ANALYSIS AND CONTROL OF DISTILLATION COLUMNS

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The emphasis in this paper is on the analysis of the dynamic behavior of distillation columns. The dynamic behavior of a distillation column is approximated with a two time constant model.

$$\begin{pmatrix} dy_D \\ dx_B \end{pmatrix} = \begin{pmatrix} \frac{g_{11}}{1+\tau_1 s} & \left(\frac{g_{11}+g_{12}}{1+\tau_2 s} - \frac{g_{11}}{1+\tau_1 s}\right) \\ \frac{g_{21}}{1+\tau_1 s} & \left(\frac{g_{21}+g_{22}}{1+\tau_2 s} - \frac{g_{21}}{1+\tau_1 s}\right) \end{pmatrix} \begin{pmatrix} dL \\ dV \end{pmatrix} \tag{7}$$

where g_{ij} is the steady-state gain. The response to changes in the external flows is approximately first order with time constant τ_1. This dominant time constant can be estimated using a simple mixing tank model for the column (Moczek et al., 1963).

$$\tau_1 \approx \frac{\Delta(\Sigma_{i=1}^{N} M_i x_i)}{D_f \Delta y_D + B_f \Delta x_B} = \frac{\text{“change in component holdup inside column(mol)”}}{\text{“imbalance in supply of this component (mol/s)”}} \tag{1a}$$

Here Δ represents the difference between two steady states. For high purity binary separations and small pertubations to the column (linear model valid, $\Delta \to 0$) Skogestad and Morari (1987) have derived an analytical expression for τ_1 from (1a)

$$\tau_1 \approx \frac{M_I}{\ln S \cdot I_s} + \frac{M_D y_D(1-y_D)}{I_s} + \frac{M_B x_B(1-x_B)}{I_s} \tag{1b}$$

Here M_I is the total holdup inside the column, and M_D and M_B are the condenser and reboiler holdups. $S = \frac{y_D(1-x_B)}{(1-y_D)x_B}$ is the separation factor and $I_s = Dy_D(1-y_D) + Bx_B(1-x_B)$ is the “impurity sum”. Note that I_s may be extremely small for columns with both products of high purity ($1-y_D$ and x_B both small) resulting in very large linearized values of τ_1.

The response to changes in the internal flows (keep D and B constant) is also first order, but its time constant τ_2 is generally significantly smaller than τ_1. It can be estimated by matching the initial response.

$$\text{Feed liquid}: \quad \left(\frac{\tau_2}{\tau_1}\right) = \frac{L}{F}\left(\frac{g_{12}}{g_{11}} - \frac{g_{22}}{g_{21}}\right) + \left(1 + \frac{g_{12}}{g_{11}}\right) \tag{24}$$

The main advantage of the simple model (7) is that it gives a good description of both the low- and high-frequency behavior of distillation columns. Such models were not available in the literature. The traditional approach has been to use a model which matches the steady-state gains, but which is not necessarily accurate for high-frequencies. The other extreme is to match the high-frequency gains (Rademaker et al., 1975, p. 137). (7) provides a link between the low- and high-frequency regions.

In the paper it is shown that the high-frequency behavior is generally much less effected by changing operating conditions than the steady-state. This partially explains why highly nonlinear distillation columns may be controlled satisfactory using linear controllers. In particular, the initial response is nearly independent of operating conditions if relative (logarithmic) compositions are used. This suggests that $\ln(1-y_D)$ and $\ln x_B$ should be used as controlled outputs for columns where y_D or x_B may vary significantly.

1. INTRODUCTION

How should we go about designing a system for composition control of a distillsation column? Three separate steps are involved: 1) Modelling the column. 2) Choice of control configuration. 3) Controller design/implementation. Step 2 involves choosing the two controlled variables which should be used for composition control, and it probably the most important step in the procedure above. Traditionally, the LV-configuration has been preferrded (Using reflux L and boilup V for composition control), but the use of "direct material balance control" (eg., the DV-configuration) has also been proposed for a number of years. More recently, the ratio configurations, in particular the $\frac{L}{D}\frac{V}{B}$-configuration has been proposed as most apllicable over the broadest range of columns (Shinskey, 1984). In another paper (Skogestad and Morari, 1987b) we discuss the issue of control configuration selection in detail. Unfortunately, we do not have space to discuss these issues here, and this paper will instead be devoted to the modelling issues. For control purposes, it is important to have a simple model, but which at the same time gives a good description of the elements which may restrict the controllability of the process. In particular, this includes issues like inverse responses and time delays (RHP-zeros) and sensitivity to model-plant mismatch (model uncertainty). The Relative Gain Array (RGA) has proven to be a simple and reliable indicator of a multivariable plants's sensitivity to model-plant mismatch (Skogestad and Morari, 1986), and Shinskey has used it for years for choosing control configurations for distillation columns. For a 2×2 plant $G = \{g_{ij}\}$ the RGA is defined as follows

