molar  $L_B = L + q_F F$  and  $V_T = V + (1 - q_F)F$ , where  $q_F$  is the fraction of liquid in the feed.

Fig. 2. Typical plot of linearized time constant as a function of product composition (Plot derived by varying feed composition ( $z_F$ ) with all flows fixed). See Skogestad and Morari (1987b).

Fig. 3. Column A. Responses to small change in external (A) and internal flows (B). Dotted lines: Approximation with simplified model (37) ( $M_i/F=0.5$  min).

Fig. 4.  $\|RGA\|_1$  as a function of frequency ( $M_i/F = 0.5$  min). Dotted line: Approximation for column A with model (37).

Column	$z_F$	$\alpha$	$N$	$N_F$	$y_D$	$x_B$	$\frac{D}{F}$	$\frac{L}{F}$	$\frac{N}{N_{min}}$	$\frac{R}{R_{min}}$
A	0.5	1.5	40	21	0.99	0.01	0.500	2.706	1.76	1.388
B	0.1	"	"	"	"	"	0.092	2.329	"	1.301
C	0.5	"	"	"	0.90	0.002	0.555	2.737	1.93	1.645
D	0.65	1.12	110	39	0.995	0.10	0.614	11.862	1.66	1.529
E	0.2	5	15	5	0.9999	0.05	0.158	0.226	1.99	1.144
F	0.5	15	10	5	0.9999	0.0001	0.500	0.227	1.47	3.183
G	0.5	1.5	80	40	0.9999	0.0001	0.500	2.635	1.76	1.318

Table 1. Steady-state data for distillation column examples. All columns have liquid feed ( $q_F = 1$ ).

Column	$G_{LV}$	$G_{DV}$	$G_d (F, z_F)$
A	$\begin{pmatrix} 0.878 & -0.864 \\ 1.082 & -1.096 \end{pmatrix}$	$\begin{pmatrix} -0.878 & 0.014 \\ -1.082 & -0.014 \end{pmatrix}$	$\begin{pmatrix} 0.394 & 0.881 \\ 0.586 & 1.119 \end{pmatrix}$
B	$\begin{pmatrix} 1.748 & -1.717 \\ .9023 & -.9054 \end{pmatrix}$	$\begin{pmatrix} -1.748 & .03088 \\ -0.9023 & -.00312 \end{pmatrix}$	$\begin{pmatrix} 0.0858 & 1.636 \\ 0.0904 & 0.936 \end{pmatrix}$
C	$\begin{pmatrix} 1.604 & -1.602 \\ .01865 & -.02148 \end{pmatrix}$	$\begin{pmatrix} -1.604 & .00227 \\ -.01865 & -.00283 \end{pmatrix}$	$\begin{pmatrix} 0.8822 & 1.790 \\ 0.0197 & 0.0170 \end{pmatrix}$
D	$\begin{pmatrix} 0.1231 & -0.1211 \\ 2.126 & -2.129 \end{pmatrix}$	$\begin{pmatrix} -0.1231 & .00192 \\ -2.126 & -.00307 \end{pmatrix}$	$\begin{pmatrix} 0.0516 & 0.1252 \\ 1.344 & 2.395 \end{pmatrix}$
E	$\begin{pmatrix} .02033 & -.01315 \\ 1.124 & -1.126 \end{pmatrix}$	$\begin{pmatrix} -.02033 & .00719 \\ -1.124 & -.00135 \end{pmatrix}$	$\begin{pmatrix} .00045 & .00949 \\ 0.1780 & 1.186 \end{pmatrix}$
F	$\begin{pmatrix} 1.074 & -1.073 \\ 0.9257 & -0.9267 \end{pmatrix}$	$\begin{pmatrix} -1.074 & .000998 \\ -0.9257 & -.000998 \end{pmatrix}$	$\begin{pmatrix} 0.5362 & 1.073 \\ 0.4636 & 0.9269 \end{pmatrix}$
G	$\begin{pmatrix} 0.8649 & -0.8646 \\ 1.135 & -1.135 \end{pmatrix}$	$\begin{pmatrix} -0.8649 & .000294 \\ -1.135 & -.000294 \end{pmatrix}$	$\begin{pmatrix} 0.4315 & 0.8647 \\ 0.5683 & 1.135 \end{pmatrix}$

Table 2. Steady-state gain matrices obtained by linearization.  $G_{DV} = G_{LV}M$  with  $M = \begin{pmatrix} -1 & 1 \\ 0 & 1 \end{pmatrix}$ .

Column	Exact	Estimates	
		(11)	Jafarey (10)
A	2.76	1.99	1.99
B	3.44	2.58	2.43
C	1.44	1.96	2.11
D	0.42	0.36	0.34
E	71.9	27.1	25.8
F	20.0	18.0	18.0
G	5.87	4.18	4.18
Average error		29.2%	32.2%

Table 3. Estimates of  $(\partial \ln S / \partial V)_D = (\partial \ln S / \partial L)_D$ . This represents the effect on  $S$  of changing the internal flows.

	Column A			Column B		
	Exact	(11)	Jafarey(10)	Exact	(11)	Jafarey(10)
$(\partial \ln S / \partial L)_V$	-21.1	1.99	12.8	85.3	2.58	64.0
$(\partial \ln S / \partial V)_L$	23.9	0	-10.8	-81.9	0	-61.6

Table 4. The shortcut models (10) and (11) yield poor estimates for the effect on  $S$  of changing the external flows.