$$RGA = \{\lambda_{ij}\} = \begin{pmatrix} \lambda_{11} & 1-\lambda_{11} \\ 1-\lambda_{11} & \lambda_{11} \end{pmatrix}, \qquad \lambda_{11} = \left(1 - \frac{g_{12}g_{21}}{g_{11}g_{22}}\right)^{-1}$$

The steady-state values of the RGA for distillation columns have been studied extensively (Shinskey, 1984). In this paper we also derive formulas for the RGA at high frequency (initial response). In summary, the main objective of this paper is to obtain basic insight into the dynamic behavior of distillation columns. All simulations in this paper are based on the assumptions of constant molar flows, immediate flow responses and constant relative volatility. The three columns in Table 1 (Column A, C and D) are used as examples. The holdup on the trays (including the condenser and the reboiler) is $M_i/F = 0.5$min for all examples.

2. DYNAMIC COMPOSITION RESPONSE OF DISTILLATION COLUMNS

The dynamic response of most distillation columns (Fig. 1) is dominated by one large time constant, which is nearly the same, regardless of where a disturbance is introduced or where composition is measured. This is well known both from plant measurements (McNeill and Sachs, 1969) and from theoretical studies (Moczek et al., 1963). Furthermore, the value of this time constant is largely unaffected by the flow dynamics. It is somewhat surprising that the response of a distillation column with, for example, 100 trays, corresponding to at least a 100th order model, may be adequately described by a simple first-order model. Skogestad and Morari (1987a) and others (Moczek et al., 1963, Wahl and Harriot, 1970) have studied this in more detail. They found that the main reason for the low-order behavior is that all the trays have essentially the same composition response. This leads to the conclusion that the distillation column can be approximated by one large mixing tank, for which the time constant τ_1 is given by

$$\tau_1 \approx \frac{\Delta(\Sigma_{i=1}^{N} M_i x_i)}{D_f \Delta y_D + B_f \Delta x_B} = \frac{\text{“change in component holdup inside column(mol)”}}{\text{“imbalance in supply of this component (mol/s)”}} \tag{1a}$$

Here Δ represents the difference between the final (subscript f) and initial (subscript 0) steady state. For example, $\Delta y_D = y_{Df} - y_{D0}$. For high-purity binary separations and small pertubations to the column (linear model valid, $\Delta \to 0$) Skogestad and Morari (1987a) have derived an analytical expression for τ_1 from (1a)

$$\tau_1 \approx \frac{M_I}{\ln S \cdot I_s} + \frac{M_D y_D(1-y_D)}{I_s} + \frac{M_B x_B(1-x_B)}{I_s} \tag{1b}$$

Here M_I is the total holdup inside the column, and M_D and M_B are the condenser and reboiler holdups. $S = \frac{y_D(1-x_B)}{(1-y_D)x_B}$ is the separation factor and $I_s = Dy_D(1-y_D) + Bx_B(1-x_B)$ is the "impurity sum". The first term in (1b), which represents the contribution from changing the holdup inside the column, dominates for columns with both products of high purity ($1-y_D$ and x_B both small). Note that I_s may be extremely small in such cases resulting in very large values of τ_1. This agrees with the observations of eg. Kapoor et al. (1985).

The agreement between (1) and observed responses is very good in many cases. This is illustrated by Fig. 2A and 2B which show the response to small increases in reflux L (V constant) and boilup V (L constant) for column A. This column has 40 theoretical trays plus a condenser and the exact model is 41st order. This response is compared with a first order response with time constant 194 minutes corresponding to the linear model

$$\begin{pmatrix} dy_D \\ dx_B \end{pmatrix} = \frac{1}{1+194s}\begin{pmatrix} 0.878 & -0.864 \\ 1.082 & -1.096 \end{pmatrix}\begin{pmatrix} dL \\ dV \end{pmatrix} \tag{2}$$

The agreement is so good that the dotted line corresponding to this approximation is hardly visible. The value of the time constant (194 min) was found using (1a) and its value in this case it is almost identical to the inverse of the smallest eigenvalue of the linearized model.

It is clear from the derivation of (1) (Skogestad and Morari, 1987a) that τ_{1c} applies only to cases when there is a change in the total holdup $\Sigma M_i x_i$ of some component in the column. Furthermore, from the total component material balance ($Fz_F = Dy_D + Bx_B$) we derive

$$D_f \Delta y_D + B_f \Delta x_B = \Delta(Fz_F) - y_{D0}\Delta D - x_{D0}\Delta B \tag{3}$$

From (3) we see that the denominator of (1a) is non-zero only if there is a change in the $\Delta(Fz_F), \Delta D$ or ΔB, that is, if there is a change in the external material balance. If we change the internal flows only (for example, increase the reflux L and the boilup V keeping the product flows B and D constant), then the numerator of (1a) will be small, and the denominator will be identically zero. Consequently, (1) does not apply in such cases.

This is indeed confirmed by simulations. Fig. 2C shows the response to a simultaneous increase in L and V (D and B constant). The response is much faster than expected from the value $\tau_1 = 194$ min. In fact, an excellent fit is obtained using a time constant $\tau_2 = 15$ min, corresponding to the linear model

$$dL = dV: \qquad \begin{pmatrix} dy_D \\ dx_B \end{pmatrix} = \frac{1}{1+15s}\begin{pmatrix} 0.014 \\ -0.014 \end{pmatrix} dV \tag{4}$$

(The gains 0.014 and -0.014 are derived from (2) using dL=dV.) Consequently, similar to what is known for the steady state (Rosenbrock, 1962), there is a fundamental difference in column behavior for changes in external and internal flows. One objective of this paper is to study this in more detail, and to develop simple column models which display this behavior.

All results in this paper (gains, RGA-values, etc.) are for reflux L and boilup V as manipulated inputs. Distillate (D) and bottom flow (B) are manipulated to keep constant holdups in the accumulator/condenser (M_D) and the column base/reboiler (M_B). This does not imply that the LV configuration is the preferred choice for control purposes. The choice is made because the column model is most naturally written in terms of L and V as manipulated inputs.

3. A MODEL BASED ON INTERNAL AND EXTERNAL FLOWS

The steady-state model using L and V as manipulated inputs is

$$\begin{pmatrix} dy_D \\ dx_B \end{pmatrix} = \begin{pmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{pmatrix}\begin{pmatrix} dL \\ dV \end{pmatrix} \tag{5}$$

In order to model explicitly the difference in dynamic behavior between internal and external flow changes we will consider V and D as manipulated inputs for the moment. To get a dynamic model we make the following assumption:

Modelling assumption. *The reponse to changes in the external flows (D) is first-order with time constant τ_1. The response to changes in internal flows (V) is first-order with time constant τ_2.*

With this assumption and assuming constant holdups (perfect level control) and constant molar flows such that $dD(s) = dV(s) - dL(s)$, we derive the following dynamic model from (5):

$$\begin{pmatrix} dy_D \\ dx_B \end{pmatrix} = \begin{pmatrix} g_{11}+g_{12} & -g_{11} \\ g_{21}+g_{22} & -g_{21} \end{pmatrix}\begin{pmatrix} dV/(1+\tau_2 s) \\ dD/(1+\tau_1 s) \end{pmatrix} \tag{6}$$

Switching back to L and V as manipulated inputs yields

$$\begin{pmatrix} dy_D \\ dx_B \end{pmatrix} = \begin{pmatrix} \frac{g_{11}}{1+\tau_1 s} & \left(\frac{g_{11}+g_{12}}{1+\tau_2 s} - \frac{g_{11}}{1+\tau_1 s}\right) \\ \frac{g_{21}}{1+\tau_1 s} & \left(\frac{g_{21}+g_{22}}{1+\tau_2 s} - \frac{g_{21}}{1+\tau_1 s}\right) \end{pmatrix}\begin{pmatrix} dL \\ dV \end{pmatrix} \tag{7}$$

This simple model is obviously not an accurate description of all distillation columns, but it is usually adequate for controller design. The model is best when the reboiler and condenser holdups are small. The model's main advantage is its simplicity and that it gives a reasonable description of both the low- and high-frequency behavior.