Column	Exact	Estimated (11)
$A$	$\begin{pmatrix} 0.878 & -0.864 \\ 1.082 & -1.096 \end{pmatrix}$	$\begin{pmatrix} 0.990 & -0.980 \\ 0.970 & -0.980 \end{pmatrix}$
$B$	$\begin{pmatrix} 1.748 & -1.717 \\ .9023 & -.9054 \end{pmatrix}$	$\begin{pmatrix} 1.0032 & -0.9800 \\ 0.9777 & -0.9800 \end{pmatrix}$
$C$	$\begin{pmatrix} 1.604 & -1.602 \\ .01865 & -.02148 \end{pmatrix}$	$\begin{pmatrix} 1.5940 & -1.5909 \\ .03145 & -.03528 \end{pmatrix}$

Table 5. Estimates of  $G_{LV}$  obtained from gain expression (17) with shortcut model (11) for  $S$ .

Column	$\lambda_{11}$ , Exact	$\lambda_{11}$ , Estimated	
		(11)	Jafarey (10)
A	35.1	50.3	50.0
B	47.5	39.3	61.0
C	7.53	9.06	3.58
D	58.7	66.7	85.9
E	2.82	1.87	3.35
F	499	558	558
G	1673	2394	2394
Average error		26.2%	36.5%

Table 6. RGA-values for the LV-configuration. The estimated values are obtained from the gains (17) using shortcut models for  $S$ . Note that the values of Jafarey are used in the book of Shinskey (1984).

$u_1$	$u_2$	M
$L$	$V$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
$L$	$D(-B)$	$\begin{bmatrix} 1 & 0 \\ \frac{1-\epsilon_L}{1-\epsilon_V} & \frac{1}{1-\epsilon_V} \end{bmatrix}$
$D(-B)$	$V$	$\begin{bmatrix} -\frac{1}{1-\epsilon_L} & \frac{1-\epsilon_V}{1-\epsilon_L} \\ 0 & 1 \end{bmatrix}$
$D$	$V/B$	$\begin{bmatrix} -\frac{1+(1-\epsilon_V)\frac{V}{B}}{1-\epsilon_L} & B\frac{1-\epsilon_V}{1-\epsilon_L} \\ -\frac{V}{B} & B \end{bmatrix}$
$L/D$	$V$	$\begin{bmatrix} \frac{D}{1+\frac{L}{B}(1-\epsilon_L)} & \frac{\frac{L}{B}(1-\epsilon_V)}{1+\frac{L}{B}(1-\epsilon_L)} \\ 0 & 1 \end{bmatrix}$
$L/D$	$V/B$	$\frac{1}{1+\frac{L}{B}(1-\epsilon_L)+\frac{V}{B}(1-\epsilon_V)} \begin{bmatrix} D(1+\frac{V}{B}(1-\epsilon_V)) & B\frac{L}{B}(1-\epsilon_V) \\ D\frac{V}{B}(1-\epsilon_L) & B(1+\frac{L}{B}(1-\epsilon_L)) \end{bmatrix}$

Table 7. Matrix M for some choices of manipulated inputs ( $u_1$  and  $u_2$ ).  $G_{u_1, u_2} = G_{LV}M$ .

<i>Column</i>	<i>Eigenvalue</i> $\tau_{1e}$	<i>Estimated</i> $\tau_{1c}(29)$	<i>Estimated shortcut</i> $\tau_{1c}(30)$
$A$	388	387	430
$B$	500	504(496)	430
$C$	49	58(60)	92
$D$	308	300(299)	388
$E$	165	142(95)	30
$F$	5992	5992	4888
$G$	40667	40664	42891

Table 8. Linearized dominant time constant (min). Values for  $\tau_{1c}$  (29) are given for small perturbations in L ( $\Delta L \rightarrow 0$ ) with V constant (and, if different, for small change in V with L constant in parantheses). All columns:  $M_i/F = M_D/F = M_B/F = 1$  min.

Configuration ( $u_1, u_2$ )	$dF$	Disturbance d		
		$dV_d - dL_d - Fdq_F$	$dD_d$	$dB_d$
$LV, \frac{L}{V}V$	$k(1 - q_F - D/F)$	$k$	0	0
$\frac{L}{D} \frac{V}{B}$	0	$\frac{k}{1 + L/D + V/B}$	$\frac{kL/D}{1 + L/D + V/B}$	$\frac{-kV/B}{1 + L/D + V/B}$
$\frac{L}{D}V$	$-k \frac{V/F}{1 + L/D}$	$\frac{k}{1 + L/D}$	$\frac{kL/D}{1 + L/D}$	0
$L \frac{V}{B}$	$k \frac{L/F}{1 + V/B}$	$\frac{k}{1 + V/B}$	0	$-\frac{kV/B}{1 + V/B}$
$DX$	$-kD/F$	0	$k$	0
$BX$	$kB/F$	0	0	$-k$

$q_F$  - fraction of liquid in feed

$X$  - denotes any other manipulated input ( $L, V, \frac{L}{D}$ , etc.) except  $D, B$  and  $\frac{D}{B}$ .

$$k = (1 + D/B)/B = F/B^2$$

subscript  $d$  denotes an additive disturbance on this flow.

Table 9.  $(\frac{\partial D/B}{\partial d})_{u_1, u_2}$  = Linearized effect of flow disturbances on  $D/B$  when both composition loops are open. Applies to steady state and constant molar flows.