τ_1 may be estimated as shown above (Eq. (1)). τ_2/τ_1 can be estimated by matching the high-frequency behavior as shown in Section 7. τ_1 is also simple to obtain from plant data or simulations. τ_2 may also be obtained from simulations (without flow dynamics) of changes in the internal flows (Fig. 2C). In most cases it will be very difficult to obtain τ_2 from plant data, since it is almost impossible, in practice, to carry out test runs for changes in the internal flows without changing the external flows (because of uncertainty and

disturbances in feed rate, boilup, etc.). Also note from Fig. 2A and B that the small time constant (τ_2) is not detectable from the response to changes in reflux (L) and boilup (V). Both these responses are almost perfectly fitted by a first order response with time constant τ_1.
Example. Column A. With the values $\tau_1 = 194$ min and $\tau_2 = 15$ min proposed in Section 2, (7) becomes

$$\begin{pmatrix} dy_D \\ dx_B \end{pmatrix} = \frac{1}{1+194s} \begin{pmatrix} 0.878 & -0.864\frac{1+12.1s}{1+15s} \\ 1.082 & -1.096\frac{1+17.3s}{1+15s} \end{pmatrix} \begin{pmatrix} dL \\ dV \end{pmatrix} \tag{8}$$

The agreement between this model and the exact 41st order model is excellent for small perturbations as seen from the simulations in Fig. 2. The relative error $\bar{\sigma}((G-\tilde{G})\tilde{G}^{-1})$ (here $\bar{\sigma}$ is the maximum singular value) between the two time constant model (8) (denoted by $\tilde{G}$) and the full linear 41st order model (denoted by G) is shown as a function of frequency in Fig. 3. It is clear that $\tilde{G}$ (8) is an excellent approximation of G up to about a frequency of 1 min^{-1}. On the other hand, the one time constant model (2) which has $\tau_1 = \tau_2 = 194$ min, gives a very poor approximation as seen from the dotted line in Fig. 3.

Note that without the seemingly negligible "correction terms" $\frac{1+12.1s}{1+15s}$ and $\frac{1+17.3s}{1+15s}$ the responses to changes in the internal flows would have a time constant of 194 min instead of the observed 15 min (Fig. 2C). In the literature each transfer matrix element in (8) is often approximated by a first-order lag with time delay $(ge^{-\theta s}/(1+\tau s))$ where g is obtained by matching steady-state data. It is clear that, unless special care is taken, it is very unlikely that such a model will be able to capture the difference in time constants between external and internal flows.

4. OBSERVATIONS OF INITIAL RESPONSE

Fig. 4 and 5 show the response in product compositions to small and large changes in the external (Fig. 4) and internal flows (Fig. 5): The initial responses $\frac{\Delta y_D}{\Delta L}$ and $\frac{\Delta x_B}{\Delta L}$ are almost independent of the magnitude of ΔL, although the steady-state behavior is entirely different. $\frac{\Delta y_D}{\Delta L}$ and $\frac{\Delta x_B}{\Delta L}$ are the responses to a unit change in ΔL, and we will call them the unit responses. Fig. 5 indicates that the initial unit response is independent of the magnitude of ΔL and ΔV.

However, are these initial unit responses also independent of operating conditions? Within a linear framework, one way of studying the effect of changing operating conditions, is to study how the linearized model changes with operating conditions. To this end consider column A and C. These actually represent the same column, but at two entirely different operating conditions. The product composition for column A are $1 - y_D = x_B = 0.01$. Column C is obtained by changing D/F from 0.500 to 0.555, which yields $1 - y_D = 0.10$ and $x_B = 0.002$. The steady-state values of the scaled gains are drastically different for columns A and C. (The unscaled gains are even more different). Fig. 6 shows the relative difference between the linearized scaled models for column A and C; the models are almost identical at higher frequencies. This implies that, even though the steady-state behavior is quite different, the initial response in terms of scaled (logarithmic) compositions is similar. The objective of the reminder of the paper is to explain these observations.

5. PREDICTED INITIAL RESPONSE

In this section we want to explain why

1) The initial unit response of Δx_i (Δx_i may be Δy_D or Δx_B) is independent of the magnitude of ΔL and ΔV.
2) The initial unit response of Δx_i is independent of the operating point if relative (logarithmic) compositions are used.

Assume constant molar flows and constant holdup. The component material balance for tray i at steady-state is

$$M_i \dot{x}_i = 0 = L_i(x_{i+1} - x_i) + V_i(y_{i-1} - y_i) \tag{9}$$

Assume a step change is made in L_i and V_i such that the flows for $t > 0$ are $L_i + \Delta L_i$ and $V_i + \Delta V_i$. Immediately following this change the values of the product compositions are unchanged. Thus we have for $t = 0^+$:

$$M_i \dot{x}_i = (L_i + \Delta L_i)(x_{i+1} - x_i) + (V_i + \Delta V_i)(y_{i1} - y_i) \tag{10}$$

Subtracting the steady-state (9) yields (Rademaker et al., 1975, p.129)

$$(t = 0^+): \quad M_i \dot{x}_i = (x_{i+1} - x_i)\Delta L_i + (y_{i-1} - y_i)\Delta V_i \tag{11}$$

$\dot{x}_i$ given by this equation is equal to the initial slope of the response for Δx_i. Note that (11) is linear in ΔL_i and ΔV_i. This explains why the initial unit responses are independent of the magnitude of ΔL and ΔV as was observed in Fig. 4 and 5.

We now want to prove claim 2) above. To this end use the steady-state relationship (9) to rewrite the expression (11) for the initial slope of the response.

$$\frac{\dot{x}_i}{x_i} = \frac{d\ln x_i}{dt} = \frac{1}{M_i}(\frac{x_{i+1}}{x_i} - 1)\left(\Delta L_i - \frac{L_i}{V_i}\Delta V_i\right) \qquad (12a)$$

Alternatively

$$\frac{\dot{x}_i}{1-x_i} = -\frac{d\ln(1-x_i)}{dt} = \frac{1}{M_i}(1 - \frac{1-x_{i+1}}{1-x_i})\left(\Delta L_i - \frac{L_i}{V_i}\Delta V_i\right) \qquad (12b)$$

Consider a binary mixture and let x_i denote the mole fraction of light component. It is easily shown that the near the bottom of the column the ratio $\frac{x_{i+1}}{x_i}$ in (12a) is 1) almost the same for any tray i and 2) only weakly dependent on operating conditions. Similarly, the ratio $\frac{1-x_{i+1}}{1-x_i}$ in (12b) is nearly constant near the top of the column. Consequently, if the logarithm of the composition is used then the initial unit responses are nearly independent of operating conditions and the entire top or bottom of the column has almost the same response. On the other hand, the slope of the initial unit response is <u>not</u> independent of the operating point if composition are measured in terms of mole fractions.

To show that the ratio $\frac{x_{i+1}}{x_i}$ is nearly constant near the bottom of the column, assume that the equilibrium line operating lines are linear.

$$y_i = K_B x_i \qquad (13)$$

$$x_{i+1} = \left(\frac{V}{L}\right)_B y_i + \frac{B}{L_B}x_B \qquad (14)$$

These assumptions are reasonable for high-purity columns. V_B and L_B denote the vapor and liquid flows in the bottom of the column. Combining (13) and (14) yields

$$\frac{x_{i+1}}{x_i} = \left(\frac{V}{L}\right)_B K_B + \frac{B}{L_B}\frac{x_B}{x_i} \qquad (15)$$

The second term is negligible as we go up the column and it is also small near the bottom for columns with $V/B > 1$. We get

$$\frac{x_{i+1}}{x_i} \approx \left(\frac{V}{L}\right)_B K_B \qquad (16)$$

Thus $\frac{x_{i+1}}{x_i}$ is 1) independent of the tray location and 2) only weakly dependent on the operating point (since K_B and $\left(\frac{V}{L}\right)_B$ are only weakly dependent on the operating point). Substituting (16) into (12a) yields

$$\begin{matrix}\text{Bottom part} \\ (t=0^+)\end{matrix} : \quad \frac{\dot{x}_i}{x_i} \approx \frac{1}{M_i}\left(\left(\frac{V}{L}\right)_B K_B - 1\right)\left(\Delta L_i - \left(\frac{L}{V}\right)_B \Delta V_i\right) \qquad (17a)$$

A similar expression is derived for the top part where $(1-y_i) \approx (1-x_i)/K_T$

$$\begin{matrix}\text{Top part} \\ (t=0^+)\end{matrix} : \quad \frac{\dot{x}_i}{1-x_i} \approx \frac{1}{M_i}\left(1 - \frac{(V/L)_T}{K_T}\right)\left(\Delta L_i - \left(\frac{L}{V}\right)_T \Delta V_i\right) \qquad (17b)$$

<u>Range of Validity</u>. From the derivation of (17) we see that the approximation is most likely to hold for high-purity columns with large reflux. Note that for the case of constant relative volatility α we have $K_B = K_T = \alpha$. This is used in the following example.

<u>Example. Column A.</u> The slopes of the initial unit response to ΔL obtained using (17a) and (17b) with $M_i/F = 0.5$ min are

$$\frac{\dot{x}_B}{\Delta L} = \frac{x_B}{M_i}\left(\left(\frac{V}{L}\right)_B \alpha - 1\right) = \frac{0.01}{0.5}(\frac{3.21}{3.71}1.5 - 1) = 0.0060 \qquad (18a)$$

$$\frac{\dot{y}_D}{\Delta L} = \frac{1-y_D}{M_i}\left(1 - \frac{(V/L)_T}{\alpha}\right) = \frac{0.01}{0.5}(1 - \frac{3.21/2.71}{1.5}) = 0.0042 \qquad (18b)$$

These are very close to the observed values in Fig. 5B.

<u>Implications for control purposes</u>. Equations (17) show that the <u>initial</u> response in terms of <u>logarithmic</u> (relative) compositions is independent of operating point. The implication of these findings for control purposes is obviously that

$$Y_D = \ln(1-y_D) \quad \text{and} \quad X_B = \ln x_B \qquad (19)$$

should be used as controlled outputs if significant variations in product compositions are expected. This has also been suggested previously by Ryskamp (1981), but without justification.

6. RGA AT HIGH FREQUENCY

We want to estimate $\lambda_{11} = \left(1 - \frac{g_{21}/g_{22}}{g_{11}/g_{12}}(j\omega)\right)^{-1}$ at high frequency. Note that

$$\frac{g_{21}}{g_{22}} = \frac{(\partial x_B/\partial L)_V}{(\partial x_B/\partial V)_L} \quad \text{and} \quad \frac{g_{11}}{g_{12}} = \frac{(\partial y_D/\partial L)_V}{(\partial y_D/\partial V)_L} \tag{20}$$

At high frequency these ratios are given by the ratio between the slopes of the initial response of x_B (and y_D) to changes in L and V. From (17a) and (17b) we get (these apply to the entire bottom and top part of the column) $\frac{g_{21}}{g_{22}}(\infty) = -\left(\frac{V}{L}\right)_B$ and $\frac{g_{11}}{g_{12}}(\infty) = -\left(\frac{V}{L}\right)_T$ and we derive

$$\frac{g_{12}g_{21}}{g_{11}g_{22}}(\infty) = \frac{(L/V)_T}{(L/V)_B} \tag{21}$$

Note that this derivation does not depend on the amount of holdup in the column. For the case of constant molar flows and feed as liquid ($L_B = L_T + F,\ V_T = V_B$) the RGA becomes

$$\text{Feed liquid :} \qquad \lambda_{11}(\infty) = 1 + \frac{L}{F} \tag{22}$$

For the three examples the agreement between the RGA-values estimated from (22) and those obtained from the full linearized model is amazing:

Column	$\lambda_{11}(\infty)$observed	$1 + \frac{L}{F}(eq.22)$
A	3.708	3.706
C	3.738	3.737
D	12.78	12.96

7. ESTIMATION OF τ_2

The ability to estimate the RGA at high frequency suggests that τ_2 may be estimated by matching the RGA-value at high frequency. The two-time constant model (7) yields

$$\frac{g_{12}g_{21}}{g_{11}g_{22}}(\infty) = \frac{\frac{g_{21}}{\tau_1}\left(\frac{g_{11}+g_{12}}{\tau_2} - \frac{g_{11}}{\tau_1}\right)}{\frac{g_{11}}{\tau_1}\left(\frac{g_{21}+g_{22}}{\tau_2} - \frac{g_{21}}{\tau_1}\right)} = \frac{(1 + \frac{g_{12}}{g_{11}}) - \frac{\tau_2}{\tau_1}}{(1 + \frac{g_{22}}{g_{21}}) - \frac{\tau_2}{\tau_1}} \tag{23}$$

The ratio τ_2/τ_1 may be estimated by equating (23) and (21). For the case of constant molar flows and feed liquid we derive

$$\text{Feed liquid :} \quad \left(\frac{\tau_2}{\tau_1}\right) = \frac{L}{F}\left(\frac{g_{12}}{g_{11}} - \frac{g_{22}}{g_{21}}\right) + \left(1 + \frac{g_{12}}{g_{11}}\right) \tag{24}$$

This ratio is 0.092, 0.420 and 0.218 for the three example columns. The ratio for column A (0.092) is reasonably close to the one ($15/194 = 0.077$) which was obtained in Section 2 and 3 when fitting the two-time constant model to observed responses.

8. DISCUSSION

The main advantage of the simple model (7) is that it gives a good description of both the low- and high-frequency behavior of distillation columns. Such models were not available in the literature. The traditional approach has been to use a model which matches the steady-state gains, but which is not necessarily accurate for high-frequencies. The other extreme is to match the high-frequency gains (Rademaker et al., 1975, p. 137). (7) provides a link between the low- and high-frequency regions. (7) was derived by considering the fundamental difference between external and internal flows, both at steady state and dynamically. The parameters in (7) are the steady state gains, the dominant first-order time constant τ_1 associated with the external flows, and the first-order time constant τ_2 associated with the internal flows. τ_1 and τ_2 can be estimated from the steady-state data using (1) and (24). (24) was derived based on matching the high-frequency behavior.

The traditional approach to modelling distillation columns is to approximate each transfer function by a first-order lag and a time delay ($ge^{-\theta s}/(1+\tau s)$) where g is obtained by matching the steady-state gains. It is very difficult to obtain a good model for high-purity columns which captures the difference between external and internal flows using this approach. Furthermore, it is unlikely that the correct behavior at high frequency (for example, the RGA) is obtained. Kapoor et al. (1986) have suggested to base the controller design on a model for the "perturbed" steady-state. This model is more likely to yield a reasonable high-frequency behavior. However, such "tricks" are unnecessary if one uses a model, for example (7), which accurately describes both the low and high frequency behavior.

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Column	z_F	α	N	N_F	$1-y_D$	x_B	D/F	L/F	G^S_{LV}	$\lambda_{11}(G_{LV})$
A	0.5	1.5	40	21	0.01	0.01	0.500	2.706	$\begin{pmatrix} 87.8 & -86.4 \\ 108.2 & -109.6 \end{pmatrix}$	35.1
C	0.5	1.5	40	21	0.10	0.002	0.555	2.737	$\begin{pmatrix} 16.023 & -16.0 \\ 9.29 & -10.7 \end{pmatrix}$	7.53
D	0.65	1.12	110	39	0.005	0.10	0.614	11.862	$\begin{pmatrix} 24.585 & -24.2 \\ 21.270 & -21.3 \end{pmatrix}$	58.7

Table 1. Steady-state data for distillation column examples. Also given: the scaled (outputs: $dy_D/1-y_D^o$ and dx_B/x_B^o) gain matrix (G^S_{LV}) and and the 1,1-element of the RGA.

Figure captions.

Figure 1. Two-product distillation column with single feed and total condenser.

Figure 2. Column A. Responses to small changes in external (A & B) and internal (C) flows. Dotted lines for A & B: First order model (2) with time constant $\tau_1 = 194$ min. Dotted line for C: First order model (4) with time constant $\tau_2 = 15$ min.

Figure 3. Column A. Relative difference between low order model $\tilde{G}$ and 41st order plant G . The two time constant model (8) provides an excellent approximation, while the one time constant model (2) is poor at high frequency.

Figure 4. Column A. Unit responses to a small and large increase in reflux L. The initial unit response is almost independent of the magnitude of ΔL, but the steady-state behavior is entirely different.

Figure 5. Column A. Unit responses to a small and large increase in internal flows.

Figure 6. The scaled linear models for column A and C are entirely different at low frequency, but almost identical at higher frequencies.